Identification and Inference for Econometric Models

Essays in Honor of Thomas Rothenberg

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CHAPTER 7

Identifying a Source of Financial Volatility
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ABSTRACT

Our primary goal is to develop and analyze a dynamic economic model that takes into account several sources of information-based trade – the markets for a stock and options on that stock. We study identification within the model, paying particular attention to assumptions about the latent trader arrival process. We also derive the stochastic properties of trade-by-trade decisions and prices. Finally, we aggregate trade-by-trade quantities and to show that data generated by the model is consistent with empirical benchmarks from exchange data.

1. INTRODUCTION

Much of Tom Rothenberg’s long and insightful career has focused on identification in econometrics. The theme is perhaps most evident in Rothenberg (1973), which has long been the standard for identification in simultaneous equation models. We analyze a market microstructure model, paying particular attention to issues of identification. (The term microstructure refers to study of asset markets at the highly disaggregated level corresponding to the arrival of individual traders.) Working from the asymmetric information model in Easley, O’Hara, and Srinivas (1998), we first detail the assumptions needed to identify the parameters. We then derive the stochastic properties of trades and squared price changes for each market and the dynamic pattern of trade across markets. Finally, we use the methods in Kelly and Steigerwald (2004) to construct aggregate trades and squared price changes and compare these to empirical benchmarks. Together, these results provide a theory-based link between asymmetric information, the behavior of market participants, and stochastic volatility.

In Section 2 we first present a model of informed trade in stock and options markets and the resultant likelihood function needed to estimate the parameters. Parameter identification requires specification of the frequency at which traders arrive. We show how misspecification of the arrival frequency imparts bias. In particular, we find that arrival frequency misspecification leads to downward bias of informed trade frequencies. Even with correct specification of the arrival frequency, the likelihood function is sensitive to aggregation and we pinpoint the difficulty. Empirical identification requires a further assumption, by which
trades are assigned to a quote. Estimates of the accuracy of the assignment rules typically find an error rate of 15 percent. We determine the bias that arises from such an error rate and again find that informed trade frequencies are biased downward.

In Section 3 we focus on the dynamic pattern of trade within and across markets. We derive (in Theorem 3.1) how frequently, in equilibrium, the informed trade in the options market. Our results rest those of Easley et al. (1998) who implicitly derive conditions under which the informed trade with constant frequency in the options market. We next derive the properties of trade-by-trade price changes. Because informed traders may choose to trade in the options market, option trades can convey information about the stock price (Black 1975; Back 1993; Biais and Hillion 1994). As a result, options are not redundant assets as assumed by the Black–Scholes pricing model (Black and Scholes 1973). We detail these linkages and, in Theorem 3.3, we show that the (conditional) variance of price changes in a market is bounded by the squared bid–ask spread for that market. As trade reveals information the bid–ask spread shrinks, thereby reducing the conditional variance. The evolution of the bid–ask spread leads to autocorrelation in the conditional variance, although not specifically of the form modeled in a GARCH process.

In Section 4, we aggregate the trade-by-trade quantities of Section 3 to study the behavior of trades and prices over calendar periods. Three empirical features of stock market data form natural benchmarks for testing the model. There is strong evidence of serial correlation in calendar period squared price changes and in the number of trades across calendar periods, and the serial correlation in the number of trades tends to be larger and to diminish more slowly than serial correlation in squared price changes (Andersen 1996; Harris 1987; Steigerwald 1997). We first show that both trades (or trading volume) and squared price changes are positively correlated. Because the conditional variance of trade-by-trade price changes shrinks as information is revealed through trading, while trade decisions are unaffected, the serial correlation in trades is larger and tends to diminish more slowly than does the serial correlation in squared price changes.

2. IDENTIFICATION IN A MICROSTRUCTURE MODEL WITH OPTIONS MARKETS

We consider a model with markets for a stock and for call and put options on the stock. We base our dual-market, sequential-trade, asymmetric information model on the market microstructure models of Easley and O'Hara (1992); Easley et al. (1998). Full details of the model and the derivations that follow are contained in Steigerwald and Vagnoni (2001).

Trade in the stock and options markets occurs over a sequence of trading days, indexed by \( m \). On trading day \( m \), the stock realizes some per share dollar value, given by the random variable \( V_m \in \{ v_{H}, v_{M}, v_{L} \} \), with \( v_{L} < v_{M} < v_{H} \). The stock takes the lower value, \( v_{L} \), with positive probability \( \delta \). Prior to the commencement of trading on day \( m \), informed traders \( X_m \), about the value of the stock \( S_m \in \{ x_{L}, x_{M}, x_{H} \} \). The informativeness of the stock. The uninformative sign of the value of the stock. Informed probability \( \theta > 0 \). Proportion of the universe of informed traders. The signal characterizes the uninformed traders as privy to the signal. At the money market makers and uninformed

The market makers set an ask either one share of stock or an option on the stock. Each option expires prior to the end of the trading signal. Consider the call option, \( v_{C} \), one share of the stock for a speciﬁc from the call option writer at the option, \( V_{C} \), max \( \{ V_m - v_{C} \} \).

As all traders are risk neutral, receive informative signals. For example one of three possibly on the stock with probability \( \epsilon_{RF} \) or buying \( \lambda \) put options with probability \( \epsilon_{PB} \). Upon receiving an informative signal that provides the largest net gain for liquidity reasons and not speciﬁc frequency in each market. For each issuer and proportion \( \epsilon_{PB} \) to the positive frequencies in each \( i \) the uninformed traders that never trade.

Traders randomly arrive to the market and for option, \( D_{i} \). The randon example, if trader \( i \) buys the stock \( \lambda \) call options on the bid, \( B_{C} \), that

\[ D_{i} = d_{i} \]

We deﬁne the sequence to publicly available information we specify the publicly available \( i+1 \) on \( m \) as \( Z_{i,m} \), with \( Z_{i,m} = 1 \).

The information set, \( Z_{i,m} \), is shrewdly market makers and uninformed.
accuracy of the assignment rules determine the bias that arises from
trade quantities of Section 3 to
end trading periods. Three empirical
benchmarks for testing the model
in calendar period squared price
measures on the serial is larger and to diminish more
changes (Andersen 1996; Harris
both trades or trading volume)
related. Because the conditional
as in news is revealed
affected, the serial correlation
ily than does the serial correlation
of trading on day \(m\), informed traders receive a randomly
determined signal, \(S_m\), about the value of the stock on \(m\). This signal
takes one of three values, \(S_m \in \{s_L, s_H, s_D\}\). The informative
signals, \(s_L\) and \(s_H\), reveal the true value of
the stock. The uninformative signal, \(s_D\),
provides no information regarding the
true value of the stock. Informed traders
learn the true value of the stock with
probability \(\theta > 0\). Proportion \(\alpha\) of the traders
receives the signal, characterizing
the universe of informed traders. The proportion of traders
that does not receive the signal characterizes the universe of
uninformed traders. Neither market
maker is privy to the signal. At the end of each
trading day, the signal is revealed
to the market makers and uninformed traders and, hence, all agree
on the value of a share of the stock.

The market makers set an ask and a bid, collectively termed the quotes, for
either one share of stock or an option contract that controls \(\lambda \geq 1\) shares of
the stock. Each option is of the European type – precluding the possibility of
exercise prior to the end of the trading day – and expires upon revelation
of the signal. Consider the call option, which provides the owner with the right to buy
one share of the stock for a specified strike price, \(K_{C_u}\), with \(K_{C_u} \in [v_{L_u}, v_{H_u}]\),
from the call option writer at the end of the trading day. The value of the call
option, \(V_{C_u}\), is \(max \{V_m - K_{C_u}, 0\}\).

As all traders are risk neutral, informed traders will trade only if they
receive an informative signal. For example, if \(S_m = s_L\), then an informed trader
implements one of three possible “bearish” strategies, selling short one share
of the stock with probability \(\epsilon_{IB}\), writing \(\lambda\) call options with probability \(\epsilon_{IBC}\),
or buying \(\lambda\) put options with probability \(\epsilon_{IPC}\). Conditional
on receiving an informative signal, the informed trader employs the strategy
that provides the largest net gain. Uninformed traders are assumed to trade
for liquidity reasons and not speculation. The uninformed trade with positive
frequency in each market. For example, proportion \(\epsilon_{UB}\) potentially sells
the stock short and proportion \(\epsilon_{UBC}\) potentially buys \(\lambda\) call options. The sum of the
positive frequencies in each market is \(\epsilon\), thus \(1 - \epsilon\) is the proportion of the
uninformed traders that never trade.

Traders randomly arrive to the markets one at a time, so we index them by
their order of arrival, \(i\). The \(i\)th trader arrives, observes the quotes, and makes
a trade decision, \(D_i\). The random variable, \(D_i\), takes on seven values. For
example, if trader \(i\) buys the stock at the ask, \(A_i\), then \(D_i = d_A\). If trader \(i\) writes
\(\lambda\) call options at the bid, \(B_i\), then \(D_i = d_B\). If trader \(i\) elects not to trade, then
\(D_i = d_N\). We define the sequence of trading decisions on \(m\) as \(\{D_i\}_{i=1}^n\). Given
all publicly available information prior to the commencement of trade on \(m\), \(Z_0\),
we specify the publicly available information set prior to the arrival of trader
\(i + 1\) on \(m\) as \(Z_i\), with \(Z_i = \{Z_0, D_k\}_{k=1}^i\).

The information set, \(Z_i\), is shared by the market makers and all traders. The
market makers (and uninformed traders) perform Bayesian updating, by which

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1 A trading day captures the interval over which asymmetric information due to a particular signal
persists in the markets and is not necessarily coincident with a calendar day.
they learn the signal received by the informed. After witnessing the \( i \)th trading decision, the market makers' beliefs regarding the signal that the informed traders received are

\[
P(S_m = s_L | Z_i) = x_i \quad \text{and} \quad P(S_m = s_R | Z_i) = y_i.
\]

Each trading decision – even if the decision is not to trade – conveys information about the signal received by the informed traders.

Quotes are determined by two equilibrium conditions. The first condition is that a market maker earns zero expected profit from each trade. From the zero expected profit condition it follows that the quotes are equal to the expected value of the asset conditional on the trade. The second condition is that the informed will trade the asset that offers the highest net gain. From the second condition it follows that the quotes are set so that an informed trader earns an equal net gain from each possible trade.

The microstructure model yields the likelihood of each trade decision \( D_i \) as a function of the parameters \( \Phi = (\alpha, \delta, \bar{e}, \theta) \), where \( \bar{e} \) is the vector of trade probabilities (for both the informed and uninformed) for each trade decision. As trader arrivals are independent, the likelihood for a sequence of \( n \) arrivals is

\[
L(\Phi | D_1, \ldots, D_n) = \prod_{i=1}^{n} P(D_i = d_i | \Phi).
\]

From the structure of the model, the probability of each trade decision is straightforward. For example, the probability of a trade at the ask in the stock market is

\[
P(D_i = d_A | \Phi) = \theta(1 - \delta)|\alpha \epsilon_{TA} + (1 - \alpha) \epsilon_{UA}| + \theta \delta(1 - \alpha) \epsilon_{UA} + (1 - \theta)(1 - \alpha) \epsilon_{UA}.
\]

If \( n = (n_A, n_B, \ldots, n_N) \) is the vector of trade counts that correspond to each trade decision, then the corresponding value of the likelihood function is

\[
L(\Phi) = \theta(1 - \delta) b_{TA}^{n_A} \cdot b_{BA}^{n_B} \cdot b_{CA}^{n_C} \cdot b_{BC}^{n_B} \cdot b_{AB}^{n_A} \cdot b_{BB}^{n_B} \cdot (1 - \alpha)(1 - \epsilon)^{n_A} + \theta \delta b_{TA}^{n_A} \cdot b_{BA}^{n_B} \cdot b_{CA}^{n_C} \cdot b_{BC}^{n_B} \cdot b_{AB}^{n_A} \cdot b_{BB}^{n_B} \cdot (1 - \alpha)(1 - \epsilon)^{n_A} + (1 - \theta)(1 - \alpha) \epsilon_{UA}^{n_A} \cdot b_{TA}^{n_A} \cdot b_{BA}^{n_B} \cdot b_{CA}^{n_C} \cdot b_{BC}^{n_B} \cdot b_{AB}^{n_A} \cdot b_{BB}^{n_B} \cdot (1 - \alpha)(1 - \epsilon)^{n_A} + \theta \delta b_{TA}^{n_A} \cdot b_{BA}^{n_B} \cdot b_{CA}^{n_C} \cdot b_{BC}^{n_B} \cdot b_{AB}^{n_A} \cdot b_{BB}^{n_B} \cdot (1 - \alpha)(1 - \epsilon)^{n_A} + (1 - \theta)(1 - \alpha) \epsilon_{UA}^{n_A} \cdot b_{TA}^{n_A} \cdot b_{BA}^{n_B} \cdot b_{CA}^{n_C} \cdot b_{BC}^{n_B} \cdot b_{AB}^{n_A} \cdot b_{BB}^{n_B} \cdot (1 - \alpha)(1 - \epsilon)^{n_A}
\]

where \( b_{ij} = \alpha \epsilon_{ij} + (1 - \alpha) \epsilon_{Uij} \) and \( p_{ij} = (1 - \alpha) \epsilon_{ij} \) with \( j \) indexing trade decisions.

Two assumptions are needed to construct the sequence of trade decisions that identify the parameters. The first assumption identifies the length of time that corresponds to a decision not to trade.\(^1\) As the no-trade decision is designed to isolate periods in which information is not present, the assumption is needed to identify \( \alpha \) and \( \epsilon \). We first investigate how misspecification of the no-trade interval affects estimation. Let \( \epsilon \) correspond to the true length of the interval and let \( \bar{\epsilon} \) correspond to the assumed length of the interval. Because all trades are observed, only \( n_N \) – the number of trades is bias.

Table 7.1. Impact of misspecification

<table>
<thead>
<tr>
<th>No-trade interval length</th>
<th>Impact</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 minute</td>
<td>.2</td>
</tr>
<tr>
<td>2 minutes</td>
<td>.6</td>
</tr>
<tr>
<td>3 minutes</td>
<td>1</td>
</tr>
<tr>
<td>4 minutes</td>
<td>1</td>
</tr>
<tr>
<td>5 minutes</td>
<td>1</td>
</tr>
<tr>
<td><strong>184. Trade misclassification</strong></td>
<td>1</td>
</tr>
</tbody>
</table>
are observed, only $n_N$ – the number of no-trade decisions, is affected by the misspecification. If $e > \hat{e}$, then the number of no trades is biased upward, while if $e < \hat{e}$ the number of no trades is biased downward (as a sequence of actual no-trade decisions are required to record an observed no trade). Given the structure of the likelihood function, it is not straightforward to analytically determine the bias on individual parameters. To measure the bias, we simulate data under $e$ and construct estimators under $\hat{e}$. We use the equal payoff condition (derived later), under which the uninformed trade frequency in each market is $\frac{2}{3}$ while the informed trade frequency in each market is $\frac{1}{3}$. For the population model we use parameter values that correspond to estimates in Easley, Kiefer, and O'Hara (1997); news arrives on half of the trading days ($\theta = .5$), bad news is slightly more prevalent than good news ($\delta = .6$), 20 percent of traders are informed ($\alpha = .2$), and the overall frequency of trade by uninformed traders is 80 percent ($\epsilon = .8$). The population model assumes a trader arrives every minute during a six-hour-trading day, for thirty trading days. The estimates are constructed under each of the alternative assumptions that a trader arrives every two, three, four, or five minutes.

As revealed in Panel A of Table 7.1, incorrectly specifying the no-trade interval underestimates the impact of informed traders ($\alpha$ is biased downward and $\epsilon$ is biased upward). The parameters governing behavior at the daily level, $\theta$ and $\delta$, are largely invariant to misspecification of the no-trade interval. For the case in which the specified no-trade interval is too long, the number of recorded no trades declines and days with and without news become more similar. To account for the greater relative frequency of trades on all days, $\epsilon$ must increase. To account for the infrequency of no-trade decisions on days without news, $\alpha$ must decline. If the specified no-trade interval is too short, the number of

<table>
<thead>
<tr>
<th>Table 7.1. Impact of misspecification on parameter estimates</th>
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<tbody>
<tr>
<td>Panel A</td>
</tr>
<tr>
<td>No-trade interval length</td>
</tr>
<tr>
<td>$\alpha$  $\epsilon$  $\theta$  $\delta$</td>
</tr>
<tr>
<td>-------------------------------------------------------------</td>
</tr>
<tr>
<td>1 minute $\alpha$  $\epsilon$  $\theta$  $\delta$</td>
</tr>
<tr>
<td>2 minutes $\alpha$  $\epsilon$  $\theta$  $\delta$</td>
</tr>
<tr>
<td>3 minutes $\alpha$  $\epsilon$  $\theta$  $\delta$</td>
</tr>
<tr>
<td>4 minutes $\alpha$  $\epsilon$  $\theta$  $\delta$</td>
</tr>
<tr>
<td>5 minutes $\alpha$  $\epsilon$  $\theta$  $\delta$</td>
</tr>
<tr>
<td>15% Trade misclassification $\alpha$  $\epsilon$  $\theta$  $\delta$</td>
</tr>
</tbody>
</table>

Panel B

| 15% Trade misclassification $\alpha$  $\epsilon$  $\theta$  $\delta$ |
recorded no trades increases and, again, days with news become more similar to days without news. Because the relative frequency of trades has declined on all days, $\varepsilon$ decreases. To account for the infrequency of trade decision on days with news, $\alpha$ declines. Incorrect specification of the no-trade interval, in either direction, biases the estimator of $\alpha$ downward and makes the presence of informed traders more difficult to detect.

Even if the no-trade interval is correctly specified, empirical identification may be problematic. The analysis of a related likelihood in Easley et al. (1997) is confined to a stock that is not heavily traded. For more heavily traded stocks, numerical difficulties prevent analysis. Rewriting the likelihood makes investigation of the numerical difficulties quite straightforward. Under the equal payoff condition, for which $p_{0j}$ equals $p_0 = (1 - \alpha) \frac{\alpha}{2} \alpha$ for all $j$, the likelihood is

$$
p_{0j}^{n-n_{\theta}} \left[1 - (1 - \alpha)(1 - \varepsilon)\right]^{n_{\theta}} \cdot \left[\frac{\theta}{3p_0} + 1\right]^{n_{\alpha}+n_{\alpha c}+n_{\alpha r}} \left[\frac{\alpha}{3p_0} + 1\right]^{n_{\alpha}+n_{\alpha c}+n_{\alpha r}} + \theta \left(1 - \theta\right) \left(\frac{\alpha}{3p_0} + 1\right)^{n_{\alpha}+n_{\alpha c}+n_{\alpha r}}.
$$

The issue concerns the three terms $\left(\frac{\alpha}{3p_0} + 1\right)^{n_{\alpha}+n_{\alpha c}+n_{\alpha r}}$, $\left(\frac{\alpha}{3p_0} + 1\right)^{n_{\alpha}+n_{\alpha c}+n_{\alpha r}}$, and $\left(\frac{\alpha}{3p_0} + 1\right)^{n_{\varepsilon}}$. For frequently traded stocks, the observed value of trade decisions is quite large. As all three terms are greater than one, these terms dominate the likelihood function when raised to a large power and render the likelihood numerically unstable. (The most common difficulty is simply overflow, the calculated value exceeds the largest number the computer is able to store.) Figures 7.1 and 7.2 reveal the issue. In Figure 7.1, a trader arrives every minute and with 360 trader arrivals in one day no numerical problems are encountered. In Figure 7.2, a trader arrives every twenty seconds, with 1,080 trader arrivals numerical difficulties are prevalent.1 Because the three terms are increasing functions of $\alpha$ and decreasing functions of $\varepsilon$, the likelihood function is correctly computed only for smaller values of $\alpha$ and larger values of $\varepsilon$. For the population values $\alpha = .2$ and $\varepsilon = .8$ the likelihood function cannot be evaluated with an arrival frequency of twenty seconds.

The second assumption regards the classification of trades. Within the model, all trades occur at a quote. In practice, many trades are recorded at prices between the quotes. To empirically identify the model, all trades must be assigned to a quote. While there are several assignment rules popular in the literature, each of the rules has an estimated error rate of 15 percent. To understand the impact of the misclassification of trades, we randomly misclassify 15 percent of trades. Panel B of Table 7.1 contains the results. Estimation of $\theta$ and $\delta$ is again largely unaffected. As misclassification of trades does not alter the relative frequency of trades, estimation of $\varepsilon$ is also unaffected. Yet random misclassification of trades does impact estimation of $\alpha$. On days with equally likely to affect trades at either news, for which there are more trades likely to affect trades at the ask quote, misclassification is more likely to affect the imbalance of trades (the number of days is reduced and the presence of info

3. INTRA-TRADING DAY DYN

The evolution of the quotes over the information revealed through trading. When the quotes have bounds that reflect market makers. We then study the frequency of the underlying parameters on this frequency the separating equilibrium derived in Easley trade only in the options market, will no

1 For ease of viewing, we set the numerically unstable values to an arbitrarily small value, to emphasize that the empirical likelihood is essentially flat.
ys with news become more similar. The frequency of trades has declined, the infrequency of trade decisions on the no-trade interval, in inward and makes the presence of

specified, empirical identification of likelihood in Easley et al. (1997) ed. For more heavily traded stocks, noting the likelihood makes investigation straightforward. Under the equal payoff $\alpha_j$, for all $j$, the likelihood is

$$\delta \left( \frac{\alpha}{\beta_0} + 1 \right)^{\alpha \beta_0 + \eta \beta_0 + \epsilon \beta},$$

$$1 - \delta \left( \frac{\alpha}{\epsilon} + 1 \right)^{\alpha \epsilon + \eta \epsilon + \epsilon \epsilon}.$$

The observed value of trade decider than one, these terms dominate the power and render the likelihood difficulty is simply overflow, the the computer is able to store.) 7.1, a trader arrives every minute numerical problems are encountered. Seconds, with 1.080 trader arrivals we the three terms are increasing of $\epsilon$, the likelihood function is more and larger values of $\epsilon$. For the the function cannot be evaluated

the distribution of trades. Within the model, trades are recorded at prices between 1, all trades must be assigned to a less popular in the literature, each percent. To understand the impact of misclassify 15 percent of trades,ation of $\beta$ and $\delta$ is again largely not tilt the relative frequency of random misclassification of trades

values to an arbitrarily small value, to 0).

3. INTRA-TRADING DAY DYNAMICS

The evolution of the quotes over the course of the trading day reflects the information revealed through trading. We show that at each point in the trading day the quotes have bounds that reflect the information asymmetry facing the market makers. We then study the frequency with which the informed trade in each market. We show that informed trade frequency in the options market generally declines over the course of the trading day and we derive the effect of the underlying parameters on this frequency. In doing so, we demonstrate that the separating equilibrium derived in Easley et al. (1998), in which the informed trade only in the options market, will not generally prevail over an entire trading
Identifying a Source of Finance

From these equations it is easy to see the respective limit values of the asset, maker is certain the informed learn the We also find that the quotes for the stock expected values of the assets, which market makers in an effort to offset ex information.

The quotes process is driven by \( x_t \) beliefs about the signal received by the according to Bayes' Rule and are determined trade frequencies. In general, init out the trading day. The dynamic behavior is intuitive: as private information is regained by the informed through trade in the analysis of variable informed trade historically relevant case in which options are symmetric, \( \lambda \left( v_{I,I} - v_{I,2} \right) = \lambda v_{I,0} \).

**Theorem 3.1.** If the options offer greater the informed trade frequencies then

(a) As \( \lambda \) increases, the informed 
and 
as \( x \) increases, the informed market.

(b) As learning evolves, the inform stock market. The rate of flow day. The rate of flow also decline.

(c) Informed trade frequencies in \( t \) if the uninform trades each as: \( \varepsilon_{\text{IRP}} > \varepsilon_{\text{IA}} \) and \( \varepsilon_{\text{IRC}} = \varepsilon_{\text{IAS}} \).

(d) The \( i \)th informed trade frequency for \( i = H, L \),

\[
\lambda < \frac{(v_{H,0} - v_{L,0})}{\beta} \left( 1 + \frac{x}{1 - x} \right)
\]

with \( \varepsilon_H = \varepsilon_{\text{IRC}} + \varepsilon_{\text{IRP}} = \varepsilon_{\text{IAS}} \).

\[b_{I,0} = x_{t-1} \text{.}
\]

**Proof.** See Appendix.

An increase in the proportion of inform of uninformed traders to informed traders makes the stock market more attractive to understand the dynamic pattern reve.
From these equations it is easy to see that each set of quotes is bounded by the respective limit values of the asset, with strict inequality unless the market maker is certain the informed learn the true value of \( V_{m} \) (no adverse selection). We also find that the quotes for the stock and the options bound the respective expected values of the assets, which illustrates the spread generated by the market makers in an effort to offset expected losses to traders with superior information.

The quotes process is driven by \( x_{t} \) and \( y_{t} \), which are the market makers' beliefs about the signal received by the informed traders. The beliefs evolve according to Bayes' Rule and are determined in large part by the equilibrium informed trade frequencies. In general, informed trade frequencies vary throughout the trading day. The dynamic behavior of the options market trade frequency is intuitive: as private information is revealed through trading, the advantage gained by the informed through trade in the options market declines. To make the analysis of variable informed trade frequencies concise, we focus on an empirically relevant case in which options offer leverage and the option payoffs are symmetric, \( (\lambda (\kappa_{m} - v_{1}) = \lambda (v_{H_{m}} - \kappa_{C_{m}}) = \lambda \beta) \).

**Theorem 3.1.** If the options offer greater leverage and have symmetric payoffs, then the informed trade frequencies behave in the following ways:

(a) As \( \lambda \) increases, the informed are less likely to trade in the stock market. As \( \alpha \) increases, the informed are more likely to trade in the stock market.

(b) As learning evolves, the informed flow from the options market to the stock market. The rate of flow declines over the course of a trading day. The rate of flow also declines as \( \alpha \) increases.

(c) Informed trade frequencies in the option market are always positive. If the uninformed trade each asset with equal frequency, then \( \epsilon_{1A_{1}} = \epsilon_{1B_{1}} > \epsilon_{1A_{1}} \), and \( \epsilon_{1B_{1}} = \epsilon_{1A_{1}} \), \( \epsilon_{1B_{1}} > \epsilon_{1R_{1}} \).

(d) The \( i \)th informed trade frequencies in the stock market are positive if, for \( j = H, L \),

\[
\alpha < \frac{(\epsilon_{H_{1}} - \epsilon_{L_{1}})}{\beta} \left( 1 + \frac{\alpha}{1 - \alpha} \epsilon_{j_{1}} \right),
\]

with \( \epsilon_{H} = \epsilon_{UA_{1}} + \epsilon_{UB_{1}}, \epsilon_{L} = \epsilon_{UA_{1}} + \epsilon_{UB_{1}}, \epsilon_{H_{1-1}} = \epsilon_{L_{1-1}}, \) and \( b_{H_{1-1}} = b_{L_{1-1}} \).

**Proof.** See Appendix.

An increase in the proportion of informed traders reduces the depth (the ratio of uninformed traders to informed traders) of the options market, which in turn makes the stock market more attractive to informed traders, as detailed in (a). To understand the dynamic pattern revealed in (b), consider a day on which
\[ S_m = s_H. \] As the informed trade and reveal their information, \( y_j \) increases. As \( y_j \) increases, the gains to trade on information shrink, as does the advantage from trading in the options market. Hence, over the course of a trading day the informed flow from the options market to the stock market. As the updating of \( y_j \) slows over the course of a trading day to reflect the reduced information content of trades, so too does the rate of flow of informed traders. In similar fashion, as \( \alpha \) increases, the information gain from each trader increases, so higher values of \( \alpha \) lead to faster learning and greater attenuation of the rate of flow of informed between markets over the course of a trading day. While the informed flow from the options market to the stock market over the course of a trading day, if the uninformed are equally likely to trade in each market then the informed trade frequency is higher in the options market uniformly over the trading day, as stated in (c).

Leverage attracts informed traders to the options market. If \( \lambda \) exceeds the separating bound in (d), then the frequency of informed trade in the stock market is zero and the equilibrium separates the markets in which the informed trade. As either \( \alpha \) decreases or \( \epsilon_j \) increases, the informed are able to hide more easily in the options market, so the separating bound in (d) decreases and informed trade is more likely to occur only in the options market. Because \( \lambda \) is fixed over the course of a trading day while \( h_j \) evolves with the trade flow, it will generally not be the case that a separating equilibrium exists in all periods.

The bid–ask spread reflects the dynamic pattern of informed trade frequencies. To illustrate the dynamic pattern of the spread, we simulate the arrival of traders over the course of 1,000 trading days on which \( S_m = s_H \). We set the information advantage of the informed at 5 percent of the initial value of the asset, so \( v_{H_m} = 105, v_{I_m} = 95 \), and \( \delta = .5 \). (We ensure that option payoffs are symmetric and set \( \kappa_{C_w} = v_{I_m} \) and \( \kappa_{P_w} = v_{H_m} \). The greater leverage afforded by options is then captured by \( \lambda > 1 \).) We further suppose that the uninformed are equally likely to trade each asset, so that the informed trade frequencies, and hence the spreads, are identical for the two options. Finally, we suppose that \( \alpha = .2 \) and \( \epsilon = .75 \). Noting that the essential features we report hold as \( \alpha \) and \( \epsilon \) vary over \([0, 1]\). In Figure 7.3 we present the average bid–ask spread over the course of a trading day. First, as \( \lambda \) increases the adverse selection problem in the options is exacerbated and forces the market maker to widen the bid–ask spreads for the call and put options, while the adverse selection problem in the stock is mitigated and allows the market maker to reduce the spread for the stock. As the trading day evolves the options spread declines more rapidly than the stock spread, reflecting the movement of informed traders into the stock market.

If the payoff from all three assets is equal, then the informed trade with constant frequency throughout the trading day. Because constant informed trade

---

\[ v_{H_m} = v_{I_m} = \lambda(v_{I_m} - v_{H_m}) \]

Equal Payoff Condition. The options have

\[ v_{H_m} = v_{I_m} = \lambda(v_{I_m} - v_{H_m}) \]

The constant informed trade frequencies greatly simplify analysis when gated into calendar periods, we make no

\[ \epsilon_{IA} = \frac{\epsilon_{IA}}{\epsilon_{UA} + \epsilon_{UAC} + \epsilon_{UBP}} \]

and

\[ \epsilon_{IBP} = \frac{\epsilon_{IBP}}{\epsilon_{UA} + \epsilon_{UAC} + \epsilon_{UBP}} \]

If the informed trade frequencies are recursive. (If the informed trade frequent to obtain a recursive structure.) With assumptions that if there were an infinite number of market makers would learn the signal, \( S_m \), converge to the strong-form efficient value and private information. As transaction these prices also converge to the respective assets.
identifying a Source of Financial Volatility

If $\lambda$ exceeds the trade in the stock which the informed traders are able to hide in (d) decreases the market. Because of the trade equilibrium exists in all frequencies greatly simplify analysis when trade-by-trade variables are aggregated into calendar periods, we make note of the condition.

Equal Payoff Condition. The options leverage and strike prices satisfy

$$v_H - v_L = \lambda(v_H - k_L) = \lambda(K - v_L).$$

The constant informed trade frequencies mirror the behavior of uninformed traders in that the informed and uninformed trade with identical relative frequency in each market

$$\epsilon_{IA} = \frac{\epsilon_{UA}}{\epsilon_{UA} + \epsilon_{UAC} + \epsilon_{UBP}},$$

$$\epsilon_{IAC} = \frac{\epsilon_{UAC}}{\epsilon_{UA} + \epsilon_{UAC} + \epsilon_{UBP}},$$

and

$$\epsilon_{IBP} = \frac{\epsilon_{UBP}}{\epsilon_{UA} + \epsilon_{UAC} + \epsilon_{UBP}}.$$
Theorem 3.2. If the equal payoff condition is satisfied, then the sequence of quotes and, hence, the sequence of transaction prices for each asset converge almost surely to the strong-form efficient value of that asset at an exponential rate. Specifically, the following results obtain as \( i \to \infty \):

If \( S_m = s_t \), then \( x_i \xrightarrow{a.s.} 1 \), \( y_i \xrightarrow{a.s.} 0 \), so \( A_i \xrightarrow{a.s.} v_{L_m} \), \( B_i \xrightarrow{a.s.} v_{L_m} \), \( C_i \xrightarrow{a.s.} v_{m} \).

If \( S_m = s_H \), then \( x_i \xrightarrow{a.s.} 0 \), \( y_i \xrightarrow{a.s.} 1 \), so \( A_i \xrightarrow{a.s.} v_{H_m} \), \( B_i \xrightarrow{a.s.} v_{H_m} \), \( C_i \xrightarrow{a.s.} v_{H_m} \) and \( A_P \xrightarrow{a.s.} v_{H_m} \), \( B_P \xrightarrow{a.s.} v_{H_m} \).

If \( S_m = s_{\mu} \), then \( x_i \xrightarrow{a.s.} 0 \), \( y_i \xrightarrow{a.s.} 0 \), so \( A_i \xrightarrow{a.s.} v_{H_m} \), \( B_i \xrightarrow{a.s.} v_{H_m} \), \( C_i \xrightarrow{a.s.} v_{H_m} \) and \( A_P \xrightarrow{a.s.} v_{H_m} \), \( B_P \xrightarrow{a.s.} v_{H_m} \), \( C_P \xrightarrow{a.s.} v_{H_m} \).

Proof. See Appendix.

Convergence of the beliefs \( (x_i)_{i \geq 0} \) and \( (y_i)_{i \geq 0} \) immediately implies that \( U_i \xrightarrow{a.s.} 0 \), so that individual trader price volatility converges to zero.

Careful analysis of individual trader price changes reveals three interesting features. First, option trades affect stock prices. Many standard option pricing models assume that the option price is derived from the stock price. Such models are misspecified when informed trade occurs in option markets. Second, price changes are predictable with respect to private information (in contrast to public information). Third, price changes are dependent and heterogeneous, and the conditional variance of each price change is bounded by the squared bid–ask spread.

Price changes reflect public information after the decision of trader \( i \) but before the arrival of trader \( i+1 \). The stock price change associated with a specific trade decision for trader \( i \) is \( U_i(D_i = d_i) = E(U_m|Z_{i-1}) - E(V_m|Z_{i-1}) \). Consider a trade at the ask in the stock. Because \( E(V_m|Z_i) = x_i v_{L_m} + y_i v_{H_m} + (1 - x_i - y_i) E V_m \), the stock price change is

\[
U_i(D_i = d_A) = |v_{H_m} - E(V_m|Z_{i-1})| \frac{\alpha e^{A_i Y_{i-1}}}{P(D_i = d_A|Z_{i-1})}.
\]

The price change reflects expected learning from the informed; if the market maker knows that the trader is uninformed, there is no learning from the trade and the price change is zero.

Because informed trade occurs in the options market, options are not redundant assets. If trader \( i \) elects to buy the call option contract, then

\[
U_i(D_i = d_AC) = |v_{H_m} - E(V_m|Z_{i-1})| \frac{\alpha e^{A_i Y_{i-1}}}{P(D_i = d_AC|Z_{i-1})}.
\]

Trade in an option affects the price of the stock.

Prices are predictable with respect to private information. Consider the stock price change expected by an informed trader with \( S_m = s_H \). The informed trader’s expectation differs from that of the market maker because the market maker is unsure of the signal. The stock trader is

\[
E(U_i|Z_{i-1}) + \alpha (1 + y_{i-1}) |e_{j} + \alpha x_{j-1} E V_m - v_{L_m}| > 0.
\]

A direct implication is that price change to private information. If \( S_m = s_H \), then informed trader is

\[
E(U_i|Z_{i-1}, S_m = s_H) = U_{i-H}
\]

Price changes are conditionally heteroskedastic.

\[
E(U_i|Z_{i-1}) = \sum_{j=A,B,AC,AP,BP} \lambda_{i-j} U_{i-j}
\]

As the conditional heteroskedasticity is \( p \) bounds. To do so, we use the effective maximum revision in price resulting from is simply the bid–ask spread. If, however and generally made by informed traders (small) then a decision not to trade can yield to trade. Hence, \( \hat{A}_i - \hat{B}_i = \max_{j=AC,AP,BP} \{ A_j, E(V) \} \)

\[
- \min_{j=AC,AP,BP} \{ B_j, E(V) \}
\]

The effective bid–ask spreads for the call \( \hat{A}_i \) and \( \hat{A}_{P_i} - \hat{B}_{P_i} \) are defined in the same way.

We find that price changes conditional and not identically distributed, although the related. An asset’s bid–ask spread drives \( \hat{A}_i \) changes, introducing autoregressive heterocorrelation.

Theorem 3.3. Price changes in economic and serially uncorrelated with respect to the

\[
E(U_i^{j^2}|Z_{i-1}) \leq \hat{A}_i - \hat{B}_i \quad \alpha
\]

for \( j = C, P \).

Proof. See Appendix.

The fact that the price change variance is an important component of the and Steigerwald (2004) in the context of.
the sequence of asset convergence at an exponential
rate 

\[ v_{Lm}, \Lambda_C \overset{as}{\rightarrow} 0, \]
\[ v_{Hm}, \Lambda_C \overset{qv}{\rightarrow} 0, \]
\[ EV_m, \Lambda_C \overset{as}{\rightarrow} 0. \]

implies that \( U_t \overset{as}{\rightarrow} \) three interesting

option pricing models. Second, price contrast to public

generous, and the squared bid-ask

1 of trader \( i \) but

associated with a

\( \epsilon^{-1}, D_t = d_j \) —

Use \( E(V_m[Z_t] =

\frac{1}{d_j}) \).

d; if the market

\( \left. \right|_{Z_{t-1}} \).

consider the stock

. The informed

use the market

maker is unsure of the signal. The stock price change expected by an informed

trader is

\[
E(U_t|Z_{t-1}) + \alpha(1 + y_{t-1})[v_{Hm} - E(V_m|Z_{t-1})]
+ \alpha x_{t-1}[EV_m - v_{Lm}] > 0.
\]

A direct implication is that price changes are serially correlated with respect
to private information. If \( S_m = s_H \), then the serial correlation expected by an
informed trader is

\[
E(U_h U_t|Z_{t-1}, S_m = s_H) = U_h E(U_t|Z_{t-1}, S_m = s_H) \neq 0.
\]

Price changes are conditionally heteroskedastic with

\[
E(U_t^2|Z_{t-1}) = \sum_{j=\{A, B, AC, BC, AP, BP, N\}} P(D_t = d_j|Z_{t-1})U_j^2(D_t = d_j).
\]

As the conditional heteroskedasticity is path dependent, we construct analytic
bounds. To do so, we use the effective bid-ask spread, \( \tilde{A}_t - \tilde{B}_t \), which is the
maximum revision in price resulting from a trade. In almost all cases, \( \tilde{A}_t - \tilde{B}_t \)
is simply the bid-ask spread. If, however, a decision not to trade is quite rare
and generally made by informed traders (when \( \epsilon \) is very large and \( \alpha \) is very small) then a decision not to trade can yield a larger price change than a decision
to trade. Hence,

\[
\tilde{A}_t - \tilde{B}_t = \max_{j \in \{AC, AP, N\}} \left[ A_j, E(V_m|Z_{t-1}, D_t = d_j) \right]
- \min_{j \in \{BC, AP, N\}} \left[ B_j, E(V_m|Z_{t-1}, D_t = d_j) \right].
\]

(The effective bid-ask spreads for the call option and the put option, \( \tilde{A}_C - \tilde{B}_C \)
and \( \tilde{A}_P - \tilde{B}_P \), are defined in the same way.)

We find that price changes conditional on public information are dependent
and not identically distributed, although they are mean zero and serially uncorrelated. An asset’s bid-ask spread drives the conditional variance of its price
changes, introducing autoregressive heteroskedasticity.

**Theorem 3.3.** Price changes in economic time for each asset are mean zero
and serially uncorrelated with respect to the public information set. In addition

\[
E(U_j^2|Z_{t-1}) \leq (\tilde{A}_j - \tilde{B}_j)^2, \quad \text{and} \quad E(U_j^2|Z_{t-1}) \leq (\tilde{A}_j - \tilde{B}_j)^2
\]

for \( j = C, P \).

**Proof.** See Appendix. 

The fact that the price change variance is bounded by the effective bid-ask
spread is an important component of the model. (This was shown in Kelly
and Steigerwald (2004) in the context of a single asset market.) Because the
Table 7.2. \( E_H(U_t^2 | Z_{t-1}) - E_0(U_t^2 | Z_{t-1}) \)

<table>
<thead>
<tr>
<th>Trader</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \epsilon = 0.9 )</td>
<td>14.40</td>
<td>1.12</td>
<td>1.80</td>
<td>0.25</td>
<td>0.23</td>
<td>0.05</td>
<td>0.04</td>
<td>0.01</td>
<td>0.01</td>
<td>0.00</td>
</tr>
<tr>
<td>( \epsilon = 0.8 )</td>
<td>15.06</td>
<td>1.21</td>
<td>1.81</td>
<td>0.20</td>
<td>0.20</td>
<td>0.04</td>
<td>0.03</td>
<td>0.01</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>( \epsilon = 0.7 )</td>
<td>15.77</td>
<td>1.24</td>
<td>1.78</td>
<td>0.21</td>
<td>0.18</td>
<td>0.04</td>
<td>0.02</td>
<td>0.01</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>( \epsilon = 0.6 )</td>
<td>16.53</td>
<td>1.22</td>
<td>1.65</td>
<td>0.23</td>
<td>0.15</td>
<td>0.04</td>
<td>0.02</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>( \epsilon = 0.5 )</td>
<td>17.34</td>
<td>1.17</td>
<td>1.43</td>
<td>0.24</td>
<td>0.10</td>
<td>0.03</td>
<td>0.01</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>( \epsilon = 0.4 )</td>
<td>18.23</td>
<td>1.11</td>
<td>1.15</td>
<td>0.27</td>
<td>0.06</td>
<td>0.04</td>
<td>0.01</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>( \epsilon = 0.3 )</td>
<td>19.17</td>
<td>1.10</td>
<td>0.82</td>
<td>0.30</td>
<td>0.05</td>
<td>0.03</td>
<td>0.01</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>( \epsilon = 0.2 )</td>
<td>20.19</td>
<td>1.20</td>
<td>0.52</td>
<td>0.31</td>
<td>0.04</td>
<td>0.01</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>( \epsilon = 0.1 )</td>
<td>21.30</td>
<td>1.52</td>
<td>0.29</td>
<td>0.22</td>
<td>0.05</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

The price uncertainty associated with informed trading widens the effective bid-ask spread. Theorem 3.3 suggests that price change behavior is systematically different on days for which the signal is informative.

To show that the price uncertainty is greater on days with an informative signal, we examine the market maker’s price uncertainty on a trading day with \( S_m = S_H \), \( E_H(U_t^2 | Z_{t-1}) \), relative to the price uncertainty on a trading day with \( S_m = S_0 \), \( E_0(U_t^2 | Z_{t-1}) \). Straightforward calculations reveal that for the first trader \( E_H(U_t^2 | Z_{t-1}) \) is larger than \( E_0(U_t^2 | Z_{t-1}) \). To determine the sign of \( E_H(U_t^2 | Z_{t-1}) - E_0(U_t^2 | Z_{t-1}) \) for \( i > 1 \), we study the behavior of \( U_t^2 \). If \( \alpha \) is large, then learning is rapid and largely occurs with the first ten traders. For illustration, in Table 7.2 we calculate \( E_H(U_t^2 | Z_{t-1}) - E_0(U_t^2 | Z_{t-1}) \) for \( \alpha = .9 \), from the exact distributions for \( U_t^2 \). We first note that as traders arrive to the market, the market maker learns and the relative price uncertainty decreases. The speed of learning increases as the proportion of uninformed traders who trade, \( \epsilon \), decreases. Most importantly, the price uncertainty during a day with an informative signal is always as large as the price uncertainty during a day with an uninformed signal.

For smaller values of \( \alpha \), learning is slowed and reduction of an asset’s bid-ask spread to zero requires many more trader arrivals. For trader \( i \), there are \( 2^i \) possible values for \( U_i \), so calculation of the distribution of \( U_t^2 \) is cumbersome for large \( i \). In Figure 7.4 we approximate \( E_H(U_t^2 | Z_{t-1}) - E_0(U_t^2 | Z_{t-1}) \) for \( \alpha = .2 \), with 1,000 simulations. We confirm the results of Table 7.2. Again, learning is more rapid if the uninformed trade with less frequency. Also, we again find that the variance of \( U_t \) is higher, uniformly, on a day with an informative signal than it is on a day with an uninformative signal.

4. CALENDAR PERIOD IMPLICATIONS

Aggregation of trader arrivals into calendar periods allows us to compare the model with three empirical benchmarks. For constant informed trade frequencies, we prove that the number of trades has positive serial correlation in each market in accord with the first benchmark serial correlation for different levels of \( \alpha \) and serial correlation is higher for data gathered at hourly intervals. For variable the formula for trade correlation and a to be positive. We then demonstrate that serially correlated, in accord with the set the model is able to satisfy the third bend in trades that is larger and diminishes not squared price changes.

To determine the serial correlation for thirty-minute intervals, we divide each trade day index, calendar periods. To understand that \( t = 1, \ldots, n \) in which \( t = 1 \) corresponds to the trading day. The sample would then consist of \( n \) consecutive trading days. E arrivals (each trader arrival can be thought of as a given trading day, we have \( r = k \) trade).

We first derive the serial correlation in the assumption of constant informed trade are positively correlated. We also find that market is less than the correlation in total markets is not a scale transformation. Our the result reported in Kelley and Steigerwa is analyzed and so informed trade frequency serial correlation directly links the param

\[ We assume that the equal payoff condition is satisfied, with \( \lambda = 1 \). \]
Identifying a Source of Financial Volatility

market in accord with the first benchmark. We are also able to compare the
serial correlation for different levels of aggregation and find that, generally, serial
correlation is higher for data gathered at five minute intervals than for data
gathered at hourly intervals. For variable informed trade frequencies, we derive
the formula for trade correlation and a sufficient condition for the correlation
to be positive. We then demonstrate that squared price changes are positively
serially correlated, in accord with the second benchmark. Last, we verify that
the model is able to satisfy the third benchmark and produce serial correlation
in trades that is larger and diminishes more slowly than the serial correlation in
squared price changes.

To determine the serial correlation properties for calendar periods, such as
thirty-minute intervals, we divide each trading day into \( k \) calendar periods. We
let \( t \) index calendar periods. To understand how \( t \) maps into \( k \) and \( m \), suppose
that \( t = 1, \ldots, a \) in which \( t = 1 \) corresponds to the first calendar period on a
trading day. The sample would then consist of the vectors of \( k \) calendar periods
drawn from \( r \) consecutive trading days. Each calendar period contains \( h \) trader
arrivals (each trader arrival can be thought of as a unit of economic time). For
a given trading day, we have \( \tau = rh \) trader arrivals.

We first derive the serial correlation in trades (per calendar period). Under
the assumption of constant informed trade frequencies, we show that trades
are positively correlated. We also find that correlation in trades in an individual
market is less than the correlation in total trades, as segmenting trades into three
markets is not a scale transformation. (Our result for total trades corresponds
to the result reported in Kelley and Steigermad (2004), in which only one market
is analyzed and so informed trade frequencies are constant.) The formula for
serial correlation directly links the parameters of the market microstructure.
model to the serial correlation pattern in trades. We analyze the more complex case, in which the informed trade frequency is not constant, in Proposition 4.6. The proposition contains both a formula for trade correlation and a condition that ensures the correlation is positive. The intuition is straightforward: Positive trade correlation arises from the entry and exit of informed traders in response to the arrival of private information.

For correlation in trades in a specific asset, we focus on trades in the call option $I_C$. (Analogous results hold for the stock and the put option.) Given $\eta$ trader arrivals in $t$, $I_C$ takes integer values between 0 and $\eta$ and so is a binomial random variable for which the number of trades in $t$ corresponds to the number of successes in $\eta$ trials. For each period on trading day $m$ we have

$$E(I_C | S_m \neq s_o) = \eta(1 - \alpha)\epsilon_U + \sum_{i = 1}^{\eta} \alpha \left[ \delta \epsilon_{I_{BC}} + (1 - \delta) \epsilon_{I_{AC}} \right] = \mu_{C1}$$

and

$$E(I_C | S_m = s_o) = \eta(1 - \alpha)\epsilon_U = \mu_{C0}.$$

In general, derivation of calendar period trades is quite complicated, as the informed trade frequencies are not constant. To begin, we assume that equal payoff condition holds so that the informed trade frequencies are constant throughout the trading day. For simplicity, we assume that the uninformative trade frequencies are equal across assets, so that each informed trade frequency is $\frac{1}{4}$. We then arrive at the following theorem and corollary in which $r > 0$.

**Theorem 4.4.** Let the equal payoff condition hold. If $r < k$, then $I_{C_{<r}}$ and $I_C$ are positively serially correlated. If $r \geq k$, then $I_{C_{<r}}$ and $I_C$ are uncorrelated. For all $r$, we have

$$\text{Cor}(I_{C_{<r}}, I_C) = \frac{\theta(1 - \theta)(\frac{1}{4} \eta)^2}{\text{Var}(I_C)} = \left[ \frac{k - \min(k, r)}{k} \right].$$

**Proof.** Straightforward, but tedious, calculations yield the formula.

The correlation in trades for each market is less than the correlation in total trades, which as derived in Kelley and Steigerwald (2004) is

$$\text{Cor}(I_{C_{<r}}, I) = \frac{\theta(1 - \theta)(\alpha \eta)^2}{\sigma^2} \left[ \frac{k - r}{k} \right].$$

The correlations differ because the probability of success for a binomial random variable is not the scale of the random variable, so the variance of trades in a specific asset is not a scale transformation of the variance of total trades.

**Corollary 4.5:** Let $r < k$. The $p_i$ increasing in $\alpha, \eta, \text{and } k$. The changing the market parameters, aggregation through $\tau = k\eta$, or $ti$ are ambiguous.

**Proof.** The comparative static result

As either the frequency of info $\eta$, or the number of calendar period increases through the heightened traders. In general, increasing the $\tau$ correlation, but if $\epsilon$ is close to one impact of the informed trader flow the probability of an informative $s$ informative signals are rare. Perhaps can compare the serial correlation in five-minute observations, higher in five-minute intervals, but longer lags, $r \geq \frac{1}{4}$, the serial correlation is higher than the serial correlation $i$.

For the case in which the informant the day, we focus on trades in the stock and we deduce that the frequency of int trading day evolves. We capture this (because informed trading frequen

$$E(I_s | S_m = s_o) = \mu_{s0}$$

and

$$E(I_s | S_m \neq s_o) = \mu_{s1}$$

with $0 < \mu_{s0} < \mu_{s1} < \mu_{s2} < \cdots$.

As any one calendar period is $dr$ day, the unconditional mean of stock

$$E(I_s) = \theta \mu_{s1} + (1 - \theta) \mu_{s0}.$$

In deriving the serial correlation prior emerges that ensures the correlation

**Positive Trade Covariance Condition** is said to hold for period $i$, with

$$\mu_{s1} > \theta \mu_{s0} + (1 - \theta) \mu_{s1}.$$

```
Corollary 4.5: Let $r < k$. The positive correlation between $I_{C_{1-\eta}}$ and $I_{C_r}$ is increasing in $\alpha$, $\eta$, and $k$. The correlation is decreasing in $r$. The effects of changing the market parameters, $\epsilon$ and $\theta$, and of altering calendar period aggregation through $\tau = k \eta$, on the positive correlation between $I_{C_{1-\eta}}$ and $I_{C_r}$ are ambiguous.

Proof. The comparative static results follow from differentiation.

As either the frequency of informed trade, $\alpha$, the number of trader arrivals, $\eta$; or the number of calendar periods, $k$, increases, the trade serial correlation increases through the heightened impact of the entry and exit of informed traders. In general, increasing the frequency of uninformed trade reduces serial correlation, but if $\epsilon$ is close to one, then further increases in $\epsilon$ can amplify the impact of the informed trader flows and increase serial correlation. Increasing the probability of an informative signal, $\theta$, leads to higher serial correlation if informative signals are rare. Perhaps most importantly for empirical work, we can compare the serial correlation in hourly observations with the serial correlation in five-minute observations. We find that serial correlation is generally higher in five-minute intervals, but that the impact is not constant across $r$. For longer lags, $r > \frac{\eta}{3}$, the serial correlation in five-minute data is unambiguously higher than the serial correlation in hourly data.

For the case in which the informed trade frequencies vary over the course of the day, we focus on trades in the stock market, $I_S$. From the results of Section 3, we deduce that the frequency of informed trade in the stock market rises as the trading day evolves. We capture this evolution with a simple structure in which (because informed trading frequencies are zero if $S_m = S_{m+1}$)

$$E(I_S | S_m = S_{m+1}) = \mu_{S0} \text{ for } j = 1, \ldots, k,$$

and

$$E(I_S | S_m \neq S_{m+1}) = \mu_{Sj} \text{ for } j = 1, \ldots, k,$$

with $0 < \mu_{S0} < \mu_{S1} < \mu_{S2} < \cdots < \mu_{Sk}$.

As any one calendar period is drawn at random from the periods of a trading day, the unconditional mean of stock trades in a calendar period is

$$EI_S = \theta \overline{\mu_{Sk}} + (1 - \theta) \mu_{S0},$$

with $\overline{\mu_{Sk}} = e \sum_{j=1}^{k} \mu_{Sj}$.

In deriving the serial correlation properties of $\{I_S\}_{t \geq 1}$, an important condition emerges that ensures the correlation is positive.

Positive Trade Covariance Condition. The positive trade covariance condition is said to hold for period $j$, with $1 \leq j \leq k$, if $j$ is the smallest value of $j$ for which

$$\mu_{Sj} > \theta \overline{\mu_{Sk}} + (1 - \theta) \mu_{S0}.$$
The positive trade covariance condition is most intuitive for the case $k = 2$. From the structure for the expectation of calendar period trades it follows that $\mu_{S0}$ lies below the unconditional mean and $\mu_{S1}$ lies above the unconditional mean. Suppose that $t - 1$ corresponds to the first calendar interval – the morning – of the trading day. For days without private news, we have $E(I_{S_k} | S_m = s_0) = \mu_{S0}$ and $E(I_{S_k} | S_m = s_0) = \mu_{S0}$. Thus, for days on which the morning observation tends to be below the unconditional mean, the afternoon observation also tends to be below the unconditional mean. For days with private news, we have $E(I_{S_k} | S_m \neq s_0) = \mu_{S1}$ and $E(I_{S_k} | S_m \neq s_0) = \mu_{S2}$. While it is clear that the afternoon observation tends to be above the unconditional mean, it is not clear whether $E I_{S_k} < \mu_{S1}$. If the positive trade covariance condition holds (for period 1), then $E I_{S_{k}} < \mu_{S3}$. As a result, on days with private news both the morning and afternoon observations tend to lie above the unconditional mean and positive serial correlation is assured.

**Proposition 4.6.** Let $r > 0$. The covariance of calendar period stock trades is

$$
\left[ \frac{k - \min(k, r)}{k} \right] \left[ \theta (1 - \theta) \sum_{j=1}^{k-r} (\mu_{S_j} - \mu_{S0}) (\mu_{S_{j+r}} - \mu_{S0}) + \theta^2 \sum_{j=1}^{k} (\mu_{S_j} - \mu_{S_j}) (\mu_{S_j} - \mu_{S_{j+r}}) \right],
$$

where the addition is wrapped at $k$. That is, if $j + r > k$, then replace $j + r$ with $j + r - k$.

If $r < k = 2$ and the positive trade covariance condition holds for period one, then

$$\text{Cov}(I_{S_k}, I_{S_k}) = \left[ \frac{2 - r}{2} \right] \left[ \theta (1 - \theta) (\mu_{S1} - \mu_{S0}) (\mu_{S2} - \mu_{S0}) + \theta^2 (\mu_{S2} - \mu_{S1}) (\mu_{S2} - \mu_{S2}) \right] \geq 0.
$$

**Proof.** See Appendix. 

Serial correlation in squared price changes follows directly from serial correlation in trades if trade-by-trade price changes are i.i.d. As trade-by-trade price changes are not i.i.d. serial correlation in squared price changes is more complex than serial correlation in trades. As a result, formulae linking the parameters of the market microstructure model to the serial correlation are intractable. Positive serial correlation in squared price changes will obtain if squared price changes are higher in periods with higher trading (due to trade by informed traders). We numerically construct the distribution of squared price changes and show that expected squared price changes are higher in periods in which the informed are trading. We then verify that squared price changes are serially correlated, satisfying the second benchmark.

![Figure 7.5. Behavior of expect](image)

Serial correlation in an asset's squared mutation content of trades, which in turn decisions in early economic time $c$ decisions. We define the (stock) pm as

$$\Delta P_i = \sum_{j=1}^{10} U_j = E (\text{...})$$

A closed-form expression for $\Delta$ changes as a function of all of the tractable. To show that squared price we compare price change volatility change volatility is systematically hi of information leads to positive serial illustrate, in Figure 7.5 we present c days with and without news. A tradiar periods, with two trader arrivals stock market and we set $\alpha = .2$ and are uniformly higher on news days, are positively serially correlated.

To show that squared price chart consider a sequence of trading days i

* Estimates of $\alpha = .17$ and $\epsilon = .33$ are obtai
Serial correlation in an asset's squared price changes stems from the information content of trades, which in turn depends on the history of trades. Trade decisions in early economic time contain more information than later trade decisions. We define the (stock) price change over calendar period $t$ on day $m$ as

$$\Delta P_t = \sum_{i=1}^{119} U_i = E (V_m|Z_{1t}) - E (V_m|Z_{1t-110}).$$

A closed-form expression for the population moments of squared price changes as a function of all of the underlying parameters is, in general, intractable. To show that squared price changes have positive serial correlation, we compare price change volatility on days with and without news. If price change volatility is systematically higher on news days, then the random arrival of information leads to positive serial correlation in squared price changes. To illustrate, in Figure 7.5 we present expected squared price changes on trading days with and without news. A trading day is assumed to consist of six calendar periods, with two trader arrivals per period. (In detail, we consider only the stock market and we set $\alpha = .2$ and $\epsilon = .5$.) Expected squared price changes are uniformly higher on news days, which implies that squared price changes are positively serially correlated.

To show that squared price changes are positively serially correlated, we consider a sequence of trading days in which $\theta = .4$ have news (with good and

---

Footnote:

6 Estimates of $\alpha = .17$ and $\epsilon = .33$ are obtained in Easley et al. (1997) for an actively traded stock.
bad news equally likely). As is evident in Figure 7.6, the interplay between sequences of squared price changes that lie above the unconditional mean (from news days) with sequences of squared price changes that lie below the unconditional mean (from days without news) leads to positive serial correlation in prices. Further, the serial correlation declines as we move from lag 1 to lag 5. As it is less likely that observations separated by five periods occur on the same trading day. Because the news arrival process is independent across trading days, it would seem that squared price changes are uncorrelated after lag 5.

Yet the nonstationarity of the process due to the signal arrival at the start of each trading day leads to correlation in squared price changes at longer lags, which is more pronounced as \( \theta \) moves away from .5. The first hour of each trading day is noisier than other hours, which leads to serial correlation at lag 6 (and at integer multiples of lag 6) that mirrors the cyclical effects in asset market data.

To verify the third benchmark, we must show that positive serial correlation in trades declines more slowly than does the positive serial correlation in squared price changes. We alter the setting slightly to more closely approximate behavior in a liquid stock traded on the NYSE. We define a trading day to be 32.5 hours, which corresponds to a normal trading week on the NYSE. We measure price and trades at thirty-minute intervals, so there are sixty-five calendar periods in a trading day. A trader arrives every five minutes, so there are six trader arrivals in each calendar period and 390 trader arrivals in a trading day. We simulate 195,000 trader arrivals over the course of 500 trading days.

The strike prices of the options are at their respective limits, \( \kappa c_0 = v_{L_0} = 95 \) and \( \kappa P = v_{H_0} = 105 \), so that \( \lambda = 1.15 \) captures the greater leverage of an option. In Figure 7.7, we find the positive serial correlation in the total number of trades declines more slowly than does the positive serial correlation in the squared (stock) price changes. As trades, rather than total trades, are reduced.

8. CONCLUSIONS

We focus on the role of private information. The model captures the link between trades among traders, given a stylized actual markets the arrival and capture, and the theoretical constraining information persists is elusive. It is not surprising, therefore, that only information it may be difficult to acquire. Further, there is widespread concern by market makers are not solely the result of multiple additional considerations and market power. Provides a theory-based explanation for so doing, establishes an economic for capture stochastic vol

6. ACKNOWLEDGMENTS

We thank Steve LeRoy and John O...
squared (stock) price changes. A similar picture emerges if we consider stock trades, rather than total trades, although the level of the trade correlation is reduced.

5. CONCLUSIONS

We focus on the role of private information in the formation of securities prices. The model captures the link between asset prices and informational asymmetries among traders, given a stylized arrival process for private information. But in actual markets the arrival and existence of private information is not easily captured, and the theoretical construct of a defined period over which asymmetric information persists is elusive. Moreover, the possibility of the occurrence of multiple, overlapping information events introduces significant complexity. It is not surprising, therefore, that without knowledge of the existence of private information it may be difficult to accurately detect such a pattern in actual data.

Further, there is widespread consensus that adverse selection problems faced by market makers are not solely responsible for bid–ask spreads; rather, they are the result of multiple additional factors, including market maker inventory considerations and market power. Nonetheless, our simple economic model provides a theory-based explanation for observed empirical phenomena and, in so doing, establishes an economic foundation for the use of statistical models employed to capture stochastic volatility in asset prices.

6. ACKNOWLEDGMENTS

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APPENDIX

Proof of Theorem 3.1. We present the analysis for $\epsilon_{1AC}$ and $\epsilon_{1A}$. Identical logic holds for the remaining informed trade frequencies in the options and stock markets, respectively.

(a) Calculation reveals that $\frac{\delta e_{k}^{1}}{\delta k} > 0$ for $j$ indexing an option trade and $\frac{\delta e_{k}^{1}}{\delta k} > 0$ for $j$ indexing a stock trade. The sign of $\frac{\delta e_{k}^{1}}{\delta k}$ is the sign of

$$
\epsilon_{1A} \left[ (v_{h_{n}} - v_{l_{n}}) - \lambda \beta \right],
$$

which is negative by the greater leverage of options. The sign of $\frac{\delta e_{k}^{1}}{\delta k}$ is the sign of

$$
(\epsilon_{1AC} + \epsilon_{UBP}) \left[ \lambda \beta - (v_{h_{n}} - v_{l_{n}}) \right],
$$

which is positive by the greater leverage of options.

(b) The sign of $\frac{\delta e_{k}^{1}}{\delta k}$ is the sign of the first displayed equation in (a) while the signs of $\frac{\delta e_{k}^{1}}{\delta k}$ and $\frac{\delta e_{k}^{1}}{\delta k}$ are opposite to the sign of $\frac{\delta e_{k}^{1}}{\delta k}$. The sign of $\frac{\delta e_{k}^{1}}{\delta k}$ is the sign of the second displayed equation in (a) while the signs of $\frac{\delta e_{k}^{1}}{\delta k}$ and $\frac{\delta e_{k}^{1}}{\delta k}$ are opposite to the sign of $\frac{\delta e_{k}^{1}}{\delta k}$.

(c) Consider $\epsilon_{1AC}$. This informed trade frequency is positive if

$$
\epsilon_{1A} \left[ \lambda \left( v_{h_{n}} - \kappa c_{n} \right) - (v_{h_{n}} - v_{l_{n}}) \right] + \epsilon_{UBP} \left[ \lambda \left( v_{h_{n}} - \kappa c_{n} \right) - \lambda \left( v_{h_{n}} - v_{l_{n}} \right) \right] > 0.
$$

The first term on the left side is positive because of the greater leverage of options. The second term on the left side is zero because of equal option payoffs. If the uninformed trade each asset with equal frequency, then the remaining inequalities are deduced by inspection of the informed trade frequencies.

(d) For informed trade in the stock market, symmetric option payoffs imply that $\epsilon_{1A}$ is positive if

$$
\alpha \gamma_{j-1} \left( v_{h_{n}} - v_{l_{n}} \right) + (1 - \alpha) \left( \epsilon_{1AC} + \epsilon_{UBP} \right) \left[ (v_{h_{n}} - v_{l_{n}}) - \lambda \beta \right] > 0.
$$

Because options offer greater leverage, the second term on the left is negative and the inequality becomes

$$
\alpha \gamma_{j-1} \left( v_{h_{n}} - v_{l_{n}} \right) > (1 - \alpha) \left( \epsilon_{1AC} + \epsilon_{UBP} \right) \left[ \lambda \beta - (v_{h_{n}} - v_{l_{n}}) \right],
$$

from which the bound in the text is easily deduced.

Proof of Theorem 3.2. The proof follows from calculations similar to those in Kelley and Steigerwald (2004) [Theorem 3.1].

Proof of Theorem 3.3. For the proof of Theorem 3.3, let $D_{j}$ represent $D_{j}$. We verify the theorem for $U_{i}$; identical logic holds for $U_{c}$ and $U_{p}$. Proof that $E(U_{i}|Z_{i-1}) = 0$ and $E(U_{i}U_{i}|Z_{i-1}) = 0$ is straightforward. The upper bound for the conditional variance is

$$
E(U_{i}^{2}|Z_{i-1}) \leq \sum_{j=A,AC,BP} I_{j} + \sum_{j=B,BC,LM} I_{j} + \sum_{j=B,BC,LM} P(D_{j}) | I_{j} \leq \sum_{j=A,AC,BP} I_{j} + \sum_{j=B,BC,LM} I_{j} \leq \left[ \hat{A}_{i} - E(V_{n}) \right] \leq \left[ \hat{A}_{i} - E(V_{n}) \right] \leq \left( \hat{A}_{i} - \hat{B}_{i} \right)^{2},
$$

where the first inequality follows from inequality follows from

Proof of Proposition 4.6. We derive general covariance expression follows first calendar period in a trading day at period. First note that

$$
Cov(I_{S_{N}}, I_{S}) = E \left[ E(I_{S_{N}}|N) \right] - \left[ E(I_{S_{N}}|N) - \sum E(I_{S_{N}} - E) \right],
$$

or the sum of the conditional covariances means. Given that

$$
E(I_{S_{N}}|N = 1) = \theta_{S_{N}} + 1
$$

and

$$
E(I_{S_{N}}|N = 2) = \theta_{S_{N}} + 1,
$$

Because $P(N = 1) = P(N = 2) = \frac{1}{2},$

$$
P(N = 1) \cdot Cov(I_{S_{N}}, I_{S}|N = 1) + P(N = 2) \cdot Cov(I_{S_{N}}, I_{S}|N = 2) = \frac{1}{2} \left[ E(I_{S_{N}}|N = 1) - E(I_{S_{N}}) \right] N = 1 + \frac{1}{2} \left[ E(I_{S_{N}}|N = 2) - E(I_{S_{N}}) \right],
$$

which simplifies to

$$
\frac{1}{2} \theta_{S_{N}} (1 - \theta_{S_{N}}) \left( \mu_{S_{N}} - \mu_{S_{0}} \right) \left( \mu_{S_{N}} - \mu_{S_{0}} \right).
$$
for the conditional variance is
\[
E \left( U_t^2 | Z_{t-1} \right) \leq \sum_{j = A, AC, BP} P(D_j) \left[ \hat{A}_t - E(V_m | Z_{t-1}) \right]^2 + \sum_{j = B, BC, AP} P(D_j) \left[ \hat{B}_t - E(V_m | Z_{t-1}) \right]^2 + P(D_N) \left[ E(V_m | Z_{t-1}) - E(V_m | Z_{t-1}) \right]^2 \leq \sum_{j = A, AC, BP, N} P(D_j) \left[ \hat{A}_t - E(V_m | Z_{t-1}) \right]^2 + \sum_{j = B, BC, AP, N} P(D_j) \left[ \hat{B}_t - E(V_m | Z_{t-1}) \right]^2 \leq \left[ \hat{A}_t - E(V_m | Z_{t-1}) \right]^2 + \left[ \hat{B}_t - E(V_m | Z_{t-1}) \right]^2 \leq \left[ (\hat{A}_t - E(V_m | Z_{t-1})) - (\hat{B}_t - E(V_m | Z_{t-1})) \right]^2 = (\hat{A}_t - \hat{B}_t)^2,
\]
where the first inequality follows from the definition of \( \hat{A}_t \) and \( \hat{B}_t \) and the fourth inequality follows from \( B_t \leq E(V_m | Z_t) \leq A_t \).

Proof of Proposition 4.6. We derive Cov\( (I_{S_{t-1}}, I_S) \) for \( k = 2 \). Derivation of the general covariance expression follows similar logic. Let \( N = 1 \) if \( t - 1 \) is the first calendar period in a trading day and \( N = 2 \) if \( t - 1 \) is the second calendar period. First note that
\[
\text{Cov}(I_{S_{t-1}}, I_S) = E \left\{ \left[ E(I_{S_{t-1}} | N) - E(I_{S_{t-1}} | N) E(I_S | N) \right] \right\} + \left[ E(I_{S_{t-1}} | N) \right] \left[ E(I_S | N) - E(I_S | N) \right],
\]

or the sum of the conditional covariance and the covariance of the conditional means. Given that
\[
E(I_{S_{t-1}} | N = 1) = \theta \mu_{S1} + (1 - \theta) \mu_{S0} = E(I_S | N = 2)
\]
and
\[
E(I_{S_{t-1}} | N = 2) = \theta \mu_{S2} + (1 - \theta) \mu_{S0} = E(I_S | N = 1).
\]
Because \( P(N = 1) = P(N = 2) = \frac{1}{2} \), the conditional covariance is
\[
P(N = 1) \cdot \text{Cov}(I_{S_{t-1}}, I_S | N = 1) + P(N = 2) \cdot \text{Cov}(I_{S_{t-1}}, I_S | N = 2) = \frac{1}{2} \left[ E(I_{S_{t-1}} | N = 1) - E(I_{S_{t-1}} | N = 1) E(I_S | N = 1) \right] + \frac{1}{2} \left[ E(I_{S_{t-1}} | N = 2) - E(I_{S_{t-1}} | N = 2) E(I_S | N = 2) \right]
\]
which simplifies to
\[
\frac{1}{2} \theta (1 - \theta) \left( \mu_{S1} - \mu_{S0} \right) \left( \mu_{S2} - \mu_{S0} \right).
\]
As \( \mu_{S0} < \mu_{S1} < \mu_{S2} \), the conditional covariance is unequivocally positive. The covariance of the conditional means,

\[
P(N = 1) \cdot \left[ EI_{S1,0} - E(I_{S1} \mid N = 1) \right] \left[ EI_{S} - E(I_{S} \mid N = 1) \right] \\
+ P(N = 2) \cdot \left[ EI_{S1,0} - E(I_{S1} \mid N = 2) \right] \left[ EI_{S} - E(I_{S} \mid N = 2) \right]
\]

simplifies to

\[
\theta^2 \left( \frac{\mu_{S1} - \mu_{S2}}{2} \right) \left( \frac{\mu_{S2} - \mu_{S1}}{2} \right).
\]

As \( \mu_{S1} < \mu_{S2} \), the covariance of the conditional means is negative. We have \( \text{Cov} (I_{S1}, I_{S}) > 0 \) if \( (1 - \theta) (\mu_{S1} - \mu_{S0}) \left( \mu_{S2} - \mu_{S0} \right) > \frac{\theta}{2} \left( \mu_{S2} - \mu_{S1} \right)^2 \). By inspection, \( \mu_{S2} - \mu_{S0} > \mu_{S2} - \mu_{S1} \), so it is enough to show that

\[
(1 - \theta) (\mu_{S1} - \mu_{S0}) > \frac{\theta}{2} (\mu_{S2} - \mu_{S1}).
\]

Now, as \( \frac{\theta}{2} (\mu_{S2} - \mu_{S1}) = \theta (\mu_{S2} - \mu_{S1}) \), this is equivalent to showing that

\[
(1 - \theta) (\mu_{S1} - \mu_{S0}) > \theta (\mu_{S2} - \mu_{S1})
\]

From the positive trade correlation condition,

\[
(1 - \theta) (\mu_{S1} - \mu_{S0}) > \theta (1 - \theta) (\mu_{S2} - \mu_{S1}).
\]

Then

\[
(1 - \theta) (\mu_{S1} - \mu_{S0}) - \theta (\mu_{S2} - \mu_{S1}) > \theta (1 - \theta) (\mu_{S2} - \mu_{S1}) - \theta (\mu_{S2} - \mu_{S1}).
\]

The right side of the preceding inequality equals

\[
\theta \left[ (\mu_{S1} - \mu_{S0}) - \theta (\mu_{S2} - \mu_{S1}) \right],
\]

which is positive by the positive trade correlation condition.

References


