

Robert Deacon

**Econ. 210C**  
**Problem Set #1**

Winter 2009

(Where the problem setup is one of partial equilibrium, you may assume that the underlying preferences make it legitimate to use partial equilibrium analysis.)

1. An economy consists of two consumers,  $i = 1, 2$ , who have preferences given by  $U_i = 2x_i^{1/2} + m_i$  where  $x_i$  is  $i$ 's consumption of an ordinary consumption good and  $m_i$  is  $i$ 's consumption of the numeraire. The economy's production technology requires that producing  $q$  units of the consumption good requires using  $2q$  units of numeraire as input. Consumers are endowed with  $\bar{w}_i$  units of the numeraire and their consumptions of the two goods must satisfy  $x_i \geq 0, m_i \geq 0$ .
  - a. Find the utility possibility frontier ( $U_1 = F(U_2)$ ) for this economy.
  - b. Suppose a tax of  $t$  per unit is imposed on consumption of  $x$  and the economy reaches a competitive equilibrium. What is the Marshallian surplus for this economy and what is the deadweight loss from the tax?
  
2. A city-owned golf course charges different prices for residents and nonresidents, with the nonresident price being exactly twice as high as the resident price. The golf course operates at zero marginal cost and has a fixed capacity of  $\bar{Q}$  rounds of golf per day. The mayor, who does not play golf, keeps any profits from golf course operation. All golfers, residents and nonresidents, have identical quasilinear utility functions, all demand functions are linear over the relevant range, and there are equal numbers of golfers in each group.
  - a. Characterize the prices the city will charge if it wishes to have zero excess demand.
  - a. Do golfers (residents and nonresidents), considered as a group, gain or lose from this pricing policy, relative to a policy of charging the same (competitive equilibrium) price to all? How does the mayor fare under the discriminatory pricing policy, relative to charging a homogeneous price.
  
3. A perfectly competitive industry is composed of numerous identical firms, each of which has the cost function  $c(q_j) = 1 + \alpha q_j^2$ , where  $q_j$  is the output of the  $j^{\text{th}}$  firm. Aggregate demand is perfectly inelastic in the relevant range, at  $X=200$ .
  - a. What is the long run (allowing for entry) equilibrium price and number of firms? (Don't worry if the number of firms is not an integer.)
  - b. Suppose the industry is in long run equilibrium with  $\alpha=1$ . If  $\alpha$  increases to 2, how will industry profits be affected in the short run situation where the number of firms cannot adjust?
  - c. What number of firms will operate in the new long run equilibrium?

4. Market demand for a particular good is given by the nonincreasing function  $X(p_c)$ , while market supply is given by the nondecreasing function  $Q(p_s)$ , where  $p_c$  and  $p_s$  indicate the prices paid by consumers and the price received by suppliers, respectively. A subsidy of  $s$  per unit is paid to consumers for each unit purchased.
- Derive an expression that characterizes the effect of the subsidy on the price producers receive, i.e.,  $dp_s(s)/ds$ , in terms of the price elasticities of supply and demand.
  - What does your expression indicate about the effect of the tax on producers' prices in the following special cases: (i) demand is perfectly price inelastic, (ii) supply is perfectly price inelastic, (iii) elasticities of supply and demand are non-zero, finite and equal in absolute value. Illustrate these cases with a diagram.
5. A market is composed of 10,000 identical consumers who all have utility functions  $U_i = \ln(x_i) + m_i$ , where  $x_i$  is  $i$ 's consumption of pizza and  $m_i$  is  $i$ 's consumption of the numeraire. Each consumer is endowed with  $\omega_i$  units of the numeraire. The supply of pizza is competitive. Consider a short run situation in which the industry producing pizza is composed of 100 identical firms, each of which takes price as given and produces with the cost function  $C(q_j) = \frac{1}{2}q_j^2$ , where  $q_j$  is firm  $j$ 's pizza production.
- What are equilibrium price of pizza, total quantity, quantity per consumer and quantity per firm? What is the level of pizza profits per firm?
  - A price ceiling is imposed,  $P \leq 5$ . What is the equilibrium quantity of pizza supplied. What is the level of excess demand?
  - Characterize an efficient allocation of the available pizza supply among consumers. Assuming an efficient allocation, do consumers gain or lose from the price ceiling? How does the price ceiling affect the total profits of producers?
  - Suppose the available product is allocated on a first come-first served basis. Each unit of time spent waiting to obtain the product reduces the consumer's endowment of the numeraire by  $\delta > 0$ . If the waiting time per unit  $x$  is  $t_w$ , what is the total (money plus time) cost per unit pizza to the consumer? What waiting time will cause supply and demand for pizza to be equal?
  - Suppose each consumer can reduce the per unit cost of waiting time to  $0.5\delta$  by spending a flat 0.1 of his or her endowment each period. Will the consumer take this action? How will this affect the equilibrium waiting time, waiting cost per unit  $x$ , and consumer utility?

**Suggested practice problems from Mas-Colell, *et al.***

Problems 10.B.1, 10.B.2, 10.C.3, 10.C.7, 10.C.9, 10.C.10, 10.C.11, 10.E.1 (assume tariff revenue is returned to consumers or producers), 10.F.2, 10.F.6.

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**Problem Set #2**

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1. Each of three consumers,  $i=1,2,3$ , has preferences represented by the utility function  $U_i = x_i g$ , where  $x_i$  is  $i$ 's consumption of a private numeraire and  $g$  is the common level of consumption for a public good. (Note that preferences are not quasi-linear.) Consumer  $i$ 's budget constraint is given by  $\omega_i = x_i + \frac{1}{3}g$ , where  $\omega_i$  is  $i$ 's endowment and  $1/3$  is  $i$ 's share of the cost of providing the public good. Endowments for our three consumers have the following ordering,  $\omega_1 < \omega_2 < \omega_3$ .
  - (a) Characterize a Pareto optimal level of provision for  $g$ .
  - (b) Characterize  $i$ 's preferred level of public good provision and the median voter equilibrium level of  $g$ . Is the median voter equilibrium level of  $g$  Pareto optimal, or does it depend on the distribution of income?
  - (c) Identify an alternative set of tax shares,  $\tau_i$ , that results in a Lindahl equilibrium.
  
2. Two consumers, A and B, derive utility from consumption of a private numeraire and from a garden that is owned by A. Let  $x$  denote the quality of the garden. Let  $m_A$  be A's consumption of the numeraire private good. A's utility is given by:
 
$$U_A = 3x^{\frac{1}{3}} + m_A.$$
 A's income is  $\omega_A$ , which she spends on gardening supplies costing  $x$  and on consumption of  $m_A$ . Hence, A's budget constraint is  $x + m_A \leq \omega_A$ .
 

B's utility, using similar notation, is  $U_B = 3x^{\frac{1}{3}} + m_B$ . B's income is  $\omega_B$ .

  - a. Assume B spends his entire income on the private numeraire. Because A owns the garden she determines its quality and incurs the cost of gardening supplies to maintain it. What is A's optimal budget allocation and resulting utility? (Assume an interior solution to A's problem.) What is B's utility, given A's choices?
  - b. Now imagine that A makes the following offer to B: If you pay the fraction  $a$  of the cost of maintaining the garden, then I will allow you to determine the garden's quality,  $x$ . Determine the cost share,  $\alpha^*$ , that will make B indifferent between accepting and rejecting A's offer. Assuming A sets the cost share at  $\alpha^*$ , is A better off making this offer?
  
3.  $N$  identical students are working on a problem set, which they will submit jointly. Each knows that the grade received by each member will be proportional to the total amount of time spent working by the entire group. Let the factor of proportionality equal 1 for simplicity, i.e., the score will equal the total time spent by the group. Let  $t_i$  indicate the time spent by individual  $i$  and  $t_{-i}$  indicate the time spent by other team members. Person  $i$ 's utility function is given by  $U_i = 2S_i^{\frac{1}{2}} + m_i$  where  $S_i$  is the score  $i$  receives and  $m_i$  is  $i$ 's consumption of the numeraire. Each student is endowed with  $\omega$  units of numeraire and spending 1 unit of time on the problem reduces this endowment by  $c$  units.
  - a. How much total time will the team spend if each member chooses her time allocation independently, taking as given the time spent by others?

- b. How much time should the group spend in total in order to maximize the sum of their utilities?
4. Person 1's choice of activity  $h$  provides a benefit (over some range) to herself, but imposes a cost on person 2. The relevant payoff functions are given by  $\phi_1(h) = h - \frac{1}{2}h^2$  and  $\phi_2 = \max(0, 2 - \frac{1}{2}h^2)$ .
- Contrast the Pareto efficient level of the externality to the level that person 1 finds privately optimal.
  - Assume person 1 has the right to determine  $h$ . Further, assume can offer person 2 an all-or-none bargain: 1 will constrain the level of  $h$  produced to a specified level in return for a lump sum payment,  $T$ . Describe the all-or-none bargain between agents 1 and 2 that would maximize 1's utility and compare the resulting level of  $h$  to the Pareto efficient level.
5. Exercise 11.B.5 from Mas-Colell, *et al.*

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**Problem Set #3**

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1. A firm enjoys a monopoly position in sales to a domestic market, but faces perfectly competitive conditions for sales abroad. Trade restrictions allow the firm to charge different prices at home and abroad. The domestic demand function is given by  $P_d = a - \frac{1}{2}bq_d$ , where  $P_d$  is the domestic price and  $q_d$  is domestic output. The foreign price is  $P_f < a$ . The firm's cost function is  $C = \frac{1}{2}c \cdot (q_d + q_f)^2$  where  $c$  is a positive constant and  $q_f$  is the amount sold abroad. Assume the firm's domestic and foreign quantities are strictly positive in all solutions examined.
  - (a) Solve for the firm's profit maximizing domestic price and domestic and foreign outputs in terms of  $P_f, c, a$  and  $b$ .
  - (b) Suppose the firm's cost parameter,  $c$ , increases slightly. How will this affect the domestic price and domestic and foreign output? (Solve for the derivative of the firm's choice variables with respect to  $c$ .) Provide an intuitive explanation for your results.
  - (c) How would an increase in the foreign price affect the domestic output and price?
  
2. Consider a Cournot oligopoly with three firms. Let  $P(q) = 150 - q$  be the inverse demand and let  $C_i(q_i) = 18q_i + q_i^2$  be the cost function of firm  $i = 1, 2, 3$ .
  - (a) Find the Cournot equilibrium price and output for each firm.
  - (b) Show that any two firms would have profit incentives to merge into one firm. (Assume that the merged firm takes the remaining firm's output as given.) You may find that it simplifies matters to start by characterizing the merged firm's cost function.
  - (c) Show that consumers would become worse off if two firms merged.
  
3. Consider an infinitely repeated Bertrand oligopoly game with discount factor  $\delta = .6$ . Demand is given by  $P(q) = 10 - q$ , where  $q$  is total output, and all firms have constant unit cost  $c = 2$ .
  - a. Assuming there are two firms in the market, what is the most profitable price that can be sustained using Nash reversion strategies? What is the most profitable price that can be sustained if there are three firms producing?
  - b. Assume there are two firms. How would a higher unit cost in all periods,  $c = 4$ , affect the most profitable price that can be sustained using NR strategies? Suppose, instead, that the unit cost is 2 in period 1, but increases to 4 in all future periods. How would this affect the highest price that can be sustained in period 1?
  
4. Consider a two-stage model of entry in which all potential entrants have a cost per unit of  $c$  and incur an entry cost of  $K$  if they enter. Assume that whenever firms enter, all firms producing form a perfect cartel. (Consumer preferences are quasi-linear.)
  - a. A social planner who wishes to maximize social surplus can control the number of firms that enter, but cannot control this cartel behavior. What number of firms should the planner allow to enter?

- b. What level of social surplus would result if the planner cannot control either the number of firms that enter or their cartel behavior?
5. A monopsony is a market with a single buyer. Suppose firm A is a monopsony buyer of its only input,  $L$ , supplied by a perfectly competitive industry, where the supply function is given by  $w(L) = 2L$  and  $w$  is the price of  $L$ . Firm A sells its output,  $y$ , competitively at a price  $p = 100$  and produces it according to the production function  $y = \log(L)$  for  $L \geq 1$ ,  $y = 0$  otherwise.
- a. Find the monopsonist's equilibrium output and the equilibrium price of  $L$ .
- b. If the monopsonist could perfectly price discriminate, i.e., pay a different price for each unit of  $L$  and thereby achieve a total input cost equal to the area under the supply curve for  $L$ , by how much would the firm's profit increase? (The answer is not an integer or fraction; just provide a correct expression.)

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1. Assume the productivity of workers,  $\theta$ , is unknown to firms and distributed uniformly on the interval  $[2,4]$ . Each worker has a reservation wage,  $r(\theta) = .8\theta$ , and chooses 'work' when indifferent. Labor is the only input, firms produce at constant returns to scale and the price of output is 1.
  - a. Characterize the supply function for labor, i.e., the set of workers who will accept employment at each wage, and the average productivity of those who choose to work at each wage.
  - b. What is the competitive equilibrium allocation and wage in this case? Is the competitive equilibrium Pareto optimal? Explain why or why not.
  
2. Suppose workers are of two types,  $L$  and  $H$ , and their productivities are given by  $\theta_L = 2$  and  $\theta_H = 4$ . The reservation wage is zero for both types. Workers can choose to acquire education,  $e$ , the cost of which is given by  $c(e, \theta) = e^2/\theta$  for the two types. Utility functions are given by  $U = w - c(e, \theta)$ .
  - a. Identify the range of possible education levels for type  $H$  that can result in separating perfect Bayesian equilibria. Which of these outcomes is Pareto dominant?
  - b. Suppose there are equal numbers of types  $L$  and  $H$  and signaling is prohibited and the reservation wage is still zero. Identify the competitive equilibria and compare the utility levels of the various parties (type  $L$ , type  $H$  and firms) to the utilities they would receive in the Pareto dominant separating PBE with signaling. (Assume people work when  $w$  equals their reservation wage.)
  
3. Firms demand labor as their only input, are competitive, risk neutral and operate with constant returns to scale. The price of output is normalized to unity. Potential workers are of 3 types,  $k=1,2,3$ , with equal numbers in each type. Their respective productivities, which are hidden to firms, are  $\theta_k = k$ . Their reservation wages are  $r(\theta_k) = \frac{3}{5}k$ .
  - a. Identify the competitive equilibrium or equilibria for this economy. (Demonstrate that the conditions for a CE are satisfied at the equilibrium or equilibria you identify.)
  - b. Suppose two firms simultaneously make wage offers for workers. Potential workers, if they accept either wage offer, accept the one offering the higher wage. Identify the SPNE for this model and compare it (them) to the allocation that is Pareto optimal with productivities known.
  
4. Suppose the reservation wage function,  $r(\theta)$ , is monotonically *decreasing* and differentiable. Let the density of workers of type  $\theta$  be  $f(\theta)$ , with  $f(\theta) > 0$  for all  $\theta$ .
  - a. Show that the *more capable* workers will choose to work at any given wage.
  - b. Show that if  $r(\theta) > \theta$  for all  $\theta$ , then the resulting competitive equilibrium is Pareto efficient.

5. A worker may be one of two types,  $L$  and  $H$ , and the productivities of the two types are given by  $\theta_L=1$  and  $\theta_H=2$ . The reservation wage is zero for both types. The worker can choose to acquire education,  $e$ , the cost of which is given by  $c(e,\theta) = e/\theta$  for the two types. The utility function is given by  $U = w - c(e,\theta)$ , where  $w$  is the wage paid by a firm (which may depend on the worker's education).
- Identify the wage and education levels for the 2 types of workers for the Pareto dominant separating perfect Bayesian equilibrium.
  - Suppose the probability of being either type is  $1/2$  (or, with the many workers interpretation, there are equal numbers of each type.) Assume the reservation wage is still zero. If signaling is prohibited, will the subgame perfect Nash equilibrium of the 2 stage wage-offer / employment choice game be Pareto superior to the separating perfect Bayesian equilibrium identified in part (a)?
  - Re-answer part b. in the case where the probability of being type  $H$  is  $1/3$ .