Bargaining Games

An Application of Sequential Move Games
The Bargaining Problem

- The “Bargaining Problem” arises in economic situations where there are gains from trade, for example, when a buyer values an item more than a seller.
- The problem is how to divide the gains, for example, what price should be charged?
- Bargaining problems arise when the size of the market is small and there are no obvious price standards because the good is unique, e.g., a house at a particular location. A custom contract to develop a web page, etc.
- We can describe bargaining games (in extensive form) that allow us to better understand the bargaining problem in various economic settings.
Bargaining Games

• A bargaining game is one in which two (or more) players bargain over how to divide the gains from trade.

• The gains from trade are represented by a sum of money, $M$, that is “on the table.”

• Players move sequentially, making alternating offers.

• Examples:
  – A Seller and a Buyer bargain over the price of a house.
  – A Labor Union and Firm bargain over wages & benefits.
  – Two countries, e.g., the U.S. and Canada bargain over the terms of a trade agreement.
The *Disagreement Value*

- If both players in a 2-player bargaining game disagree as to how to divide the sum of money M, (and walk away from the game) then each receives their *disagreement value*.
- Let \( a \)=the disagreement value to the first player and let \( b \)=the disagreement value to the second player.
- In many cases, \( a=b=0 \), e.g., if a movie star and film company cannot come to terms, the movie star doesn’t get the work and the film company doesn’t get the movie star.
- The disagreement value is know by some other terms, e.g., the best alternative to negotiated agreement “BATNA.”
- By *gains from trade we mean that* \( M>a+b \).
Take it or Leave it Bargaining Games

• “Take-it-or-leave-it” is the simplest sequential move bargaining game between two players; each player makes one move.
• Player 1 moves first and proposes a division of M.
  – For example, x for player 1 and M-x for player 2.
• Player 2 moves second and must decide whether to accept or reject Player 1’s proposal.
• If Player 2 accepts, the proposal is implemented: Player 1 gets x and Player 2 gets M-x.
• If Player 2 rejects, then both players receive their disagreement values, a for Player 1 and b for Player 2.
• This game has a simple “rollback” equilibrium:
  – Player 2 accepts if M-x ≥ b, her disagreement value.
Used Car Example

- Buyer is willing to pay a maximum price of $8,500.
- Seller will not sell for a price less than $8,000.
- $M = 8500 - 8000 = 500, a = b = 0.$
- Suppose the seller moves first and there is *perfect information*: the seller knows the maximum value the buyer attaches to the car. Then the seller knows the buyer will reject any price $p > 8500$, and will accept any price $p \leq 8500$.
- The seller maximizes his profits by proposing $p = 8500$, or $x = 500$. The buyer accepts, since $M - x \geq b$.
- The seller gets the entire amount, $M = 500$.
- What happens if the buyer moves first?
“Ultimatum Game”
Discrete Version of Take it or Leave it Bargaining

• Player 1 moves first and proposes a division of $1.00. Suppose there are just 3 possible discrete divisions, limited to $0.25 increments:
  – Player 1 can propose $x = $0.25, $x = $0.50, or $x = $0.75 for himself, with the remainder, $1-x$ going to Player 2.

• Player 2 moves second and can either accept or reject Player 1’s proposal.

• If Player 2 accepts, the proposal is implemented.

• If Player 2 rejects, both players get $0 each. The $1.00 gains from trade vanish.
Problems with Take-it-or-Leave-it

• Take-it-or-leave-it games are too trivial; there is no back-and-forth bargaining.

• Another problem is the credibility of take-it-or-leave-it proposals.
  – If player 2 rejects player 1’s offer, *is it really believable* that both players walk away even though there are potential gains from trade?
  – Or do they continue bargaining? Recall that M>a+b.

• What about fairness? Is it really likely that Player 1 will keep as much of M as possible for himself?
The Dictator Game

- Are Player 1’s concerned about *fairness*, or are they concerned that Player 2’s will reject their proposals? The Dictator Game gets at this issue.
The Alternating Offers Model of Bargaining

• A sequential move game where players have perfect information at each move.
• Players take turns making alternating offers, with one offer per round, i.e., this is real back-and-forth bargaining.
• Round numbers $t = 1, 2, 3, \ldots$
• Let $0 \leq x(t) \leq 1$ be the fraction of M that player 1 asks for in bargaining round $t$, and let $0 \leq y(t) \leq 1$ be the fraction of M that player 2 asks for in bargaining round $t$. 
Alternating Offer Rules

• Player 1 begins in the first round by proposing to keep $x(1)M$ for himself and giving Player 2 $[1-x(1)]M$.

• If Player 2 accepts, the deal is struck. If Player 2 rejects, another bargaining round may be played. In round 2, player 2 proposes to keep $y(2)M$ for herself, and giving $[1-y(2)]M$ to player 1.

• If Player 1 accepts, the deal is struck, otherwise, it is round 3 and Player 1 gets to make another proposal.

• Bargaining continues in this manner until a deal is struck or no agreement is reached (an impasse is declared by one player – a “holdout”).

• If no agreement is reached, Player 1 earns $a$, and Player 2 earns $b$ (the disagreement values).
Alternating Offers in Extensive Form

Round 1
- Player 1
  - Impasse
  - a, b
    - Player 2
      - x(1)M, [1-x(1)]M
        - Accept
        - Reject
          - Impasse
            - [1-y(2)]M, y(2)M
              - a, b
                - Player 1
                  - x(3)M, [1-x(3)]M
                    - Accept
                    - Reject
                      - [1-y(2)]M, y(2)M
                        - a, b
                          - Player 2
                            - x(1)M, [1-x(1)]M
                              - Impasse
                                - [1-y(2)]M, y(2)M
                                  - Round 3
                                    - Etc.
When Does it End??

• Alternating offer bargaining games could continue indefinitely. In reality they do not.

• Why not?
  – Both sides have agreed to a deadline in advance (or M=0 at a certain date).
  – The gains from trade, M, *diminish* in value over time, and may fall below a+b.
  – The players are *impatient* (time is money!).

• Take-it-or-leave-it has 1 round deadline. Let’s focus on the last two possibilities.
Decreasing Gains from Trade

• Suppose there are 2 rounds, Player 1 proposes a division of M first, player 2 accepts/rejects.
• If accepted the proposal is implemented.
• If rejected, player 2 gets to make a counter-proposal for how to split a reduced amount of money \( \lambda M \), where \( 0 < \lambda < 1 \) is the rate of decay.
• Player 1 can then accept or reject this final proposal. The bargaining game is then over with certainty.
• Example: Suppose that \( M = $12 \), \( \lambda = 1/3 \).
Shrinking Gains Equilibrium

• What is the equilibrium?
• Note first that $M = $12 in round 1, but only $4 in round 2.
• Working backward, player 2 knows s/he can get $3.99 in round 2 as the first mover since player 1 will strictly prefer $0.01 to nothing in round 2, and will accept that division (this follows from our assumption that players are forward looking, rational, payoff maximizers).
• Knowing this, player 1 must offer player 2 $4.00 > $3.99 in round 1 keeping $12 - 4 = 8 for himself. As player 2 recognizes this as a better payoff than can be had by waiting, she accept player 1’s first round proposal.
• Player 1’s first stage offer equals the amount at stake at the start of the final round, $\lambda M$. This generalizes to any finite n-round bargaining game, where $2 < n < \infty$. 
**Impatience** as a Reason for Ending Bargaining: The Period Discount Factor, $\delta$

- The period discount factor, $0 < \delta < 1$, provides a means of evaluating future money amounts in terms of current equivalent money amounts.

- Suppose a player values a $1 offer now as equivalent to $1(1+r)$ one period later. The discount factor in this case is: $\delta=1/(1+r)$, since $\delta \times \$1 = \$1/(1+r)$ now = $\$1$ later.

- If $r$ is high, $\delta$ is low: players discount future money amounts heavily, and are therefore very impatient.

- If $r$ is low, $\delta$ is high; players regard future money almost the same as current amounts of money and are more patient (less impatient).
Example: Bargaining over a House

• Suppose the minimum price a seller will sell her house for is $150,000, and the maximum price the buyer will pay for the house is $160,000. Therefore, M=$10,000.

• Suppose both players have the exact same discount factor, $\delta=.80$. (This implies that $r=.25$).

• Suppose that there are just two rounds of bargaining. Why? The Seller has to sell by a certain date (buying another house or the Buyer has to start a new job and needs a house.

• Suppose the buyer makes a proposal in the first round, and the seller makes a proposal in the second round.

• Work backward starting in the second (last) round of bargaining and apply backward induction.
Two period bargaining over a house, continued

• Work backward. From the perspective of today, the value of the gain from trade in the second and final round is $\delta M$. In that round, the seller has to make a counterproposal.

• In that second and final round, the seller’s proposal will be to keep $\delta M$ for herself and the buyer must accept or reject. Since he is indifferent between accepting and rejecting (he gets 0 in either case), let’s suppose he will accept the proposal.

• Knowing this, the buyer must offer the seller $\delta M$ in the first period and, since in this case the seller is made indifferent between waiting and accepting, the seller accepts immediately.

• In our example, where $\delta =.8$ and $M=\$10,000$, the buyer offers $.8M$ to the seller, or $\$8,000$, keeping $\$2,000$ for himself. The sale price of the home is thus $\$150,000+\$8,000=\$158,000$.

• So, the same logic as in the decreasing gain case.
Infinitely Repeated Analysis

• Now suppose there is no end to the number of bargaining rounds; bargaining can go on forever (an infinitely repeated game)

• If the Buyer’s moves first, the amount he proposes to keep for himself, $x(1)M$, must leave the Seller an amount that is equivalent to that which the Seller can get in the next round, 2, by rejecting and proposing $y(2)M$ for herself next round. The equivalent amount now, in period 1, has value to the seller of $\delta y(2)M$, where $\delta$ is the period discount factor.

• Dropping the time indexes, the Buyer offers $(1-x)M=\delta yM$ to the Seller, and so $x=1-\delta y$

• By a similar argument, the Seller must offer $(1-y)M=\delta xM$ to the buyer, and so $y=1-\delta x$.

$$x=1-\delta (1-\delta x) \text{ and } y=1-\delta (1-\delta y)$$
Infinitely Repeated Analysis (Continued)

• \( x = y = \frac{(1-\delta)}{(1-\delta^2)} \). (Note that \( x+y > 1! \)) What is \( x \) and \( y \)?
• \( x \) is the amount the Buyer gets if he makes the first proposal in the very first round.
• \( y \) is the amount the Seller gets if she makes the first proposal in the very first round.
• If the Buyer is the first proposer, he gets \( xM \), and the Seller gets \((1-x)M\). Price is \$150,000+(1-x)M\.
• If the Seller is the first proposer, she gets \( yM \) and the Buyer gets \((1-y)M\). Price is \$150,000+yM\.
• In our example, the Buyer was the first proposer: \( x = \frac{1}{1+0.8} = \frac{1}{1.8} = 0.556M \). The Seller gets \((1-x)M=(1-0.556)M=0.444M\). Since \( M=10,000 \), the price of the house is \$154,440. \((150,000+0.444*10,000)\).
Differing Discount Factors

• Suppose the two players have different discount factors, for example the buyer’s discount factor $\delta_b$ is less than the seller’s discount factor $\delta_s$.
  • Buyer is less patient than the seller. Who gets more in this case?
• When Buyer is the first mover, he now offers $(1-x)M= \delta_s yM$ to the Seller, and when Seller is the first mover she offers $(1-y)M= \delta_b xM$ to the Buyer.
  • $x=1-\delta_s y$ and $y=1-\delta_b x$. $x^{\text{new}}=(1-\delta)/(1-\delta_s \delta_b)$.
  • It is easy to show that $x^{\text{new}} < (1-\delta)/(1-\delta^2) = x$ when both buyer and seller had the same discount factors.
• Example: $\delta_b = .5$, $\delta_s = .8$; $x=(1-\delta_s)/(1-\delta_s \delta_b) = .2/.6 = .333$. Recall that when $\delta_b = \delta_s = \delta = .8$ that $x = (1-\delta)/(1- \delta^2) = .2/.36 = .556$. 
Practical Lessons

• In reality, bargainers do not know one another’s discount factors, $\delta$, (or their relative levels of patience), but may try to guess these values.

• Signal that you are patient, even if you are not. For example, do not respond with counteroffers right away. Act unconcerned that time is passing—have a “poker face.”

• Remember that our bargaining model indicates that the more patient player gets the higher fraction of the amount M that is on the table.