

Robert Deacon

**Econ. 210C**  
**Problem Set #1**

Winter 2008

In the following problems you may assume that the underlying preferences make it legitimate to use partial equilibrium analysis.

1. Suppose that in a partial equilibrium context there are  $J$  identical firms that use labor and capital to produce good  $l$  with a cost function  $c(w,r,q)$ , where  $w$  is the price of labor,  $r$  is the price of capital services and  $q$  is output. Assume labor is an inferior input. Show that in the short run, where the number of firms is fixed, an increase in  $w$  will reduce the price of the product, while in the long run (allowing for entry and exit) it will increase the price of the product. Illustrate your answer with a diagram.
  
2. A perfectly competitive industry is composed of numerous firms with identical cost functions. In order to produce any positive output a firm must use a fixed amount of one input, e.g., entrepreneurship, but all other inputs are variable. As a result each firm's cost function is given by:  
 $c(q_j) = 9 + q_j^2$  for  $q_j > 0$ , and  $c(q_j) = 0$  for  $q_j = 0$ , where  $q_j$  is  $j^{\text{th}}$  firm's output. Aggregate demand (in inverse form) is given by  $P = 36 - .1q$ , where  $q$  is the total quantity consumed.
  - a. What is the long run (allowing for entry and exit) equilibrium price, output per firm and number of firms?
  - b. Suppose a tax of 6 per unit is imposed on all units produced. Calculate the equilibrium price and output in a short-run situation where the number of firms remains fixed but the fixed cost cannot be avoided by shutting down. Calculate the new long run equilibrium price, output and number of firms.
  
3. Two consumers,  $i = 1, 2$ , have preferences given by  $U_i = \ln(x_i + a_i) + m_i$  where  $a_1 = \frac{1}{3}$ ,  $a_2 = \frac{2}{3}$  and  $x_i \geq 0$ ,  $m_i \geq 0$  are  $i$ 's consumption of an ordinary good and a numeraire. Consumer  $i$  is endowed with  $\omega_i$  units of numeraire. The price of the ordinary (non-numeraire) good is denoted  $p$  and its supply function is given by  $q=p$ , where  $q$  is aggregate supply. The market is competitive. (You may consider the two consumers as representing large numbers of consumers of two types.)
  - a. Find the equilibrium price, total output, and consumption by each consumer.
  - b. Suppose a regulatory agency imposes a price ceiling  $p \leq \frac{1}{3}$ . The supply function remains unchanged and the regulator allocates the resulting quantity supplied among the two consumers in a way that maximizes their aggregate utility. Is consumer 2 better or worse off as a result? How about consumer 1? (It is okay to answer by setting up the utility comparison but without carrying out the calculation.) Explain.

4.  $N$  identical consumers all have utility functions  $U_i = \ln(x_i) + m_i$ , where  $x_i$  is  $i$ 's consumption of an ordinary consumption good and  $m_i$  is  $i$ 's consumption of numeraire. Each consumer is endowed with  $\mu$  units of the numeraire, which can be allocated to consumption of  $x$  and  $m$ . The market for  $x$  is perfectly competitive and it is in perfectly elastic supply at price  $\theta w$ . The term  $\theta$  is a technological parameter and  $w$  is the price of the sole input used in production. The supply of the sole input is affected by weather conditions that cause  $w$  to fluctuate. In even numbered years  $w=2$ , while in odd numbered years  $w$  alternates between the values of 1 and 3; i.e.,  $w_1=1, w_2=2, w_3=3, w_4=2, w_5=1$  and so forth, where subscripts indicate years.
- (i) Calculate the consumer's utility in years 1, 2, and 3.
- (ii) Would the consumer's utility, averaged over an indefinitely long number of periods, be enhanced by a government policy that stabilizes  $w$  at its average value? Answer rigorously and provide a brief verbal explanation. (When answering, ignore discounting; that is, simply average utility over time with and without the policy.)
5. Problem 10.C.2 from Mas-Colell, *et al.*

**Suggested practice problems from Mas-Colell, *et al.***

Problems 10.B.1, 10.B.2, 10.C.3, 10.C.7, 10.C.9, 10.C.10, 10.C.11, 10.E.1 (assume tariff revenue is returned to consumers or producers), 10.F.2, 10.F.6.

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**Problem Set #2**

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1. Three consumers have identical utility functions,  $U_i = \ln(x) + m_i$  where  $x$  is the level of a public good and  $m_i$  is consumption of a numeraire. Let  $x_i$  indicate the amount of public good voluntarily provided by consumer  $i$ , where  $\sum_i x_i = x$ . Assume each individual has an endowment  $\omega > 0$  of the private numeraire and each takes the provision of other consumers as given. The public good can be produced at a constant marginal cost of  $c=1$ .
  - a. What level of utility results in a free-rider equilibrium, with voluntary provision of the public good?
  - b. Suppose 2 of the consumers join to form a club. The club pools its endowment and allocates it between consumption (equally divided) and a voluntary contribution to the public good, which is set to maximize their summed utility. Will club members benefit from this action? How will the consumer who does not join fare, compared to members of the club; compared to the situation where all 3 act independently?
  
2. A community of  $N$  individuals lives on the shores of a lake, which they use for fishing. Consumer  $i$ 's utility is given by  $U_i = Q_i^{1/2} + m_i$ , where  $Q_i$  is  $i$ 's catch during the year and  $m_i$  is  $i$ 's consumption of a (private) numeraire. Although fish do not propagate naturally in the lake, individuals can release fish into the lake at a cost of  $c$  per unit. Each person's catch is directly proportional to the *total* number of fish released into the lake by all individuals, denoted  $F$ , where the factor of proportionality is denoted  $\alpha$ , i.e.,  $Q_i = \alpha F$ . Each individual is endowed with  $\mu$  units of the numeraire and the number of fish  $i$  releases is denoted  $R_i$ .
  - a. What value of  $F$  would be Pareto efficient? State your answer in terms of the parameters of the problem. Explain.
  - b. What total number of fish will be released by individuals acting independently, taking the releases of others as given, i.e., what is  $F$  in the 'free rider' equilibrium? Explain.
  - c. Suppose the factor of proportionality is related to the number of individuals fishing, specifically  $\alpha = \beta/N$ , where  $0 < \beta < 1$ . How would the number of fish released in the Pareto efficient and free rider solutions respond to a doubling of  $N$ ? Explain.
  
3. Consider a situation of bilateral externality, where agent 1's choice of some activity  $h$  provides a benefit to 1 but imposes a cost on 2. The relevant payoff functions are given by  $\Phi_1 = \min\{h^\alpha, 1\}$ ,  $\alpha < 1$  and  $\Phi_2 = \max\{1 - h^\beta, 0\}$ ,  $\beta > 1$ . Assume  $\alpha = \frac{1}{2}$  and  $\beta = \frac{3}{2}$ .
  - a. What is the Pareto efficient level of the externality?
  - b. What level of  $h$  would be privately optimal for agent 1? For agent 2?
  - c. What per unit tax on agent 1's choice of  $h$ ,  $t_h$ , would result in Pareto efficiency?

- d. Is there an all-or-none bargain between agents 1 and 2 that would yield the efficient level of  $h$ , assuming 2 has the right to set  $h$ ? If so, describe it, determining the level of the all-or-none bribe paid. If not, explain why none exists.
  - e. Suppose a pollution control authority imposes the per unit tax  $t_h$ , effectively giving agent 1 the right to pollute so long as the associated tax is paid and externality level  $h(t_h)$  emerges in equilibrium. If agents 1 and 2 can now bargain over control of the externality, what level of  $h$  will emerge in the new equilibrium? Is this new level of externality Pareto efficient?
4. Exercise 11.B.5 from Mas-Colell, *et al.*
5. Each of three consumers,  $i=1,2,3$ , has preferences represented by the utility function  $U_i = x_i g$ , where  $x_i$  is  $i$ 's consumption of a private numeraire and  $g$  is the common level of consumption for a public good. Consumer  $i$ 's budget constraint is given by  $\omega_i = x_i + \frac{1}{3} g$ , where  $\omega_i$  is  $i$ 's endowment and  $1/3$  is  $i$ 's share of the cost of providing the public good, i.e., a "tax price". Endowments for our three consumers have the following ordering,  $\omega_1 < \omega_2 < \omega_3$ .
- a. Characterize  $i$ 's preferred level of public good provision. Are  $i$ 's preferences over alternative levels of  $g$  single peaked? Explain and illustrate.
  - b. Is there a "Bowen" or "median voter equilibrium" level of  $g$  in this economy? If so, characterize it.
  - c. Characterize a Pareto Optimal level of provision for  $g$ .
  - d. Is there a restriction on the distribution of income that will ensure that the median voter equilibrium level of  $g$  is Pareto Optimal? If so, describe it.

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**Problem Set #3**

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1. A monopolist produces a product at a constant marginal cost of  $c > 0$ . The utility functions of  $N$  identical consumers who demand the monopolist's product are given by  $U_i = x_i^{\frac{1}{2}} + m_i$ , where  $x_i$  is  $i$ 's consumption of the monopolist's good and  $m_i$  is  $i$ 's consumption of a numeraire. Each consumer is endowed with  $\mu$  units of the numeraire. Assume the monopolist can make all or none offers to consumers, i.e., the monopolist can offer any level of  $x$  she chooses in exchange for any lump sum payment,  $T$  she chooses.
  - a. What offer will the monopolist choose to make in equilibrium, i.e., what specific payment,  $T$ , and what quantity of the good,  $x$ , will the monopolist offer to each consumer? How will the resulting allocation compare to a Pareto efficient allocation of resources?
  - b. How would consumer welfare be affected by a law requiring the monopolist to charge a fixed price, chosen by the monopolist, for any number of units the consumer wishes to purchase?
  
2. Consider an infinitely repeated Bertrand oligopoly game with discount factor  $\delta$  and  $J$  firms producing. Demand is given by  $P(q) = 10 - q$ , where  $q$  is total output, and all firms have constant unit cost  $c=2$ . Assume firms play Nash reversion strategies as described in the text (charge the monopolistic price if the entire price history contains only monopolistic prices, and the single-stage Bertrand equilibrium price otherwise.)
  - a. What is the largest number of firms, expressed as a function of  $\delta$ , that can sustain the monopolistic price? Assuming the number of firms is smaller than this level, can the price  $P=3$  be sustained as a SPNE? Explain.
  - b. Suppose each firm assigns a fixed, (common) positive probability,  $\pi$ , to the market being discontinued in any future period and firms are risk neutral. What is the largest number of firms, expressed as a function of  $\delta$  and  $\pi$ , that can sustain a monopolistic price in this circumstance?
  
3. Problem 12.C.3 from Mas-Colell, *et al.*
  
4. Suppose all potential firms can produce output at constant marginal cost,  $c=2$ , and that entering a market to begin production requires a setup cost,  $k=5$ . Demand for the product is given by  $P(q) = 10 - q$ , where  $q$  is the combined output of all firms producing. Firms play a two-stage entry game. In the first stage, firms decide "enter" or "don't enter", and incur the cost  $k=5$  if they choose "enter". In the second stage, all firms in the market play a static Cournot or Bertrand game.
  - a. Identify the SPNE to this game if second stage competition is Cournot.
  - b. Identify the SPNE to this game if second stage competition is Bertrand.

5. Two firms,  $j=1,2$ , are Cournot oligopolists and produce a homogeneous product. Demand is given by  $p = a - b(q_1 + q_2)$ , where  $q_j$  is firm  $j$ 's output. Each firm's cost function exhibits constant costs,  $c_j(q_j) = c_j q_j$ , where  $c_1 > c_2 > 0$ .
- Determine the Cournot equilibrium of this model. Under what conditions on the parameters of the problem ( $a, b, c_1, c_2$ ) will only one firm produce output?
  - Assuming both firms produce positive output in equilibrium, how would a slight decrease in  $c_2$  affect the Cournot equilibrium level of  $q_1$ ?

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1. Assume the productivity of workers,  $\theta$ , is unknown to firms and distributed uniformly on the interval  $[2,4]$ . Each worker has a reservation wage,  $r(\theta) = .8\theta$ , and chooses 'work' when indifferent. Labor is the only input, firms produce at constant returns to scale and the price of output is 1.
  - (a) Characterize the supply function for labor, i.e., the set of workers who will accept employment at each wage, and the average productivity of those who choose to work at each wage.
  - (b) What is the competitive equilibrium allocation and wage in this case? Is the competitive equilibrium Pareto optimal? Explain why or why not.
  
2. In the labor market model with firms unable to observe worker productivity, consider a case where the reservation wage,  $r$ , is the same for all individuals. Productivity is uniformly distributed on the interval  $[\underline{\theta}, \bar{\theta}]$ . Assume that when the wage is below  $r$ , so no individual will accept employment, firms believe that any worker who might accept employment is of the lowest quality, i.e.,  $E(\theta | \Theta = \text{null}) = \underline{\theta}$ , where  $\Theta$  is the set of worker types accepting employment. Assume workers accept employment when indifferent between working and not.
  - (a) Suppose the interval over which productivities are distributed is  $[0,1]$  and  $r = .25$ . Identify two competitive equilibria for this case and show that one of the two Pareto dominates the other.
  - (b) Suppose two firms simultaneously make wage offers for workers. Potential workers, if they accept either wage offer, accept the one offering the higher wage. Identify the SPNE for this model.
  
3. Firms demand labor as their only input, are competitive, risk neutral and operate with constant returns to scale. The price of output is normalized to unity. Potential workers are of two types,  $L$  and  $H$ , with productivities  $\theta_L = 1$  and  $\theta_H = 2$ . There are equal numbers of workers of each type. Each type has a reservation wage below which she will not work. Reservation wages for the two types are  $r_L = \frac{7}{8}$  and  $r_H = \frac{7}{4}$ . Each type may purchase education,  $e$ , at some cost. The cost functions for education for the two types are  $c(e | \theta_L) = \alpha_L e^2$ ,  $\alpha_L > 1$ , and  $c(e | \theta_H) = e^2$ .
  - a. Suppose firms cannot observe worker productivity. Find the competitive equilibrium wage and allocation for a situation in which neither type has an opportunity to acquire education. (If more than one competitive equilibrium exists, find the equilibrium that corresponds to a subgame perfect Nash equilibrium to the two-stage game-theoretic model in which firms simultaneously announce wage offers in the first stage, and workers then make employment decisions in the second stage.) How does the competitive equilibrium allocation compare to a Pareto optimal allocation in which productivities are observed?

- b. Suppose workers can now acquire education if they wish, with costs as given above. Describe a separating perfect Bayesian equilibrium for this economy, and verify that the conditions for a PBE are satisfied. (If there is more than one PBE, describe the one that Pareto dominates the others. To simplify, confine attention to equilibria in which firms' beliefs are such that the wage functions,  $w^*(e)$ , are simple step functions.)
- a. Compare the welfare of the various parties (type  $L$ , type  $H$  and firms) in this separating equilibrium to their welfare in the competitive equilibrium without signaling. How does your answer depend on  $\alpha_L$ ?
4. The reservation wage function,  $r(\theta)$ , is monotonically increasing and differentiable and there exists a  $\hat{\theta} \in (\underline{\theta}, \bar{\theta})$  such that  $r(\theta) > \theta$  for  $\theta > \hat{\theta}$  and  $r(\theta) < \theta$  for  $\theta < \hat{\theta}$ . The density of workers of type  $\theta$  is  $f(\theta)$ , with  $f(\theta) > 0$  for all  $\theta$ . Evaluate the following statement, as to whether it is true or false: A competitive equilibrium (with worker types unobserved by employers) is Pareto inefficient.
5. Suppose the reservation wage function,  $r(\theta)$ , is monotonically *decreasing* and differentiable. Let the density of workers of type  $\theta$  be  $f(\theta)$ , with  $f(\theta) > 0$  for all  $\theta$ .
- a. Show that the *more capable* workers will choose to work at any given wage.
- b. Show that if  $r(\theta) > \theta$  for all  $\theta$ , then the resulting competitive equilibrium is Pareto efficient.