Choosing Partners:
A Classroom Experiment

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July 29, 2011

Abstract

We describe classroom experiments designed to test and illustrate matching mechanisms outlined in Gale and Shapley (1962). Two methods are presented, one in which rankings are provided and one in which they are supplied by the students.

1 Introduction

Suppose there are n men and n women. Each person has a personal ranking over all members of the opposite sex and there are no ties. Can we match the men and women together in such a way that no two people of opposite sex who would both rather have each other than their current partners? This is the marriage problem.

The marriage problem has been studied extensively over the years, with extensions to more complex markets with complementarities between positions or applicants (see Roth and Sotomayor, 1990). The main result is the Gale-Shapley algorithm which answers the question of the preceding paragraph in the affirmative. In fact, it goes further by establishing procedures under which stable matches are obtained that are most preferred by the men in one instance,

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and most preferred by the women in the other. The Gale-Shapley algorithm is designed to be implemented by a computer (the code is only a dozen lines), but it is simple enough to act out with live subjects. Doing so illustrates the power of mechanism design and initiates students to the burgeoning field of two-sided matching theory.

2 Experiments

The first set of experiments involve a two-sided matching problem where n differently colored squares must match with n differently colored circles. Each color of square is given a personal ranking of the n colors of circles and each color of circle is given a personal ranking of the n colors of squares. We define matching payoffs by assigning points to individuals according to the reverse ranking of their match. If an individual matches with their most preferred color then they get n points, their second favorite is worth $n-1$ points, etc.

Actual colors and point values used in our experiments are shown in Table 1.

		SQUARE								
		green	purple	blue	orange	lime	grey	red	yellow	pink
	green	(4,9)	(9,4)	(5,8)	(7,6)	(3,6)	(1,7)	(8,5)	(6,7)	(2,5)
C	purple	(5,8)	(4,9)	(6,7)	(8,5)	(1,5)	(2,6)	(9,4)	(7,6)	(3,4)
I	blue	(9,4)	(8,5)	(4,9)	(6,7)	(2,7)	(3,8)	(7,6)	(5,8)	(1,6)
R	orange	(7,6)	(6,7)	(8,5)	(4,9)	(3,9)	(1,4)	(5,8)	(9,4)	(2,8)
C	lime	(9,1)	(1,2)	(2,3)	(8,1)	(7,2)	(5,3)	(4,2)	(3,2)	(6,3)
L	grey	(4,2)	(3,3)	(5,1)	(1,2)	(9,1)	(7,1)	(2,1)	(6,3)	(8,1)
E	red	(6,7)	(5,8)	(7,6)	(9,4)	(2,4)	(3,5)	(4,9)	(8,5)	(1,9)
	yellow	(8,5)	(7,6)	(9,4)	(5,8)	(1,8)	(2,9)	(6,7)	(4,9)	(3,7)
	pink	(1,3)	(7,1)	(9,2)	(3,3)	(4,3)	(5,2)	(8,3)	(2,1)	(6,2)

Table 1: Matching payoffs for $n = 9$: The first number each pair in the matrix gives the points earned by the circle in the match. The second number of each pair in the matrix gives the points earned by the square in the match.

The first experiment involves unstructured matching. Students are given colored shapes with their ranking written on the back. Cards for the red circle and red square are shown in Figure 1. Students are asked to move about

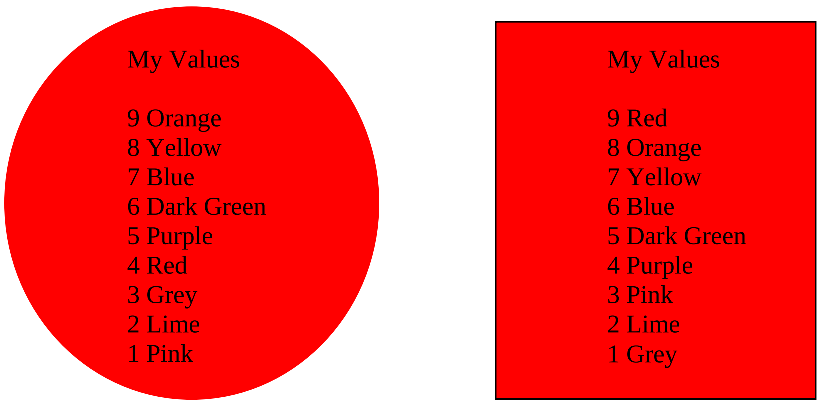

Figure 1: Sample game cards.

and find matches. They are told that circles must with squares and squares must match with circles. They can string people along while trying to find a better match, but once they report the match to the instructor it is final. The instructor writes reported matches on the board.

Table 2 shows the results of a session with 22 students.[1] One should not expect to see a stable match. In fact, at this point the students should be allowed to wonder if there necessarily is a stable match. In Table 2 there are multiple instances of unstable matches. These can be found and demonstrated to the class using the following procedure.

Start at the top. Ask: "Do any squares prefer the green circle to your current match? If so, please hold up your colored squares." Then ask the green circle: "Do you prefer any of these squares to your current match?" If the answer is yes then point out that the matching arrangement shown in Table 2 is not stable. Otherwise, proceed down the list. Chances are you will find multiple instance of unstable matches. In Table 2 there are multiple unstable

[1]The experiment is designed for $n = 9$ (i.e., 18 subjects). We use duplicates to run the experiment for larger n. In this case we duplicated blue and purple to get $n = 11$. If you want to play with $n = 8$ get rid of lime. If you want to play with $n = 7$ keep lime, and get rid of pink and grey.

CIRCLE	SQUARE
green	purple
purple	blue
blue	orange
blue	yellow
purple	blue
orange	green
lime	pink
gray	lime
red	red
yellow	purple
pink	gray

Table 2: Unstructured Matching: Round 1

CIRCLE	SQUARE
lime	lime
blue	purple
green	yellow
yellow	purple
purple	blue
blue	orange
purple	blue
orange	red
red	green
pink	grey
grey	pink

Table 3: Unstructured Matching: Round 2

matches.

When we did this last, we repeated the above process and achieved the matching arrangement shown in Table 3. This time there was only one unstable match.

At this point we raised the question whether a stable match even exists. To show the students that a stable match does exist we ran the iterative procedure proposed by Gale and Shapley to prove the existence of a stable match.

Have all the student who are designated as squares line up at the front of the room, holding their colored squares in front of them.

Stage 1. Ask the circles to line up in front of their favorite square. Then ask the squares to pick their favorite among the circles who are lined up in front of them. The circles who are not selected must step to the back of the room.

Stage 2. Ask the circles at the back of the room to go and line up in front of their second favorite square. Then ask the squares to pick their favorite among the circles who are lined up in front of them. The circles who are not selected must step to the back of the room.

Repeat...

Notice that circles that were selected in an early stage may be turned away in a later stage. Matches are not final until each square has one and only one circle in front of them. As Gale and Shapley point out, this can take at most $n_2 - 2n + 2$ stages.

When we conducted the circles-propose iterative procedure we achieved the match shown in Table 4. This match is, in fact, stable. We provide the students with a simple (verbal) proof of this fact. Suppose there are two people,

CIRCLE	SQUARE
yellow	blue
green	purple
purple	red
blue	purple
orange	blue
purple	orange
blue	green
lime	lime
pink	grey
grey	pink
red	yellow

Table 4: Gale and Shapley Iterative Procedure: Circles Propose

say orange circle and purple square that would prefer to break their existing matches and join together. Since orange circle prefers purple square to blue square, under the iterative procedure, it must have been the case that at some point orange circle lined up in front of purple square. Moreover, orange circle must have been turned away by purple square. However, this is a contradiction, since it implies purple square must have at one point had a match she preferred to orange circle, but ended up with a less preferred match. Under the iterative procedure the matches retained by squares can only improve.

In fact it is, as the theory predicts, the most preferred stable match of the circles. When we conducted this experiment a student noticed that circles seemed to well under this procedure. To illustrate this it is useful to show students the matching payoffs. We did so by amending table 4 at the front of the room to look like Table 5.

CIRCLE	SQUARE
yellow, 9	blue, 4
green, 9	purple, 4
purple, 9	red, 4
blue, 8	purple, 5
orange, 8	blue, 5
purple, 8	orange, 5
blue, 9	green, 4
lime, 7	lime, 2
pink, 5	grey, 2
grey, 8	pink, 1
red, 8	yellow, 5

Table 5: Gale and Shapley iterative procedure: Matches and corresponding payoffs when circles propose.

Next we reversed the iterative procedure and had the circles line up at the front of the room. The resulting match is shown (with payoffs) in Table 6. At this point it is worthwhile to point out that all the squares prefer the squares propose match in table 6 to the stable match in Table 5, and all the circles prefer the stable match in table 5 to the stable match in Table 6.

CIRCLE	SQUARE
orange, 4	orange, 9
red, 4	red, 9
purple, 4	purple, 9
blue, 4	blue, 9
blue, 4	blue, 9
green, 4	green, 9
grey, 8	pink, 1
pink, 5	grey, 2
yellow, 4	yellow, 9
lime, 7	lime, 2
purple, 4	purple, 9

Table 6: Gale and Shapley iterative procedure: Matches and corresponding payoffs when squares propose.

The Gale and Shapley iterative procedure works for any equal number of men and women, provided each individual assigns a unique ranking to each member of the opposite sex. Hence, the payoff table provided above can be thrown away and the experiment can be run in a more free-form method. At the start of class hand out n colored circles and n colored squares with the rankings left blank, as shown in figure 2 for the case of $n = 10$. Tell the

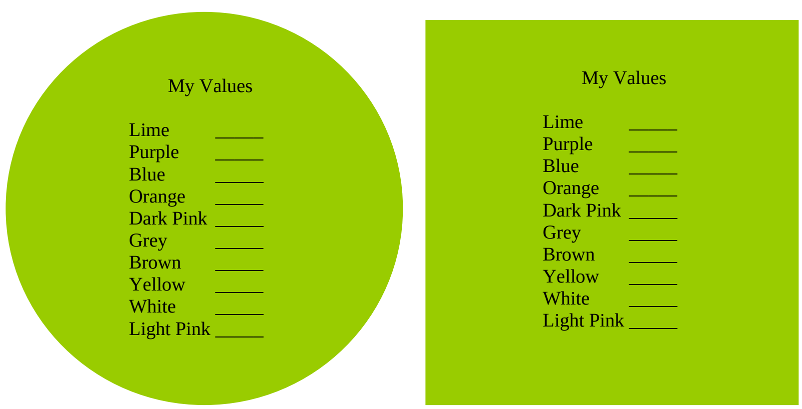

Figure 2: Blank sample game cards.

students to place unique integer values from 1 to 10 next to the colors shown in accordance with their preferences. Remind them to give the score of 10 to their favorite color, 9 to their second favorite color and so on. This method of ranking seems to work well, and is consistent with the conventional notion that an ideal mate is a perfect 10! There are a few advantages to this free-form approach. First, the students will not suspect that the payoffs are rigged. Second, students have direct ownership of their ranking, it is not imposed upon them. Finally, preferences over colors overlap in much the same way preferences over members of the opposite sex do, creating realistic competition for matches. We have had success running the free-form method and recommend it.

3 Discussion

After running the experiment, it is worthwhile to have a discussion with students about the results relate to real world mechanisms. There is an immediate connection to traditional courtship rules whereby men propose to women. This raises quite a reaction from students when they realize that this procedure looks very much like the version of the algorithm that produced the preferred match for men!

It is also interesting to discuss the generalization of the algorithm to more complicated matching markets such as college admissions or the medical internship market. The Gale-Shapley algorithm extends to many-to-one matching problems in which one side of the market demands multiple matches from the other side, so long as preferences are still strict. A very interesting paper by Al Roth and Elliot Peranson (Roth and Peranson, 1999) discusses the case of the National Resident Matching Program (NRMP), which has operated as a clearinghouse for assigning interns to hospitals since the early 1950s. In the past, this program utilized a procedure that was "functionally equivalent" to a hospital-propose algorithm, in which hospitals received their preferred matches. However, growing discontent over matching outcomes by students caused the

NRMP to commission the design of a new applicant-propose algorithm that was adopted in 1998.

4 References

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