Lecture 5: Understanding Interest Rates
Measuring Interest Rates

• Present Value:
  • A dollar paid to you one year from now is less valuable than a dollar paid to you today

• Why?
  – A dollar deposited today can earn interest and become $1 \times (1+i)$ one year from today.
Discounting the Future

Let $i = .10$

In one year  $100 \times (1 + 0.10) = 110$
In two years  $110 \times (1 + 0.10) = 121$
   or $100 \times (1 + 0.10)^2$
In three years  $121 \times (1 + 0.10) = 133$
   or $100 \times (1 + 0.10)^3$

In $n$ years  
$100 \times (1 + i)^n$
Simple Present Value

$PV = \text{today's (present) value}$

$CF = \text{future cash flow (payment)}$

$i = \text{the interest rate}$

$PV = \frac{CF}{(1 + i)^n}$
Time Line

• Cannot directly compare payments scheduled in different points in the time line

Year 0 1 2 n
PV $100 100/(1+i) 100/(1+i)^2 100/(1+i)^n
The NHL's Biggest Contracts
Wayne Gretzky | 1979: 21 years, $21 million ($1m per)

The Great One's first big contract, signed on his 18th birthday — after he was sold by the WHA's Indianapolis Racers — was a personal services deal with Oilers owner Peter Pocklington designed to keep Gretzky out of a draft pool when the WHA collapsed and some of its teams were absorbed by the NHL. The pact made Gretzky a great bargain during his record-shattering years in Edmonton. After Gretzky was traded to the Los Angeles in 1988, Kings owner Bruce McNall gave him a new three-year deal worth a reported $25.5 million.

Photo: Manny Millan/SI
Market yield on 1-year U.S. Treasury securities in 1979

0.1065

Contract: 21 years, $1 million per year

<table>
<thead>
<tr>
<th>x</th>
<th>PV of year x payment</th>
<th>PV of contract</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.9038</td>
<td>8.2685 SUM(C7:C27)</td>
</tr>
<tr>
<td>2</td>
<td>0.8168</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.7382</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.6671</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.6029</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.5449</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>0.4924</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>0.4450</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>0.4022</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>0.3635 1/(1+.1065)^10</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>0.3285</td>
<td></td>
</tr>
<tr>
<td>12</td>
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<td>0.2191</td>
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<td>16</td>
<td>0.1981</td>
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<td>17</td>
<td>0.1790</td>
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<tr>
<td>18</td>
<td>0.1618</td>
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<td>19</td>
<td>0.1462</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>0.1321</td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>0.1194</td>
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</table>
Four Types of Credit Market Instruments

- Simple Loan
- Fixed Payment Loan
- Coupon Bond
- Discount Bond
Yield to Maturity

- The interest rate that equates the present value of cash flow payments received from a debt instrument with its value today
**Simple Loan**

$PV = \text{amount borrowed} = $100$

$CF = \text{cash flow in one year} = $110$

$n = \text{number of years} = 1$

\[ $100 = \frac{110}{(1 + i)^1} \]

\[ (1 + i) \times $100 = $110 \]

\[ (1 + i) = \frac{110}{100} \]

\[ i = 0.10 = 10\% \]

For simple loans, the simple interest rate equals the yield to maturity.
Fixed Payment Loan

The same cash flow payment every period throughout the life of the loan

LV = loan value

FP = fixed yearly payment

n = number of years until maturity

\[ LV = \frac{FP}{1 + i} + \frac{FP}{(1 + i)^2} + \frac{FP}{(1 + i)^3} + \ldots + \frac{FP}{(1 + i)^n} \]
Using the same strategy used for the fixed-payment loan:

\[
P = \text{price of coupon bond}
\]

\[
C = \text{yearly coupon payment}
\]

\[
F = \text{face value of the bond}
\]

\[
n = \text{years to maturity date}
\]

\[
P = \frac{C}{1+i} + \frac{C}{(1+i)^2} + \frac{C}{(1+i)^3} + \ldots + \frac{C}{(1+i)^n} + \frac{F}{(1+i)^n}
\]
Table 1  Yields to Maturity on a 10%-Coupon-Rate Bond Maturing in Ten Years (Face Value = $1,000)

<table>
<thead>
<tr>
<th>Price of Bond ($)</th>
<th>Yield to Maturity (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,200</td>
<td>7.13</td>
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<tr>
<td>1,100</td>
<td>8.48</td>
</tr>
<tr>
<td>1,000</td>
<td>10.00</td>
</tr>
<tr>
<td>900</td>
<td>11.75</td>
</tr>
<tr>
<td>800</td>
<td>13.81</td>
</tr>
</tbody>
</table>

- When the coupon bond is priced at its face value, the yield to maturity equals the coupon rate.
- The price of a coupon bond and the yield to maturity are negatively related.
- The yield to maturity is greater than the coupon rate when the bond price is below its face value.
Consol or Perpetuity

- A bond with no maturity date that does not repay principal but pays fixed coupon payments forever

\[ P = \frac{C}{i_c} \]

- \( P_c \) = price of the consol
- \( C \) = yearly interest payment
- \( i_c \) = yield to maturity of the consol

Can rewrite above equation as this: \( i_c = \frac{C}{P_c} \)

For coupon bonds, this equation gives the current yield, an easy to calculate approximation to the yield to maturity.
Discount Bond

For any one year discount bond

\[ i = \frac{F - P}{P} \]

F = Face value of the discount bond
P = current price of the discount bond

The yield to maturity equals the increase in price over the year divided by the initial price. As with a coupon bond, the yield to maturity is negatively related to the current bond price.
The Distinction Between Interest Rates and Returns

- Rate of Return:

  The payments to the owner plus the change in value expressed as a fraction of the purchase price
  
  \[
  \text{RET} = \frac{C}{P_t} + \frac{P_{t+1} - P_t}{P_t}
  \]
  
  \( \text{RET} \) = return from holding the bond from time \( t \) to time \( t + 1 \)
  
  \( P_t \) = price of bond at time \( t \)
  
  \( P_{t+1} \) = price of the bond at time \( t + 1 \)
  
  \( C \) = coupon payment
  
  \( \frac{C}{P_t} \) = current yield \( = i_c \)
  
  \( \frac{P_{t+1} - P_t}{P_t} \) = rate of capital gain \( = g \)
The Distinction Between Interest Rates and Returns (cont’d)

• The return equals the yield to maturity only if the holding period equals the time to maturity

• A rise in interest rates is associated with a fall in bond prices, resulting in a capital loss if time to maturity is longer than the holding period

• The more distant a bond’s maturity, the greater the size of the percentage price change associated with an interest-rate change
The Distinction Between Interest Rates and Returns (cont’d)

- The more distant a bond’s maturity, the lower the rate of return occurs as a result of an increase in the interest rate.
- Even if a bond has a substantial initial interest rate, its return can be negative if interest rates rise.
Table 2 One-Year Returns on Different-Maturity 10%-Coupon-Rate Bonds When Interest Rates Rise from 10% to 20%

<table>
<thead>
<tr>
<th>(1) Years to Maturity When Bond Is Purchased</th>
<th>(2) Initial Current Yield (%)</th>
<th>(3) Initial Price ($)</th>
<th>(4) Price Next Year* ($)</th>
<th>(5) Rate of Capital Gain (%)</th>
<th>(6) Rate of Return (2 + 5) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>10</td>
<td>1,000</td>
<td>503</td>
<td>-49.7</td>
<td>-39.7</td>
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<tr>
<td>20</td>
<td>10</td>
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<td>516</td>
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<td>5</td>
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<td>741</td>
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<td>2</td>
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<td>917</td>
<td>-8.3</td>
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<tr>
<td>1</td>
<td>10</td>
<td>1,000</td>
<td>1,000</td>
<td>0.0</td>
<td>+10.0</td>
</tr>
</tbody>
</table>

*Calculated with a financial calculator using Equation 3.
Interest-Rate Risk

• Prices and returns for long-term bonds are more volatile than those for shorter-term bonds.

• There is no interest-rate risk for any bond whose time to maturity matches the holding period.
The Distinction Between Real and Nominal Interest Rates

• **Nominal interest rate** makes no allowance for inflation

• **Real interest rate** is adjusted for changes in price level so it more accurately reflects the cost of borrowing

• Ex ante real interest rate is adjusted for expected changes in the price level

• Ex post real interest rate is adjusted for actual changes in the price level
Fisher Equation

\[ i = i_r + \pi^e \]

\(i\) = nominal interest rate

\(i_r\) = real interest rate

\(\pi^e\) = expected inflation rate

When the real interest rate is low, there are greater incentives to borrow and fewer incentives to lend. The real interest rate is a better indicator of the incentives to borrow and lend.
Figure 1 Real and Nominal Interest Rates (Three-Month Treasury Bill), 1953–2011