

“Personal Influence”: Social Context and Political Competition

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Abstract

We study the role of interpersonal communication between voters in a model of informative political advertising. We show that when advertising costs are low, an increase in the level of interpersonal communication causes polarization: the more voters exchange political information, the more often extremist candidates are elected. This result is reinforced when interpersonal communication occurs more frequently among ideologically homogeneous individuals and parties can target political advertising.

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“The mass do not now take their opinions from dignitaries in Church or State, from ostensible leaders, or from books. Their thinking is done for them by men much like themselves, addressing them or speaking in their name, on the spur of the moment.” J. S. Mill (1869). On Liberty.

“Rational citizens will seek to obtain their free political information from other persons if they can. This expectations seems to be borne out by the existing evidence.” A. Downs (1957). An Economic Theory of Democracy.

1 Introduction

In modern societies a large majority of individuals rely on others in order to obtain most of their political information. Evidence of the importance of information sharing among voters in shaping individuals’ political choices dates back to the early 1950s, when, through a series of pioneering field experiments, Columbia sociologist Paul Lazarsfeld and his coauthors demonstrated the primacy of face-to-face interaction in spreading political information, and documented that this information was more likely to reach undecided voters.¹

In spite of the predominant role of the mass media in political advertising, recent empirical work shows that interpersonal communication is still fundamental in the process of political information sharing and acquisition. For example, in an empirical study of the 1992 American presidential election campaign, Beck et al. [2] conclude that interpersonal discussions outweigh the media in affecting voting behavior. In a recent study on political disagreement within communication networks, Huckfeldt et al. [17] observe that: “Democratic electorates are composed of individually interdependent, politically interconnected decision makers. [...] they depend on one another

¹See, e.g., Lazarsfeld et al. [20], Berelson et al. [3], and Katz and Lazarsfeld [19]. The work of the Columbia sociologists is the “existing evidence” to which Downs refers in the quotation. See Downs [9] pages 222 and 229.

for political information and guidance [...]”². Moreover, there is evidence that interpersonal communication occurs more frequently among ideologically similar individuals, as documented by McPherson et al. [26].

In light of this evidence, understanding the relationship between interpersonal communication, individuals’ voting behavior and political outcomes is of considerable interest. However, very little theoretical work has been done on this topic. This paper proposes a framework in which interpersonal communication between voters is embedded in a standard model of electoral competition, and addresses the following questions: How does the level of interpersonal communication between voters affect political equilibrium outcomes? In particular, does it increase the likelihood that moderate policies are implemented or, on the contrary, does it increase polarization? How does the tendency of voters to communicate within ideologically homogeneous groups affect strategic electoral competition and political outcomes?

The first part of the paper addresses the first two questions within a citizen-candidate model where the policy space is unidimensional. There are three groups of citizens: leftists and rightists (henceforth the “partisans”), and independent voters. Citizens have distance preferences over policy, independents are decisive in the election, and the identity of the median independent voter is ex-ante uncertain. Two policy-motivated parties, representative of the leftist and rightist groups, select one of their members to run in an election. The candidate that wins a simple majority of votes is elected and implements his (preferred) policy.

Voters cast their votes on the basis of the information they possess about candidates’

²Apparently, not much has changed since W. Lippmann’s [22] treatise on public opinion where he states: “Inevitably our opinions cover a bigger space, a longer reach of time, a greater number of things, than we can directly observe. They have, therefore, to be pieced together out of what others have reported and what we can imagine”. The recent use of internet in political campaign and, in particular, of on-line social networking sites (e.g., MySpace, Flickr, YouTube, Facebook), suggests that interpersonal communication is becoming a crucial aspect of political advertising. See, for example, <http://politicalinsider.com/2007/05/the.facebook.campaign.html>.

policy positions. This information depends on their exposure to political advertising and it derives from two channels. First, each party may disclose costly information about the political ideology of the candidate running for election. Advertising is truthful, and reaches a randomly selected fraction of independents, who become informed about the candidate’s policy position. We call this information channel *direct exposure*.³ The second channel is interpersonal communication, which we refer to as *contextual exposure*. In particular, each independent randomly samples a finite number of other independents, who truthfully report the information they have obtained, if any, from parties’ advertising.⁴ Clearly, the effectiveness of interpersonal communication in spreading political information depends both on the level of contextual exposure (the sample size) and the intensity of parties’ political advertising.

Our model has a unique symmetric equilibrium. When marginal costs of advertising are high, parties always select extremist candidates and do not disclose any political information. Otherwise, parties select a moderate candidate with positive probability and disclose political information only when the candidate is a moderate.

We then show that when marginal costs of advertising are low, an increase in the level of interpersonal communication between voters increases the equilibrium probability that extreme policies are implemented. The intuition behind this result is that, from the viewpoint of a party, interpersonal communications and informative advertising are substitutes, so that an increase in the former reduces parties’ incentive to disclose political information. This affects the composition of voters who are informed

³For evidence about the importance of political advertising in providing voters with information see, e.g., Lodge et al. [23], and Coleman and Manna [7]. See also Zaller [37] for evidence on the effect of media content on policy preferences.

⁴Our model of acquisition of political information formalizes the idea of “two-step flow communication” which played a central role in the analysis of the Columbia sociologists. They describe the two-step flow communication as a relay function of interpersonal relations, where political information flows directly from mass media to a subset of voters, the “opinion leaders” (which corresponds to direct exposure), and from them to other voters they are in contact with (which corresponds to contextual exposure). See Lazarsfeld et al. [20].

and uninformed as well as the beliefs of uninformed voters. In particular, average voters' beliefs of facing an extremist candidate are reinforced, implying that the difference between the probability of winning the election when a party selects a moderate as compared to an extremist decreases. Hence, electoral competition decreases and policy-motivated parties select extremist candidates more often.

The second part of the paper addresses our third question: How does the tendency of voters to communicate within ideologically homogenous groups affects strategic electoral competition and political outcomes? To do so, we extend our benchmark model to consider the empirically relevant case in which i) parties can target political advertising to ideologically similar voters, and ii) interpersonal communication occurs more frequently among ideologically similar individuals.⁵ The latter assumption is a very simple form of the so-called “value homophily,” according to the original formulation of Lazarsfeld and Merton [21].⁶ In our framework, absence of homophily corresponds to a situation in which the frequency of interpersonal communication between voters does not depend on their ideological similarity. In contrast, pure homophily characterizes a society in which there are ideologically-homogeneous groups and interpersonal communication occurs only between voters belonging to the same group. In this sense, the level of homophily can also be interpreted as the level of segregation between different

⁵The fact that parties target their political advertising to specific subset of the population is well-known. As Ken Mehlman, Chairman of the RNC, recently argued: “[...] does this person have children, owns a gun [...] that can be predictive of vote behavior and issues that voters care about [...] it helps you figure out the best way to reach out to them. [...] one of the reasons we in Ohio doubled our support among African Americans in 2004 is because we were able to reach out in places where before you couldn't [...] before you would mail 10 people and get one [...] it was unaffordable [...] it becomes affordable if you can find out one family that's willing to consider voting Republican.” See also Gerber and Green [13] for experimental results on the effectiveness of targeting.

⁶The word “homophily” literally means “love of the same”. The presence of homophily in social relations is a robust observation which applies very broadly. See, e.g., McPherson et al. [26] for a survey of research on homophily, and Myatt [24] for evidence on the effect of homophily on voting decisions. See also Currarini et al. [8] for a simple model in which homophily emerges as an equilibrium outcome.

ideologically-homogeneous groups.

We show that in the extended model there exists an equilibrium in which parties choose to target only their ideologically closer subset of independents, both parties select a moderate candidate with positive probability, and they disclose political information only when the candidate is a moderate. In this equilibrium, an increase in homophily increases the probability that parties select extremist candidates, as well as the expected probability that an extremist candidate wins the election. Intuitively, with targeted political advertising, an increase in the level of homophily decreases the probability that voters will observe both parties' candidates, and therefore decreases electoral competition.

This paper builds on two different strands of theoretical literature. The first strand focuses on the effects of political advertising on electoral competition and voters' welfare, e.g., Coate [5], [6] and Prat [30], [31]. The second strand studies interpersonal communication and learning.⁷ We model electoral competition and direct exposure to political information following Coate [6], while the model of interpersonal communication follows the approach of Ellison and Fudenberg [10], [11], and Galeotti and Goyal [12]. To the best of our knowledge, the present paper is the first to embed informal communication between voters in a political economy framework. Our results also relate to the existing empirical literature on polarization in US politics, e.g., Poole and Rosenthal [29] and McCarthy et al. [25]. This literature documents an increase in polarization of the Democratic and Republican parties in the last thirty years, an increase which was not accompanied by a corresponding polarization in the preferences of the electorate. Our analysis shows that changes in social context – an increase in the level of interpersonal communication and in the frequency of communication between ideologically similar voters – may be important to understand these empirical findings.

The paper is organized as follows. Section 2 presents the model. Section 3 studies

⁷For a survey of the existing literature on interpersonal communication and local learning see Goyal [15].

the effect of the level of interpersonal communication on political outcomes. Section 4 extends the model to the case in which interpersonal communication are more frequent between ideologically similar individuals and parties can target political advertising. Section 6 concludes.

2 Model

Citizens and Parties: Ideologies. There is a continuum of citizens of unit measure. The policy space is unidimensional, and citizens are exogenously divided into three groups: leftists, rightists, and independents. Partisans represent an equal fraction of the population, and their ideology is symmetrically distributed on $[0, m]$ and $[1 - m, 1]$, respectively. The ideology of independents is uniformly distributed on the interval $[\mu - \tau, \mu + \tau]$, where $\tau > 0$, and μ is drawn from a uniform distribution with support $[1/2 - m, 1/2 + m]$. Hence, the identity of the median independent is ex-ante uncertain. We assume that $m < 1/4 - \tau/2$ so that ideologies of independents are always between those of partisans.

There are two policy-motivated political parties: party L and party R . Party $L(R)$ consists of a representative subgroup of the leftist (rightist) group. A representative of each party is selected to be a candidate in an election and, in the spirit of the citizen-candidate model, the candidate that wins a simple majority of votes is elected and implements his ideology.⁸ For simplicity, we restrict the candidates' type space to be $T = \{e, m\}$, where $e \equiv m/2$. Let $\mathbf{t} = (t_L, t_R) \in T \times T$ be a profile of types, where $t_L \in \{e, m\}$ denotes the ideology of party L 's candidate, and $1 - t_R$ denotes the ideology of party R 's candidate. Henceforth, a candidate is an extremist if his type is

⁸An alternative specification, more in the spirit of the Downsian tradition, would be to consider that parties choose policies instead of candidates, and they can perfectly commit. The citizen-candidate approach (see Besley and Coate [4], and Osborne and Slivinski [27]) provides a natural rationale for the commitment assumption.

$t = e$, while a candidate is a moderate if his type is $t = m$.⁹

Citizens have distance preferences over ideology and, in particular, a citizen with ideology i gets utility $-|t - i|$ if a candidate of ideology t wins the election. Partisans always vote for their party, while independents cast their votes on the basis of the information they possess about candidates' types.¹⁰ Independents are *ex-ante* ignorant about candidates' types, but receive information from two sources: parties' advertising (direct exposure) and interaction with other voters (contextual exposure).

Direct Exposure. Each party $j = \{L, R\}$, after having selected its candidate, chooses a level $x_j \in [0, 1]$ of political advertising. Advertising is truthful and fully informative. In particular, if a party chooses x_j , then a random fraction x_j of independents observe party j candidate's position. The cost of informing x_j voters is $C(\alpha, x_j) = \alpha x_j$, where α is a positive constant measuring the efficiency of the advertising technology.¹¹

Contextual Exposure. In addition to direct exposure, independents obtain information via interpersonal communication, which we model as follows. Each independent samples a finite number $k > 0$ of other independents and each sampled independent reports truthfully the information, if any, he has obtained directly from parties' advertising.¹²

⁹When m is small, the assumption that $t \in \{e, m\}$ is without loss of generality. Indeed, a party, which maximizes the expected utility of its median voter, will never select a candidate that is more extreme than its median member e . Moreover, as the uncertainty about the median voter is sufficiently small, i.e., m is sufficiently small, it is possible to show that a party will never select a candidate with ideology lying in the interior of the interval $[e, m]$.

¹⁰For evidence about the fact that partisans tend to be little affected by campaigns, see, e.g., Zaller [36], and Huckfeldt et al. [17].

¹¹The assumption of linear advertising costs is made for convenience. Indeed, our results hold for cost functions which are increasing and convex in x_j and α , respectively.

¹²Two remarks are in order. First, we assume that independents only communicate with other independents. It is easy to extend the model to allow independents to sample other voters from the entire population of citizens. Depending on what we assume about the report of a partisan (truthful or not), this would change the specification of the posterior beliefs of independents, but it would not affect qualitatively the results. Second, we assume that interpersonal communication only travels one

Hence, k parameterizes the level of contextual exposure among voters, where higher k is equivalent to greater contextual exposure. In Section 3.3 we generalize the framework to allow for the possibility that k may vary across individuals. As it is common in models of local externalities, we take the communication structure, which in our case simply corresponds to the parameter k , as exogenously given.¹³

We analyze the following Bayesian Game. Parties choose simultaneously their own candidate and their advertising intensity conditional on the candidate selected, in order to maximize the expected utility of their median voter. Independents do not observe these choices, but they may be exposed to political information either directly or via interpersonal communication. Based on the information received, independents update their beliefs about candidates' types and cast their vote to maximize their expected utility.

Parties' Strategies and Parties' Utilities. A strategy for party j is a probability distribution over candidates' types and an intensity of informative political advertising for each candidate's type. Formally, let $\sigma_j : T \rightarrow [0, 1]$, where $\sigma_j(t)$ denotes the probability that party j selects a candidate of type t , and $\sigma_j(e) + \sigma_j(m) = 1$. Analogously, let $x_j : T \rightarrow [0, 1]$, where $x_j(t)$ denotes the intensity of informative advertising of party j when candidate t is selected. A strategy for party j is denoted by $s_j = (\sigma_j, x_j)$; $s = (s_L, s_R)$ denotes a strategy profile.

Let $\pi_L(s|\mathbf{t})$ denote the expected probability of winning of party L , given that the electoral candidates are specified by the profile \mathbf{t} , and that parties are playing according step in the underlying social structure. In principle, communication may travel $r \geq 1$ steps. As it is clear in the proof of the results, an increase in the radius of information affects political outcomes similarly to an increase in k .

¹³To this end, Huckfeldt and Sprague [16] point out: "People often choose their associates and the content of their conversations, but each of these choices is, in turn, bounded by an environment that for many purposes must be taken as given rather than chosen." Also, Huckfeldt et al. [17] report that: "[...] individuals exercise limited discretion in the selection of informants. [...] the construction of a communication network occurs with pronounced constraints on supply."

to s . Thus, the expected payoff to party L when its candidate is t_L can be written as follows,

$$U_L(s|t_L) = \sum_{t_R \in \{e, m\}} \sigma_R(t_R) [\pi_L(s|\mathbf{t}) (1 - t_R - t_L) - (1 - t_R - e)] - \alpha x_L.$$

Voting Behavior of Independent Voters. *Ex-post*, the information of an independent about party j 's candidate can be summarized by $I_{k,j} \in T \cup \emptyset$, where $I_{k,j} = t$ means that the independent, after having sampled k other voters, knows that party j 's candidate is t , while $I_{k,j} = \emptyset$ indicates that the independent is uninformed about party j 's candidate.

Let $\rho_j(t|I_{k,j}, s, k)$ denote the belief of an independent that party j 's candidate is t , given $I_{k,j}$ and s . Whenever possible, $\rho_j(t|I_{k,j}, s, k)$ is derived using Bayes' rule. Hence, $\rho_j(t|t, s, k) = 1$, $\rho_j(t|t', s, k) = 0$, for $t \neq t'$, and

$$\rho_j(t|\emptyset, s, k) = \frac{\sigma_j(t)(1 - x_j(t))^{k+1}}{\sum_{t' \in T} \sigma_j(t')(1 - x_j(t'))^{k+1}}, \quad (1)$$

for every $t \in T$ such that $\sigma_j(t) > 0$ and $x_j(t) > 0$. We also assume that equation (1) holds at zero probability events, i.e., when $\sigma_j(t') = 0$ and/or $x_j(t') = 0$.¹⁴

Each independent votes as if he is pivotal. Hence, an independent with ideology i and information $(I_{k,L}, I_{k,R})$ votes for party L if and only if $i < i^*(I_{k,L}, I_{k,R})$, where $i^*(I_{k,L}, I_{k,R})$ is the identity of the indifferent independent voter. Given (t_L, t_R) and s , party L 's candidate gets at least half of the independents' votes if and only if $\mu < \mu_L^*(s|t_L, t_R)$.¹⁵ Therefore,

$$\pi_L(s|\mathbf{t}) = \begin{cases} 0 & \text{if } \mu_L^*(s|\mathbf{t}) \leq \frac{1}{2} - m \\ \frac{\mu_L^*(s|\mathbf{t}) + m - \frac{1}{2}}{2m} & \text{if } \mu_L^*(s|\mathbf{t}) \in (\frac{1}{2} - m, \frac{1}{2} + m) \\ 1 & \text{if } \mu_L^*(s|\mathbf{t}) \geq \frac{1}{2} + m. \end{cases}$$

¹⁴Note that this is a necessary condition for a Bayesian equilibrium to be a sequential equilibrium. See also Footnote 16 for a discussion of the role of this condition in the characterization of equilibria.

¹⁵Formally, $i^*(I_{k,L}, I_{k,R}) = \frac{1}{2} + \frac{\sum_{t \in T} \rho_L(t|I_{k,L}, s, k)t - \sum_{t \in T} \rho_R(t|I_{k,R}, s, k)t}{2}$ and $\mu_L^*(s|t_L, t_R) = \sum_{(I_{k,L}, I_{k,R}) \in \{\{t_L\} \cup \emptyset\} \times \{\{t_R\} \cup \emptyset\}} i^*(I_{k,L}, I_{k,R}) \Pr(I_{k,L}|s, t_L) \Pr(I_{k,R}|s, t_R)$.

Political Equilibrium. A political equilibrium consists of (i) parties' strategies, $s^* = (s_L^*, s_R^*)$; (ii) voter belief functions $\rho_j^*(\cdot)$, $j = L, R$ and indifferent independent voters $i^*(\cdot)$ such that:

- 1 (s_L^*, s_R^*) are mutual best responses given subsequent voting behavior;
- 2 $\rho_j^*(\cdot)$ are consistent with s^* for all $j = L, R$, and $i^*(\cdot)$ are consistent with $\rho_j^*(\cdot)$ and s , $j = L, R$.

3 Characterization of Political Equilibrium

Our first result provides a complete characterization of symmetric political equilibria. The nature of political equilibria is pinned down by a simple measure of *misperception* about the types of candidates running for election. Before stating the result formally, we define this measure of misperception.

Consider a symmetric strategy profile, s . We define the misperception of an independent about the candidate t of a party, say party L , as the probability that a (randomly selected) independent believes that the leftist candidate is $t' \neq t$ when in fact the candidate is of type t . Formally,

$$Q[t'|t, s, k] = (1 - x(t))^{k+1} \rho(t'|\emptyset, s, k), \quad (2)$$

where $(1 - x(t))^{k+1}$ is the probability that an independent does not observe that the candidate is of type t , and $\rho(t'|\emptyset, s, k)$ (defined in equation (1)) is the probability that an uninformed independent will place on the event that the leftist candidate is of type t' . We can then define the overall level of misperception as the *ex-ante* probability that a randomly selected independent misperceives the type of the candidate running for election. Formally,

$$\Psi(s, k) \equiv Q[e|m, s, k] + Q[m|e, s, k]. \quad (3)$$

Clearly, in every pure strategy equilibrium the level of misperception is zero; in general, a mixed-strategy equilibrium will always entail some positive level of misperception. We are now ready to provide a complete characterization of symmetric political equilibria.

Proposition 1 *A symmetric political equilibrium always exists and it is unique. For every k , there exists a critical level of the marginal costs of advertising $\alpha^*(k) > 0$ such that in the symmetric political outcome (I) if $\alpha \geq \alpha^*(k)$ parties always select an extremist candidate and they do not advertise, i.e., $\sigma^*(e) = 1$, and $x^*(e) = 0$; (II) if $\alpha < \alpha^*(k)$ parties randomize between selecting an extremist candidate and a moderate candidate, and they only advertise moderate candidates. In particular, $x^*(e) = 0$, and $x^*(m)$ and $\sigma^*(e)$ jointly solve:*

$$-\frac{\partial Q[e|m, s^*, k]}{\partial x(m)} \frac{2 - 4m + \sigma^*(e)m}{16} = \alpha \quad (4)$$

$$1 - \Psi(s^*, k) = \frac{4m + 16\alpha x^*(m)}{2 - 3m}. \quad (5)$$

When choosing its political strategy, a party faces the following trade-off. Since parties are policy motivated, they enjoy higher utility when selecting an extremist candidate conditional on winning the election. However, since independents are decisive, a moderate has a higher chance of winning the election relative to an extremist and, intuitively, this advantage is higher the lower is independents' misperception. Therefore, selecting a moderate candidate is worthwhile only when independents' misperception is low, which, for given contextual exposure, comes at the cost of higher level of informative advertising. Proposition 1 summarizes how these trade-offs are solved in equilibrium.¹⁶

¹⁶Two remarks are in order. First, pure strategy asymmetric equilibria do not exist in this model. Second, Proposition 1 characterizes symmetric equilibria in which voters' beliefs are constant at zero probability events. There is only one equilibrium which does not satisfy this condition: parties always select a moderate, they set $x(m) = 1$ and, out-of-equilibrium, an uninformed voter believes that the candidate is an extremist with strictly positive probability. This equilibrium exists for a value of α sufficiently low, and it is not robust to small imperfections of the advertising technology. In particular,

For given level of contextual exposure k , if marginal costs of advertising are sufficiently high, parties always choose extremist candidates and never advertise (Part I of Proposition 1). In line with the intuition above, selecting a moderate is profitable only if independents' misperception is sufficiently low, which implies that the party must choose a high level of political advertising. When the advertising technology is very inefficient this is not optimal. In contrast, if the costs of advertising are low enough and one party always selects an extremist candidate, the opponent party will find it profitable to select a moderate candidate and inform the independents about it. Thus, in equilibrium parties have to randomize between the two candidates' types and they will choose to advertise only moderate candidates (Part II of Proposition 1).¹⁷

Condition (4) requires that parties advertise moderate candidates so that marginal returns from advertising equal the marginal cost α . By (marginally) increasing the level of informative advertising, a party lowers the misperception of independents and therefore increases its probability of winning the election. The marginal returns from advertising a moderate equal the decrease in misperception multiplied by the gains of winning the election.

Condition (5) requires that parties are indifferent between selecting a moderate and an extremist candidate. The right hand side of this equation represents the expected costs of proposing a moderate instead of an extremist, which are composed of the advertising costs, and of the costs of implementing a less preferred policy. The left hand side of equation (5) is the benefit for the party of choosing a moderate instead of an extremist candidate, which is materialized in facing a higher probability of winning the election. This is inversely proportional to the level of independents' misperception. To see this, suppose that the right party follows the strategy prescribed in the second part of Proposition 1. The expected probability of winning of the left party when

it does not survive if there is positive probability that a voter remains uninformed on the equilibrium path.

¹⁷It is easy to show that the threshold $\alpha^*(k)$ is increasing in k . Hence, the region where the symmetric equilibrium is in mixed strategies becomes larger as k increases.

choosing candidate t is $\hat{\pi}_L(s^*|t) = (1 - \sigma^*(e))\pi_L(s^*|t, m) + \sigma^*(e)\pi_L(s^*|t, e)$. Since, in a symmetric equilibrium, $\pi_L(s^*|t, t) = 1/2$ and $\pi_L(s^*|m, e) = 1 - \pi_L(s^*|e, m)$, it follows that

$$\hat{\pi}_L(s^*|m) - \hat{\pi}_L(s^*|e) = \frac{1}{2} - \pi_L(s^*|e, m) = \frac{1 - \Psi(s^*, k)}{8},$$

where the last equality is easily checked.

3.1 The Equilibrium Effect of Contextual Exposure

We now investigate the effect of an increase in contextual exposure on the political outcome. In particular, we explore the equilibrium relation between the level of interpersonal communication and policy polarization. In order to do so, we compare the political equilibrium when the level of contextual exposure is k with the political equilibrium when independents communicate with $k + 1$ other voters. We focus on the case $\alpha < \alpha^*(k)$ described in part II of Proposition 1 since this is the only non trivial situation.

As a measure of policy polarization we define the *ex-ante* expected probability that in equilibrium an extremist candidate is elected. This is denoted by $\Pi(s^*)$ and it is equal to:

$$\Pi(s^*) = \sigma^*(e)^2 + 2\sigma^*(e)(1 - \sigma^*(e))\pi(s^*|e, m), \quad (6)$$

where the first term of the right hand side of (6) is the probability that two extremist candidates compete in the election, and the last term is the probability that an extremist candidate wins against a moderate candidate.

The next proposition summarizes the results. We denote by $s_k^* = (\sigma_k^*(e), x_k^*)$ the parties' strategy profile in a symmetric political equilibrium when the level of contextual exposure is equal to k .

Proposition 2 *For every k , there exists $\hat{\alpha}(k) \in (0, \alpha^*(k)]$ such that, for every $\alpha < \hat{\alpha}(k)$, it follows that $x_{k+1}^*(m) < x_k^*(m)$, $\sigma_{k+1}^*(e) > \sigma_k^*(e)$, and $\Pi(s_{k+1}^*) > \Pi(s_k^*)$.*

Proposition 2 establishes that, for sufficiently low marginal costs of advertising, greater contextual exposure decreases the level of informative advertising, and it increases the probability that extremist candidates run in an election. Ultimately, as independents can exchange political information with more voters, policy polarization increases, i.e., the likelihood that an extremist candidate is elected increases.¹⁸

An increase in contextual exposure decreases marginal returns from disclosing political information because independents are more likely to become informed via interpersonal communication. That is, in equilibrium advertising and contextual exposure are substitutes and therefore, when interpersonal communication increases, the level of direct exposure decreases. When this substitution effect is strong enough, despite the increase in the level of interpersonal communication, voters are more likely to be uninformed. Hence, voters' misperception is higher and policy motivated parties find it profitable to select extremist candidates more often. If, on the contrary, the substitution effect is weak, voters' misperception decreases and parties select moderate candidates more often. Intuitively, the magnitude of the substitution effect is increasing in the amount of parties' political advertising which is inversely related to the marginal cost of advertising.¹⁹

¹⁸An alternative measure of policy polarization is the probability that in equilibrium an extremist candidate defeats a moderate candidate. Formally, $\pi_L(s^*|e, m) = \pi_R(s^*|m, e) = \frac{1}{2} - \frac{1 - \Psi(s^*, k)}{8}$. When $\alpha < \hat{\alpha}(k)$, the effect of an increase in k on $\Pi(s^*)$ is equivalent to the effect of an increase in k on $\pi_L(s^*|e, m)$.

¹⁹In our model total informative advertising and the level of interpersonal communication are substitutes. While in recent years we observed an increase in both interpersonal communication and in total campaign spending, only a fraction of this spending is devoted to informative advertising, which is the form of political advertising we are focusing on. In fact, part of the increase in campaign advertising is accounted for by an increased use of negative political advertising, see, e.g., Sabato [33]. In his recent survey on political advertising, Prat [32] states: "often ads convey verifiable information on the political record of the candidate [...] it is also true that some commercials are extremely expensive but appear to contain little new information." See Gerber et al. [14] for mixed evidence on the effectiveness of media advertising, and in particular Ansolabehere and Iyengar [1] for experimental evidence supporting the claim that political advertising is effective even when it is not directly informative.

We conclude this section with three observations. First, the comparative static result on polarization is not an artifact of the mixed-strategy nature of the equilibrium. In fact, suppose that each party can be either a “high-cost” type, with probability σ , or a “low-cost” type with the remaining probability. A “high-cost” party has high marginal costs of advertising, say α_H , while the marginal costs of a “low-cost” party is $\alpha \ll \alpha_H$. Each party observes its own type, but it does not observe the type of the opposite party. For simplicity, let α_H be sufficiently large so that the best response of a “high-cost” party is always to select an extremist candidate. For α sufficiently low a “low-cost” party will always select a moderate and advertise with intensity x^* which is the solution to equation (4). Suppose now that $\alpha = 0.03$, $m = 0.1$ and $\sigma = 0.4$. Simple calculation reveals that an increase of the level of interpersonal communication from $k = 1$ to $k = 2$ leads “low-cost” parties to decrease the level of informative advertising from 0.84 to 0.67. In turn, voters’ misperception as well as polarization increases of about 41%.

Second, for sufficiently inefficient technology, an increase in the level of contextual exposure can lead to the opposite comparative statics result of Proposition 2. For example, suppose that $\alpha = 0.1$ and $m = 0.1$. For $k = 8$, it is possible to show that in equilibrium parties select extremist candidates with probability $\sigma_8^*(e) = 0.28$, and they advertise moderate candidates with intensity $x_8^*(m) = 0.11$. Consequently, in expectation an extremist candidate is elected with probability $\Pi_8^* = 0.26$. Suppose now that $k = 9$. Simple calculations show that at the new level of contextual exposure parties select extremist candidates less often, $\sigma_9^*(e) = 0.26$, they advertise moderate candidates less, $x_9^*(m) = 0.10$ and, as a result, the political outcome is less polarized, $\Pi_9^* = 0.24$.

Our final observation is that the effects of an increase in the level of interpersonal communication on equilibrium are rather different from the effects of a decrease in the marginal costs of advertising α (i.e., direct exposure becomes less expensive). Indeed, it is possible to show that a decrease in the costs of direct exposure unambiguously

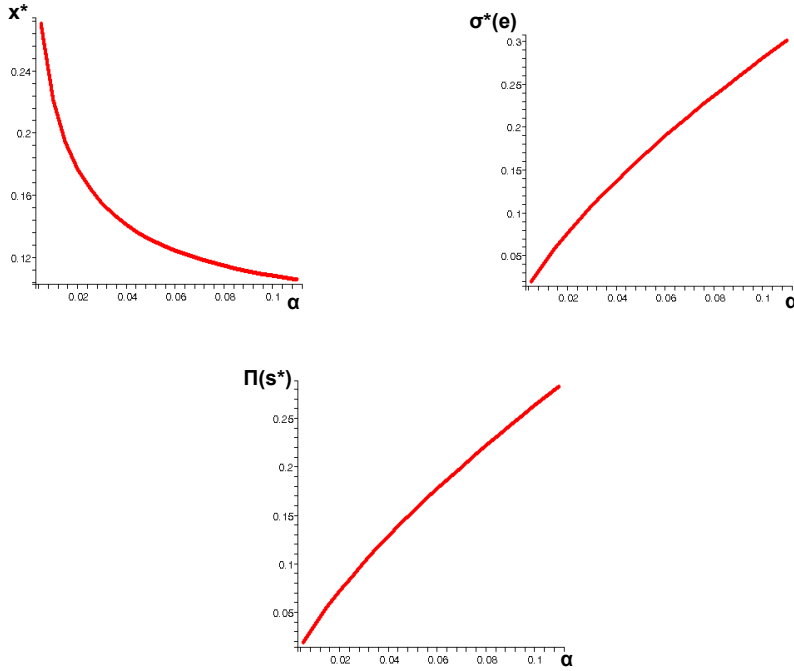


Figure 1: Comparative statics on advertising costs, $\alpha = 0 \dots 0.15$, $m = 0.1$, $k = 8$.

increases the intensity of advertising. As an illustration, let $k = 8$ and $m = 0.1$. Figure 1 plots the equilibrium advertising intensity, the probability that parties select an extremist candidate and the ex-ante expected probability that in equilibrium an extremist candidate is elected, for different values of $\alpha \in (0, \alpha^*(8)]$. Intuitively, when the costs of informative advertising become lower, it is always optimal for parties to advertise their moderate candidate with higher intensity. This increases electoral competition and therefore parties find it profitable to select extremist candidates less often. Consequently, political outcomes become less polarized.²⁰

3.2 Heterogeneity in Contextual Exposure

So far, we assumed that voters have the same level of interpersonal communication. However, empirical evidence suggests that the structure of interpersonal communication is characterized by a high level of heterogeneity, with some individuals having few

²⁰The formal proof of the comparative static result with respect to α is straightforward and therefore omitted.

interpersonal relations, while others many.²¹ To extend our model in this direction we consider that independents are divided in \bar{k} groups, and an independent belonging to group h samples randomly h other citizens among the entire set of independents. Formally, let $I = \{1, 2, \dots, \bar{k}\}$, and let $P : I \rightarrow [0, 1]$, where $P(h)$ gives the fraction of independent voters that sample $h \in I$ other voters. The model presented in the previous section is a special case where the distribution P is such that $P(k) = 1$ and $P(h) = 0$, for all $h \in I \setminus \{k\}$, and $k \in I$.

Proposition 5 (in Appendix A) generalizes the characterization of symmetric political equilibria presented in Proposition 1 to arbitrary distributions P . In summary, for a given distribution P , there exists a threshold α_P such that, if marginal costs of advertising exceed α_P , then parties select extremist candidates and do not advertise; otherwise parties randomize between extremists and moderates and advertise only moderate candidates. Figure 2 depicts the equilibrium for particular values of the parameters. The decreasing curve represents the points $(\sigma(e), x(m))$ which equate the marginal costs of advertising a moderate with the marginal benefits (i.e., the solution to equation (8) in Appendix A); the increasing curve depicts points $(\sigma(e), x(m))$ which make a party indifferent between selecting either of the two candidates (i.e., the solution to equation (9) in Appendix A). The intersection of the two schedules pins down the equilibrium values of $\sigma^*(e)$ and $x^*(m)$.

The next example shows that the results in Proposition (2) are robust to heterogeneity between voters in contextual exposure.

Example: Effects of higher contextual exposure on political outcomes. Suppose that $m = 0.2$ and assume that independents sample either one or two other voters, i.e., $P(1) = p$, $P(2) = 1 - p$. Note that a decrease of p implies a first order stochastic dominance shift in the distribution of contextual exposure, which we interpret as an increase in the level of interpersonal communication. Figure 3 depicts the political

²¹See Jackson and Rogers [18] for an extensive discussion of the empirical features of a variety of social networks.

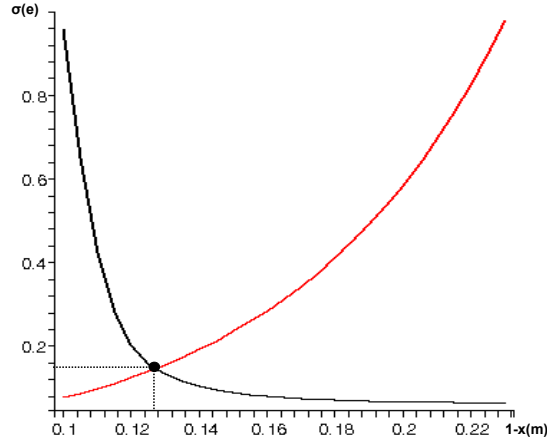


Figure 2: Equilibrium under Heterogeneity in Contextual Exposure, $P(1) = P(2) = 0.5$, $\alpha = 0.01$, $m = 0.2$

equilibrium for different values of p and α .

First, consider that advertising is sufficiently inexpensive, $\alpha = 0.01$; Figure 3A illustrates this case. A first order stochastic shift in the distribution of contextual exposure decreases parties advertising, and increases the likelihood that extremist candidates are selected. As a result, the political outcome is more polarized. Figure 3B illustrates the same exercise when advertising is sufficiently expensive, $\alpha = 0.1$. In contrast to the previous case, here a first order stochastic shift decreases the likelihood that extremist candidates are selected, and the political outcome is less polarized.

A new insight that comes from this extension is that, *ex-post*, independents with a different level of interpersonal communication have different beliefs about the candidates running in the election. In other words, heterogeneity in contextual exposure maps into heterogeneity in expectations between otherwise equally informed voters. In particular, the higher the contextual exposure of an independent, the more precise are (on average) his beliefs about political candidates. However, conditional on remaining uninformed, independents who are more exposed to interpersonal communication have stronger beliefs that the candidates running for election are extremist candidates.

Figure 3A. Low Costs $\alpha = 0.01$

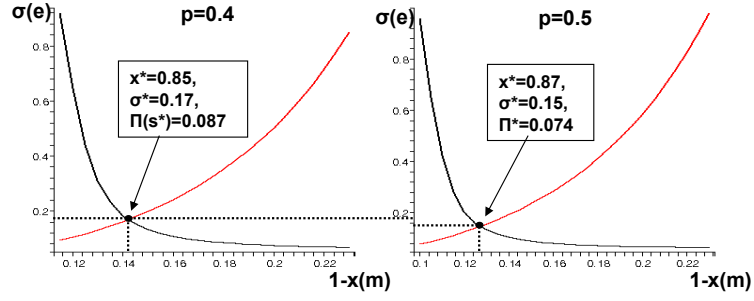


Figure 3B. High Costs $\alpha = 0.1$

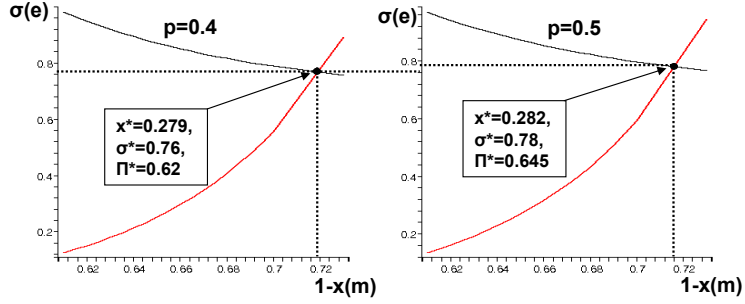


Figure 3: FOSD shifts in Contextual Exposure, $P(1) = p, P(2) = 1 - p, m = 0.2$.

4 Homophily and Targeted Political Advertising

A well-known empirical fact is that interpersonal communication occurs more frequently among similar individuals.²² In our context, this phenomenon would entail a higher probability of interpersonal communication between voters with closer political ideologies. Note that, if parties cannot target political advertising, our results will be unaffected by any form of homophily. However, if parties can target advertising to ideologically similar voters, homophily will affect electoral competition and political outcomes. In this section we modify our benchmark model in order to capture these additional features in a simple and parsimonious way.

First, we define group l as the group of independents in the interval $[\mu - \tau, \mu]$; analogously we call group r the remaining group of independents. Each independent samples k other voters. For each draw, a group- l member samples a voter in his own group with probability β , while with the remaining probability he samples a group- r

²²See, e.g., McPherson et al. [26].

voter. We assume that $\beta \in [1/2, 1)$, and we interpret this parameter as the level of homophily in the society. Absence of homophily corresponds to $\beta = 1/2$. When instead $\beta = 1$, voters only communicate with members of their own group, which exemplifies a society in which ideology-based groups are totally segregated. Since the focus of this section is to study the effect of homophily on political outcomes, we set $k = 1$, hereafter.²³

Second, we assume that a party can choose either to advertise at a cost $\alpha > 0$ or not to advertise. If Party L (R) chooses to advertise its candidate, then all members of group l (r) observe perfectly the candidate of party L (R).²⁴ In other words, we are assuming that advertising of a party is targeted to its closer group of independents. A more general model is one where a party can choose to target advertising either to one of the two groups (at a cost α), or to disclose political information to both groups (for example at a cost 2α). In Appendix B (Proposition 6) we show that the equilibria we characterize by assuming that advertisement of a party is targeted to its closer group of independents (see Proposition 3 below) are indeed equilibria in the more general model where parties can choose where to target advertising. The next proposition characterizes the equilibrium.

Proposition 3 *A symmetric political equilibrium always exists and it is unique. For every β , there exists $\underline{\alpha}(\beta) < \bar{\alpha}$ such that in equilibrium: (I) for all $\alpha > \bar{\alpha}$ parties select extremist candidates and they do not advertise; (II) for all $\alpha \in (0, \max[0, \underline{\alpha}(\beta)])$, parties select moderates and they advertise; (III) for all $\alpha \in (\max[0, \underline{\alpha}(\beta)], \bar{\alpha})$ parties select extremist candidates with probability*

$$\sigma^*(e) = 1 - \frac{2 - 7m - 16\alpha}{\beta(2 - 3m)} \quad (7)$$

²³We assume that $k = 1$ merely for expositional reasons; all the results hold for arbitrary finite k .

²⁴Here, we assume that advertising is discrete, i.e., a party can decide whether or not to advertise but not how much to advertise. This assumption is needed for tractability. The fact that when a party advertises its candidate then *all* its close independents observe perfectly the candidate is not crucial for our results. Moreover, note that, abusing notation, we are denoting by α the total cost of advertising, while in the benchmark model α was the marginal cost of advertising.

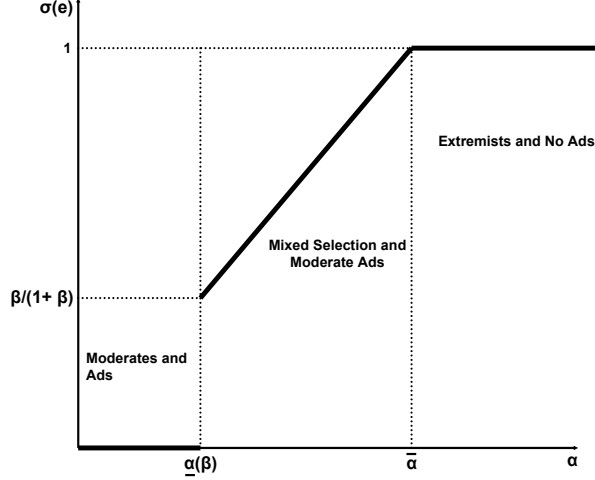


Figure 4: Equilibria under Homophily

and they advertise when they select a moderate. Furthermore, $\underline{\alpha}(\beta) > 0$ if and only if $\beta < (2 - 7m)/(4m)$.

Figure 4 provides a graphical illustration of the equilibrium in the $(\alpha, \sigma(e))$ parameter space. The intuitions behind part (I) and part (III) of Proposition 3 are analogous to the intuitions behind Proposition 1. When the level of homophily is sufficiently high these are the only equilibria. However, when the level of homophily is low, i.e., $\beta < (2 - 7m)/(4m)$, for low values of α a party cannot be indifferent between selecting the two candidates. In fact, since information travels across groups, it is likely that voters are informed if a moderate candidate is running. This increases political competition and it reduces the probability that an extremist candidate wins election. Hence, in equilibrium parties always select moderates and they advertise.²⁵

Our final result shows how the level of homophily affects political outcomes.

Proposition 4 *Suppose $\alpha \in (\max[0, \underline{\alpha}(\beta)], \bar{\alpha}(\beta))$. If β increases then parties select extremist candidates with (strictly) higher probability and the policy outcome is more*

²⁵This equilibrium is sustained under specific out-of-the-equilibrium beliefs: when a voter in group $l(r)$ is uninformed about party's $L(R)$ candidate, then she believes that party $L(R)$ has selected an extremist.

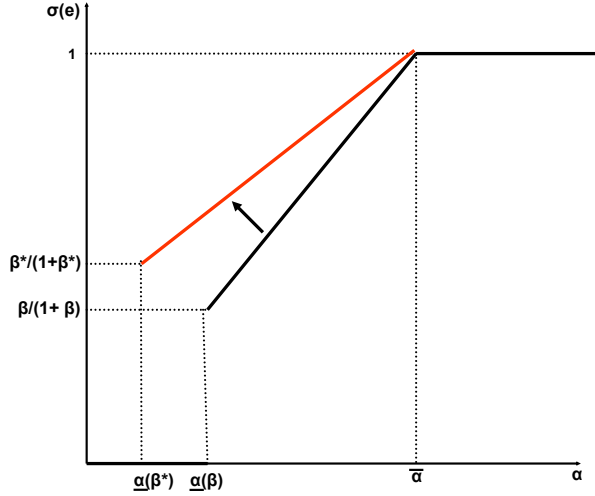


Figure 5: Comparative statics under Homophily

polarized, i.e., the (ex-ante) expected probability that an extremist candidate wins the election is higher.

Figure 5 illustrates the comparative statics with respect to β . Since parties target political information to distinct groups, an increase in homophily leads to a lower probability that a voter will observe both candidates' types. This decreases political competition and therefore policy motivated parties select extremist candidates more often.²⁶

5 Conclusion

The importance of interpersonal communication in affecting voters' choices has been empirically documented in economics, political science, and sociology. To the best of

²⁶In the context of the mixed-strategy equilibrium described in Proposition 3, we can compute the *ex-ante* equilibrium probability that a randomly selected left(right)-leaning independent votes for the Left(Right) party. This probability is increasing in β , meaning that the higher is the level of homophily, the higher is the correlation between vote choice and ideology. This result is consistent with empirical evidence suggesting that receiving information about the ideologically closer candidate reduces the probability that voters switch their votes from their initial disposition. See Velazquez Nunez [35] and Pattie and Johnston [28].

our knowledge there is no theoretical model that examines the effects of voters' communication on electoral competition and consequently on political equilibrium outcomes. This paper proposes a model that provides novel insights on the equilibrium relation between relevant aspects of interpersonal communication and electoral outcomes.

In our model interpersonal communication is truthful. A first step toward relaxing this assumption is to consider the case in which voters can strategically choose to omit information. In this case, a voter who knows that the right (left) candidate is a moderate and who strictly prefers to vote for the left (right) candidate, will choose to omit information about the right (left) candidate. Preliminary work suggests that this extension does not affect qualitatively our results. In fact, if any, strategic communication increases the incentive for parties to target advertising to the closer group of independents.

Another assumption of the model is that voters cannot abstain. Given that in our model nobody is pivotal, one possible way to analyze the effect of interpersonal communication on turnout within our framework is to consider an expressive theory of voting. That is, each voter votes as if he were pivotal, provided that the increase in his expected utility when his most preferred candidate gets elected is above some private cost of voting. To see how interpersonal communication can affect the decision of turning out to vote, consider the case of a voter informed only about party L 's candidate. Conditionally on remaining uninformed, an increase in the level of interpersonal communication increases this voter's posterior beliefs that the right candidate is an extremist. This reduces the voter expected utility were this candidate being elected, which makes it more likely that this voter would choose to turn out and vote. Clearly, the overall effect on turnout will depend on the equilibrium distribution of voters in each information set. These and other extensions are the object of ongoing research.

Appendix A: Proofs of Section 3. In this appendix we provide the results of section 3. We first provide Proposition 5 which is a generalization of Proposition 1. Next, we complete the proof of Proposition 1 by proving uniqueness of symmetric equilibria. Finally, we provide the proof of Proposition 2.

Proposition 5 *A symmetric political equilibrium always exists for any distribution P . In particular, for every P , there exists a $\alpha_P^* > 0$ such that*

I If $\alpha \geq \alpha_P^$ in the unique symmetric political equilibrium parties always select an extremist candidate and they do not advertise, i.e., $\sigma^*(e) = 1$, and $x^*(e) = 0$;*

II If $\alpha < \alpha_P^$ in every symmetric political equilibrium parties randomize between selecting an extremist candidate and a moderate candidate and they only advertise moderate candidates. In particular, $x^*(e) = 0$, and $x^*(m)$ and $\sigma^*(e)$ jointly solve:*

$$-\sum_{k=1}^{\bar{k}} P(k) \frac{\partial Q[s^*|e, m, k]}{\partial x(m)} \frac{2 - 4m + \sigma^*(e)m}{16} = \alpha \quad (8)$$

$$\sum_{k=1}^{\bar{k}} P(k) [1 - \Psi(s^*, k)] = \frac{4m + 16\alpha x^*(m)}{2 - 3m}. \quad (9)$$

Proof. Proposition 5. We first characterize symmetric political equilibria in pure strategies. Let $s^* = (s_L^*, s_R^*)$ be part of a symmetric pure-strategy equilibrium. Then s_j^* prescribes to select a candidate $t^* \in T$ with probability 1 and to advertise that candidate with intensity $x^*(t^*)$, $\forall j \in \{L, R\}$. We start by noticing that for s^* to be part of an equilibrium it has to be the case that $x^*(t^*) = 0$. Hence, $U_L(s^*|t^*) = -(1 - m)/2$. There are two possibilities, which we now analyze.

One possibility is that $t^* = m$. Let party L deviate by selecting $t_L = e$. It is easy to see that the best advertising strategy is $x_L(e) = 0$. Let's denote this strategy by \tilde{s}_L . Then, $U_L(\tilde{s}_L, s_R^*|e) = -(2 - 3m)/4 > U_L(s^*|m)$, which contradicts our hypothesis that s^* is an equilibrium.

The other possibility is that $t^* = e$. Let party L deviate by selecting $t_L = m$ and $x_L(m)$; call this strategy \tilde{s}_L . Observe that such deviation is profitable only if

$x_L(m) \notin \{0, 1\}$. Thus, assume that $x_L(m) \in (0, 1)$. We now derive the optimal advertising level, given s_R^* , which we denote by $x_L^*(m)$. To do this, we start by observing that

$$\mu_L^*(\tilde{s}_L, s_R^*|m, e) = \frac{1}{2} + \frac{m}{4} - \frac{m}{4} \sum_{k=1}^{\bar{k}} P(k)(1 - x_L(m))^{k+1},$$

and it is readily seen that $\mu_L^*(\tilde{s}_L, s_R^*|m, e) \in [1/2 - m, 1/2 + m]$, $\forall x_L(m) \in (0, 1)$, which implies that $\pi_L(\tilde{s}_L, s_R^*|m, e) \in (0, 1)$, $\forall x_L(m) \in (0, 1)$. Next, since $\pi_L(\tilde{s}_L, s_R^*|m, e) \in (0, 1)$, it follows that the expected utility of party L by playing \tilde{s}_L against s_R^* , is,

$$U_L(\tilde{s}_L, s_R^*|m) = \left[\left(\frac{5 - \sum_{k=1}^{\bar{k}} P(k)(1 - x_L(m))^{k+1}}{8} \right) \left(1 - \frac{3}{2}m \right) - (1 - m) \right] - \alpha x_L(m).$$

Hence, the optimal $x_L^*(m) \in (0, 1)$ solves

$$\sum_{k=1}^{\bar{k}} P(k)(k+1)(1 - x_L^*(m))^k = \frac{16\alpha}{2 - 3m}. \quad (10)$$

Clearly $x_L^*(m)$ is decreasing in α . Moreover, $x_L^*(m) \geq 0$ if and only if $\alpha \leq (2 - 3m)(\hat{k} + 1)/16$, where $\hat{k} = \sum_{k=1}^{\bar{k}} P(k)k$, and $x_L^*(m) = 1$ if and only if $\alpha = 0$. Thus, if $\alpha \geq (2 - 3m)(\hat{k} + 1)/16$, then $x_L^*(m) = 0$ and a pure strategy equilibrium exists.

We can then assume that $\alpha < (2 - 3m)(\hat{k} + 1)/16$. In this case, party L will not deviate from s_L^* if and only if $U_L(s^*|e) \geq U_L(\tilde{s}_L, s_R^*|m)$, where abusing notation \tilde{s}_L prescribes to advertise a moderate candidate with intensity $x_L^*(m)$. The latter inequality is satisfied if and only if

$$\sum_{k=1}^{\bar{k}} P(k)(1 - x_L^*(m))^{k+1} \geq \frac{2 - 7m - 16\alpha}{2 - 3m}. \quad (11)$$

As we shall show later, there exists $\alpha_P^* \in (0, (2 - 3m)(\hat{k} + 1)/16)$ such that condition (11) holds if and only if $\alpha \geq \alpha_P^*$. We will also formally derive the expression of α_P^* . These observations show that a symmetric political equilibrium in pure strategies exists if and only if $\alpha \geq \alpha_P^*$. Furthermore, in a symmetric pure-strategy political equilibrium each party selects an extremist candidate with probability one and the extremist candidate is not advertised.

We now characterize symmetric mixed-strategy equilibria. For convenience we use the notation $\sigma \equiv \sigma(e)$. First, assume that a symmetric mixed-strategy equilibrium exists, and let $s_j^* = (\sigma^*, x^*(e))$, $j = L, R$, denote the equilibrium strategy profile. It is easy to see that in equilibrium $x^*(e) = 0$. Given a profile $s = (s_L, s_R)$ with $x_j(e) = 0$, $j = L, R$, we have that

$$U_L(s|e) = \sigma_R(1 - m)[\pi_L(s|e, e) - 1] + (1 - \sigma_R) \left(1 - \frac{3m}{2}\right) [\pi(s|e, m) - 1],$$

and

$$\begin{aligned} U_L(s|m) &= \sigma_R \left[\pi_L(s|m, e) \left(1 - \frac{3m}{2}\right) - (1 - m) \right] + \\ &+ (1 - \sigma_R) \left[\pi_L(s|m, m)(1 - 2m) - \left(1 - \frac{3m}{2}\right) \right] - \alpha x_L(m). \end{aligned} \quad (12)$$

Moreover,

$$\begin{aligned} \frac{\partial \pi_L(s|m, e)}{\partial x_L(m)} &= \frac{\partial \pi_L(s|m, m)}{\partial x_L(m)} = \\ &= \frac{1}{8} \sum_{k=1}^{\bar{k}} P(k)(k+1)(1 - x_L(m))^k [\rho_L(e|\emptyset, s, k)]^2. \end{aligned}$$

Hence, in a symmetric equilibrium, $\frac{\partial U_L(m, x_L(m); s_R)}{\partial x_L(m)}|_{s^*} = 0$ if and only if

$$\sum_{k=1}^{\bar{k}} P(k)(k+1)(1 - x^*(m))^k [\rho(e|\emptyset, s^*, k)]^2 = \frac{16\alpha}{2 - 4m + \sigma m}, \quad (13)$$

where

$$\rho(e|\emptyset, s^*, k) = \frac{\sigma}{\sigma + (1 - \sigma)(1 - x^*(m))^{k+1}}. \quad (14)$$

It is easy to verify that condition (13) is equivalent to condition (8) stated in Proposition 5.

Next, in a symmetric mixed-strategy political equilibrium each party is indifferent between selecting a moderate candidate and selecting an extremist candidate, i.e.,

$U_L(s^*|e) = U_L(s^*|m)$. Since in a symmetric equilibrium we have that

$$\pi_L(s^*|e, e) = \pi(s^*|m, m) = \frac{1}{2}$$

$$\pi(s^*|e, m) = \frac{1}{2} + \frac{1}{4m} \sum_{k=1}^{\bar{k}} P(k)(\bar{t}_k - m)(1 - (1 - x^*(m))^{k+1}) = 1 - \pi(s^*|m, e),$$

where $\bar{t}_k = m - \frac{m}{2}\rho(e|\emptyset, s^*, k)$, it follows that $U_L(s^*|e) = U_L(s^*|m)$ if and only if

$$\sum_{k=1}^{\bar{k}} P(k)\rho(e|\emptyset, s^*, k)(1 - (1 - x^*(m))^{k+1}) = \frac{4m + 16\alpha x}{2 - 3m}. \quad (15)$$

Note that condition (15) is equivalent to condition (9) stated in Proposition 5. We have then proved that if a symmetric mixed-strategy equilibrium exists then it is characterized by conditions (8) and (9).

The final step of the proof is to show existence of equilibria. Here, we start by showing that a mixed strategy equilibrium exists if and only if $\alpha < \alpha_p^*$, and we determine the value of α_p^* . For convenience, we use the notation $p = 1 - x(m)$, hereafter.

Define

$$f(\sigma, p) = \sum_{k=1}^{\bar{k}} P(k) \frac{(k+1)p^k \sigma^2 (2 - 4m + \sigma m)}{[\sigma + (1 - \sigma)p^{k+1}]^2},$$

and note that the equilibrium condition (13) holds if and only if (σ, p) are such that $f(\sigma, p) = 16\alpha$. The following properties of $f(\cdot, \cdot)$ will prove useful for the proof.

Property 1: $f(0, p) = 0$;

Property 2: $f(1, p) = \sum_{k=1}^{\bar{k}} P(k)(k+1)p^k(2 - 3m)$ and it is increasing in p ;

Property 3:

$$\frac{\partial f(\sigma, p)}{\partial \sigma} = \sum_{k=1}^{\bar{k}} P(k) \frac{(k+1)p^k \sigma [2p^{k+1}(2 - 4m + \sigma m) + \sigma m(\sigma + (1 - \sigma)p^{k+1})]}{[\sigma + (1 - \sigma)p^{k+1}]^3} > 0.$$

Properties 1, 2, and 3 imply that $\tilde{\sigma}(p) : f(\tilde{\sigma}(p), p) = 16\alpha$ is a well defined function of p for all $p \in [\underline{p}, 1]$, where \underline{p} solves $f(1, \underline{p}) = 16\alpha$, i.e.,

$$\sum_{k=1}^{\bar{k}} P(k)(k+1)\underline{p}^k = \frac{16\alpha}{2 - 3m}.$$

Note that $\underline{p} \in (0, 1)$ if and only if $\alpha < (2 - 3m)(\bar{k} + 1)/16$.

We now study how $\tilde{\sigma}(p)$ behaves in $p \in [\underline{p}, 1]$. The following properties of $\tilde{\sigma}(\cdot)$ are useful:

Property 4: $\tilde{\sigma}(\underline{p}) = 1$;

Property 5: $\tilde{\sigma}(1) \in (0, 1)$ solves

$$\tilde{\sigma}(1)^2(2 - 4m + \tilde{\sigma}(1)m) = \frac{16\alpha}{\bar{k} + 1};$$

Property 6: $\partial f(p, \sigma)/\partial p$ may change sign only once, and

$$\frac{\partial f(\sigma, p)}{\partial p} \Big|_{\underline{p}, \tilde{\sigma}(\underline{p})} > 0.$$

Note that Property 6 follows from Property 4 and inspection of

$$\frac{\partial f(\sigma, p)}{\partial p} = \sum_{k=1}^{\bar{k}} P(k) \frac{(k+1)p^{k-1}\sigma^2(2-4m+\sigma m)(k\sigma - p^{k+1}(1-\sigma)(k+2))}{[\sigma + (1-\sigma)p^{k+1}]^3},$$

Using the implicit function theorem and invoking properties 3, 4, 5, and 6 it follows that $\tilde{\sigma}(p)$ is either always decreasing in p for all $p \in [\underline{p}, 1]$, or there exists a $\tilde{p} > \underline{p}$ such that $\tilde{\sigma}(p)$ is decreasing in p for all $p \in [\underline{p}, \tilde{p}]$, while it is increasing in p for all $p \in (\tilde{p}, 1]$.

We now define

$$g(\sigma, p) = \sum_{k=1}^{\bar{k}} P(k) \frac{(1-p^{k+1})\sigma}{\sigma + (1-\sigma)p^{k+1}} + \frac{16\alpha(1-p)}{2-3m},$$

so that the equilibrium condition (15) holds if and only if $g(\sigma, p) = 4m/(2-3m)$. The following properties of $g(\cdot, \cdot)$ are useful:

Property 1': $g(0, p) = -16\alpha p/(2-3m)$;

Property 2': $g(1, p) = \sum_{k=1}^{\bar{k}} P(k)(1-p^{k+1})$ is decreasing in p for all $p \geq \underline{p}$;

Property 3':

$$\frac{\partial g(\sigma, p)}{\partial \sigma} = \sum_{k=1}^{\bar{k}} P(k) \frac{(1-p^{k+1})p^{k+1}}{[\sigma + (1-\sigma)p^{k+1}]^2} > 0;$$

Property 4': $\partial g(\sigma, p)/\partial p < 0$, which is easily checked.

Properties 1', 2', and 3' imply that $\bar{\sigma}(p) : g(\bar{\sigma}(p), p) = 4m/(2 - 3m)$ is a well defined function of p for all $p \in (0, \bar{p}]$, where $\bar{p} : g(1, \bar{p}) = 4m/(2 - 3m)$, i.e.

$$\sum_{k=1}^{\bar{k}} P(k)(1 - \bar{p}^{k+1}) = \frac{4m + 16\alpha(1 - \bar{p})}{2 - 3m}. \quad (16)$$

It is easy to check that $\bar{p} \in (0, 1)$. Furthermore, using the implicit function theorem and invoking properties 3' and 4' it follows that $\bar{\sigma}(p)$ is increasing in p , for all $p \in (0, \bar{p}]$. Also, $\bar{\sigma}(\bar{p}) = 1$ and $\bar{\sigma}(\underline{p}) < 1$.

Summarizing $\tilde{\sigma}(p)$ is first decreasing and then possibly increasing in p and it is defined for all $p \in [\underline{p}, 1]$, while $\bar{\sigma}(p)$ is increasing in p and it is defined for all $p \in (0, \bar{p}]$. Furthermore, $\tilde{\sigma}(\underline{p}) = 1$, while $\bar{\sigma}(\underline{p}) < 1$. Since a symmetric mixed strategy equilibrium is given by p^* and σ^* such that $\sigma^* = \tilde{\sigma}(p^*) = \bar{\sigma}(p^*)$, an equilibrium exists if and only if $\underline{p} < \bar{p}$. This holds, if and only if $\alpha < \alpha_P^*$, where α_P^* is the unique solution to

$$\sum_{k=1}^{\bar{k}} P(k)(k + 1)\underline{p}^k = \frac{16\alpha_P^*}{2 - 3m} \quad (17)$$

$$\sum_{k=1}^{\bar{k}} P(k)(1 - \underline{p}^{k+1}) = \frac{4m + 16\alpha_P^*(1 - \underline{p})}{2 - 3m}. \quad (18)$$

Note that $\alpha_P^* \in (0, (2 - 3m)(\hat{k} + 1)/16)$, and that if $\alpha = \alpha_P^*$, then $\underline{p} = \bar{p}$. In this latter case, $\sigma^* = 1$ and condition (11) holds with equality. Similarly, for all $\alpha > \alpha_P^*$ condition 11 holds with strict inequality. We have proved that a symmetric equilibrium always exists and that if $\alpha \geq \alpha_P^*$ there is a unique equilibrium in pure strategy, otherwise there exists an equilibrium in mixed strategy. This concludes the proof of Proposition 5. ■

Proof. Proposition 1 The characterization of equilibria in Proposition 1 is a special case of Proposition 5. So, to complete the proof of Proposition 1 we only need to show that when P is such that $P(k) = 1$ and $P(h) = 0$, for all $h \in I \setminus \{k\}$, and $k \in I$, there exists a unique symmetric equilibrium. For such distribution denote $\alpha_P^* = \alpha^*(k)$. If $\alpha \geq \alpha^*(k)$ the claim is obviously true. Assume then that $\alpha < \alpha^*(k)$,

and recall that $p = 1 - x(m)$ and $\sigma(e) = \sigma$. We can rewrite the equilibrium condition (5) as follows:

$$\sigma = \frac{p^{k+1}(4m + 16\alpha(1 - p))}{(1 - p^{k+1})(2 - 7m - 16\alpha(1 - p))}, \quad (19)$$

and we know from the proof of Proposition 5 that, in equilibrium, σ is increasing in p . Equilibrium condition (4) is equivalent to:

$$(2 - 4m + \sigma m)(k + 1)p^k \rho^2(e|\emptyset, s^*, k) = 16\alpha, \quad (20)$$

where σ is given by expression (19). To establish uniqueness is then sufficient to prove that, in equilibrium, the left hand side of (20) is increasing in p (where we must take into account that σ is a function of p). To see this note that since in equilibrium σ is increasing in p , it follows that $(2 - 4m + \sigma m)(k + 1)p^k$ is also increasing in p . Therefore, it is enough to show that in equilibrium $\rho(e|\emptyset, s^*, k)$ is also increasing in p . In order to prove this, we first use condition (5) to rewrite the expression of $\rho(e|\emptyset, s^*, k)$ in equilibrium, and we obtain

$$\rho(e|\emptyset, s^*, k) = \frac{4m + 16\alpha(1 - p^*)}{(2 - 3m)(1 - p^{*k+1})}.$$

Therefore,

$$\frac{d\rho(e|\emptyset, s^*, k)}{dp} = \frac{(k + 1)p^{*k}(4m + 16\alpha(1 - p^*))}{(2 - 3m)(1 - p^{*k+1})^2} - \frac{16\alpha}{(2 - 3m)(1 - p^{*k+1})} > 0$$

if and only if

$$\frac{(k + 1)p^{*k}(4m + 16\alpha(1 - p^*))}{(1 - p^{*k+1})} - 16\alpha > 0. \quad (21)$$

Note that the equilibrium condition (20) is the same as

$$(2 - 4m + \sigma m)(k + 1)p^{*k} \left(\frac{4m + 16\alpha(1 - p^*)}{(2 - 3m)(1 - p^{*k+1})} \right)^2 = 16\alpha,$$

which implies that

$$\frac{(k + 1)p^{*k}[4m + 16\alpha(1 - p^*)]}{(2 - 3m)(1 - p^{*k+1})} = \frac{16\alpha(2 - 3m)}{(2 - 4m + \sigma m)(4m + 16\alpha(1 - p^*))}.$$

Using the last equation, we have that inequality (21) is satisfied if and only if:

$$(1 - p^{*k+1})(2 - 3m)^2 - (2 - 4m + \sigma m)(4m + 16\alpha(1 - p^*)) > 0,$$

which is always satisfied for every $\alpha < \alpha^*(k)$. Indeed,

$$(1 - p^{*k+1})(2 - 3m)^2 - (2 - 4m + \sigma m)(4m + 16\alpha(1 - p^*)) > \\ (2 - 3m) \left((1 - p^{*k+1})(2 - 3m) - (4m + 16\alpha(1 - p^*)) \right),$$

because $(1 - p^{*k+1})(2 - 3m)^2 - (2 - 4m + \sigma m)(4m + 16\alpha(1 - p^*))$ is decreasing in σ . Further,

$$(2 - 3m) \left((1 - p^{*k+1})(2 - 3m) - (4m + 16\alpha(1 - p^*)) \right) \\ > (2 - 3m) \left((1 - \bar{p}^{*k+1})(2 - 3m) - (4m + 16\alpha(1 - \bar{p}^*)) \right) = 0,$$

because for all $\alpha < \alpha^*(k)$ and for all $p \in [p, \bar{p}]$, the left hand side of the inequality is decreasing in p , and the last equality follows by the definition of \bar{p} (see equation (16)).

■

Proof. Proposition 2 Let $A = 4m + 16\alpha(1 - p)$, $B = 2 - 7m - 16\alpha(1 - p)$ and $C = 2 - 4m + \sigma m$. Recall that in equilibrium

$$f(p, \sigma(p)) - \alpha = \frac{(k+1)p^k A^2 C}{(2-3m)^2(1-p^{k+1})} - 16\alpha = 0 \\ \sigma(p) = \frac{Ap^{k+1}}{B(1-p^{k+1})}.$$

We start by showing that if k increases then p increases, i.e., $x(m)$ decreases. We first derive the following expressions:

$$\frac{\partial f(p, \sigma(p))}{\partial k} = \frac{A^2 p^k \left[C + C(k+1) \ln(p) + (k+1)m \frac{\partial \sigma(p)}{\partial k} + \frac{2(k+1)Cp^{k+1} \ln(p)}{(1-p^{k+1})} \right]}{(2-3d)^2(1-p^{k+1})^2} \\ \frac{\partial \sigma(p)}{\partial k} = \frac{Ap^{k+1}}{B(1-p^{k+1})^2} \ln(p).$$

It is easy to see that for p sufficiently low then $\partial f(p, \sigma(p))/\partial k < 0$. Therefore, there exists $\hat{\alpha} > 0$ such that for all $\alpha < \hat{\alpha}$, in equilibrium, $\partial f(p, \sigma(p))/\partial k < 0$. Since we

know that in equilibrium $\partial f(p, \sigma(p))/\partial p > 0$, using the implicit function theorem it follows that for all $\alpha < \hat{\alpha}$, if k increases then p increases, i.e., $x(m)$ decreases.

Next, we show that if k increases then σ increases. To see this note that:

$$\frac{d\sigma}{dk} = \frac{\partial f(p, \sigma(p))}{\partial p} \frac{\partial \sigma(p)}{\partial k} - \frac{\partial f(p, \sigma(p))}{\partial k} \frac{\partial \sigma(p)}{\partial p},$$

where

$$\begin{aligned} \frac{\partial f(p, \sigma(p))}{\partial p} &= \frac{(k+1)p^{k-1} \left[kCA^2 - 32pAC\alpha + mpA^2 \frac{\partial \sigma(p)}{\partial p} + \frac{2CA^2(k+1)p^{k+1}}{(1-p^{k+1})} \right]}{(2-3m)^2(1-p^{k+1})^2} \\ \frac{\partial \sigma(p)}{\partial p} &= \frac{A(k+1)p^k}{B(1-p^{k+1})^2} - \frac{16\alpha(2-3m)p^{k+1}}{B^2(1-p^{k+1})}. \end{aligned}$$

Using these expressions, it follows that:

$$\lim_{p \rightarrow 0} \frac{d\sigma}{dk} = \lim_{p \rightarrow 0} \left[-\frac{(k+1)CA^3}{B(2-3m)^2(1-p^{k+1})^4} p^{2k} \ln(p) \right] = 0^+,$$

which implies that for low α (i.e. low p), an increase in k increases σ .

We finally show that for sufficiently small α ,

$$\Pi(s^*) = \sigma^{*2} + 2\sigma^*(1-\sigma^*)\pi(s^*|e, m),$$

is increasing in k . Note that for small α , $\sigma < 1/2$ and therefore $\Pi(s^*, k)$ is increasing in σ , keeping constant $\pi(s^*|e, m)$. So, it is sufficient to show that $\pi(s^*|e, m)$ is increasing in k . Since $\pi(s^*|e, m) = 1/2 - (1 - \Psi(s^*, k))/8$, it is sufficient to show that $(1 - \Psi(s^*, k))$ is decreasing in k . This follows immediately by equilibrium condition (5). Indeed, we know that if k increases, then $x(m)$ decreases (and so the right hand side of condition (5) decreases), which implies that in the new equilibrium $(1 - \Psi(s^*, k))$ must decrease. This concludes the proof of the proposition. ■

Appendix B. Proofs of Section 4. This appendix contains the proofs of Section 4. We also provide an additional result in which we show that the equilibrium described in Proposition 3 is also an equilibrium of a more general model in which parties can target political advertising to different groups of voters (see Proposition (6) below).

With some abuse of notation, we denote by $i_{j,in}$ the indifferent group- j voter that has sampled only voters belonging to group j , $j = l, r$; analogously, $i_{j,out}$ is the indifferent group- j voter that has sampled at least one voter in group j' , where $j, j' = l, r$ and $j \neq j'$.

Proof. Proposition 3 We start by considering symmetric pure strategy equilibria. Note that in equilibrium a party never advertise an extremist candidate. Also note that a (pure) strategy profile in which each party selects a moderate and do not advertise cannot be part of equilibrium; for otherwise, a party, by switching to an extremist candidate, would win the election with the same probability and would obtain a higher expected payoff, which contradicts optimality. We are then left with two possible candidates: (1.) parties select extremist candidates and do not advertise and (2.) parties select moderates and do advertise. We now analyze these two possibilities.

(1.) Consider a strategy profile s^* such that $\sigma = 1$ and $x = 0$. In this case the utility of party L is $U_L^*(s^*) = -(1 - m)/2$. It is easy to check that the best deviation of party L is to select a moderate and to advertise. Let s_L be such strategy profile. The utility of party L is then

$$U_L(s_L, s_R^*) = \pi_L(s_L, s_R^* | m, e) \left(1 - \frac{3m}{2} \right) - (1 - m) - \alpha.$$

We now derive $\pi_L(s_L, s_R^* | m, e)$. First, all group- l voters observe that $t_L = m$, and, regardless of the realization of the sampling, they believe that $t_R = e$. Therefore, $i_{l,in} = i_{l,out} = 1/2 + m/4$. Second, all group- r voters believe that $t_R = e$, a fraction β always sample group- r voters so that they believe that $t_L = e$, while the remaining group- r voters have sampled at least one voter in group l and therefore they know that $t_L = m$. Thus, $i_{r,in} = 1/2$, while $i_{r,out} = 1/2 + m/4 = i_{l,in}$. Putting together these facts it follows that for all $\mu > i_{l,in} = i_{r,out}$ party L never wins, and that for all $\mu \leq i_{l,in}$ party L wins with probability 1. Hence, $\pi_L(s_L, s_R^* | m, e) = \Pr[\mu \leq i_{l,in}] = 5/8$. Therefore, s^* is equilibrium if and only if

$$U_L(s_L, s_R^*) = \frac{5}{8} \left(1 - \frac{3m}{2} \right) - (1 - m) - \alpha \leq U_L^*(s^*) = -(1 - m)/2,$$

which is satisfied if and only if

$$\alpha \geq \frac{2 - 7m}{16} = \bar{\alpha}. \quad (22)$$

(2.) Next, consider a strategy profile s^* such that $\sigma = 0$ and $x = 1$. In this case $U_L^*(s^*) = -(1 - m)/2 - \alpha$. Suppose that if a group- l voter does not observe the ads from party L he believes that $t_L = e$; analogously for group- r voters. Clearly, the best deviation of party L is to select an extremist candidate and do not advertise. Let s_L be such strategy. The utility of party L is

$$U_L(s_L, s_R^*) = \left(1 - \frac{3m}{2}\right) [\pi_L(s_L, s_R^*|e, m) - 1].$$

We now derive $\pi_L(s_L, s_R^*|e, m)$. First, all group- l voters observe that party L does not advertise and therefore they believe that $t_L = e$; also, regardless of the sampling, all group- l voters believe that $t_R = m$. Hence, $i_{l,in} = i_{l,out} = 1/2 - m/4$. Second, all group- r voters believe that $t_R = m$, a fraction β of group- r voters believe that $t_L = m$, while the remaining voters believe that $t_L = e$. Hence, $i_{r,in} = 1/2$, while $i_{r,out} = 1/2 - m/4 = i_{l,in}$. Using these considerations, three facts follow. One, for all $\mu \geq i_{r,in}$ party L never wins; two, for all $\mu \leq i_{r,out}$ party L wins with probability 1. Three, for all $\mu \in [i_{r,out}, i_{r,in}]$, total votes for party L are

$$TV_L = \frac{1}{2} + \frac{1 + \beta}{2\tau} \left(\frac{1}{2} - \mu\right) - \frac{1}{\tau} \frac{m}{8},$$

and $TV_L > 1/2$ if and only if

$$\mu \leq \frac{1}{2} - \frac{m}{4} \frac{1}{1 + \beta} = \mu^*,$$

where it is easy to check that $\mu^* \in [i_{r,out}, i_{r,in}]$. Combining these three facts we have that

$$\pi_L(s_L, s_R^*|e, m) = \Pr \left[\mu \leq \frac{1}{2} - \frac{m}{4} \frac{1}{1 + \beta} \right] = \frac{3 + 4\beta}{8(1 + \beta)}.$$

Hence, s^* is an equilibrium if and only if

$$U_L(s_L, s_R^*) = \left(1 - \frac{3m}{2}\right) \left(\frac{3 + 4\beta}{8(1 + \beta)} - 1\right) \leq -\frac{1 - m}{2} - \alpha,$$

which is satisfied if and only if

$$\alpha \leq \frac{2 - 7m - 4m\beta}{16(1 + \beta)} = \underline{\alpha}(\beta). \quad (23)$$

It is easy to verify that $\underline{\alpha}(\beta) < \bar{\alpha}$ and that $\underline{\alpha}(\beta) \geq 0$ if and only if $\beta \leq (2 - 7m)/(4m)$.

We now turn to symmetric mixed-strategy equilibria. Clearly, the only candidate is a strategy profile s^* in which each party selects an extremist candidate with probability $\sigma \in (0, 1)$ and advertises only moderates. Randomization implies that a party is indifferent between selecting an extremist and a moderate, which holds if and only if:

$$\pi_L(s^*|m, e) = \frac{1 - m + 2\alpha}{2 - 3m}. \quad (24)$$

We now derive the expression for $\pi_L(s^*|m, e)$. Suppose that $t_L = m$ and $t_R = e$. First, a fraction β of group- l voters believe that $t_L = m$ and, with probability σ , that $t_R = e$. The remaining fraction $1 - \beta$ believe that $t_L = m$ and that $t_R = e$. Hence, $i_{l,in} = 1/2 + m\sigma/4$, while $i_{l,out} = 1/2 + m/4$. Second, all group- r voters believe that $t_R = e$, a fraction β believe that $t_L = e$ with probability σ , and a fraction $1 - \beta$ believe that $t_L = m$. Hence, $i_{r,in} = 1/2 + m(1 - \sigma)/4$, while $i_{r,out} = 1/2 + m/4 = i_{l,out}$.

Given these observations there are two relevant cases to be considered: (1.) $\sigma \leq \frac{1}{2}$ and (2.) $\sigma \geq \frac{1}{2}$.

Case 1. Suppose $\sigma \leq 1/2$; then $i_{l,in} \leq i_{r,in} < i_{l,out} = i_{r,out}$. It follows that for all $\mu \leq i_{l,in}$ party L wins with probability 1. For all $\mu \in [i_{l,in}, i_{r,in}]$, total votes of party L are

$$TV_L = \frac{1}{2} + \frac{1 + \beta}{2\tau} \left(\frac{1}{2} - \mu \right) + \frac{1}{2\tau} \frac{m}{4},$$

and $TV_L \geq 1/2$ if and only if

$$\mu \leq \frac{1}{2} + \frac{m}{4} \frac{1}{1 + \beta} = \mu_1^*,$$

where $\mu_1^* \geq i_{l,in}$, and $\mu_1^* \leq i_{r,in}$ if and only if $\sigma \leq \beta/(1 + \beta)$. Therefore, for all $\mu \in [i_{l,in}, i_{r,in}]$, if $\sigma \leq \beta/(1 + \beta)$ party L wins with probability $\Pr(\mu \leq \mu_1^*) =$

$(5 + 4\beta) / (8(1 + \beta))$, while if $\sigma \geq \beta/(1 + \beta)$ then party L wins with probability 1. For all $\mu \in [i_{r,in}, i_{l,out}]$, total votes of party L are

$$TV_L = \frac{1}{2} + \frac{1}{2\tau} \left(\frac{1}{2} - \mu \right) + \frac{1}{2\tau} \frac{m}{4} (1 - \beta(1 - \sigma)),$$

and $TV_L \geq 1/2$ if and only if

$$\mu \leq \frac{1}{2} + \frac{m}{4} (1 - \beta(1 - \sigma)) = \mu_2^*.$$

Clearly, $\mu_2^* \leq i_{l,out}$, and $\mu_2^* \geq i_{r,in}$ if and only if $\sigma \geq \beta/(1 + \beta)$. Hence, for all $\mu \in [i_{r,in}, i_{l,out}]$, if $\sigma \leq \beta/(1 + \beta)$, party L never wins, while if $\sigma \geq \beta/(1 + \beta)$, party L wins with probability $\Pr(\mu \leq \mu_2^*) = (5 - \beta(1 - \sigma)) / 8$. Finally, for all $\mu \geq i_{l,out}$ party L never wins. Summarizing, we have that: if $\sigma \leq \beta/(1 + \beta)$ then $\pi_L(m, e|s^*) = (5 + 4\beta) / (8(1 + \beta))$, while if $\sigma \in [\beta/(1 + \beta), 1/2]$ then $\pi_L(m, e|s^*) = (5 - \beta(1 - \sigma)) / 8$.

Case 2. Suppose $\sigma \geq 1/2$; then $i_{r,in} \leq i_{l,in} < i_{l,out} = i_{r,out}$. Here note that for all $\mu \leq i_{l,in}$ party L wins with probability 1. In contrast, for all $\mu \geq i_{l,out}$ party L never wins. Finally, for all $\mu \in [i_{l,in}, i_{l,out}]$, total votes of party L are

$$TV_L = \frac{1}{2} + \frac{1}{2\tau} \left(\frac{1}{2} - \mu \right) + \frac{1}{2\tau} \frac{m}{4} (1 - \beta(1 - \sigma)),$$

and $TV_L \geq 1/2$ if and only if

$$\mu \leq \mu_2^*,$$

where it is easy to check that $\mu_2^* \in [i_{l,in}, i_{l,out}]$. Hence, for all $\mu \in [i_{l,in}, i_{l,out}]$, party L wins with probability $\Pr(\mu \leq \mu_2^*) = (5 - \beta(1 - \sigma)) / 8$. Combining these observations it follows that if $\sigma \in [1/2, 1)$ then $\pi_L(s^*|m, e) = (5 - \beta(1 - \sigma)) / 8$.

By combining case 1 and case 2, it follows that if $\sigma \in (0, \beta/(1 + \beta)]$ then $\pi_L(s^*|m, e) = (5 + 4\beta) / (8(1 + \beta))$, and if $\sigma \in [\beta/(1 + \beta), 1)$ then $\pi_L(s^*|m, e) = (5 - \beta(1 - \sigma)) / 8$.

Next, note that a mixed-strategy equilibrium exists only if $\sigma \in [\beta/(1 + \beta), 1)$. Indeed, if $\sigma < \beta/(1 + \beta)$, then $\pi_L(s^*|m, e)$ does not depend on σ . Therefore, the equilibrium condition (24) cannot be satisfied generically. Therefore, it must be that $\sigma \in [\beta/(1 + \beta), 1)$, and the equilibrium condition (24) holds if and only if

$$\sigma^* = 1 - \frac{2 - 7m - 16\alpha}{\beta(2 - 3m)}. \quad (25)$$

Note that σ^* is increasing in α and, when $\alpha = \bar{\alpha}$ then $\sigma^* = 1$. Also if $\underline{\alpha}(\beta) \geq 0$, we have that at $\alpha = \underline{\alpha}(\beta)$, $\sigma^* = \beta/(1 + \beta)$. If instead $\underline{\alpha}(\beta) \leq 0$ then as α approaches 0, σ^* converges to $1 - (2 - 7m) / (\beta(2 - 3m)) \geq \beta/(1 + \beta)$, where the last inequality follows from the fact that $\underline{\alpha}(\beta) \leq 0$ (i.e., $\beta < (2 - 7m)/(4m)$). This concludes the proof of Proposition 3. ■

Proof. Proposition 4 It is immediate to see that an increase in β , increases the probability that a party selects an extremist candidate. We now show that an increase in β it increases the ex-ante expected probability that an extremist candidate wins the election. To see this note that:

$$\Pi(s^*) = [\sigma^*]^2 + 2\sigma^*(1 - \sigma^*)\pi(s^*|e, m),$$

and therefore

$$\frac{d\Pi(s^*)}{d\beta} = 2 \left((\sigma^* + \pi(s^*|e, m)(1 - 2\sigma^*)) \frac{d\sigma^*}{d\beta} + \sigma^*(1 - \sigma^*) \frac{d\pi(s^*|e, m)}{d\beta} \right).$$

Since in equilibrium $\pi_L(s^*|m, e) = (1 - m + 2\alpha) / (2 - 3m)$, and $\pi(s^*|e, m) = 1 - \pi(s^*|m, e)$, it must be that $d\pi(s^*|e, m)/d\beta = 0$. Hence, it follows that:

$$\begin{aligned} \frac{d\Pi(s^*)}{d\beta} &= 2 \frac{d\sigma^*}{d\beta} (\sigma^* + \pi(s^*|e, m)(1 - 2\sigma^*)) \\ &= 2 \frac{d\sigma^*}{d\beta} (\sigma^*(1 - \pi(s^*|e, m)) + \pi(s^*|e, m)(1 - \sigma^*)) > 0. \end{aligned}$$

■

We now consider a model where parties choose whether to advertise to group l , to group r , to both groups, or not to advertise. Abusing notation, assume that advertising to one group costs α , while advertising to both groups costs 2α . The following proposition shows that the equilibrium described in Proposition 3 is also an equilibrium of this model.

Proposition 6 *Suppose parties can target advertisement to group l , or group r , or to both groups. The following holds: (I.) If $\alpha > \bar{\alpha}$ there exists an equilibrium where*

parties select extremist candidates with probability one and they never advertise. (II.) If $\alpha \in (0, \max[0, \underline{\alpha}(\beta)])$ there exists an equilibrium in which parties select moderates and they only advertise to their closer group of independents. (III.) For every β , there exists $\alpha^* \in (\max[0, \underline{\alpha}(\beta)], \bar{\alpha})$, such that if $\alpha \in (\alpha^*, \bar{\alpha})$, in equilibrium parties select extremist candidates with probability σ^* and they advertise a moderate only to their closer group of independents, where σ^* is given by equation (7).

Proof. Proposition 6 Consider the strategy s^* in which parties select extremist candidates with probability one and they never advertise. From Proposition 3 we know that for all $\alpha > \bar{\alpha}$ the following deviation is not profitable: a party selects a moderate and advertises only to its own group. It is then sufficient to show that this is indeed the best deviation for a party. To see this, first note that if party L selects a moderate and advertises only to its own group, the probability of winning is $\Pr(\mu \leq 1/2 + m/4)$. Second, suppose that party L deviates by selecting a moderate and by advertising to group r only. It is immediate to check that in this case the probability of winning of party L will be lower than $\Pr(\mu \leq 1/2 + m/4)$. Therefore, the latter deviation is at least as good as the deviation considered above. The other possible deviation is one in which party L selects a moderate and advertises to both groups. Again, it is a matter of simple algebra to check that the probability of winning under this deviation cannot be higher than $\Pr(\mu \leq 1/2 + m/4)$. However, this deviation involves higher costs of advertising.

Consider now the strategy s^* in which parties select moderates and advertise to their own group. If $\underline{\alpha}(\beta) \leq 0$ then Proposition 3 implies that this is not an equilibrium. If instead $\underline{\alpha}(\beta) > 0$, Proposition 3 implies that for all $\alpha \leq \underline{\alpha}(\beta)$ the following deviation is not profitable: a party selects an extremist candidate and does not advertise. Moreover, if a party deviates from s^* by advertising to both groups, then simple algebra delivers that the party will face the same probability of winning as in s^* , and it will face an higher cost. Hence, we only need to check the following deviation s^d : party L selects a moderate and advertises only to group- r voters. However, note that s^* and s^d are cost

equivalent, but the probability of winning of party L under s^d is lower than under s^* .

We now consider the mixed-strategy equilibrium defined in Proposition 3. We already know that for all $\alpha \in (\max[0, \underline{\alpha}(\beta)], \bar{\alpha})$ there exists a σ , defined by equation (7), such that parties are indifferent between selecting an extremist and selecting a moderate which they advertise only to their closer group. Suppose party R follows this strategy s_R^* . We consider the possible deviations of party L .

Deviation 1: consider the following strategy s_L : party L selects a moderate and advertises to group r only. In order to derive $\pi_L(s_L, s_R^*|m, e)$, note that a fraction β of group- l voters believe that $t_L = e$ and, with probability σ , that $t_R = e$, while the remaining fraction of group- l voters believe that $t_L = m$ and $t_R = e$. Hence, $i_{l,in} = 1/2 - m(1 - \sigma)/4$ and $i_{l,out} = 1/2 + m/4$. All group- r voters believe that $t_L = m$ and $t_R = e$, so that $i_{r,in} = i_{r,out} = 1/2 + m/4$. It follows that $\pi_L(s_L, s_R^*|m, e) = \Pr(\mu \leq 1/2 + (m/4)(1 - \beta(1 - \sigma)) / (1 + \beta)) < \pi_L(s^*|m, e) = \Pr(\mu \leq 1/2 + (m/4)(1 - \beta(1 - \sigma)))$. Next, we derive $\pi_L(s_L, s_R^*|m, m)$. In this case, a fraction β of group- l voters believe that $t_L = e$ and, with probability σ , that $t_R = e$. The remaining fraction of group- l voters believe that $t_L = t_R = m$. Hence $i_{l,in} = 1/2 - m(1 - \sigma)/4$ and $i_{l,out} = 1/2$. All group- r voters believe that $t_L = t_R = m$ and therefore $i_{r,in} = i_{r,out} = 1/2$. It is now easy to check that $\pi_L(s_L, s_R^*|m, m) \leq 1/2 = \pi_L(s^*|m, m)$. Putting together these two facts, it follows that deviation 1 is not profitable.

Deviation 2: consider the following strategy s_L : party L selects a moderate and advertises to both groups. We first derive $\pi_L(s_L, s_R^*|m, e)$. A fraction β of group- l voters believe that $t_L = m$ and, with probability σ , that $t_R = e$. The remaining fraction believe that $t_L = m$ and $t_R = e$. Hence, $i_{l,in} = 1/2 + m\sigma/4$ and $i_{l,out} = 1/2 + m/4$. All group- r voters believe that $t_L = m$ and $t_R = e$; therefore $i_{r,in} = i_{r,out} = 1/2 + m/4$. It is now easy to check that $\pi_L(s_L, s_R^*|m, e) = \Pr(\mu \leq 1/2 + (m/4)(1 + \beta\sigma) / (1 + \beta))$, which is now bigger than $\pi_L(s^*|m, e)$. We now derive $\pi_L(s_L, s_R^*|m, m)$. A fraction β of group- l voters believe that $t_L = m$ and, with probability σ , that $t_R = e$. The remaining fraction believe that $t_L = t_R = m$. Hence, $i_{l,in} = 1/2 + m\sigma/4$ and $i_{l,out} = 1/2$. All group- r

voters believe that $t_L = t_R = m$ and therefore $i_{r,in} = i_{r,out} = 1/2$. It follows that $\pi_L(s_L, s_R^*|m, m) = 1/2 = \pi_L(s^*|m, m)$. Using these two facts we have that deviation 2 is not profitable if and only if

$$\alpha \geq \sigma^* \left(1 - \frac{3m}{2}\right) [\pi_L(s_L, s_R^*|m, e) - \pi_L(s^*|m, e)].$$

Given that the right hand side of the above inequality, which is a function of α , vanishes as α approaches $\bar{\alpha}$, there exists $\alpha^* \in (\max[0, \underline{\alpha}(\beta)], \bar{\alpha})$ such that the inequality holds if $\alpha > \alpha^*$. ■

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