You can earn up to 50 points on this exam. You have 2 hours to complete this exam. Explain everything that needs explaining. Good Luck!

1. Two identical firms each simultaneously choose a non-negative quantity of output. There are no costs. The payoff to firm $i$ as a function of the outputs is $\Pi_i(q_i, q_{-i}) = (100 - q_i - q_{-i})q_i$. Show that for each firm, any level of output greater than 50 is strictly dominated. (5 points)

2. On the hit TV show Survivor Thailand two tribes competed in a game they called Thai 21. The game starts with 21 flags in a circle. The teams take turns removing flags from the circle until they are all gone. The team that removes the last flag wins immunity. At each turn teams must remove 1, 2 or 3 flags. Team 1 moves first. How many flags should Team 1 take on its first move? (5 points)

3. Consider the two-player Bayesian game in which $S_1 = \{T, B\}$ and $S_2 = \{L, R\}$, each player has two types $\Theta_1 = \Theta_2 = \{0, 1\}$, and each type profile is equally likely, i.e., $p(0,0) = p(0,1) = p(1,0) = p(1,1) = \frac{1}{4}$. Payoffs are given below:

\[
\begin{array}{c|cc}
 & L & R \\
\hline
T_1 = 0 & T & 0, 0 & 1, 2 \\
 & B & 2, 1 & 0, 0 \\
T_1 = 1 & T & 2, 0 & 0, 2 \\
 & B & 0, 1 & 1, 0 \\
\end{array}
\]

The type profile determines which of the four matrices maps actions to payoffs. If, for example, $t_1 = 0$ and $t_2 = 1$, then the upper right matrix shows how player payoffs depend on actions. In that case, if player 1 chooses $T$ and player 2 chooses $R$ then their payoffs are 1 and 0, respectively.

a. Is $(s_1(0), s_1(1)) = (T, B)$ part of a Bayesian equilibrium? (4 points)

b. Is $(s_2(0), s_2(1)) = (L, L)$ part of a Bayesian equilibrium? (4 points)
4. Find all the pure-strategy perfect Bayesian equilibria of the following game in extensive form. **(5 points)**

![Extensive Form Game Diagram]

5. Once upon a time a signaling game was played between a princesses and a frog. The frog was the “Sender.” He could either say he was a ‘prince’ or a ‘frog.’ The princess was the receiver. She could either kiss the frog, in which case he might turn into a prince. Or, she could eat him. It was well known that 10% of the frogs in the kingdom would turn into princes when kissed by a princess. For reasons that arise from commonly known facts about frogs and princesses, the payoffs of the signaling game are as presented below.

![Signaling Game Diagram]

a. Find all the pure-strategy perfect Bayesian equilibria. Are there any separating equilibria? Are there any pooling equilibria? **(12 points)**

b. Do any of the pooling equilibria fail the (weak) intuitive criterion? **(4 points)**
6. An agent has effort levels $a_1 = 1$ and $a_2 = 3$. The probability distribution over sales for each level of effort is given below.

<table>
<thead>
<tr>
<th>Effort</th>
<th>$s_1 = $100$</th>
<th>$s_2 = $150$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1 = 1$</td>
<td>.8</td>
<td>.2</td>
</tr>
<tr>
<td>$a_2 = 3$</td>
<td>.4</td>
<td>.6</td>
</tr>
</tbody>
</table>

The expected profit for the principal when the agent takes action $a_i$ is expected sales conditional on $a_i$ minus expected costs conditional on $a_i$. The agent has utility over wages and actions given by $U(w, a_i) = w^{1/2} - a_i$ and a reservation utility of 1. Contracts can be written in terms of sales, but not actions.

a. Find the minimum cost of implementing each action. (**8 points**)  

b. Which action does the principal implement to maximize profit? (**3 points**)