Initial Question

How should firms pay workers to elicit maximum effort?

- economics - effort increases as a worker’s wage rises
- psychology - workers withhold effort if they perceive they have not been paid a fair wage
  - fairness depends on coworker wages
- firm response
  - compress wages; keep wages secret

How would you determine if firm responses are working?
Does worker effort depend on coworker wages?

- Random variables of interest
  - $E$ - worker effort
  - $W$ - worker wage
  - $C$ - coworker wage
  - $U$ - forces other than wages that affect effort

- Stochastic Model

$$E = \beta_0 + \beta_1 W + \beta_2 C + U$$

- stochastic - effort depends on random variables, so effort is not deterministic
Stochastic Model

\[ E = \beta_0 + \beta_1 W + \beta_2 C + U \]

*linear* model - linear in coefficients

- \( \beta_0 \) - the average effort level due to non-wage factors
- \( \beta_1 \), a partial derivative \( \beta_1 = \partial E / \partial W \)
  - effect of own wage on effort, holding coworker wage constant
- \( \beta_2 \), a partial derivative \( \beta_2 = \partial E / \partial C \)
  - effect of coworker wage on effort, holding own wage constant
Initial Question Answered

- $\beta_2$
  - a partial derivative $\beta_2 = \partial E / \partial C$
  - effect of coworker wage on effort, holding own wage constant

- $\beta_2 = 0$  coworker wage doesn’t impact effort
- $\beta_2 < 0$  firm policy: compression, secrecy
- $\beta_2 > 0$  firm policy: expand differentials, public
- Firm responses are working if $\beta_2 < 0$
Stochastic Model

\[ E = \beta_0 + \beta_1 W + \beta_2 C + U \]

- Can we use the model to predict the effort level of a worker?

- To answer these questions, we must be able to determine

\[ \mathbb{E}(E) = \mathbb{E}(\beta_0 + \beta_1 W + \beta_2 C + U) \]

We need to simplify \[ \mathbb{E}(\beta_0 + \beta_1 W + \beta_2 C + U) \]
**Framework: Expectations Operator Properties**

**Constancy**

- \( \beta_0 \) is a constant
  - \( \mathbb{E}(\beta_0) = \beta_0 \)

- \( \beta_1 \) \( W \) \( W \) takes \( J \) values
  - \( \mathbb{E}(\beta_1 W) = \beta_1 \mathbb{E}(W) \)

\[
\sum_{j=1}^{J} \beta_1 w_j \cdot \mathbb{P}(W = w_j) = \beta_1 \sum_{j=1}^{J} w_j \cdot \mathbb{P}(W = w_j)
\]
Framework: Expectations Operator Properties

Linearity

Fact: \[ \sum_k P(W = w_j, C = c_k) = P(W = w_j) \]

- \((\beta_1 W + \beta_2 C)\) \(C\) takes \(K\) values
- \(E(\beta_1 W + \beta_2 C) = \beta_1 E(W) + \beta_2 E(C)\)

\[
\sum_j \sum_k (\beta_1 w_j + \beta_2 c_k) \cdot P(W = w_j, C = c_k)
\]

\[
= \sum_j \beta_1 w_j \sum_k P(w_j, c_k) + \sum_k \beta_2 c_k \sum_j P(w_j, c_k)
\]

\[
= \sum_j \beta_1 w_j P(W = w_j) + \sum_k \beta_2 c_k P(C = c_k)
\]

\[
= \beta_1 E(W) + \beta_2 E(C)
\]
\[ \mathbb{E}(E) = \mathbb{E}(\beta_0 + \beta_1 W + \beta_2 C + U) \\
= \beta_0 + \beta_1 \mathbb{E}(W) + \beta_2 \mathbb{E}(C) + \mathbb{E}(U) \]

- Can we use the model to predict the effort level of a worker?
  - model predicts the expected effort level of a worker

- Intercept is expected effort if wages are zero
To predict effort levels for a worker, need to link random variables with individual workers

- **Stochastic Process**

  A sequence of random variables is a stochastic process

- **Examples:** let \( i = 1, \ldots, n \) index workers
  - \( \{E_i\}_{i=1}^n \)
  - \( \{(E_i, W_i, C_i)\}_{i=1}^n \)

- **Stochastic Model:** For \( i = 1, \ldots, n \)

\[
E_i = \beta_0 + \beta_1 W_i + \beta_2 C_i + U_i
\]
Does China have an abnormally low number of female births?

- let $i$ index countries
  - $M_i$ - fraction of male births
  - $C_i$ - indicator variable for China

$$C_i = \begin{cases} 
1 & \text{if country } i \text{ is China} \\
0 & \text{if country } i \text{ is not China}
\end{cases}$$

Example: $i = 1, \ldots, 3$ \{France, Germany, China\}

- $i = 1$ France $C_1 = 0$
- $i = 2$ Germany $C_2 = 0$
- $i = 3$ China $C_3 = 1$
Review

Stochastic Model

\[ M_i = \beta_0 + \beta_1 C_i + U_i \]

- Interpret \( \beta_0 \)
  - the average fraction of male births in countries other than China

- Interpret \( \beta_1 \)
  - the difference between the fraction of male births in China and the average fraction of male births in other countries

- How can you use the model to determine if: *China has an abnormally low number of female births*
  - \( \beta_1 > 0 \)
  - need to estimate \( \beta_1 \)