Strategic Communication Networks *

Jeanne HAGENBACH[†] Frédéric KOESSLER[‡]

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(Preliminary. Comments Welcome.)

Abstract

In this paper, we consider situations in which individuals want to choose an action close to others' actions as well as close to a payoff relevant state of nature with the ideal proximity to the common state varying across the agents. Before this coordination game with incomplete information and heterogeneous preferences is played, a cheap talk communication stage is offered to players who decide to whom they reveal the private information they hold about the state. The strategic information transmission taking place in the communication stage is characterized by a *strategic communication network*. We provide a direct link between individuals' heterogeneous preferences and the strategic communication network emerging at equilibrium, depending on the type of communication that is allowed (private, public or group communication), the strength of the coordination motive and the prior information structure. Equilibrium strategic communication networks are characterized in a very tractable way and compared in term of efficiency. In general, a maximal strategic communication network may not exist and communication networks cannot be ordered in the sense of Pareto. However, expected social welfare always increases when the communication network expands. Strategic information transmission can be improved when group or public communication is allowed, or when information is certifiable.

KEYWORDS: Cheap talk, coordination, partially verifiable types, public and private communication.

JEL CLASSIFICATION: C72; D82; D83; D85.

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†Université de Paris 1, Centre d'Économie de la Sorbonne, 106-112 Boulevard de l'Hôpital, 75647 Cedex 13, France, *E-mail :* `hagenbach@univ-paris1.fr`

‡Paris School of Economics (PSE). *E-mail :* `myname@pse.ens.fr`

1 Introduction

For social scientists, pressure to conform is a central instance of social influence. Since the work of Jones (1984), economists have acknowledged that, in many situations, the cost of non-conformist behavior can shape economic interactions. In the present paper, following Bernheim (1994) and Akerlof (1997), the need for conformity is directly incorporated into individual preferences assuming that agents can be directly penalized for departing from behaviors accepted in their social group. In addition, the agents who have an interest to take a decision coordinated to that of others have heterogeneous preferences toward this decision. More precisely, we analyze situations in which individuals want to choose an action that is close to others' actions as well as close to a payoff relevant state of nature with the ideal proximity to this common state varying across the agents.

In the two stages game that we consider, players have access to independent sources of partial information about the true state of nature. They choose to whom they want to transmit their private information before a payoff-relevant coordination game with incomplete information is played. Within this stylized framework, our main object of study is the *strategic information transmission* that takes place during the one-shot communication stage preceding the decision stage. The way agents strategically communicate with each other is at the origin of a *strategic communication network* in the sense that a connection is formed from one individual to another if the former correctly transmits his private information to the latter. We provide a direct link between individuals' heterogeneous preferences and the emerging strategic communication network, depending on the type of communication that is allowed (private, public or group communication), the strength of the coordination motive and the prior information structure.

The game through which we build the network is completely different from usual non-cooperative network formation games starting with Jackson et Wolinsky (1996) [1]. In such games, every player's strategy consists in listing wished contacts whereas we establish connections regarding communication strategies. A common point of both network formation approaches lies in the fact that nodes are players whose payoff depends on the communication network effectively formed. Since it is now largely admitted that much of the information useful to economic and social decision making (information about job opportunities, state of the market, environment of the firms, ...) is exchanged via networks of relationships, gains associated to network structures are often interpreted in term of information. In network formation games *a la* Jackson et Wolinsky (1996) focusing on communication networks, agents create links to maximize their utility with the informational costs and benefits of direct and indirect connections being usually exogenous. In this work, the information structure is explicitly formalized and the way a player benefits from informing or being informed is endogeneously given by the equilibrium outcome of the decision stage. As a consequence, our model enables to examine the incentives to misrepresent or hide information that circulates trough network links.

The situations in which agents have different "ideal action" but an interest to coordinate their decisions with each other have the relevant features of many economic and social situations. Examples of actions taken within a social group and having bad social consequences if they turn out to be isolated include demand for education or effort towards environmental problems [2] . One can

[1] See Jacskon (2007) for an extensive survey of such models.

[2] Bernheim (1994) and Akerlof (1997) consider such examples.

also think of financial analysts having, for one part, an interest to make predictions similar to that of others to be credible and, for another part, heterogeneous preferences towards such announcements [3]. Inside a firm, decisions should be adapted to the market conditions and information about these conditions is often distributed among the members of the organization due to their specialization. On the one hand, the different divisions of the organization have to coordinate their decisions to maximize the firm's profit, but, on the other hand, each division may be biased in its decision because of career concerns, effort aversion or local adaptation costs [4] . In a market, firms have to take decisions, such as investment in order to launch a new product or amount of advertising expenses, that are the most appropriate to the underlying fundamentals. In addition, such firms may also have a "beauty contest" coordination motive arising from the strategic complementarities in the actions of all the firms on the market considered. In all these settings, a question arises about how players strategically share their private information, and whether some physical communication links are worthless due to a lack of incentives between the sender(s) and the receiver(s) to correctly transmit it. When social welfare increases with information transmission, one could search for communication protocols that stimulate strategic communication.

When individuals only differ in terms of knowledge, but not in term of preferences, it is theoretically well known how coordination and welfare is affected by the information structure, and in particular by the public or private nature of individuals' signals (see, e.g., Morris et Shin, 2002 and Angeletos et Pavan, 2007). The most efficient way to disseminate information about the fundamentals can therefore be investigated. With agents' goals aligned but physical or cost constraints on the number of communication links between agents, another object of study is to identify the most efficient communication structures. This problem has been analyzed in different settings in team theory and in coordination games with incomplete information by, among others, Marschak et Radner (1972), Radner (1993), Jehiel (1999), Chwe (2000), Calvó-Armengol et Martí (2007), and Morris et Shin (2007).

A common feature of papers cited above is that there is no conflict of interests between agents regarding the ideal state-contingent action profile. As a consequence, efficient networks are characterized under *physical* communication constraints. On the contrary, coordination situations we are interested in involve some conflicts of interests which is why we focus on networks arising in equilibrium under *strategic* communication constraints. Since a single stage of cheap talk communication is offered to players before they take their actions, our paper is methodologically related to the literature on strategic information transmission built on Crawford et Sobel (1982) (see, for example, the survey by Farrell et Rabin, 1996). Our model includes multiple and interdependent decision-makers, all of them being endowed with private information, whereas most extensions of Crawford et Sobel's sender-receiver game with more than two players involve multiple senders (with no decision) but one uninformed receiver [5]

One exception in the literature on cheap talk with multiple receivers (and only one informed sender) is the paper by Farrell et Gibbons (1989) and some economic and accounting applications by Newman et Sanssing (1993), Gigler (1994), Evans et Sridhar (2002) and Levy et Razin (2004).[6]

[3] Desgranges et Rochon (2007) develops this example.

[4] The framework of firm internal organization is adopted by Alonso, Dessein, et Matouschek (2008).

[5] See, among others, Austen-Smith (1993), Battaglini (2002), Gilligan et Krehbiel (1989), Krishna et Morgan (2001a,b), and Ambrus et Takahashi (2007).

[6] Of course, several game theoretical, but more abstract, research papers deal with general cheap talk games

In Farrell et Gibbons (1989)'s setting, the main question addressed is whether sending private or public messages to the receivers makes a difference. Farrell et Gibbons indeed illustrate a situation, called *mutual discipline of public communication*, in which information is revealed to neither decision-maker when communication is private but a fully revealing equilibrium is played when communication takes place publicly. Such an effect also arises in our setting, but, contrary to Farrell et Gibbons (1989), the receivers we consider are not independent decisionmakers whose actions are separable in the sender's utility function. This enables us to identify another mutual discipline effect which is absent in Farrell et Gibbons (1989) and that we call *mutual discipline of coordination*. This effect lies in the fact that, for a fix sender, when the set of his receivers gets larger, the incentive constraints to reveal his information to the original receivers become weaker. This implies that when the informational incentive constraints are satisfied for information revelation to a set of receivers, these constraints are not necessarily satisfied for information revelation to a strict subset of these receivers. In particular, complete information revelation consisting in every player revealing his information to the whole set of players that he faces, can be the unique informative equilibrium.

Two recent papers are very closely related to the present work in that they consider incentive conflicts over decisions and therefore endogenize communication between agents. Alonso et al. (2008) and Rantakaraki (2006) both analyze strategic communication, modeled as a cheap talk game, in a two-divisions organization in which the decisions of the divisions must be responsive to local particularities as well as coordinated with each other. They compare different governance structures such as Decentralization, a case in which division managers communicate horizontally and make their respective decisions, and Centralization, a case in which decision managers communicate vertically with an independent headquarter who issues its decisions orders. Decisions makers' payoffs considered in Alonso et al. (2008) and Rantakaraki (2006) are similar to the ones we consider but conflicts of interest regarding decisions are modeled in a different way. In Alonso et al. (2008) and Rantakaraki (2006), each division manager has an "ideal action" that depends on an idiosyncratic state of nature and maximizes a weighted sum of his own division's profit and the other division's one. These weights capture how biased each manager is towards his own division's profit. The focus is on determining the best organizational arrangement driven by these biases and by the relative importance of coordination need.

Our model, presented in Section 2, is a n-player coordination game with continuous, one-dimensional action spaces. As in Morris et Shin (2002) and Calvó-Armengol et Martí (2007), each player i has a private signal about the fundamentals and incurs losses from a mismatch between his action and (i) his "ideal action" given by a parameter that depends on the underlying fundamentals, and (ii) others' actions. As in Crawford et Sobel (1982) or Dessein (2002), each individual's ideal action is characterized by a systematic positive or negative bias. Biases vary across individuals and the profile of biases in the population is a measure of the conflict of interests faced by agents. Before players choose their action, they are offered a single stage to send costless messages to each other.

In Section 3, we first characterize the second-stage equilibrium decisions depending on the

with many players, but the focus is mainly on characterizing conditions under which a (mediated) communication equilibrium can be decentralized with multilateral and multistage communication (see, e.g., the references in Forges, 2007).

communication network induced by the first-stage communication strategy profile. This enables us to compare communication networks in terms of efficiency : while communication networks cannot be ordered in the sense of Pareto, even at the ex ante stage, expected social welfare always increases when the communication network expands. Next, we investigate the conditions for a communication network to be an equilibrium of the cheap talk extension of the game. In short, the incentive constraints for some player i to reveal his type to some subset of players R_i are satisfied when player i's bias is close enough to the average bias of every subset of players in R_i. Surprisingly, no maximal equilibrium network may exist meaning that there may be an equilibrium in which player i reveals his type to players in R_i, another equilibrium in which he reveals his type to players in R'_i, but no equilibrium in which he reveals his type to players in $R_i \cup R'_i$. The tractable equilibrium characterization that we get also directly provides necessary and sufficient conditions for the complete, social welfare maximizing, network to be an equilibrium of the communication game.

In Section 4, the informational incentive constraints are weakened by considering other communication protocols. In Section 4.1, players are required to send the same message either to all the other players (public communication) or to a subset of these (group communication). With such communication forms, the informational incentive constraints for player i to reveal his type to a subset of players R_i are satisfied whenever player i's bias is close enough to the average bias of players in R_i, which is a weaker requirement than under private communication. Finally, in Section 4.2, we allow players to use messages that completely or partially certify their type. By providing sufficient conditions for complete information revelation in this case, we extend some results from the literature on strategic information revelation by Okuno-Fujiwara, Postlewaite, et Suzumura (1990), Seidmann et Winter (1997) and Van Zandt et Vives (2007). When types are completely certifiable, the conditions for complete information revelation do not depend on the communication protocol, whereas when types are only partially certifiable, public communication appears more efficient than private communication.

We conclude and provide further links to the literature in the concluding section. All proofs are relegated to the appendix.

2 Model

2.1 A Class of Coordination Games with Incomplete Information

Let $N = \{1, ..., n\}$ be a finite set of agents. Each agent chooses an action $a_i \in A_i = \mathbb{R}$. The action profile over all agents is denoted $a = (a_1, \dots, a_n)$. Each agent's payoff depends on the action profile and a state of nature θ. Before the game starts, nobody knows the state of nature, but each agent $i \in N$ receives a private signal $s_i \in S_i$ about θ. We assume that agents' types are independent and denote $q_i \in \Delta(S_i)$ the prior probability distribution over agent i's set of types, for every $i \in N$. When the type profile is $s = (s_1, ..., s_n)$, the underlying state of nature is $\theta(s)$ and agent i's payoff function is given by

$$u_i(a_1, ..., a_n; \theta(s)) = -(1-\alpha)(a_i - \theta(s) - b_i)^2 - \frac{\alpha}{n-1} \sum_{j \neq i} (a_i - a_j)^2. \quad (1)$$

The first component of agent i's utility function is a quadratic loss in the distance between his action a_i and his ideal action $\theta(s) + b_i$. The second component is a miscoordination quadratic loss which increases in the average distance between i's action and other agents' actions. The constant $\alpha \in (0, 1)$ weights both sources of quadratic loss, i.e., it parameterizes agents' coordination motives arising from the strategic complementarity in their actions. The constant $b_i \in \mathbb{R}$ parameterizes agent i's preference regarding his ideal action in the first component of his utility function. We allow the bias parameter b_i to vary across individuals to reflect agents' conflict of interests with respect to their ideal actions. If all b_i were equal, there would be no informational incentive problem and the problem of strategic information transmission would therefore be trivial.

2.2 Communication Game

Before the coordination game described below is played, but after each player has learnt his type, a communication stage is introduced in which players can send costless and private messages to each other. More precisely, every player i can send a different message $m_i^j \in M_i$ to every other player $j \neq i$, M_i denoting the (nonempty) set of messages available to player i. Let $m_i = (m_i^j)_{j \neq i} \in (M_i)^{n-1}$ be the vector of messages sent by player i, and $m^i = (m_j^i)_{j \neq i} \in \prod_{j \neq i} M_j \equiv M_{-i}$ be the vector of messages received by player i.

In this communication game, player i's first stage communication strategy is a profile $\sigma_i = (\sigma_i^j)_{j \neq i}$ with

$$\sigma_i^j : S_i \to M_i.$$

Let $\sigma_i^j(m_i^j \mid s_i)$ be the probability (0 or 1) that player i sends message m_i^j to player j according to his strategy σ_i when his type is s_i.

Player i's second-stage decision strategy is a mapping

$$\tau_i : S_i \times (M_i)^{n-1} \times M_{-i} \to A_i,$$

where $\tau_i(s_i, m_i, m^i)$ is the action chosen by player i when his type is $s_i \in S_i$, he sent the vector of private messages $m_i = (m_i^j)_{j \neq i} \in (M_i)^{n-1}$ and received the vector of private messages $m^i = (m_j^i)_{j \neq i} \in M_{-i}$. Let $\tau(s, (m_i)_{i \in N}) = (\tau_i(s_i, m_i, m^i))_{i \in N}$ be the corresponding action profile.

At the end of the communication stage, a *belief system* is a profile $\mu = (\mu_i^j)_{i \neq j}$, where $\mu_i^j : M_j \to \Delta(S_j)$ for every $i \in N$ and $j \neq i$. Given player j's message m_j^i to player i, $\mu_i^j(s_j \mid m_j^i)$ is player i's belief about player j's type.

A *perfect Bayesian equilibrium* (PBE)[7] of the communication game is a strategy profile $(\sigma, \tau) = ((\sigma_i)_{i \in N}, (\tau_i)_{i \in N})$ and a belief system μ satisfying the following properties:

(i) *Sequential rationality in the communication stage.* For all $i, j \in N$, $i \neq j$, and $s_i \in S_i$,

$$\sigma_i^j(s_i) \in \arg \max_{m_i^j \in M_i} \sum_{s_{-i} \in S_{-i}} q_{-i}(s_{-i}) \, u_i \left(\tau(s, (\sigma_{-i}(s_{-i}), \sigma_i^{-j}(s_i), m_i^j)); \theta(s) \right),$$

[7]See Fudenberg et Tirole (1991b). Notice that, in the cheap talk game, this definition yields the same equilibrium outcomes as the Nash equilibrium definition, but we already require sequential rationality and belief consistency here for consistency with the solution concept used in the case of certifiable information in Section 4.2.

where $q_{-i}(s_{-i}) = \prod_{j \neq i} q_j(s_j)$.

(ii) *Sequential rationality in the action stage.* For all $i \in N$, $m_i \in (M_i)^{n-1}$ and $m^i \in M_{-i}$,

$$\tau_i(s_i, m_i, m^i) \in \arg\max_{a_i \in A_i} \sum_{s_{-i} \in S_{-i}} \mu_i(s_{-i} \mid m^i)\ u_i\left((\tau_j(s_j, (\sigma_j^{-i}(s_j), m_j^i), (\sigma_{-i}^j(s_{-i}), m_i^j)))_{j \neq i}, a_i; \theta(s)\right),$$

where $\mu_i(s_{-i} \mid m^i) = \prod_{j \neq i} \mu_i^j(s_j \mid m_j^i)$.

(iii) *Belief consistency.* For all $i, j \in N$, $i \neq j$, and $m_j^i \in \text{supp}[\sigma_j^i]$,

$$\mu_i^j(s_j \mid m_j^i) = \frac{\sigma_j^i(m_j^i \mid s_j) q_j(s_j)}{\sum_{t_j \in S_j} \sigma_j^i(m_j^i \mid t_j) q_j(t_j)}.$$

3 Results

In this section, in order to characterize information transmission networks that emerge from the cheap talk extension of the game as (directed) hypergraphs over the set of players, we assume that each player i can only have two possible types, $S_i = \{\underline{s}_i, \overline{s}_i\}$ with $\underline{s}_i < \overline{s}_i$. Thus, any message from player i to player j is either fully revealing or non-revealing, and a communication link is said to be formed from i to j when i's message to j is fully revealing. Without further loss of generality, we can restrict ourselves to binary messages spaces, $M_i = \{\underline{m}, \overline{m}\}$. In addition, to get explicit and tractable equilibrium characterizations, we assume that the state of nature is additive in types: $\theta(s) = \sum_{i \in N} s_i$. These assumptions on the number of types and the additivity of the state of nature will be relaxed in Section 4.2 when focusing on the conditions for complete information revelation (by all players to all the other players) with certifiable types.

With some abuse of language, the difference between the two possible signals of player i, $\overline{s}_i - \underline{s}_i$, is called the value of the private information for player i. The idea is that when this value is high, player i's private information has a large impact on the fundamentals.

3.1 Second-Stage Equilibrium Characterization

With only two possible types for each player, every communication strategy profile $(\sigma_i)_{i \in N}$ can be characterized by a *communication network* $(R_i)_{i \in N}$, where, for every player i,

$$R_i = \{j \in N \backslash \{i\} : \sigma_i^j(\underline{s}_i) \neq \sigma_i^j(\overline{s}_i)\},$$

is the set of players to which player i completely reveals his type.[8] Let $r_i = |R_i|$ be the number of players who learn player i's type after the communication stage.

Given a profile of types $(s_i)_{i \in N}$ and a communication strategy profile characterized by $(R_i)_{i \in N}$,

[8] A communication network is a directed hypergraph in which the set of nodes is the set of players, and an hyperarc is a pair $(\{i\}, R_i)$ where the origin $\{i\}$ is always a singleton.

the second stage equilibrium action of each player $i \in N$ is given by (see Appendix 6.1),

$$a_i = \sum_{j \in I_i} \frac{\alpha(n-1-r_j)E(s_j) + (1-\alpha)(n-1)s_j}{n-1-\alpha r_j} + \sum_{j \in \overline{I}_i} E(s_j) + \frac{[(n-1)-(n-2)\alpha]b_i + \alpha \sum_{j \neq i} b_j}{n+\alpha-1}, \tag{2}$$

where $I_i = \{k : i \in R_k\} \cup \{i\}$ is the set of signals which are known by player i after the communication stage, and $\overline{I}_i = \{k : i \notin R_k\} \backslash \{i\}$ is the set of signals which are unknown by player i after the communication stage.

Hence, i's optimal action has three components. The first component is a weighted sum of j's actual type, s_j, and the expected value of j's type, $E(s_j)$, for each player j whose type is known by player i (including himself). Note that more relative weight is put on the actual type of player j when the coordination motive, α, is low and when the number of players who know j's type, r_j, is high. The second component corresponds to the sum of the expected values of j's type for each player j whose type is unknown by player i. The last component adjusts the action of player i with respect to the bias profile. It is increasing with all players biases, with more relative weight being put on player i's own bias, b_i, when the coordination motive decreases.

This explicit characterization of optimal actions as a function of the information structure and players' preference allows us to characterize in a very tractable way the efficient and equilibrium communication strategy profiles, as shown in the following subsections.

3.2 Efficient Networks

The next proposition compares players' ex ante expected payoffs when the communication network that arises from the communication stage expands.[9] While an increase in the set of receivers who learn player i's type is always strictly beneficial for player i and for these receivers, such an increase makes players who don't learn player i's type always strictly worse off.

Proposition 1 *Consider two communication networks $R = (R_i, R_{-i})$ and $R' = (R'_i, R_{-i})$ such that $R_i \subsetneq R'_i$.*

i) Player i is strictly better off*, ex-ante, with the communication network R' than with the communication network R;*

ii) Every player $j \in R'_i$ (with $j \in R_i$ or $j \notin R_i$) is strictly better off*, ex-ante, with the communication network R' than with the communication network R;*

iii) Every player $j \in N \backslash (\{i\} \cup R'_i)$ is strictly worse off*, ex-ante, with the communication network R' than with the communication network R.*

Proof. See Appendix 6.2. ■

This result implies that, in general, communication networks cannot be ordered in the sense of Pareto, except the complete communication network ($R_i = N \backslash \{i\}$ for all $i \in N$) that Pareto dominates every other network.

[9] As in Crawford et Sobel (1982), it is not possible to compare players' expected payoffs at the interim stage.

The next proposition shows, however, that the overall effect of an increase in information transmission is positive. We define the *social welfare* as the sum of individual utilities, $w(a;\theta) = \sum_{i\in N} u_i(a;\theta)$, and consider that a communication network $R' = (R'_i)_{i\in N}$ is *larger* than a communication network $R = (R_i)_{i\in N}$ when $R_i \subseteq R'_i$ for all $i \in N$ (with at least one strict inclusion).

Proposition 2 *The welfare is always strictly larger, ex-ante, with the communication network R' than with the communication network R if R' is larger than R.*

Proof. See Appendix 6.3. ■

3.3 Equilibrium Networks

The next proposition provides a complete characterization of communication networks that may arise as a perfect Bayesian equilibrium outcome of the cheap talk extension of the game.

Proposition 3 *There exists a perfect Bayesian equilibrium in which every player i completely reveals his private information to every player in $R_i \subseteq N\backslash\{i\}$ iff for all $i \in N$ and $R'_i \subseteq R_i$, with $|R'_i| = r'_i$, we have*

$$\left| b_i - \frac{\sum_{j\in R'_i} b_j}{r'_i} \right| \leq \frac{(n-1+\alpha)(n-1-\alpha r'_i)}{2(n-1)(n-1-\alpha r_i)}(\overline{s}_i - \underline{s}_i). \tag{3}$$

Proof. See Appendix 6.4. ■

Hence, the proposition tells us that, in equilibrium, player i transmits his information to a set of players $R_i \subseteq N\backslash\{i\}$ if player i's bias is close enough to the average bias of the players who belong to *every subset* of players in R_i. As it can be seen from the threshold on the RHS of Inequality (3), this condition for information transmission is weaker when the weight on coordination motives, α, increases,[10] when the value of player i's private information, $\overline{s}_i - \underline{s}_i$, increases, and when the total number of players, n, decreases.[11] Note that the condition for player i to transmit his information to players in R_i does not depend on the communication strategies used by players different from i, which highly simplifies the analysis.

From the previous proposition, a corollary is deduced that gives the necessary and sufficient condition for the efficient communication network, the complete one, to be an equilibrium of the communication game.

Corollary 1 *There is a fully revealing equilibrium, leading to the complete communication network, if and only if for all $i \in N$ and $R_i \subseteq N\backslash\{i\}$,*

$$\left| b_i - \frac{\sum_{j\in R_i} b_j}{r_i} \right| \leq \frac{(n-1+\alpha)(n-1-\alpha r_i)}{2(n-1)^2(1-\alpha)}(\overline{s}_i - \underline{s}_i). \tag{4}$$

[10] The RHS of Equation (3) is increasing in α because $\frac{\partial}{\partial\alpha}\frac{(n-1-\alpha r'_i)}{(n-1-\alpha r_i)} = \frac{(n-1)(r_i - r'_i)}{(n-1-\alpha r_i)^2} \geq 0$.

[11] The RHS of Equation (3) is decreasing in n since the sign of its derivative with respect to n is $2\alpha(n-1)r'_i - \alpha^2 r_i r'_i - (n-1)^2(r_i + 1 - r'_i)$, which is always negative.

As an illustration, consider a game with $n = 4$ players and $\alpha = 1/2$. Without loss of generality, players are numbered so that their biases are arranged in increasing order. We examine the incentives for information transmission of a player $i \in N$ whose value of private information is $\overline{s}_i - \underline{s}_i = \frac{13\times 3}{7}$, with a null bias $b_i = 0$. Then, the RHS of Equation (3) in Proposition 3 simplifies to $3\frac{6-r_i'}{6-r_i}$. It follows that player i reveals his type to all the other players if for all $k, l \in N\backslash\{i\}$,

$$\left|\frac{\sum_{j\neq i} b_j}{3}\right| \leq 3, \quad \left|\frac{b_k + b_l}{2}\right| \leq 4, \quad \text{and} \quad |b_k| \leq 5. \tag{5}$$

Similarly, player i reveals his type to players in $\{j,k\} \subsetneq N\backslash\{i\}$ if

$$\left|\frac{b_j + b_k}{2}\right| \leq 3, \quad \text{and} \quad |b_j|, |b_k| \leq 3.75. \tag{6}$$

Finally, player i reveals his type only to player $j \neq i$ if $|b_j| \leq 3$.

Let $b_1 \simeq -4.5 \simeq -b_4$ and $b_3 \simeq 3.2$. Then, the only informative equilibrium strategy for player $i = 2$ is to reveal his type to players in $R_2 = \{1, 3, 4\}$. This illustrates that there may be *no* equilibrium in which player 2 transmits his information to any strict subset of R_2 only, but there may be an equilibrium in which player 2 transmits his information to all players in R_2. More generally, we have an effect that we may call *mutual discipline of coordination*, reflecting the fact that information transmission from some player to another one depends on whether the former's information is also transmitted to other players or not. Indeed, the conditions on the proximity between i's bias and the average bias of the strict subsets $R_i' \subsetneq R_i$ of receivers, $|b_i - \frac{\sum_{j\in R_i'} b_j}{r_i'}|$, become weaker as the set of all receivers, R_i, increases. This effect is absent in Farrell et Gibbons (1989) where the payoff of each decisionmaker only depends on his own action, while in our model players want to coordinate their actions.

Despite this positive effect for information transmission of larger sets of receivers, a maximal equilibrium communication network may *not* exist. To see this, let $b_1 \simeq 0$, $b_2 \simeq 2.2$ and $b_3 \simeq b_4 \simeq 3.5$. Then, there is an equilibrium in which player $i = 1$ reveals his type to $R_1 = \{2, 3\}$ and an equilibrium in which he reveals his type to $\tilde{R}_1 = \{2, 4\}$, but there is no equilibrium in which player 1 reveals his type to players in $R_1 \cup \tilde{R}_1 = \{2, 3, 4\}$. More generally, when there is an equilibrium in which some player i transmits his information to R_i and an equilibrium in which he transmits his information to $\tilde{R}_i$, a sufficient condition to get an equilibrium in which player i also transmits his information to $R_i \cup \tilde{R}_i$ is that R_i and $\tilde{R}_i$ do not overlap.

By looking at the overlapping of the conditions under which a communication strategy is an equilibrium one, further observations can be made on the way agents can be connected at equilibrium. For instance, the condition that must be satisfied so that a player completely reveals his private information to a single other player enables to deduce what follows: if there exists an equilibrium in which player i transmits his information to a single player k, then there is also an equilibrium in which player i transmits his information to R_i, for all $R_i \subseteq \{i+1\ldots, k-1, k\}$ with $k \geq i$. Eventually, one could also note that the directed connection that is built from player i to player k, when player i transmit his private information to player k, can be reciprocal, in the sense that it also exists an equilibrium in which player k reveals his information to player i, if players i and k both have the same value of information.

4 Extensions

Since larger communication networks are always beneficial in terms of welfare, we are led to investigate how other types of (strategic and decentralized) communication extensions of the game may allow more effective information transmission than private cheap talk. In this section we first show how communication can be improved by considering *group communication*, where each player i is required to send the same message to all players in a group $\bar{R}_i \subseteq N \backslash \{i\}$. This includes as a particular case public communication, where $\bar{R}_i = N \backslash \{i\}$ for all $i \in N$. In the second subsection we show that, even when the conditions for a fully revealing equilibrium to exist in the private or public cheap talk case (conditions (4) or (7)) are not satisfied, complete information revelation becomes possible whatever the bias profile and the communication protocol (public or private) when players are able to certify their types. When types can only be partially certified, the condition for complete information revelation depends again on the communication protocol and the bias profile, but not on the magnitude of players' biases.

4.1 Group and Public Communication

It is well known since Farrell et Gibbons (1989) that the credibility of a sender's claim may radically depend on whether this claim has been made publicly or privately. In our model, in order to investigate how players' incentive to transmit their information is affected in by the communication protocol, we consider group communication games in which each player is required to send the same message to a fixed subset of players. Of course, the public communication game is a particular group communication game in which all players are required to send the same message to all the other players.

Formally, in the *public communication game*, each player i's communication strategy is simply a mapping $\sigma_i : S_i \to M_i$, where $\sigma_i(s_i)$ is the message publicly observed by all players in $N \backslash \{i\}$ when player i's type is s_i. When there is more than two potential audiences, each player i may also be required to send the same message to a subset $\bar{R}_i$ of players in $N \backslash \{i\}$, for $i = 1, \ldots, n$. This communication extension of the game is called the *group $\bar{R}$-communication game*, where $\bar{R} = (\bar{R}_i)_{i \in N}$. In this game, each player i's communication strategy is a mapping $\sigma_i : S_i \to M_i$, where each player i is required to send the same message $\sigma_i(s_i) = m_i \in M_i$ to all players in $\bar{R}_i$.

The definition of a perfect Bayesian equilibrium for the group (and public) communication games is similar to the definition for the private communication game. When focusing on equilibrium outcomes in which each player i fully reveals his type to the subset of players $\bar{R}_i$, the only difference between the group and private communication protocols is that the informational incentive constraints are weaker in the former one: the only possible deviation from a common message sent to players in $\bar{R}_i$ is to jointly lie to all of them, while player i can choose to lie to any subset of $\bar{R}_i$ when the messages are private.

Proposition 4 *In the group $\bar{R}$-communication game, there exists a perfect Bayesian equilibrium in which every player i completely reveals his private information to every player in $\bar{R}_i \subseteq N \backslash \{i\}$ iff for all $i \in N$, Inequality (3) holds for all $i \in N$, with $R'_i = R_i = \bar{R}_i$.*

Proof. See Appendix 6.4. ■

In particular, in the public communication game, there exists a perfect Bayesian equilibrium in which player i completely reveals his private information to all the other players iff Inequality (3) holds for $R'_i = R_i = N \backslash \{i\}$, and there is a fully revealing equilibrium if and only if for all $i \in N$,

$$\left| b_i - \frac{\sum_{j \neq i} b_j}{n-1} \right| \leq \frac{(n-1+\alpha)}{2(n-1)} (\overline{s}_i - \underline{s}_i). \tag{7}$$

Notice that if there is an equilibrium in which player i transmits his information to players in R_i in the private communication game, then there is also an equilibrium in which player i transmits his information to players in R_i in the group R-communication game. In particular, the set of all private strategic communication networks is included (and may be strictly included) in the set of all group strategic communication networks. This is a generalization of the *mutual discipline effect of public communication* observed by Farrell et Gibbons (1989, Proposition 1).

As an illustration, consider again the 4-player example of Section 3. We have seen that player i reveals his type to all the other players in private if for all $k, l \in N \backslash \{i\}$, all conditions of Inequality (5) are satisfied. On the contrary, under public communication, only the first inequality of (5) is required. Similarly, in the group $\{j, k\}$-communication game, player i reveals his type to players in $\{j, k\}$ whenever the first inequality of Equation (6) is satisfied. Hence, there is a mutual discipline effect of public communication if, e.g., $b_1 \in (-9, -5)$ and $b_3 = b_4 = -b_1$, since in that case there is no informative equilibrium from player $i = 2$ in private, while in public there are equilibria in which player 2 reveals his type to players in $R_2 = \{1, 3, 4\}$, $R_2 = \{1, 3\}$ or $R_2 = \{1, 4\}$ (but not $R_2 = \{3, 4\}$). Finally, using the same example as under private communication, it can be seen that a maximal equilibrium communication network may not exist even if communication takes place publicly.

4.2 Certifiable Information

In this section we extent our communication game by allowing the set of messages available to each player to depend on his private information. Following the terminology of Grossman (1981), Milgrom (1981), Green et Laffont (1986), Okuno-Fujiwara et al. (1990), Bull et Watson (2004), Forges et Koessler (2005) or Giovannoni et Seidmann (2007), this means that players are able to provide hard, verifiable, or certifiable information about their type.

Formally, the model is equivalent to the cheap talk model analyzed in section 2, except that each player i can send messages in $M_i(s_i)$, where $M_i(s_i) \neq \emptyset$ is a type dependent set of messages. In this subsection, the set of types S_i of player i is any finite set, and the function $\theta(s)$ is not required to be additive in types anymore; we only assume that it is weakly increasing with s_i for all $i \in N$. Without further loss of generality, assume that types in $S_i \subset \mathbb{R}$ are increasingly ordered.

The communication game and perfect Bayesian equilibria are defined exactly as in Section 2 except that the belief consistency condition (iii) on page 7 is stronger. In the private communication game, for all $i, j \in N$, $i \neq j$, and for all $s_j \in S_j$, we have the following additional condition: $\mu_i^j(s_j \mid m_j^i) = 0$ if $m_j^i \notin M_j(s_j)$. Similarly, in the public communication game, for all $i, j, k \in N$, with $k \neq i \neq j \neq k$, and for all $s_j \in S_j$, we have $\mu_i^j(s_j \mid m_j) = \mu_k^j(s_j \mid m_j) = 0$ if $m_j \notin M_j(s_j)$.[12]

[12] We do not consider the other group communication games here since we obtain full revelation of information (a

We say that type $s_i \in S_i$ is *certifiable* if there exists a message $c_i(s_i) \in M_i \equiv \bigcup_{t_i \in S_i} M_i(t_i)$ such that $M_i^{-1}(c_i(s_i)) \equiv \{t_i \in S_i : c_i(s_i) \in M_i(t_i)\} = \{s_i\}$. The following proposition shows that whatever the communication protocol (public or private), if every player can certify his type, then there exists a fully revealing equilibrium in which all players reveal their type to all the other players.

Proposition 5 *Whatever the communication protocol (public or private) and the bias profile, $(b_i)_{i\in N}$, if each type of each player is certifiable, then the communication game has a perfect Bayesian equilibrium which is fully revealing.*

Proof. See Appendix 6.5. ∎

This proposition extends the results of the literature in several aspects. First, in Okuno-Fujiwara et al. (1990), the class of n-person games with $n > 2$ is restricted to the following class of linear-quadratic utility functions for player i:[13]

$$a_i[\beta_i(s) - d \sum_{j \neq i} a_j - a_i], \tag{8}$$

where $d \in (0, 2)$ and $\beta_i(s_1, \ldots, s_n)$ is increasing with s_i and decreasing with s_{-i}. Developing the utility function in our model (see Equation (1)) we get instead (minus a constant):

$$a_i[2(1-\alpha)(\theta(s) + b_i) + \frac{2\alpha}{n-1} \sum_{j \neq i} a_j - a_i] - \frac{\alpha}{n-1} \sum_{j \neq i} (a_j)^2. \tag{9}$$

Equation (9) cannot be rewritten as Equation (8) for three important reasons:

1. In our model, $\beta_i(s) = 2(1-\alpha)(\theta(s) + b_i)$, which is increasing with s_j for all $j \in N$;
2. Our model involves strategic complementarities because $d = -\frac{2\alpha}{n-1}$ is negative, while Okuno-Fujiwara et al. (1990) assume strategic substitutes ($d > 0$);
3. Equation (9) contains the additional term $-\frac{\alpha}{n-1} \sum_{j \neq i} (a_j)^2$ which is absent from Equation (8).[14]

Second, Van Zandt et Vives (2007) also prove the existence of a fully revealing equilibrium in a class of games with strategic complementarities, but they assume that each player's utility function is increasing in the actions of the other players. This assumption of positive externalities in actions is clearly not satisfied in our model.

Third, our proposition shows that full revelation of information holds in the public and private communication games while Okuno-Fujiwara et al. (1990) and Van Zandt et Vives (2007) only consider public communication.

Finally, with the exception of some sender-receiver games considered, e.g, by Seidmann et Winter (1997) or Koessler (2003), fully revealing equilibria found in the literature are usually robust

complete communication network) in both the private and public settings.

[13]Like us, they consider finite sets of types and assume that players' types are independent.

[14]This term does not modify the second stage equilibrium strategies but affects players' incentives to communicate.

to a simple inference that either always puts probability one on the lowest type consistent with the sender's report, or always puts probability one on the highest type. Here, as shown in Appendix 6.5, to support full revelation of information, the form of players' beliefs off the equilibrium path depends on the parameters of the game (the profile of biases $(b_1, \dots, b_n)$), on the player who deviates, and on the players who observe this deviation (which depends on whether the communication game is public or private). More precisely, in the private communication game, when player j receives a private message m_i^j from player i and his bias is higher than player i's bias ($b_j \geq b_i$), then his belief off the equilibrium path consists in believing that player i's type is the highest type compatible with i's message (i.e., player j believes that player i's type is $\max\{t_i \in S_i : m_i^j \in M_i(t_i)\}$). On the contrary, when player j's bias is lower than player i's bias, then he believes the lowest type compatible with i's message. In the public communication game, players' inferences depend on whether the bias of the player who deviates is lower or higher than the average bias $\bar{b} = \sum_{i \in N} b_i / n$. When $\bar{b} \geq b_i$, players in $N \backslash \{i\}$ believe the highest type compatible with player i's report, and when $\bar{b} \leq b_i$ they believe the lowest type.

The last observation has implications on the certifiability requirements for complete information revelation.

Proposition 6 (Fully revealing equilibrium with partially certifiable types)

In the public communication game, if each player i with a lower bias than the average bias (i.e., $b_i \leq \bar{b}$) can certify whatever his actual type s_i that his type is at most s_i (i.e., there exists $m_i \in M_i$ such that $s_i = \max M_i^{-1}(m_i)$), and if each player i with a higher bias than the average bias (i.e., $b_i \geq \bar{b}$) can certify whatever his actual type s_i that his type is at least s_i (i.e., there exists $m_i \in M_i$ such that $s_i = \min M_i^{-1}(m_i)$), then there is a perfect Bayesian equilibrium which is fully revealing.

In the private communication game, if each player i with the lowest bias (i.e., $b_i \leq b_j$ for all $j \in N$) can certify whatever his actual type s_i that his type is at most s_i, if each player i with the highest bias (i.e., $b_i \geq b_j$ for all $j \in N$) can certify whatever his actual type s_i that his type is at least s_i, and the other players can completely certify their types, then there is a perfect Bayesian equilibrium which is fully revealing.

Hence, as in the cheap talk case, the sufficient conditions for full information revelation are stronger in the private than in the public communication game. As in the cheap talk game, this is because in the public communication game, less deviations in the communication stage are possible. It is however important to notice that, on the other hand, when communication is private, different receivers can make different inferences from the same deviation, while in the public communication game belief consistency requires all receivers to make the same inferences.[15] These two differences between public and private communication are exactly to potential sources of "mutual discipline" (full information revelation in public but not in private) in Farrell et Gibbons (1989) cheap talk game and of "mutual subversion" (full information revelation in private but not in public) in Koessler (2007) information certification game. Here, mutual subversion is never possible, and when types of completely certifiable, mutual discipline is also impossible since full revelation of information always occurs in both the public and private cases. However, mutual discipline is again possible with partially certifiable information, as shown in the next example. More precisely, the example

[15] See Fudenberg et Tirole (1991a, condition B(iv) on page 332).

gives a simple instance where the previous proposition applies for the public communication game but there is no fully revealing equilibrium in the private one.

Example 1 (Mutual Discipline under Partial Certifiability) Consider the 3-player game in which only player 1 knows the state $\theta \in \{\theta_1, \theta_2, \theta_3\}$, players' biases satisfy $b_2 \leq b_1 \leq b_3$ and $b_1 \leq \frac{b_2+b_3}{2}$, and the messages available to player 1 depending on the state are:

$$M(\theta_1) = \{m_1, m_2, m_3\}, \quad M(\theta_2) = \{m_2, m_3\}, \quad M(\theta_3) = \{m_3\}.$$

By Proposition 6, these assumptions imply that there exists a fully revealing equilibrium in the public communication. Hence, in equilibrium, players' actions in state θ are given by

$$a_i(\theta) = \theta + \frac{3b_i + \sum_{j \neq i} b_j}{5}.$$

Consider now the fully revealing communication strategy in the private communication game. When the real state is θ_1 and player 1 deviates by sending message m_2 instead of m_1 to player 2 (without deviating towards player 3), his best response is to choose action $a_1' = \frac{2(\theta_1+b_1)+a_2(\theta_2)+a_3(\theta_1)}{4} = \frac{3\theta_1+\theta_2}{4} + \frac{3b_1+b_2+b_3}{5}$. After some simplifications, the condition for this deviation to be profitable for player 1 is

$$b_1 - b_2 > \frac{15(\theta_2 - \theta_1)}{16}.$$

Hence, under this condition there is no fully revealing equilibrium in the private communication game, while a fully revealing equilibrium exists in the public one whatever the distance between the possible fundamentals and the distance between player 1 and player 2's biases (as long as $b_1 \leq \frac{b_2+b_3}{2}$).

5 Conclusion

In the present work, information on a common state of nature is dispersed among some players. These players must choose an action by balancing the benefit of choosing it close to their "ideal action", depending on the state and on an idiosyncratic bias, with that of choosing actions close to each other. In such a setting, we investigate the way individuals' heterogeneity affects strategic information transmission that takes place during a cheap talk stage offered to players before they take a decision. We first show that expected social welfare always increases when communication expands but that the strategic communication networks cannot be ordered in the sense of Pareto even at the ex ante stage. Next, we provide conditions on the proximity of players' biases to get every possible communication structure as an equilibrium of the cheap talk game and extend results to communication protocols that enable larger networks to emerge under weaker conditions, namely the use of group communication and certifiable messages.

Considering the private communication protocol, one main result is that a player perfectly reveals his information to a group of players if this player's bias is close enough to the average bias of every subgroup of players in the considered group. In short, agents are more prone to communicate when their ideal actions present some alignments. In social sciences, homophily is a well-documented tendency of individuals to associate with similar others (see McPherson, Smith-Lovin, et Cook (2001) for an extensive review paper). In particular, this pervasive social fact induces that communication more likely takes place between agents whose individual characteristics are related, in the sense that their goals are not too different for instance. Even if the interpretation of the proximity between one's bias and the "average bias" in a group of receivers is delicate, one can consider that communication networks emerging at equilibrium in the present work exhibit homophily.

In addition, as it has already been mentioned, our approach of network formation completely departs from usual non-cooperative network formation games mainly because we derive connections from the equilibrium strategy profile of the cheap talk stage of the game. More generally, we embed the formation of a network in a game in which this formation is not itself the purpose, in the sense that players are not aware of building ties. Within our particular framework, it appears quite reasonable to materialize the strategic information revelation between two players by the existence of a connection between them. Nevertheless, linking players regarding strategies that do not consist in choosing contacts will raise, in many settings, the fussy question of the kind of interaction that can be considered as a network "link". In sociology, the right way to measure whether a tie exists or not constitutes one of the main questions addressed especially when trying to elicit networks from questionnaires about relationships [16].

6 Appendix

6.1 Second-Stage Equilibrium Characterization

First, we characterize the unique equilibrium action profile under complete information. The best response of each player i to a_{-i} solves $\frac{\partial u_i(a_i,a_{-i};\theta)}{\partial a_i} = 0$, i.e.,

$$a_i(a_{-i};\theta) = (1-\alpha)(\theta + b_i) + \frac{\alpha}{n-1}\sum_{j\neq i} a_j. \tag{10}$$

If a_i is a best response to a_{-i}, then it follows from Equations (9) and (10) that player i's utility takes the following simple form (minus a constant):

$$u_i(a_i(a_{-i};\theta), a_{-i};\theta) = (a_i(a_{-i};\theta))^2 - \frac{\alpha}{n-1}\sum_{j\neq i}(a_j)^2. \tag{11}$$

[16]See, for instance, the questions used to get network data in *1985 General Social Survey* and the related work of Mardsen (1987)

The system of equations formed by Equation (10) leads to:

$$\begin{pmatrix} a_1 \\ \vdots \\ \vdots \\ a_n \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & -\frac{\alpha}{(n-1)} & \cdots & -\frac{\alpha}{(n-1)} \\ -\frac{\alpha}{(n-1)} & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & -\frac{\alpha}{(n-1)} \\ -\frac{\alpha}{(n-1)} & \cdots & -\frac{\alpha}{(n-1)} & 1 \end{pmatrix}}_{I}^{-1} \begin{pmatrix} (1-\alpha_1)(\theta+b_1) \\ \vdots \\ \vdots \\ (1-\alpha_n)(\theta+b_n) \end{pmatrix}.$$

Simple algebra yields:

$$I^{-1} = \frac{1}{(n-1)-(n-2)\alpha-\alpha^2} \begin{pmatrix} (n-1)-(n-2)\alpha & \alpha & \cdots & \alpha \\ \alpha & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \alpha \\ \alpha & \cdots & \alpha & (n-1)-(n-2)\alpha \end{pmatrix}.$$

Therefore, when every player knows the state of nature, the equilibrium actions are given by:

$$a_i(\theta) = \theta + \frac{[(n-1)-(n-2)\alpha]b_i + \alpha\sum_{j\neq i} b_j}{n+\alpha-1} \equiv \theta + B_i, \text{ for every } i \in N. \tag{12}$$

Since players' best responses are linear, exactly the same algebra shows that, under incomplete information and whatever the information structure generated by the communication strategy profile, expected equilibrium actions are uniquely characterized by

$$E(a_i) = E(\theta) + B_i, \text{ for every } i \in N. \tag{13}$$

By explicitly solving some particular incomplete information situations as above, it is possible to guess the general form of the second-stage equilibrium actions. To check that the solution given by Equation (2) is indeed the equilibrium when the communication strategy profile is characterized by $(R_i)_{i\in N}$, fix some player $l \in N$ and suppose that the second stage equilibrium action of every player $i \neq l$ is given by Equation (2). We show that player l's best response to the profile of second stage actions $(a_i)_{i\neq l}$ also takes the form of Equation (2).

After the communication stage, for all $i \in N$, recall that $I_i = \{k : i \in R_k\} \cup \{i\}$ is the set of players whose signals are known by player i, $\overline{I}_i = \{k : i \notin R_k\}\backslash\{i\}$ the set of players whose signals are unknown by player i, and let $E_i(\cdot) = E(\cdot \mid \{s_l : \ l \in I_i\})$ be player i's expectation operator conditional to the set of signals that he knows.

The expected payoff of player l after the communication stage takes the following form:

$$-(1-\alpha)E_l\big[(a_l - \sum_{j\in N} s_j - b_l)^2\big] - \frac{\alpha}{n-1}\sum_{j\neq l} E_l\big[(a_l - a_j)^2\big], \tag{14}$$

so his best-response is given by:

$$a_l = (1-\alpha)\left(\sum_{j\in I_l} s_j \;+ \sum_{j\in \overline{I}_l} E(s_j) \;+ b_l\right) + \frac{\alpha}{n-1}\sum_{j\neq l} E_l(a_j). \tag{15}$$

Using Equation (2) for $i \neq l$, player l's conditional expectation of player i's action is given by:

$$\begin{aligned} E_l(a_i) &= \sum_{j\in I_i} \frac{\alpha(n-1-r_j)E(s_j)}{n-1-\alpha r_j} + \sum_{j\in I_i\cap I_l} \frac{(1-\alpha)(n-1)s_j}{n-1-\alpha r_j} \\ &\quad + \sum_{j\in I_i\cap \overline{I}_l} \frac{(1-\alpha)(n-1)E(s_j)}{n-1-\alpha r_j} + \sum_{j\in \overline{I}_i} E(s_j) + B_i. \end{aligned}$$

Summing over all agents different from l, we can write:

$$\begin{aligned} &\sum_{i\neq l} E_l(a_i) = \\ &= \sum_{i\neq l}\sum_{j\in I_i\cap I_l} \frac{\alpha(n-1-r_j)E(s_j)}{n-1-\alpha r_j} + \sum_{i\neq l}\sum_{j\in I_i\cap \overline{I}_l} \frac{\alpha(n-1-r_j)E(s_j)}{n-1-\alpha r_j} + \sum_{i\neq l}\sum_{j\in I_i\cap I_l} \frac{(1-\alpha)(n-1)s_j}{n-1-\alpha r_j} \\ &+ \sum_{i\neq l}\sum_{j\in I_i\cap \overline{I}_l} \frac{(1-\alpha)(n-1)E(s_j)}{n-1-\alpha r_j} + \sum_{i\neq l}\sum_{j\in \overline{I}_i\cap I_l} E(s_j) + \sum_{i\neq l}\sum_{j\in \overline{I}_i\cap \overline{I}_l} E(s_j) + \sum_{i\neq l} B_i. \end{aligned} \tag{16}$$

Every signal s_j known by player l is known by r_j players different from l and unknown by $n-1-r_j$ players different from l; every signal s_j unknown by player l is known by r_j+1 players different from l and unknown by $n-2-r_j$ players different from l. This enables to deduce:

$$\begin{aligned} \sum_{i\neq l} E_l(a_i) &= \sum_{j\in I_l} r_j \frac{\alpha(n-1-r_j)E(s_j)}{n-1-\alpha r_j} + \sum_{j\in \overline{I}_l} (r_j+1) \frac{\alpha(n-1-r_j)E(s_j)}{n-1-\alpha r_j} \\ &\quad + \sum_{j\in I_l} r_j \frac{(1-\alpha)(n-1)s_j}{n-1-\alpha r_j} + \sum_{j\in \overline{I}_l} (r_j+1) \frac{(1-\alpha)(n-1)E(s_j)}{n-1-\alpha r_j} \\ &\quad + \sum_{j\in I_l} (n-1-r_j)\, E(s_j) + \sum_{j\in \overline{I}_l} (n-2-r_j)\, E(s_j) + \sum_{i\neq l} B_i. \end{aligned} \tag{17}$$

In addition, we have:

$$\sum_{i\neq l} B_i = \frac{\alpha(n-1)b_l + (n-1)\sum_{i\neq l} b_i}{n+\alpha-1}. \tag{18}$$

Plugging (18) and (17) into (15) and simplifying, we get player l's optimal action, which takes exactly the same form as in Equation (2).

6.2 Proof of Proposition 1

The ex ante equilibrium payoff of player $j \in N$ is given by:

$$\begin{aligned} U_j &= -(1-\alpha)V(a_j - \sum_{i\in N} s_i - b_j) - (1-\alpha)[E(a_j - \sum_{i\in N} s_i - b_j)]^2 \\ &\quad -\frac{\alpha}{n-1}\sum_{m\neq j} V(a_j - a_m) - \frac{\alpha}{n-1}\sum_{m\neq j}[E(a_j - a_m)]^2. \end{aligned}$$

It follows from (13) that $E(a_j) = \sum_{i\in N} E(s_i) + B_j$, so we get:

$$U_j = -(1-\alpha)V(a_j - \sum_{i\in N} s_i) - \frac{\alpha}{n-1}\sum_{m\neq j} V(a_j - a_m) - (1-\alpha)[B_j - b_j]^2 - \frac{\alpha}{n-1}\sum_{m\neq j}[B_j - B_m]^2.$$

We consider two communication networks $R = (R_k)_{k\in N}$ and $R' = (R'_k)_{k\in N}$ such that $R_i = R'_i\backslash\{t\}$ and $R_k = R'_k$ for all $k \in N\backslash\{i\}$. That is, R and R' are the same except that player i has one additional receiver (player t) in R'. Players i and t are fixed throughout the analysis. We denote $|R_i| = r_i$ and $|R'_i| = r'_i = r_i + 1$. The ex ante equilibrium payoff of every player $j \in N$ with the communication network R (R', resp.) is denoted U_j (U'_j, resp.). Given the communication network R (R', resp.), the second stage equilibrium action of every player $j \in N$ is denoted a_j (a'_j, resp.). For all $j \in N$, given a strategic communication network R (R', resp.), let $I_j = \{k : j \in R_k\} \cup \{j\}$ ($I'_j = \{k \in N : j \in R'_k\} \cup \{j\}$, resp.) denote the set of players whose signals are known by player j, and $\overline{I}_j = \{k : j \notin R_k\}\backslash\{j\}$ ($\overline{I'}_j = \{k : j \notin R'_k\}\backslash\{j\}$, resp.) the set of players whose signals are unknown by player j.

For every player $j \in N$, we have:

$$U_j - U'_j = (1-\alpha)\left(V(a'_j - \sum_{i\in N} s_i) - V(a_j - \sum_{i\in N} s_i)\right) + \frac{\alpha}{n-1}\left(\sum_{m\neq j} V(a'_j - a'_m) - \sum_{m\neq j} V(a_j - a_m)\right). \tag{19}$$

The second-stage equilibrium action a_j given by (2) enables to write:

$$V(a_j - \sum_{i\in N} s_i) = V\left(\sum_{l\in I_j}\frac{\alpha(n-1-r_l)[E(s_l) - s_l]}{n-1-\alpha r_l} + \sum_{l\in \overline{I}_j}[E(s_l) - s_l] + B_j\right).$$

The independence of signals yields:

$$\begin{aligned} &V(a_j - \sum_{i\in N} s_i) \\ &= \sum_{l\in I_j} V\left(\frac{\alpha(n-1-r_l)s_l}{n-1-\alpha r_l}\right) + \sum_{l\in \overline{I}_j} V(s_l) = \sum_{l\in I_j}\left(\frac{\alpha(n-1-r_l)}{n-1-\alpha r_l}\right)^2 V(s_l) + \sum_{l\in \overline{I}_j} V(s_l) \\ &= \sum_{l\in I_j\backslash\{i\}}\left(\frac{\alpha(n-1-r_l)}{n-1-\alpha r_l}\right)^2 V(s_l) + \sum_{l\in \overline{I}_j\backslash\{i\}} V(s_l) + V(s_i)\left(\mathbf{1}[i\in I_j]\left(\frac{\alpha(n-1-r_i)}{n-1-\alpha r_i}\right)^2 + \mathbf{1}[i\in \overline{I}_j]\right), \end{aligned}$$

where $\mathbf{1}[i \in I_j]$ is the indicator function that equals 1 when player j knows the signal s_i and $\mathbf{1}[i \in \overline{I}_j]$ is the indicator function that equals 1 when player j does not know the signal s_i. A similar equation holds for $V(a'_j - \sum_{i \in N} s_i)$, when the communication network is R'.

The two communication networks R and R' that we consider are such that $I_j \backslash \{i\} = I'_j \backslash \{i\}$ and $\overline{I}_j \backslash \{i\} = \overline{I'}_j \backslash \{i\}$, so for all $j \in N$, we have:

$$\begin{aligned} & V(a'_j - \sum_{i \in N} s_i) - V(a_j - \sum_{i \in N} s_i) \\ & = V(s_i) \left(\mathbf{1}[i \in I'_j] \left(\frac{\alpha(n-1-r'_i)}{n-1-\alpha r'_i} \right)^2 + \mathbf{1}[i \in \overline{I'}_j] - \mathbf{1}[i \in I_j] \left(\frac{\alpha(n-1-r_i)}{n-1-\alpha r_i} \right)^2 - \mathbf{1}[i \in \overline{I}_j] \right). \end{aligned} \tag{20}$$

When the communication network is R, for all $j \in N$ and $m \neq j$, we have, from (2):

$$\begin{aligned} V(a_j - a_m) &= \sum_{l \in I_j \cap \overline{I}_m} \left(\frac{(1-\alpha)(n-1)}{n-1-\alpha r_l} \right)^2 V(s_l) + \sum_{l \in \overline{I}_j \cap I_m} \left(\frac{(1-\alpha)(n-1)}{n-1-\alpha r_l} \right)^2 V(s_l) \\ &= \sum_{l \in (I_j \cap \overline{I}_m) \backslash \{i\}} \left(\frac{(1-\alpha)(n-1)}{n-1-\alpha r_l} \right)^2 V(s_l) + \sum_{l \in (\overline{I}_j \cap I_m) \backslash \{i\}} \left(\frac{(1-\alpha)(n-1)}{n-1-\alpha r_l} \right)^2 V(s_l) \\ &\quad + \left(\frac{(1-\alpha)(n-1)}{n-1-\alpha r_i} \right)^2 V(s_i) \left(\mathbf{1}[i \in I_j \cap \overline{I}_m] + \mathbf{1}[i \in \overline{I}_j \cap I_m] \right). \end{aligned}$$

A similar equation holds for $V(a'_j - a'_m)$, when the communication network is R'.

The two communication networks R and R' are such that $(I_j \cap \overline{I}_m) \backslash \{i\} = (I'_j \cap \overline{I'}_m) \backslash \{i\}$ and $(\overline{I}_j \cap I_m) \backslash \{i\} = (\overline{I'}_j \cap I'_m) \backslash \{i\}$, so for all $j \in N$ and $m \neq j$ we have:

$$\begin{aligned} & V(a'_j - a'_m) - V(a_j - a_m) \\ & = ((1-\alpha)(n-1))^2 V(s_i) \left[\frac{\mathbf{1}[i \in I'_j \cap \overline{I'}_m] + \mathbf{1}[i \in \overline{I'}_j \cap I'_m]}{(n-1-\alpha r'_i)^2} - \frac{\mathbf{1}[i \in I_j \cap \overline{I}_m] + \mathbf{1}[i \in \overline{I}_j \cap I_m]}{(n-1-\alpha r_i)^2} \right]. \end{aligned} \tag{21}$$

Plugging (20) and (21) into (19), we get:

$$\begin{aligned} U_j - U'_j = (1-\alpha) V(s_i) & \left(\mathbf{1}[i \in I'_j] \left(\frac{\alpha(n-1-r'_i)}{n-1-\alpha r'_i} \right)^2 + \mathbf{1}[i \in \overline{I'}_j] - \mathbf{1}[i \in I_j] \left(\frac{\alpha(n-1-r_i)}{n-1-\alpha r_i} \right)^2 - \mathbf{1}[i \in \overline{I}_j] \right. \\ & \left. + \alpha(1-\alpha)(n-1) \sum_{m \neq j} \left(\frac{\mathbf{1}[i \in I'_j \cap \overline{I'}_m] + \mathbf{1}[i \in \overline{I'}_j \cap I'_m]}{(n-1-\alpha r'_i)^2} - \frac{\mathbf{1}[i \in I_j \cap \overline{I}_m] + \mathbf{1}[i \in \overline{I}_j \cap I_m]}{(n-1-\alpha r_i)^2} \right) \right). \end{aligned} \tag{22}$$

To evaluate the sign of $U_j - U'_j$ in order to know who is better off and who is worse off depending on the communication network, we distinguish four types of players:

- *(i)*: Players who belong both to R_i and to R'_i. For every such player $j \in R_i = R'_i \backslash \{t\}$, we have $i \in I_j$ and $i \in I'_j$.

- *(ii)*: Players different from player i who belong neither to R_i nor to R'_i. For every such player $j \in N \backslash (R'_i \cup \{i\}) = N \backslash (R_i \cup \{i, t\})$, we have $i \in \overline{I}_j$ and $i \in \overline{I'}_j$.
- *(iii)*: Player t who belongs to R'_i but not to R_i. For this player we have $i \in I'_t$ and $i \in \overline{I}_t$.
- *(iv)*: Player i, for whom we have $i \in I_i$ and $i \in I'_i$.

(i) For every player $j \in R'_i \backslash \{t\}$, the set of players different from j can be divided into three sets of players: $\{i\} \cup (R'_i \backslash \{j, t\})$, $N \backslash (R'_i \cup \{i\})$ and $\{t\}$. We have:

- for every player $m \in \{i\} \cup (R'_i \backslash \{j, t\})$, $i \in I_m$ and $i \in I'_m$,
- for every player $m \in N \backslash (R'_i \cup \{i\})$, $i \in \overline{I}_m$ and $i \in \overline{I'}_m$,
- for player t, $i \in \overline{I}_t$ but $i \in I'_t$.

Since $i \in I_j$, $i \in I'_j$, and $|N \backslash (R'_i \cup \{i\})| = (n - 1 - r'_i)$, Equation (22) simplifies to:

$$U_j - U'_j \quad = \quad \alpha(1-\alpha)V(s_i)\left(\frac{(n-1-r'_i)}{(n-1-\alpha r'_i)} - \frac{(n-1-r_i)}{(n-1-\alpha r_i)}\right). \tag{23}$$

Using $r'_i = r_i + 1$, we get $U_j - U'_j = -\left(\frac{\alpha(1-\alpha)^2(n-1)}{(n-1-\alpha r'_i)(n-1-\alpha r_i)}\right)V(s_i) < 0$. Hence, for all $j \in R'_i \backslash \{t\}$, we have $U_j < U'_j$.

(ii) For every player $j \in N \backslash (R'_i \cup \{i\})$, the set of players different from j can be divided into three sets of players: $\{i\} \cup (R'_i \backslash \{t\})$, $N \backslash (R'_i \cup \{i, j\})$ and $\{t\}$. We have:

- for every player $m \in \{i\} \cup (R'_i \backslash \{t\})$, $i \in I_m$ and $i \in I'_m$,
- for every player $m \in N \backslash (R'_i \cup \{i, j\})$, $i \in \overline{I}_m$ and $i \in \overline{I'}_m$,
- for player t, $i \in \overline{I}_t$ but $i \in I'_t$.

Since $i \in \overline{I}_j$, $i \in \overline{I'}_j$, and $|\{i\} \cup (R'_i \backslash \{t\})| = r'_i$, Equation (22) gives:

$$U_j - U'_j \quad = \quad \alpha(1-\alpha)^2(n-1)V(s_i)\left(\frac{r'_i + 1}{(n-1-\alpha r'_i)^2} - \frac{r'_i}{(n-1-\alpha r_i)^2}\right). \tag{24}$$

Since $r_i = r'_i - 1$, we have $\left[\frac{r'_i+1}{(n-1-\alpha r'_i)^2} - \frac{r'_i}{(n-1-\alpha r_i)^2}\right] > 0$. Hence, for all $j \in N \backslash (R'_i \cup \{i\})$, we have $U_j > U'_j$.

(iii) The set of players different from t can be divided into two sets of players: $\{i\} \cup (R'_i \backslash \{t\})$ and $N \backslash (R'_i \cup \{i\})$. We have:

- for every player $m \in \{i\} \cup (R'_i \backslash \{t\})$, $i \in I_m$ and $i \in I'_m$,
- for every player $m \in N \backslash (R'_i \cup \{i\})$, $i \in \overline{I}_m$ and $i \in \overline{I'}_m$,

Since $i \in \overline{I}_t$, $i \in I'_t$, $|\{i\} \cup (R'_i \backslash \{t\})| = r'_i$, and $|N \backslash (R'_i \cup \{i\})| = n - 1 - r'_i$, Equation (22) gives:

$$U_t - U'_t \quad = \quad -(1-\alpha)^2 (n-1) V(s_i) \left(\frac{1}{n-1-\alpha r'_i} + \frac{\alpha\, r'_i}{(n-1-\alpha r_i)^2} \right) < 0. \qquad (25)$$

Hence, for player t who belongs to R'_i but not to R_i we have $U_t < U'_t$.

(iv) The set of players different from i can be divided into three sets of players: $R'_i \backslash \{t\}$, $N \backslash (R'_i \cup \{i\})$ and $\{t\}$. We have:

- for every player $m \in R'_i \backslash \{t\}$, $i \in I_m$ and $i \in I'_m$,
- for every player $m \in N \backslash (R'_i \cup \{i\})$, $i \in \overline{I}_m$ and $i \in \overline{I'}_m$,
- for player t, $i \in \overline{I}_t$ but $i \in I'_t$.

Since $i \in I_i$, $i \in I'_i$, and $|N \backslash (R'_i \cup \{i\})| = (n-1-r'_i)$, Equation (22) gives exactly the same difference as in Equation (23). Hence, for player i such that $R_i = R'_i \backslash \{t\}$, we have $U_i < U'_i$. This completes the proof of Proposition 1.

6.3 Proof of Proposition 2

As in the proof of Proposition 1, we consider two communication networks $R = (R_i, R_{-i})$ and $R' = (R'_i, R_{-i})$ such that $R_i = R'_i \backslash \{t\}$. Again, player t is such that $t \in R'_i$ but $t \notin R_i$.

Ex ante expected welfare is the sum of ex ante expected utilities. When the communication network is R', it is given by:

$$W' \quad = \quad \sum_{j \in R'_i \backslash \{t\}} U'_j + \sum_{j \in N \backslash (R'_i \cup \{i\})} U'_j \ + \ U'_t \ + \ U'_i.$$

When the communication network is R, it is given by:

$$W \quad = \quad \sum_{j \in R_i} U_j + \sum_{j \in N \backslash (R_i \cup \{i,t\})} U_j \ + \ U_t \ + \ U_i.$$

Using $R'_i \backslash \{t\} = R_i$, $N \backslash (R'_i \cup \{i\}) = N \backslash (R_i \cup \{i,t\})$, and the fact that for all $j \in R'_i \backslash \{t\}$, $U_j - U'_j = U_i - U'_i$, the difference $W - W'$ can be written as follows:

$$W - W' \quad = \quad \sum_{j \in \{i\} \cup (R'_i \backslash \{t\})} [U_j - U'_j] + \sum_{j \in N \backslash (R'_i \cup \{i\})} [U_j - U'_j] \ + \ [U_t - U'_t].$$

We have $|\{i\} \cup (R'_i \backslash \{t\})| = r'_i$ and $|N \backslash (R'_i \cup \{i\})| = n - 1 - r'_i$. Using equations (23), (24) and (25), we get:

$$\begin{aligned} W - W' &= \alpha(1-\alpha)r_i' \left[\frac{(n-1-r_i')}{(n-1-\alpha r_i')} - \frac{(n-1-r_i)}{(n-1-\alpha r_i)} \right] V[s_i] \\ &+ \alpha(1-\alpha)^2(n-1)(n-1-r_i') \left[\frac{r_i'+1}{(n-1-\alpha r_i')^2} - \frac{r_i'}{(n-1-\alpha r_i)^2} \right] V[s_i] \\ &- (1-\alpha)^2(n-1) \left[\frac{1}{n-1-\alpha r_i'} + \frac{\alpha\, r_i'}{(n-1-\alpha r_i)^2} \right] V[s_i]. \end{aligned}$$

After some simplifications and using the fact that $r_i' = r_i + 1$, we get:

$$W - W' = \underbrace{- \frac{(1-\alpha)^3(n-1)^2 V[s_i]}{(n-1-\alpha r_i')^2(n-1-\alpha r_i)^2}}_{<0} \underbrace{\left[\alpha^2(1-r_i'-r_i'^2) + 2\alpha(n-1) + (n-1)^2\right]}_{x}.$$

Solving $x = 0$ in α gives the following discriminant: $4(n-1)^2(r_i' + r_i'^2) \geq 0$. We have $x \geq 0$ if and only if $\alpha \in [\alpha_1, \alpha_2]$, with $\alpha_1 = \frac{(n-1)(1-\sqrt{r_i'+r_i'^2})}{r_i'+r_i'^2-1}$ and $\alpha_2 = \frac{(n-1)(1+\sqrt{r_i'+r_i'^2})}{r_i'+r_i'^2-1}$. From $r_i' \geq 1$, we deduce that $\alpha_1 < 0$. From $r_i' \leq n-1$ and the fact that α_2 is decreasing in r_i', we deduce that $\alpha_2 > 1$. Since $\alpha \in (0,1)$, x is always strictly positive. Hence, $W < W'$.

6.4 Proof of Propositions 3 and 4

Consider an equilibrium of the private communication game in which each player i reveals his type to players in $R_i \subseteq N\backslash\{i\}$. Without loss of generality, assume that player i sends to every player $j \in R_i$ the message $m_i^j = \overline{m}$ when his type is $\overline{s}_i$ and the message $m_i^j = \underline{m}$ when his type is $\underline{s}_i$, and sends the same message whatever his type to players outside R_i. Given $(R_i)_{i\in N}$, second stage equilibrium actions are given by (2).

Without loss of generality, we look for the conditions under which player 1 does not deviate from his equilibrium communication strategy above. First, assume that player 1's true type is $s_1 = \overline{s}_1$. In equilibrium, using Equation (2), the second-stage action of every player $i \in R_1 \cup \{1\}$ is given by

$$\begin{aligned} \overline{a}_i &= \sum_{j\in I_i\backslash\{1\}} \frac{\alpha(n-r_j-1)E(s_j) + (1-\alpha)(n-1)s_j}{n-1-\alpha r_j} + \sum_{j\in \overline{I}_i} E(s_j) + B_i \\ &+ \frac{\alpha(n-r_1-1)E(s_1) + (1-\alpha)(n-1)\overline{s}_1}{n-1-\alpha r_1}, \end{aligned} \tag{26}$$

and the second-stage action of every player $i \notin R_1 \cup \{1\}$ is given by

$$a_i = \sum_{j\in I_i} \frac{\alpha(n-r_j-1)E(s_j) + (1-\alpha)(n-1)s_j}{n-1-\alpha r_j} + \sum_{j\in \overline{I}_i\backslash\{1\}} E(s_j) + B_i + E(s_1). \tag{27}$$

The relevant deviations for player 1 in the communication stage consist in lying to a subset of players $M \subseteq R_1$, i.e., sending message $\underline{m}$ instead of $\overline{m}$ to players in M (and not deviating towards the other players). Let $m = |M|$, and denote by $(a'_i)_{i\in N}$ the profile of players' actions after this deviation. Every player $i \in M$ chooses action $a'_i = \underline{a}_i$, which is given by (26) by replacing $\overline{s}_1$ by $\underline{s}_1$. The action a'_i of every player $i \in N\backslash(M \cup \{1\})$ is the same as in the original equilibrium. Player 1's optimal action in the second stage is obtained from the best response of Equation (15) to $(a'_i)_{i\neq 1}$, and takes the following form:

$$a'_1 = (1-\alpha)\left(\sum_{j\in I_1\backslash\{1\}} s_j \; + \overline{s}_1 \; + \sum_{j\in \overline{I}_1} E(s_j) \; + b_1\right) + \frac{\alpha}{n-1}\sum_{i\neq 1} E_1(a'_i). \tag{28}$$

Using the same reasoning as the one used to get expression (17), we get:

$$\begin{aligned}\sum_{i\neq 1} E_1(a'_i) \quad &= \quad \sum_{j\in I_1} r_j \,\frac{\alpha(n-r_j-1)E(s_j)}{n-1-\alpha r_j} + \sum_{j\in \overline{I}_1} (r_j+1)\,\frac{\alpha(n-r_j-1)E(s_j)}{n-1-\alpha r_j} \\ &\quad + \sum_{j\in I_1\backslash\{1\}} r_j\,\frac{(1-\alpha)(n-1)s_j}{n-1-\alpha r_j} + \frac{m(1-\alpha)(n-1)\underline{s}_1}{n-1-\alpha r_1} \\ &\quad + \frac{(r_1-m)(1-\alpha)(n-1)\overline{s}_1}{n-1-\alpha r_1} + \sum_{j\in \overline{I}_1} (r_j+1)\,\frac{(1-\alpha)(n-1)E(s_j)}{n-1-\alpha r_j} \\ &\quad + \sum_{j\in I_1} (n-1-r_j)E(s_j) + \sum_{j\in \overline{I}_1} (n-2-r_j)\,E(s_j) + \sum_{i\neq 1} B_i. \end{aligned} \tag{29}$$

Plugging (29) into (28), using (18) and simplifying, we get:

$$\begin{aligned} a'_1 \quad &= \quad \sum_{j\in I_1\backslash\{1\}} \frac{\alpha(n-r_j-1)E(s_j) + (1-\alpha)(n-1)s_j}{n-1-\alpha r_j} + \sum_{j\in \overline{I}_1} E(s_j) + B_1 \\ &\quad + \frac{\alpha m(1-\alpha)\underline{s}_1 + (n-1-\alpha m)(1-\alpha)\overline{s}_1 + \alpha(n-r_1-1)E(s_1)}{n-1-\alpha r_1}. \end{aligned} \tag{30}$$

We denote by V_1 the expected payoff of player 1 conditional to signal s_1 under the original equilibrium, and V'_1 his expected payoff conditional to signal s_1 when he deviates by lying to players in M (and thus plays action a'_1 in the second-stage game. Player 1 does not deviate by lying to players in M if $V'_1 - V_1 \leq 0$. We have:

$$\begin{aligned} & V'_1 - V_1 \\ = \quad & (1-\alpha)E\big[(\overline{a}_1 - \sum_{i\in N} s_i - b_1)^2 - (a'_1 - \sum_{i\in N} s_i - b_1)^2 \mid s_1\big] + \frac{\alpha}{n-1}\left(\sum_{i\in M} E\big[(\overline{a}_1 - \overline{a}_i)^2 - (a'_1 - \underline{a}_i)^2 \mid s_1\big]\right. \\ & \left. + \sum_{i\in R_1\backslash M} E\big[(\overline{a}_1 - \overline{a}_i)^2 - (a'_1 - \overline{a}_i)^2 \mid s_1\big] + \sum_{i\in N\backslash(R_1\cup\{1\})} E\big[(\overline{a}_1 - a_i)^2 - (a'_1 - a_i)^2 \mid s_1\big]\right). \end{aligned}$$

For the sake of simplicity, we examine separately the elements of the difference $V'_1 - V_1$ and use the following notation for $i \neq 1$:

$$z_i = \sum_{j \in (I_1 \cap \bar{I}_i) \setminus \{1\}} \frac{(1-\alpha)(n-1)(s_j - E(s_j))}{n-1-\alpha r_j} + \sum_{j \in (\bar{I}_1 \cap I_i) \setminus \{1\}} \frac{(1-\alpha)(n-1)(E(s_j) - s_j)}{n-1-\alpha r_j} + B_1 - B_i.$$

Using (26), (27) and (30) and the fact that $E[z_i \mid s_1] = B_1 - B_i$, we get:

$$\sum_{i \in M} E\left[(\bar{a}_1 - \bar{a}_i)^2 - (a_1' - \underline{a}_i)^2 \mid s_1\right] = \sum_{i \in M} E\left[z_i^2 - \left(z_i + \frac{(1-\alpha)(n-1-\alpha m)(\overline{s}_1 - \underline{s}_1)}{n-1-\alpha r_1} \right)^2 \middle| s_1 \right]$$
$$= -2\left(\frac{(1-\alpha)(n-1-\alpha m)(\overline{s}_1 - \underline{s}_1)}{n-1-\alpha r_1}\right) \sum_{i \in M} (B_1 - B_i) - m \left(\frac{(1-\alpha)(n-1-\alpha m)(\overline{s}_1 - \underline{s}_1)}{n-1-\alpha r_1}\right)^2 . \quad (31)$$

$$\sum_{i \in R_1 \setminus M} E\left[(\bar{a}_1 - \bar{a}_i)^2 - (a_1' - \bar{a}_i)^2 \mid s_1\right] = \sum_{i \in R_1 \setminus M} E\left[z_i^2 - \left(z_i - \frac{(1-\alpha)\alpha m(\overline{s}_1 - \underline{s}_1)}{n-1-\alpha r_1} \right)^2 \middle| s_1 \right]$$
$$= 2\left(\frac{(1-\alpha)\alpha m(\overline{s}_1 - \underline{s}_1)}{n-1-\alpha r_1}\right) \sum_{i \in R_1 \setminus M} (B_1 - B_i) - (r_1 - m)\left(\frac{(1-\alpha)\alpha m(\overline{s}_1 - \underline{s}_1)}{n-1-\alpha r_1}\right)^2 . \quad (32)$$

$$\sum_{i \in N \setminus (R_1 \cup \{1\})} E\left[(\bar{a}_1 - a_i)^2 - (a_1' - a_i')^2 \mid s_1\right] = \sum_{i \in N \setminus (R_1 \cup \{1\})} E\left[\left(z_i + \frac{(1-\alpha)(n-1)(\overline{s}_1 - E(s_1))}{n-1-\alpha r_1} \right)^2\right.$$
$$\left. - \left(z_i + \frac{(1-\alpha)\alpha m \underline{s}_1 + (1-\alpha)(n-1-\alpha m)\overline{s}_1 - (1-\alpha)(n-1)E(s_1)}{n-1-\alpha r_1} \right)^2 \middle| s_1 \right]$$
$$= 2\left(\frac{(1-\alpha)\alpha m(\overline{s}_1 - \underline{s}_1)}{n-1-\alpha r_1}\right) \sum_{i \in N \setminus (R_1 \cup \{1\})} (B_1 - B_i) + (n-1-r_1)\left(\frac{(1-\alpha)(n-1)(\overline{s}_1 - E(s_1))}{n-1-\alpha r_1}\right)^2$$
$$- (n-1-r_1)\left(\frac{(1-\alpha)\alpha m \underline{s}_1 + (1-\alpha)(n-1-\alpha m)\overline{s}_1 - (1-\alpha)(n-1)E(s_1)}{n-1-\alpha r_1}\right)^2 . \quad (33)$$

In addition, using

$$\bar{a}_1 - a_1' = \frac{(1-\alpha)\alpha m(\overline{s}_1 - \underline{s}_1)}{n-1-\alpha r_1},$$

and

$$\bar{a}_1^2 - {a_1'}^2 = \left(\frac{\alpha(n-r_1-1)E(s_1) + (1-\alpha)(n-1)\overline{s}_1}{n-1-\alpha r_l}\right)^2$$
$$- \left(\frac{\alpha m(1-\alpha)\underline{s}_1 + (n-1-\alpha m)(1-\alpha)\overline{s}_1 + \alpha(n-r_1-1)E(s_1)}{n-1-\alpha r_1}\right)^2$$
$$+ 2\left(\sum_{j \in I_1 \setminus \{1\}} \frac{\alpha(n-r_j-1)E(s_j) + (1-\alpha)(n-1)s_j}{n-1-\alpha r_j} + \sum_{j \in \bar{I}_1} E(s_j) + B_1 \right)\left(\frac{(1-\alpha)\alpha m(\overline{s}_1 - \underline{s}_1)}{n-1-r_1}\right),$$

we get:

$$
\begin{aligned}
&E\left[(\bar{a}_1 - \sum_{i\in N} s_i - b_1)^2 - (a_1' - \sum_{i\in N} s_i - b_1)^2 \middle| s_1\right] \\
&= E\left[\bar{a}_1^2 - {a_1'}^2 \middle| s_1\right] - 2E\left[(\bar{a}_1 - a_1')(\sum_{i\in N\setminus\{1\}} s_i + s_1 + b_1) \middle| s_1\right] \\
&= \left(\frac{\alpha(n-r_1-1)E(s_1) + (1-\alpha)(n-1)\overline{s}_1}{n-1-\alpha r_1}\right)^2 + 2\left(B_1 - b_1 - \overline{s}_1\right)\left(\frac{(1-\alpha)\alpha m(\overline{s}_1 - \underline{s}_1)}{n-1-\alpha r_1}\right) \\
&- \left(\frac{\alpha m(1-\alpha)\underline{s}_1 + (n-1-\alpha m)(1-\alpha)\overline{s}_1 + \alpha(n-r_1-1)E(s_1)}{n-1-\alpha r_1}\right)^2. \qquad (34)
\end{aligned}
$$

Next, we plug (31), (32), (33) and (34) into $V_1' - V_1$ and simplify. To simplify the part of the difference $V_1' - V_1$ that deals with biases, one should note that:

$$B_1 - B_i = \frac{(1-\alpha)(n-1)(b_1 - b_i)}{n+\alpha-1},$$

and

$$B_1 - b_1 = \frac{-\alpha(n-1)b_1 + \sum_{j\neq 1} b_j}{n+\alpha-1}.$$

Finally, we get:

$$V_1' - V_1 = \frac{2\alpha(1-\alpha)^2(n-1)(\overline{s}_1 - \underline{s}_1)}{(n+\alpha-1)(n-1-\alpha r_1)}\left(\sum_{i\in M} b_i - m b_1\right) - \frac{\alpha(1-\alpha)^2 m(n-1-\alpha m)(\overline{s}_1 - \underline{s}_1)^2}{(n-1-\alpha r_1)^2}.$$

Hence, in the private communication game, player 1 of type $s_1 = \overline{s}_1$ does not deviate by lying to players in $M \subseteq R_1$ if $V_1' - V_1 \leq 0$, i.e.,

$$-\left(b_1 - \frac{\sum_{i\in M} b_i}{m}\right) \leq \frac{(n-1+\alpha)(n-1-\alpha m)}{2(n-1)(n-1-\alpha r_1)}(\overline{s}_1 - \underline{s}_1). \qquad (35)$$

Applying the same reasoning, player 1 of type $s_1 = \underline{s}_1$ has no profitable deviation if, for all $M \subseteq R_1$, the following condition holds:

$$b_1 - \frac{\sum_{i\in M} b_i}{m} \leq \frac{(n-1+\alpha)(n-1-\alpha m)}{2(n-1)(n-1-\alpha r_1)}(\overline{s}_1 - \underline{s}_1). \qquad (36)$$

Condition (3) is obtained from (35) and (36).

In a group $\bar{R}$-communication game, every player i is required to send the same message to all players in $\bar{R}_i$. Consider an equilibrium in which player 1 sends to all the players in $\bar{R}_1$ the message $m_1 = \overline{m}$ when his type is $\overline{s}_1$ and the message $m_1 = \underline{m}$ when his type is $\underline{s}_1$. The only possible deviation for player 1 in the communication stage consists in lying to all the players in $\bar{R}_1$, i.e., sending the message $\underline{m}$ instead of $\overline{m}$ to all the players in $\bar{R}_1$. Therefore, condition (3) for $R_1' = R_1 = \bar{R}_1$ is the condition under which player 1 does not deviate from his equilibrium communication strategy above in the group $\bar{R}$-communication game.

6.5 Proof of Proposition 5

When types are fully certifiable, the simplest way to support a fully revealing equilibrium is to consider the communication strategy profile in which every player completely certifies his type to all the other players whatever his type. That is, $\sigma_i(s_i) = c_i(s_i)$ for all $i \in N$ and $s_i \in S_i$ in the public communication game, and $\sigma_i^j(s_i) = c_i(s_i)$ for all $i \in N$, $j \neq i$ and $s_i \in S_i$ in the private communication game, where $c_i(s_i) \in M_i$ is such that $M_i^{-1}(c_i(s_i)) = \{s_i\}$. When such strategies are used in the first stage, then each player knows the state θ in the decision stage, so the second stage equilibrium actions are given by Equation (12).

We prove the existence of a fully revealing equilibrium in the public and private communication games separately.

- *Public Communication.* We start from a fully revealing communication strategy profile $\sigma_i(s_i) = c_i(s_i)$ for all $i \in N$ and $s_i \in S_i$, and consider a deviation by player i to a message $m_i \neq c_i(s_i)$ when his type is s_i. To support this equilibrium, we consider the degenerate *common* belief $\mu_j^i(m_i) = \mu^i(m_i) = \max\{t_i \in S_i : m_i \in M_i(t_i)\}$ for every $j \neq i$ when $b_i \leq \bar{b}$, and $\mu_j^i(m_i) = \mu^i(m_i) = \min\{t_i \in S_i : m_i \in M_i(t_i)\}$ for every $j \neq i$ when $b_i \geq \bar{b}$. By Equation (11), a sufficient (but not necessary) condition for player i's deviation not to be profitable is that for all $s_{-i} \in S_{-i}$,

$$
\begin{aligned}
&\left[a_i(a_{-i}(\theta(\mu^i(m_i), s_{-i})); \theta(s))\right]^2 - \frac{\alpha}{n-1}\sum_{j\neq i}\left[a_j(\theta(\mu^i(m_i), s_{-i}))\right]^2 \\
&\leq \left[a_i(a_{-i}(\theta(s)); \theta(s))\right]^2 - \frac{\alpha}{n-1}\sum_{j\neq i}\left[a_j(\theta(s))\right]^2.
\end{aligned}
\tag{37}
$$

Given player i's best response (10), this is equivalent to

$$
\begin{aligned}
&\left[(1-\alpha)(\theta+b_i) + \frac{\alpha}{n-1}\sum_{j\neq i} a_j(\theta(\mu^i(m_i), s_{-i}))\right]^2 - \frac{\alpha}{n-1}\sum_{j\neq i}\left[a_j(\theta(\mu^i(m_i), s_{-i}))\right]^2 \\
\leq &\left[(1-\alpha)(\theta+b_i) + \frac{\alpha}{n-1}\sum_{j\neq i} a_j(\theta(s))\right]^2 - \frac{\alpha}{n-1}\sum_{j\neq i}\left[a_j(\theta(s))\right]^2.
\end{aligned}
\tag{38}
$$

By replacing the equilibrium action of every player $j \neq i$ given by Equation (12) in the last inequality we get (after some simplifications):

$$
\left[\theta(\mu^i(m_i), s_{-i}) - \theta(s)\right]\left[\theta(s) - \theta(\mu^i(m_i), s_{-i}) + 2\frac{(n-1)b_i - \sum_{j\neq i} b_j}{n+\alpha-1}\right] \leq 0. \tag{39}
$$

Since $\theta(s)$ is increasing in s_i, a sufficient condition for this inequality to be satisfied is $\mu^i(m_i) = \max\{t_i \in S_i : m_i \in M_i(t_i)\}$ when $b_i \leq \bar{b}$, and $\mu^i(m_i) = \min\{t_i \in S_i : m_i \in M_i(t_i)\}$ when $b_i \geq \bar{b}$.

- *Private Communication.* We start from a fully revealing communication strategy profile $\sigma_i^j(s_i) = c_i(s_i)$ for all $i \in N$, $j \neq i$ and $s_i \in S_i$, and consider a deviation by player i to a vector of messages $m_i \neq (c_i(s_i), \ldots, c_i(s_i))$ when his type is s_i. To support this equilibrium, we consider the degenerate *private* beliefs $\mu_j^i(m_i^j) = \max\{t_i \in S_i : m_i^j \in M_i(t_i)\}$ when $b_i \le b_j$, and $\mu_j^i(m_i^j) = \min\{t_i \in S_i : m_i^j \in M_i(t_i)\}$ when $b_i \ge b_j$.

The analogue of Equation (38) for the private communication game is:

$$\begin{aligned} &\Big[(1-\alpha)(\theta+b_i) + \frac{\alpha}{n-1}\sum_{j\neq i} a_j(\theta(\mu_j^i(m_i^j), s_{-i}))\Big]^2 - \frac{\alpha}{n-1}\sum_{j\neq i}\Big[a_j(\theta(\mu_j^i(m_i^j), s_{-i}))\Big]^2 \\ \le &\Big[(1-\alpha)(\theta+b_i) + \frac{\alpha}{n-1}\sum_{j\neq i} a_j(\theta(s))\Big]^2 - \frac{\alpha}{n-1}\sum_{j\neq i}[a_j(\theta(s))]^2, \end{aligned} \tag{40}$$

i.e., by (12),

$$\begin{aligned} &\Big[(1-\alpha)\theta + \frac{\alpha}{n-1}\sum_{j\neq i}\theta(\mu_j^i(m_i^j), s_{-i}) + B_i\Big]^2 - \Big[\theta + B_i\Big]^2 \\ &+ \frac{\alpha}{n-1}\Big[\sum_{j\neq i}\theta^2 - \big(\theta(\mu_j^i(m_i^j), s_{-i})\big)^2 + 2B_j(\theta - \theta(\mu_j^i(m_i^j), s_{-i}))\Big] \le 0. \end{aligned}$$

Letting

$$\begin{aligned} T \equiv &\Big(\frac{\alpha}{n-1}\Big)^2\Big(\sum_{j\neq i}\theta(\mu_j^i(m_i^j), s_{-i})\Big)^2 + \frac{2\alpha(1-\alpha)}{n-1}\theta\sum_{j\neq i}\theta(\mu_j^i(m_i^j), s_{-i}) \\ &- \frac{\alpha}{n-1}\sum_{j\neq i}[\theta(\mu_j^i(m_i^j), s_{-i})]^2 - \alpha(1-\alpha)\theta^2, \end{aligned}$$

the condition further simplifies to

$$\begin{aligned} &T + 2\alpha\theta\left(\frac{\sum_{j\neq i} b_j + \alpha b_i}{n+\alpha-1} - B_i\right) + \frac{2\alpha}{n-1}\sum_{j\neq i}\theta(\mu_j^i(m_i^j), s_{-i})(B_i - B_j) \le 0 \\ \Leftrightarrow\ &T + \frac{2\alpha(1-\alpha)}{n+\alpha-1}\sum_{j\neq i}\big[b_i - b_j\big]\big[\theta(\mu_j^i(m_i^j), s_{-i}) - \theta\big] \le 0. \end{aligned}$$

By the construction of players' beliefs, and since $\theta(s)$ is increasing in s_i, we have

$$\big[b_i - b_j\big]\big[\theta(\mu_j^i(m_i^j), s_{-i}) - \theta\big] \le 0, \text{ for all } j \neq i.$$

Finally, to show that the condition for no deviation is satisfied, it suffices to remark that T is always negative. Indeed, solving $T = 0$ in θ gives the following discriminant:

$$\frac{4\alpha^2(1-\alpha)}{(n-1)^2}\Big(\big[\sum_{j\neq i}\theta(\mu_j^i(m_i^j), s_{-i})\big]^2 - (n-1)\sum_{j\neq i}\big[\theta(\mu_j^i(m_i^j), s_{-i})\big]^2\Big),$$

which can be checked to be always negative.[17]

[17]By the property $(x_1 + \cdots + x_m)^2 \le m((x_1)^2 + \cdots + (x_m)^2) \Leftrightarrow (m-1)((x_1)^2 + \cdots + (x_m)^2) - \sum_{i\neq j} x_i x_j \ge 0 \Leftrightarrow \sum_{i\neq j}(x_i - x_j)^2 \ge 0$ for all $(x_1, \ldots, x_m) \in \mathbb{R}^m$ and $m \in \mathbb{N}_+$.

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