Second Chance Offer

$n$ regular bidders and 1 shill bidder

shill bidder bids $v_{\text{max}}$

equilibrium bidding strategy of regular bidders is to bid

$$b^l(v) = \frac{n - 1}{n} v$$

True because high bidder wins and pays her bid!
Suppose it is uncertain whether second chance offer will appear

Suppose this occurs according to the empirical frequency $\phi$

Suppose bidder 1 expects all other bidders to value bid.

Then payoff to bid $b$ is

$$(1 - \phi)G(b)(v_1 - E(y | y < b)) + \phi G(b)(v_1 - b)$$
\[ E[y \mid y < b] = \int_0^b y \frac{g(y)}{G(b)} \, dy \]

So...

Previous expression is equal to

\[
(1 - \phi)G(b)v_1 - (1 - \phi) \int_0^b yg(y) \, dy + \phi G(b)(v_1 - b) \\
= G(b)v_1 - (1 - \phi) \int_0^b yg(y) \, dy - \phi G(b)b
\]
Suppose $n=2$ and $F(v)=v$

\[
   b v_1 - (1 - \phi) \int_0^b y dy - \phi b^2 \\
   = b v_1 - (1 - \phi) \frac{b^2}{2} - \phi b^2 \\
   = b v_1 - (1 + \phi) \frac{b^2}{2}
\]
Take derivative w.r.t. $b$

$$v_1 - (1 + \phi)b = 0$$

$$b = \frac{v}{1 + \phi}$$

Note: $\phi = 0$ implies $b = v$, $\phi = 1$ implies $b = v/2$
Find equilibrium

Suppose opponent bids \( b = \frac{v}{1+\phi} \)

Expected payoff to bid \( b \) is

\[
(1 - \phi)(1 + \phi)b(v_1 - E\left(\frac{y}{1+\phi} \mid \frac{y}{1+\phi} < b\right)) + \phi(1 + \phi)b(v_1 - b)
\]

\[
= (1 - \phi)(1 + \phi)b(v_1 - \frac{y}{1+\phi}E(y \mid y < (1 + \phi)b)) + \phi(1 + \phi)b(v_1 - b)
\]

\[
= (1 - \phi)(1 + \phi)b(v_1 - \frac{b}{2}) + \phi(1 + \phi)b(v_1 - b)
\]

\[
= (1 + \phi)b \left[ (v_1 - \frac{b}{2}) - \phi(v_1 - \frac{b}{2}) + \phi(v_1 - b) \right]
\]

\[
= (1 + \phi)b \left[ v_1 - \frac{b}{2} - \phi\frac{b}{2} \right]
\]

\[
= (1 + \phi)b \left[ v_1 b - (1 + \phi)\frac{b}{2} \right]
\]
Take derivative w.r.t. $b$ and set equal to 0

$v_1 - (1 + \phi)b = 0$

$b = \frac{v_1}{1 + \phi}$