

**Econ 100B (Grossman)—Winter 2010**  
**Practice questions for final exam**  
**With answers**  
**March 9, 2010**

*If you can do the blue-book questions from the two midterms, plus these practice questions, you should be in good shape for the blue-book/free-response section of the final. The questions that will be on the exam have been designed so that the numbers typically work out relatively “nicely”. I haven’t taken such care here, so don’t be surprised or scared if you get less “nice” numbers.*

1. Consider an exchange economy consisting of two people,  $A$  and  $B$ , endowed with two goods, 1 and 2. The endowments are  $\omega^A = (0, 5)$  and  $\omega^B = (5, 5)$ . Preferences are given by  $U^A(x_1, x_2) = x_1^2 x_2$  and  $U^B(x_1, x_2) = x_1 x_2$ . Find the competitive equilibrium prices and allocations.

*Answer:*  $x^A = (10/7, 5/3)$ ,  $x^B = (25/7, 25/3)$  and  $p = (7/3, 1)$  (or any scalar multiple of these prices).

2. Firm 1 produces good  $x$  and firm 2 produces good  $y$ . Both operate in competitive markets in which  $p_x = 50$  and  $p_y = 50$ . Firm 1’s costs are given by  $c_1(x) = 100 + x^2$  and firm 2’s costs are  $c_2(y) = 50 + 2y^2 + xy$ .

- (a) How much of each good will be produced and what are the profits for each firm?

*Answer:*  $x = 25$ ,  $y = 25/4$ ,  $\pi_1 = 525$ ,  $\pi_2 = 225/8 \approx 28$ ,  $\Pi = 4425/8 \approx 553$

- (b) What are the socially optimal quantities and what are the resulting profits?

*Answer:*  $x = 150/7$ ,  $y = 50/7$ ,  $\pi_1 = 25100/49 \approx 512$ ,  $\pi_2 = 2650/49 \approx 54$ ,  $\Pi = 27750/49 \approx 566$

- (c) What are the optimal Pigouvian tax rates for each good?

*Answer:* tax on  $x$  is  $50/7$  and tax on  $y$  is zero

- (d) Suppose that the firms can negotiate costlessly and that firm 1 is assigned the right to produce  $x$  as it sees fit. What is the resulting quantity of each good and profits for each firm?

*Answer:* Quantities are the same as the socially optimal quantity. Because firm 1 decreases output from 25 to  $150/7 \approx 21.5$ , firm 2 must pay firm 1  $50/7$  for each unit that it decreases. Thus firm 2 pays  $\frac{50}{7}(25 - 150/7)$ . Profits are the same as the social optimum, except firm 1’s profits are higher by this amount and firm 2’s profits are lower by this amount.

- (e) Now suppose that instead firm 2 is given the right to compensation for any harm inflicted by firm 1. What is the resulting quantity of each good and profits for each firm?

*Answer:* Same as above, except now firm 1 pays firm 2 for the damage done, which is  $xy$ , evaluated at the social optimum, or  $\frac{150}{7} \times \frac{50}{7}$ . Profits are almost the same as social optimum, except firm 1’s profits are lower by this amount and firm 2’s profits are higher by this amount.

3. The ten residents of Barrow, Alaska are each willing to pay \$2 for a campfire, regardless of how many campfires are provided. If the cost of providing  $f$  campfires is  $c(f) = f^2$ , what is the Pareto optimal number of campfires to build?

*Answer:* The sum of the MRS or Marginal Benefits is  $10 \times 2 = 20$ . Setting this equal Marginal Cost yields  $f = 10$ .

4. During the winter, it is impossible to plow the snow clear from the street in front of Marit’s house without also clearing the snow in front of her neighbor, Justina’s house, and vice versa. Marit’s marginal benefit of snowplowing is  $12 - s$ , where  $s$  is the number of times the snowplow clears the street, and

Justina's marginal benefit is  $8 - 2s$ . The marginal cost of plowing is \$16. What is the efficient level of provision of snowplowing services?

*Answer:* Add marginal benefits and set equal to MC. So  $20 - 3s = 16$  and  $s = 4/3$