What is a Game?

- There are many types of games, board games, card games, video games, field games (e.g. football), etc.

- We focus on games where:
  - There are 2 or more *players*.
  - There is some choice of action where *strategy* matters.
  - The game has one or more *outcomes*, e.g., someone wins, someone loses.
  - The outcome depends on the strategies chosen by all players; there is *strategic interaction*.

- What does this rule out?
  - Games of pure chance, e.g., bingo, slot machines. (Strategies don't matter).
  - Games without strategic interaction between players, e.g., Solitaire.
Why Do Economists Study Games?

- Games are a convenient way in which to model strategic interactions among economic agents.
- Many economic issues involve strategic interaction.
  - Behavior in imperfectly competitive markets, e.g. Coca-Cola and Pepsi.
  - Behavior in auctions, e.g. when to bid in dynamic internet auctions.
  - Behavior in economic negotiations, e.g. trade.
- Game theory is not limited to Economics.
Three Elements of a Game:

1. The *players*
   - how many players are there?
   - does nature/chance play a role?

2. A complete description of the *strategies* of each player

3. A description of the *consequences* (*payoffs*) for each player for every possible profile of strategy choices of all players.
Illustration: The Prisoner’s Dilemma Game

• Two players, “prisoners” 1, 2.
• Each is questioned by detectives separately and without communication.
• Each has two strategies.
  – Prisoner 1: Don't Confess, Confess
  – Prisoner 2: Don't Confess, Confess
  – Payoff consequences quantified in prison years (negative numbers): fewer years (less negative payoffs) = greater satisfaction.
  – In strategic/normal form game depiction, Prisoner 1 payoff first, followed by professor 2 payoff.
Prisoners’ Dilemma Game in “Normal” or “Strategic” Form

<table>
<thead>
<tr>
<th>Prisoner 1’s Strategies</th>
<th>Don't Confess</th>
<th>Confess</th>
</tr>
</thead>
<tbody>
<tr>
<td>Don't Confess</td>
<td>-1, -1</td>
<td>-15, 0</td>
</tr>
<tr>
<td>Confess</td>
<td>0, -15</td>
<td>-5, -5</td>
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</tbody>
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Dilbert Illustration

http://www.gametheory.net/media/Dilbert.wmv
Prisoners' Dilemma Game in “Extensive” Form

Payoffs are: Prisoner 1 payoff, Prisoner 2 payoff.
Prisoners' Dilemma is an example of a Non-Zero Sum Game

- A zero-sum game is one in which the players' interests are in direct conflict, e.g., in football, one team wins and the other loses. If a win = +1 and a loss = -1, the sum is zero.
  - More generally, the sum at stake can be a constant.
- A game is non-zero/non-constant sum, if players' interests are not always in direct conflict, so that there are opportunities for both to gain.
- For example, when both players choose Don't Confess in Prisoners' Dilemma.
The Prisoners' Dilemma is applicable to many other situations.

• Nuclear arms races.
• Settle a divorce amicably or hire a lawyer?
• Immediately merge left when signs indicate the right lane ahead is closed or continue driving in right lane up to the merge point?
• Full price menu or value price menu?
• Can you think of other applications?
Labels or “Framing”, Non-Binding, Pregame Communication or “Cheap Talk”: should/does it matter in the PD?

http://www.youtube.com/watch?v=p3Uos2fzIJ0

<table>
<thead>
<tr>
<th></th>
<th>Split</th>
<th>Steal</th>
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<tr>
<td>Split</td>
<td>£50,075, £50,075</td>
<td>£0, £100,150</td>
</tr>
<tr>
<td>Steal</td>
<td>£100,150, £0</td>
<td>£0, £0</td>
</tr>
</tbody>
</table>
Simultaneous v. Sequential Move Games

• Games where players choose actions simultaneously are simultaneous move games.
  – Examples: Prisoners' Dilemma, Sealed-Bid Auctions.
  – Must anticipate what your opponent will do right now, recognizing that your opponent is doing the same.

• Games where players choose actions in a particular sequence are sequential move games.
  – Must look ahead in order to know what action to choose now.
  – Many sequential move games have deadlines/ time limits on moves.

• Many strategic situations involve both sequential and simultaneous moves.
The Investment Game is a Sequential Move Game

The investment game is depicted in “extensive” form
One-Shot versus Repeated Games

• One-shot: play of the game occurs once.
  – Players likely to not know much about one another.
  – Example - tipping on your vacation

• Repeated: play of the game is repeated with the same players.
  – Indefinitely versus finitely repeated games
  – Reputational concerns matter; opportunities for cooperative behavior may arise.

• Advise: If you plan to pursue an aggressive strategy, ask yourself whether you are in a one-shot or in a repeated game. If a repeated game, think again.
Strategies

- A *strategy* must be a “comprehensive plan of action”, a decision rule or set of instructions about which actions a player should take.

- It is the equivalent of a memo, left behind when you go on vacation, that specifies the actions you want taken in every situation which could conceivably arise during your absence.

- Strategies will depend on whether the game is one-shot or repeated.

- **Examples of one-shot strategies**
  - *Prisoners' Dilemma*: Don't Confess, Confess
  - *Investment Game*:
    - Sender: Don't Send, Send
    - Receiver: Keep, Return

- How do strategies change when the game is repeated?
Repeated Game Strategies

• In repeated games, the sequential nature of the relationship allows for the adoption of strategies that are contingent on the actions chosen in previous plays of the game.

• Most contingent strategies are of the type known as "trigger" strategies.

• Example trigger strategies
  – In prisoners' dilemma: Initially play Don't confess. If your opponent plays Confess, then play Confess in the next round. If your opponent plays Don't confess, then play Don't confess in the next round. This is known as the "tit for tat" strategy.
  – In the investment game, if you are the sender: Initially play Send. Play Send as long as the receiver plays Return. If the receiver plays Keep, never play Send again. This is known as the "grim trigger" strategy.
Information

• Players have *perfect information* if they know exactly what has happened every time a decision needs to be made, e.g. in Chess.

• Otherwise, the game is one of *imperfect information*

  – Example: In the repeated investment game, the sender and receiver might be differentially informed about the investment outcome. For example, the receiver may know that the amount invested is always tripled, but the sender may not be aware of this fact.
Assumptions Game Theorists Make

✓ Payoffs are known and fixed. People treat expected payoffs the same as certain payoffs (they are risk neutral).
  – Example: a risk neutral person is indifferent between $25 for certain or a 25% chance of earning $100 and a 75% chance of earning 0.
  – We can relax this assumption to capture risk averse behavior.

✓ All players behave rationally.
  – They understand and seek to maximize their own payoffs.
  – They are flawless in calculating which actions will maximize their payoffs.

✓ The rules of the game are common knowledge:
  – Each player knows the set of players, strategies and payoffs from all possible combinations of strategies: call this information “X.”
  – *Common knowledge* means that each player knows that all players know X, that all players know that all players know X, that all players know that all players know that all players know X and so on,..., *ad infinitum.*
Common Knowledge?

Common knowledge can be a strong assumption, but is necessary for analysis of a game. While it rules out completely surprising actions/outcomes from a strategic setting (e.g., the events of 9/11/01), most settings can be reformulated as ones where all possible outcomes have some probability of occurrence, however small.

Examples of statements that may be regarded as common knowledge: Everyone knows that everyone knows that: 1) there are four seasons, 2) there are seven days in a week; 3) right from left (e.g. what side of the road to drive on);

Examples of things that are not common knowledge: 1) The capital of Botswana is Gaborone. 2) Henry the VIII’s fifth wife was Catherine Howard (beheaded after two years of marriage); 3) The average annual catch of fish by Philippine fishermen exceeds 2 million metric tons.
Equilibrium

- The interaction of all (rational) players' strategies results in an outcome that we call "equilibrium."
- In equilibrium, each player is playing the strategy that is a "best response" to the strategies of the other players, that is the strategy that yields the highest payoff given the strategies of the other players.
- Thus, *in equilibrium, no one has an incentive to change his strategy given the strategy choices of all others.*
- Equilibrium is not:
  - The best possible outcome. Equilibrium in the one-shot prisoners' dilemma is for both players to confess.
  - A situation where players always choose the same action. Sometimes equilibrium will involve changing action choices (known as a *mixed strategy* equilibrium).