Indivisibilities, Gambles and Giffen Goods*

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Abstract  
It is shown that if a consumer is allowed to undertake a gamble in money prior to making a consumption decision then her expected demand curve for an indivisible good can be upward sloping. The result is contrary to standard theory that defines a reservation price for the indivisible good, below which the good is consumed and above which it is not.

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1 Introduction  
According to standard theory, a consumer’s demand for an indivisible good is defined by her reservation price for the good. For prices below her reservation price she demands the good and for prices above her reservation price she does not. There is no sense in which the quantity she demands of the indivisible good can vary directly with price. When considering the purchase of an

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indivisible good, however, consumers can sometimes increase utility ex ante by randomizing over final consumption bundles. They do so by undertaking gambles in money before their consumption decisions are made. A feature of an optimum gamble is that the indivisible good is always purchased for one outcome of the gamble and not purchased for the other outcome of the gamble. Thus a consumer’s expected demand for the indivisible good is equal to the probability of one outcome of the gamble. The question arises whether there can be a Giffen effect in a consumer’s expected demand for an indivisible good.\footnote{This matters because then it would be possible for an increase in price to lead to an increase in (aggregate) market demand.}

In this paper, the possibility of a Giffen effect in a consumer’s expected demand for an indivisible good is demonstrated. An example is provided in which the probability chosen by the consumer of consuming an indivisible good increases with the price of the indivisible good.

## 2 Model

Consider a consumer who chooses between a perfectly divisible composite good, and an indivisible good. Let \( x \in \mathbb{R}_+ \) denote the quantity consumed of the divisible good and \( e = 0 \) or \( 1 \) denote the quantity consumed of the indivisible good. Suppose utility for the consumer is given by the function \( u(e, x) : \{0, 1\} \times \mathbb{R}_+ \to \mathbb{R} \). It is assumed that \( u(1, x) > u(0, x) \) for all \( x \), and that \( u \) is continuous, strictly increasing and strictly concave in \( x \).

Suppose the divisible good is the numeraire, the price of the indivisible good is \( q \), and endowed income of the consumer is \( y \). Any income that is not spent on the indivisible good is spent on the divisible good. Thus the consumer’s problem is simply to choose between two alternatives, \((0, y)\) and \((1, y - q)\), picking the alternative with the highest utility.

Given our assumptions on utility it is easy to establish the existence of a reservation price for the indivisible good \( \hat{q} \) such that below \( \hat{q} \) the indivisible good is always consumed and above \( \hat{q} \) it is not. In cases where the consumer chooses to purchase the indivisible good at any affordable price, the reservation price is \( \hat{q} = y \). Otherwise it is defined as the price that satisfies the equation

\[
u(0, y) = u(1, y - \hat{q}).\tag{1}\]
The existence of a reservation price seems to imply the indivisible good cannot be a Giffen good. However, this conclusion is misleading since the analysis fails to take account of the fact that when considering the purchase of an indivisible good consumers may desire to undertake gambles in money prior to making their consumption decisions.\footnote{Garratt and Marshall (1994) demonstrate the desire for gambles when the indivisible good is college education.} Next, it is shown that if gambles are allowed then the consumer’s expected demand for the indivisible good may vary directly with its price.

\section{Expected Demand for the Indivisible Good}

Suppose the consumer is a von Neumann - Morgenstern expected utility maximizer. The consumer chooses a gamble by selecting an income $y_0$ corresponding to the state in which the indivisible good is not consumed, an income $y_1$ corresponding to the state in which the indivisible good is consumed, and a probability $\lambda$ of receiving the income $y_1$. The income $y_0$ is received with probability $(1 - \lambda)$. The probability $\lambda$ is the consumer’s expected demand for the indivisible good. Actuarial fairness implies that

$$
\lambda = \frac{y - y_0}{y_1 - y_0}.
$$

(2)

The consumer’s optimization problem now involves three choice variables and is written formally as

$$
\max_{\lambda, y_0, y_1} (1 - \lambda)u(0, y_0) + \lambda u(1, y_1 - q)
$$

(3)

subject to $(1 - \lambda)y_0 + \lambda y_1 = y$

(4)

$$
0 \leq \lambda \leq 1
$$

(5)

$$
y_0 \geq 0, \quad y_1 \geq 0.
$$

(6)

The first order conditions for an interior solution are given by (4), together with

$$
u_2(0, y^*_0) = u_2(1, y^*_1 - q),
$$

(7)  

$$
u(1, y^*_1 - q) = u(0, y^*_0) + u_2(0, y^*_0)[y^*_1 - y^*_0].
$$

(8)
Despite the non-concavity of the objective function and the nonlinearity of the constraint (4), the values of $y^*_0$, $y^*_1$ and $\lambda^*$ that solve equations (4), (7) and (8) represent a global maximum.  

It is now possible to analyze how the optimum $\lambda$ responds to changes in $q$. Let $r(0, y) = -u_{22}(0, y)/u_2(0, y)$ and $r(1, y) = -u_{22}(1, (y-q))/u_2(1, y-q)$ denote the Arrow-Pratt measures of absolute risk aversion at points on each of the curves, $u(0, y)$ and $u(1, y-q)$ respectively. From the first order equations,

$$\frac{\partial \lambda^*}{\partial q} = -\left(\frac{1 - \lambda^*}{r(0, y^*_0)(y^*_1 - y^*_0)^2} + \frac{\lambda^*}{r(1, y^*_1)(y^*_1 - y^*_0)^2}\right) - \frac{\lambda^*}{(y^*_1 - y^*_0)}.$$  

Equation (9) shows the Slutsky decomposition of the effect of a price change. For an interior solution $\partial y^*_0/\partial y = \partial y^*_1/\partial y = 0$. Thus the term

$$\frac{\lambda^*}{(y^*_1 - y^*_0)}$$  

on the right-hand-side of (9) is equal to $\partial \lambda^*/\partial y \cdot \lambda^*$, which is the part of the Slutsky equation that identifies the income effect. The remaining term on the right-hand-side of (9),

$$-\left(\frac{1 - \lambda^*}{r(0, y^*_0)(y^*_1 - y^*_0)^2} + \frac{\lambda^*}{r(1, y^*_1)(y^*_1 - y^*_0)^2}\right)$$  

identifies the substitution effect.\(^4\)

The substitution effect is always negative, and hence, for a Giffen effect in the consumer’s expected demand for the indivisible good to be possible, the payoffs in the optimum gamble must satisfy $y^*_1 < y^*_0$. That is, the indivisible good must be an inferior good.\(^5\) The section concludes with an example of

\(^3\)Verification of the second order conditions for similar problems is found in Marshall (1984) and Bergstrom (1986). Also see Garratt and Marshall (1994) for details.

\(^4\)For very large values of $r(0, y^*_0)$ and $r(1, y^*_1)$ the substitution effect is close to zero and the total effect of a price change on the consumer’s expected demand for the indivisible good is approximately equal to the negative of the income effect. For the extreme case, suppose the utility curves $u(0, y)$ and $u(1, y-q)$ are kinked at $y^*_0$ and $y^*_1$ respectively. Then, for interior solutions, $\partial y^*_0/\partial q = 0$ and $\partial y^*_1/\partial q = 1$. Thus, the total effect is equal to the negative of the income effect, that is, $\partial \lambda^*/\partial q = -\lambda^*/(y^*_1 - y^*_0)$.

\(^5\)The indivisible good is called inferior when $y^*_1 < y^*_0$ because then the income effect on the consumer’s expected demand for the indivisible good is negative. In order to obtain a solution with $y^*_1 < y^*_0$ the indivisible good must also be inferior in the sense of Cook and Graham (1977). Namely, without gambles, the consumer’s reservation price for the good decreases with income.
differentiable functions $u(0, x)$ and $u(1, x)$ that produce an upward sloping expected demand curve for the indivisible good.

Let
\[ u(0, x) = \begin{cases} 
\frac{.75}{(7.425/8.9)^{10}} x^{.99} & \text{if } 0 \leq x \leq 7.425/8.9 \\
(x - .75)^{10} & \text{if } x > 7.425/8.9 
\end{cases} \]

and $u(1, x) = .4 + .5x^9$. At the change point $x = 7.425/8.9$ the two parts of the function $u(0, x)$ meet and have the same slope. Thus, the function $u(0, x)$ is continuous and differentiable. Suppose endowed income is $y = .5$. It is not possible to obtain a closed form solution for $\lambda^*$. Therefore, the values of $\lambda^*$ are obtained numerically using Newton’s method, as $q$ varies in increments of .01. For $q \leq .07$, $\lambda^* = 1$ and for $q \geq .43$, $\lambda^* = 0$. For values of $q$ in between .07 and .43 an interior solution occurs. The values of $\lambda^*$ are shown in Figure 1. Over a range of prices $\lambda^*$ varies directly with $q$.

## 4 Concluding Remark

The possibility of Giffen effects in the expected demand for an indivisible good depends upon consumers undertaking gambles in money prior to making their consumption decisions. Literal interpretation of the theory seems to require that consumers travel to Las Vegas before deciding on the purchase of an indivisible good. In the real world we typically do not imagine gamblers in Las Vegas having indivisible good consumption as the express purpose for their trip. However such behavior — though plausible — is not required to corroborate the theory. Consumers can approximate their desired gambles by investing in the stock market or by purchasing other risky assets such as real estate. Giffen effects are possible if consumers structure their holdings of risky assets in accordance with the prices of the indivisible goods they seek to purchase.

## References

