In all the questions that follow you may assume each of the $i = 1, \ldots, n$ bidders’ values are drawn independently from the uniform distribution on $[0,100]$, which is defined as follows

$$F(v) = \Pr[\tilde{v}_i \leq v] = \frac{v}{100}.$$ 

1. What is the probability that individual $i$’s value draw is equal to 60, is less than 60, is less than or equal to 60?

2. Let $\tilde{v}_{(1)}$ denote the random variable for the highest of the $n$ bidders’ use-value draws.
   
   (a) Compute $F_{(1)}(\cdot)$, the distribution of $\tilde{v}_{(1)}$.
   
   (b) Compute $f_{(1)}(\cdot)$, the density function of $\tilde{v}_{(1)}$.

3. Let $\tilde{v}_{(2)}$ denote the random variable for the second-highest of the $n$ bidders’ use-value draws.
   
   (a) Compute $F_{(2)}(\cdot)$, the distribution of $\tilde{v}_{(2)}$.
   
   (b) Compute $f_{(2)}(\cdot)$, the density function of $\tilde{v}_{(2)}$.

4. Compute the expected value of $\tilde{v}_{(1)}$.

5. Evaluate the following claim: In a SPA you should bid more than your value because you increase your probability of winning the auction and it doesn’t cost you anything since you pay the second-highest bid.

6. Consider a second-price auction involving $n$ bidders, reserve price $r$, and entry fee $c$. 
(a) Write down the equation that defines the minimum value, call it $v_0$, that you would have to possess in order to be willing to pay the entry fee and enter a bid. Explain.

(b) State your bid function for realizations of values greater than $v_0$.

(c) Compute $v_0$ for $n = 2$, $r = 20$, and $c = 5$.

7. Let $b^I(\cdot)$ denote the bid function used by all bidders in the symmetric Nash equilibrium of the FPA with no reserve and no entry fee.

(a) Write down the bid function $b^I(\cdot)$ for the case where there are two bidders.

(b) Write down the bid function $b^I(\cdot)$ for the case where there are $n$ bidders.

(c) Verify that the bid function you wrote down for the first-price auction with $n$ bidders is, in fact, an equilibrium bid function.

8. Compute the expected revenue for a FPA with no reserve and no entry fee.

9. Evaluate the following claim: A bidder who pays an entry fee to enter an auction should try to recoup some of that entry fee by reducing her bid.

10. Evaluate the following claim: A seller who is interested in maximizing her sale price would be crazy to sell something using a SPA rather than a FPA. In a SPA the seller is only paid the second-highest bid, but in a FPA she is paid the highest bid!

11. Evaluate the following claim: The only reason why a seller would choose to use a SPA over a FPA would be that she wants to make sure the item she is selling is allocated efficiently (i.e., goes to the person with the highest value).

12. In theory the seller can maximize expected revenue by holding either a first-price auction or a second-price auction, provided in each case there is an optimal entry fee. Discuss what happened when we tested this claim using class data.