ENERGY ECONOMICS and POLICY

JAMES M. GRIFFIN
Texas A & M University

HENRY B. STEELE
University of Houston

ACADEMIC PRESS COLLEGE DIVISION
(Harcourt Brace Jovanovich, Publishers)
Orlando San Diego San Francisco New York London
Toronto Montreal Sydney Tokyo São Paulo

1986
INTRODUCTION

The analysis in Chapter 2 is purely static; that is, production and consumption are assumed to occur in one particular time period. In many cases, there is no reason to introduce dynamic considerations except to recognize that policies which would generate welfare gains or losses during the current period would also generate welfare gains or losses in future periods, thus providing an even stronger rationale for or against the policy change. For example, static analysis is more than adequate to analyze the welfare effects of a tax on hot dog vendors. Each week the vendor purchases new supplies of mustard, hot dogs, and buns and rents his equipment. Moreover, the effects of one week’s purchases on next week’s prices are likely to be minimal. Thus each week can be viewed as a separate event, independent from past or future market periods.

In the case of nonrenewable natural resources, each period is not independent of the other. With a nonrenewable resource base, today’s consumption reduces the resource base available for tomorrow. Consequently, both the firm and society must attempt to optimize production over many periods, recognizing that actions in one period affect opportunities in future periods. Since our dominant energy forms—oil, natural gas, coal, and uranium—fit into this category, we must be concerned that energy consumption is efficient not only in a static sense, but also over time, that is, in a dynamic sense.

Intertemporal Profit Maximization

The first and most fundamental assumption underlying the economics of nonrenewable resources is that firms attempt to maximize long-run profits over time, or intertemporally. The profit maximization motive that is so essential to static economic analysis is equally important in the dynamic case. In static analysis, the production decisions in one period do not affect profits in other periods, so that firms simply pro-
ceed to equate marginal cost with marginal revenue in each period, obtaining the maximum profit ($\Pi_i$) for any period $i$. The result is a production plan resulting in an optimal stream of profits ($\Pi_i$) over time ($\Pi_0, \Pi_1, \Pi_2, \ldots, \Pi_n$), which maximizes the firms' long-run profits over time.

However, with production using nonrenewable resources, the decision to produce and earn profits of $\Pi_0$ today necessarily precludes the ability to produce and earn profits in future time periods. How then can the firm select the optimal tradeoff between current and future profits? Basically, the firm must apply a discount rate to future profits, since a dollar today is worth more than a dollar in the future. Normally, the firm's discount rate will closely approximate the market rate of interest, differing only by an adjustment for risk. Thus, the rate of interest provides the firm with guidance as to the value of one dollar's profit today in comparison, for example, with a profit of $1.50 in 5 years. In effect, if the market rate of interest is 10 percent per year ($r = 0.1$), then a profit of $1.10 next year has a present value of $1.00. In fact, it is possible to calculate the present value of the profit stream ($\Pi_i$) beginning in the current period 0 and terminating at some distant period $n$ in the future:

\begin{equation}
(3.1) \quad \text{Present value} = \Pi_0 + \frac{\Pi_1}{1 + r} + \frac{\Pi_2}{(1 + r)^2} + \frac{\Pi_3}{(1 + r)^3} + \cdots + \frac{\Pi_n}{(1 + r)^n}
\end{equation}

Equation (3.1) provides the firm with guidance as to what is meant by long-run profit maximization. By choosing a profile of profits ($\Pi_0, \Pi_1, \Pi_2, \ldots, \Pi_n$), the present value of future profits can be maximized. By maximizing the present value of future profits, one maximizes the value of the firm, since stockholders' perception of the value of existing shares is based on the anticipated stream of profits over the future $n$ periods plus the salvage value of the firm in the last period. Any other alternative stream of profits ($\Pi_0^a, \Pi_1^a, \Pi_2^a, \ldots, \Pi_n^a$) will lead to a lower present value.

What forces are at work to compel the firm to maximize the present value of future profits? The answer is market forces. If the firm finds itself in a competitive market, its very corporate survival depends on its ability to maximize profits. Even in markets insulated from competitive forces, factors are at work promoting profit maximization. Normally, managers will seek to maximize the value of the firm's shares since they typically receive stock options and/or profit-related bonuses. If the firm does not select the optimal stream of profits ($\Pi_0, \Pi_1, \Pi_2, \ldots, \Pi_n$), it becomes a candidate for corporate takeover. The existence of suboptimal profits provides an incentive for an acquiring firm to rectify the profit performance. By restoring the firm to a production time path that maximizes long-run profits, the value of the

*CRITERIA FOR EFFICIENT DYNAMIC RESOURCE ALLOCATION*
firm's shares can be raised, providing an above normal return to the acquiring firm.

**QUESTIONS**

1. Suppose you own a shut-in gas well with reserves of 1 billion cubic feet. You know with certainty that natural gas will be much more valuable in 10 years and will sell for $10 per thousand cubic feet. Assuming the market rate of interest is 10 percent, what price would you sell the gas reserves for today? (Hint: $3.85 million.)

2. Suppose in question 1 that you do not know future gas prices with certainty. To compensate for the risk attached to natural gas as an asset, you adopt a discount rate of 12 percent. In effect, if the risk-free return on treasury bonds is 10 percent, you add a 2 percent risk premium. Now what is the present value of the gas well, applying a 12 percent discount rate? (Hint: $3.22 million.)

**USER COSTS**

While equation (3.1) correctly describes the present value calculation, it lacks the economic insight provided by static analysis. For example, in static analysis, profit maximization automatically implies selecting the production level where marginal cost equals marginal revenue. In the nonrenewable resource case, can the firm arrive at \( \Pi_0, \Pi_1, \Pi_2, \ldots, \Pi_n \) in equation (3.1) by simply setting marginal cost equal to marginal revenue in each period? If so, we arrive at a surprising result. Knowing that price equals marginal cost in a competitive market, one might be tempted to reason that the competitive world price of oil should be about $2.25 per barrel since the marginal cost of producing oil is approximately $2.25 per barrel in most Persian Gulf countries. This fallacy arises from an inadequate understanding of marginal costs. In the static case of a firm using renewable resource inputs, the decision to produce today in no way affects the costs of producing in the future. Thus marginal cost (MC) consists entirely of the marginal production cost (MC\(^P\)), that is, the capital, labor, and material costs of producing the last unit of output. In cases involving the use of nonrenewable products, the decision to produce a barrel of oil today precludes the possibility of producing it at some time in the future. The resource owner must trade off the opportunity value of selling the resource today versus the opportunity value of selling it at some future time. This is precisely the notion captured by the user value, or as it is most commonly called, the user cost. The user cost in period \( i \) reflects the opportunity value of producing a unit of output in that period. While there is no tax collector present to collect these user costs, the firm clearly foregoes the current opportunity value of the resource when it chooses to leave it for later production. Thus, in the nonrenewable case, marginal cost (MC) in period \( i \) is modified to in-
clude both the conventional marginal production cost \((MC_i^p)\) and the user cost \((U_i)\):

\[
MC_i = MC_i^p + U_i
\]

(3.2)

The method of calculating the user cost is predicated on long-run profit maximization (i.e., setting marginal revenue, \(MR_i\), equal to marginal cost, \(MC_i\)). The user cost, which shows the opportunity value of producing a marginal unit today, is calculated by simply subtracting marginal production costs \((MC_i^p)\) from marginal revenue \((MR_i)\):

\[
U_i = MR_i - MC_i^p
\]

(3.3)

Combining equations (3.2) and (3.3) we reaffirm the standard price theory result that marginal costs (including user costs) equal marginal revenue even for nonrenewable resources.

Since \(U_i\) represents the opportunity value of selling a barrel in period \(i\), the producer may elect to switch part of his production in period \(i\) to some other period \(i'\) where user costs are higher. In fact, if the producer is to maximize his long run profit, he should be indifferent between producing the last barrel in any given time period. That is, user costs should be equated across all production periods, so that:

\[
U_0 = U_1 = U_2 = U_3 = \cdots = U_n
\]

(3.4)

But equation (3.4) overlooks the fact that a dollar received today is worth far more today than is a dollar received ten years from now. Thus, wealth-maximizing producing agents must in reality discount future user costs at the market rate of interest \((r)\), implying that discounted user costs must be equated over time:

\[
U_0 = \frac{U_1}{1 + r} = \frac{U_2}{(1 + r)^2} = \frac{U_3}{(1 + r)^3} = \cdots = \frac{U_n}{(1 + r)^n}
\]

(3.5)

This means that the user costs must rise at the rate of interest if the net present value of the resource is to be maximized, that is:

\[
U_1 = U_0 (1 + r)
\]

\[
U_2 = U_1 (1 + r)
\]

\[
U_n = U_{n-1} (1 + r)
\]

(3.5a)

As an example, if the interest rate is 10 percent \((r = .10)\) and \(U_0 = \$1\), the user cost \((U_i)\) must be \$1.10 in period 1, \$1.21 in period 2, \$1.33 in period 3 and so forth, in order for the discounted user costs to be equal over time. If, for example, \(U_3 = \$2\), the producer has not chosen an
optimal production strategy since the discounted value of oil produced in period 3 would be $1.50 (U_3/(1.1)^3)$ and yet the discounted value for all other periods is $1. The producer can increase the present value of its reserves by allocating more production to period 3 and reducing production in the other periods. Thus, the problem facing the wealth-maximizing producer is to schedule production over time such that equation (3.5) holds for the last barrel produced in any period.

A more formal proof of the result that equality of discounted user costs (equation (3.5)) follows from maximization of the present value of the resource (equation (3.1)) can be shown as follows. For simplicity, consider a model with two periods rather than $n$ periods. According to equation (3.1),

$$PV = \Pi_0 + \frac{1}{1 + r} \Pi_1$$

Now consider the effect of moving one unit of production from period 0 to 1 (i.e., $-\Delta Q_0 = \Delta Q_1$):

$$\Delta PV = \Delta \Pi_0 (\Delta Q_0) + \frac{1}{1 + r} \Delta \Pi_1 (\Delta Q_1)$$

But the change in profit ($\Delta \Pi_0$) resulting from rescheduling production of the last barrel is simply the difference between marginal revenue and marginal production cost ($MR_0 - MC_0^p$). Thus equation (3.7) can be written equivalently as:

$$\Delta PV = (MR_0 - MC_0^p) (\Delta Q_0) + \frac{1}{1 + r} (MR_1 - MC_1^p) (\Delta Q_1)$$

By definition [see equation (3.2)], we can replace the difference between marginal revenue and marginal production cost with the appropriate user cost:

$$\Delta PV = U_0 (\Delta Q_0) + \frac{1}{1 + r} U_1 (\Delta Q_1)$$

But if $PV$ is truly at a maximum, a small change of $\Delta Q_0$ will not affect $PV$. Setting $\Delta PV = 0$ in equation (3.9) and utilizing the fact that the production increase in one period equals the decrease in the other period ($-\Delta Q_0 = \Delta Q_1$), we obtain the familiar result.

$$U_0 = \frac{U_1}{1 + r}$$
Thus, equating the present value of user costs necessarily implies that the firm has selected a production strategy that maximizes the present value of the resource.

1. Can you explain why you cannot use equation (3.2) to compute the user cost? You must first compute user cost from equation (3.3) and then, knowing the user cost, it is possible to compute marginal cost in equation (3.2).

2. Explain why a profit-maximizing producer must reorder production until the present values of the user costs are equated over time. If \( U_1/(1 + r) > U_0 \), what must happen to production in \( Q_0 \) and \( Q_1 \)?

3. Suppose you have 3 grades of coal, with respective user costs per ton of \(-\$5, \$1, \) and \$10. Will coal grade 1 be mined? How about grades 2 and 3? Explain why the low-cost mines will be exploited first.

**USER COSTS AND EXPECTATIONS ABOUT THE FUTURE**

The notion of user costs is extremely complex because they are not easily identified and calculated as marginal production cost might be. But it is precisely their complexity that makes user costs so fascinating. In understanding user costs, there are a few key points to keep in mind. First, a set of user costs \((U_0, U_1, U_2, \ldots, U_n)\) is conditional on a given set of expectations about the future. As long as the "old expectations" (that set of expectations under which the profit-maximizing production schedule was originally made) are fulfilled, the user costs will rise at rate \( r \) over time as seen in equation (3.5a). For example, in Figure 3.1, over the time period 0 to \( t_0 \) user costs rise along a given path at rate \( r \). Suddenly, at time period \( t_0 \), producers revise their original expectations, probably due to new information about the future, and form "new expectations." User costs shift upward, rising along a new path, as producers judge that the opportunity value of the reserves has become much greater. Second, these expectations depend on demand as well as supply conditions, and future as well as present conditions. Remember from equation (3.8) that it is necessary to form expectations about future marginal revenues and thus future demand conditions as well as production costs. These expectations are no doubt further influenced by the expected size of the resource base. Large additions to resource supplies will drive down both current and expected future market prices and marginal revenues. Curiously, even when present demand conditions develop exactly as expected, if pro-
producers become convinced that future demand will be much greater than previously expected, user costs will be revised upward as in Figure 3.1, switching user costs onto a new path. The mechanism is simple—at time period $t_0$ producers revised upward their expectation of user costs in some future period. Thus based on the old production plan, the discounted future user cost now exceeds the user cost of today's production, leading to a reduction in current production and a planned increase in future production. As current production is reduced, user costs rise to the new user cost path.

Implications for the Pricing of Nonrenewable Resources

The previous section establishes the key result that the user cost will tend to rise with the rate of interest for a given set of expectations. But our concern is more with the implications for the pricing of nonrenewable energy sources than with the change in user costs. As we shall see in the subsequent section, the linkage between user costs and prices can differ widely, depending on the set of assumptions made.

HOTELLING'S r PERCENT PRICE PATH

According to Harold Hotelling, the linkage between user costs and prices is direct. Prices, like user costs, would also tend to rise at the rate of interest. In his famous 1931 article in the *Journal of Political Economy*, Hotelling set forth this provocative idea. In view of the trivial marginal production cost of most Persian Gulf oil, it is remark-
able that Hotelling chose to simplify his analysis to a case where marginal production costs are zero ($MC^p = 0$), even though these reserves were at that time undiscovered. Together with the assumption of perfect competition (i.e., the firm's marginal revenue equals the market price), one can substitute prices ($P_i$) for user costs ($U_i$) in equation (3.5) as follows:

$$P_0 = \frac{P_1}{1 + r} = \frac{P_2}{(1 + r)^2} = \ldots = \frac{P_n}{(1 + r)^n}$$

(3.11)

Equivalently, prices will rise at the rate of interest:

$$P_1 = P_0(1 + r)$$
$$P_2 = P_1(1 + r)$$

(3.11a)
$$\vdots$$
$$P_n = P_{n-1}(1 + r)$$

Hotelling's result is captivating because of both its simplicity and its underlying economic sophistication. By making just three key assumptions—(1) long-run profit maximization, (2) zero marginal production costs, and (3) perfect competition—we arrive at the result that prices will tend to rise in a smooth predictable manner with the rate of interest. Hotelling's $r$ percent rule seems sensible because deposits of oil, coal, or natural gas are irreplaceable assets of value, and in order to entice producers to hold oil for future periods, they must receive a return of exactly $r$ percent per year. Prices cannot systematically rise faster than $r$ percent per year, since current production would cease in anticipation of a return greater than $r$ percent, driving up current prices, which would induce shifting of production toward the current period and restore the rule. Conversely, systematic price increases of less than $r$ percent per year would lead producers to raise current production, thereby lowering the present price, which would induce shifting of production to future periods, again restoring the equilibrium implied by the rule.

While prices must rise at $r$ percent per year to establish a market equilibrium, there are an infinite number of price paths that will rise at $r$ percent per year. For example, at $r = .10$, oil prices could move along a path of 1.00, 1.10, 1.21, 1.33 \ldots or another path of $30.00, 33.00, 36.30, 39.93$ and so forth. How can we establish the market-clearing price path? Or, put another way, what is the optimal initial price?

Mathematically, the solution can become quite complicated with $n$ time periods. But if we restrict the number of time periods to two (the present and the future), a graphical solution is possible. Let us con-
sider the case in which 60 barrels of oil are to be consumed between the current period 0 and the future period 1. Assume for illustrative purposes that the expected demand curves in the two periods are \( P_0 = 50 - .5Q_0 \) in period 0 and \( P_1 = 50 - .33Q_1 \) in period 1. Furthermore, the rate of interest is taken to be \( r = .10 \). To preserve the zero marginal production cost assumption, we assume that the oil is being stored in the backyard swimming pools of our competitive oil producers.

Figure 3.2 provides a graphical description of how the optimal price path is determined. Beginning on the left-hand vertical axis, we measure price in the current period; the price is $50 per barrel when current output is zero. We draw the demand curve, \( D_0 \), emanating from \( P_0 = $50 \) and sloping downward to the right with a slope of \(-.5\).

Similarly, the demand schedule for the future period emanates from \( P_1 = $50 \) on the right-hand vertical axis and slopes downward to the left with a slope of \(-.33\). The prices of oil in the two periods can be determined by inspection of the two demand curves for any particular output division between the two periods. For example, if \( Q_0 = 40 \) and \( Q_1 = 20 \), the two demand curves imply prices of \( P_0 = $30 \) and \( P_1 = $43.34 \).

The next step is to determine the user cost schedules \( U_0 \) and \( U_1 \) for the two periods. Recall from equation (3.11) that the user cost is identical to price in Hotelling’s case. Thus for the current period, \( U_0 \) is identical to the demand schedule. Similarly, for the future period, \( U_1 = P_1 \); thus the user cost and demand schedule are identical. But since \( U_0 \) is equated with the discounted future period’s user cost, we

\[ \text{FIGURE 3.2 Solution of the Hotelling price path and production rates.} \]
must consider the dashed line, labelled $U_1/1.1$. The discounted user
cost schedule is obtained by discounting the intercept of $50$ to give a
$45.45$ intercept. Likewise the slope is discounted by $1.1$, giving a
slope of $-0.3$. As a consequence, $U_1/1.1$ shows, for any output level, the
original market price in period $1$ divided by $1.1$, the discount factor.

According to equation (3.5), the intersection of $U_0$ and $U_1/1.1$ shows
the optimal output allocation between the two periods which equates
the discounted user costs. Furthermore, by reference to the demand
schedules for these output levels, we obtain the solution for the prices
in the two periods. According to Figure 3.2, $28.2$ barrels will be pro-
duced in the first period and $31.8$ barrels will be produced in the fu-
ture. Corresponding to $28.2$ barrels in the current period is a market
price of $35.90$. Corresponding to the $31.8$ barrels produced in the fu-
ture period is a market price of $39.50$ per barrel—a price exactly $10
percent higher than the $35.90$ price in the current period.\(^4\)

It is easy to see that no other output combination and price path
will bring about a long-run profit maximizing equilibrium. Consider
an output level of less than $28.2$ barrels in the current period. At that
output combination, $U_0$ exceeds $U_1/1.1$, which will encourage produc-
ers to increase current production and reduce future consumption.
But the increase in current production only lowers $U_0$ and raises
$U_1/1.1$. Nevertheless, producers will find it profitable to expand cur-
tent production until $U_0 = U_1/1.1$. Similarly, one can show that if cur-
tent production were greater than $28.2$ barrels, market forces would
drive current production down to the optimal level.

The graphical analysis in Figure 3.2 serves to emphasize the numer-
ous determinants of the user costs and price path. As noted earlier,
both future and current demand expectations are important. Marginal
production costs are also potentially important. Also, if oil reserves
had been $100$ instead of $60$, the user costs would have been much
smaller, and the price path would have started from a lower initial
price.

1. What three assumptions give rise to equation (3.11a)? Explain the
role of each.

2. Would you think that Hotelling's rule would apply to coal mining?
Why or why not?

3. Suppose in Figure 3.2 that expectations about future demand im-
plied a $10$ higher price at every output level. Show graphically
how you would solve for the new optimal output levels and price
path.

4. Using Figure 3.2, and assuming instead that reserves were $100$ bar-
rels, how would you solve for the new optimal output levels and the
price path?
5. Assuming a constant demand schedule over time, what would the output price path look like, that corresponds to the rising price path? Draw the output path.

THE PREDICTIVE POWER OF HOTELLING'S MODEL

Critics of Hotelling's model are quick to point out that observed oil prices do not move along a smooth price path, rising with the rate of interest. Table 1.4 and Figure 4.1 show that the path of oil prices bears little resemblance to Hotelling's predictions.

Before discarding Hotelling's simple model, let us first recall the three critical assumptions: (1) long-run profit maximization, (2) zero marginal production cost, and (3) perfect competition. Finally, there is the hidden caveat that any one price path is conditional upon a unique set of expectations about future and present supply and demand conditions. This last caveat alone might explain why prices failed to move along a smooth path. For example, if producers were continually revising their expectations, one might observe frequent jumping from one price path to another. Still other explanations will result from relaxing the assumptions of zero production costs or perfect competition. Let us proceed to extend Hotelling's simple model by considering the effects on the observed oil price path of: (1) a monopolistic market, (2) the size of the resource base, (3) the presence of a backstop fuel, (4) the rate of discount, (5) the magnitude of the long-run price elasticity of oil demand, (6) the rate of world economic growth, and (7) increasing marginal production costs.

A Monopolistic Market Structure

For the monopolist facing zero production costs, user costs will be identical to marginal revenue, which then must rise with the rate of interest:

\[
(3.12) \quad MR_0 = \frac{MR_1}{1 + r} = \frac{MR_2}{(1 + r)^2} = \frac{MR_3}{(1 + r)^3} = \cdots = \frac{MR_n}{(1 + r)^n}
\]

This result follows directly from equation (3.3) when \(MC^p = 0\). Obviously, with marginal revenues rising, prices will also rise, but the rate of increase depends on the nature of the demand curve. In the usual textbook case of a linear demand curve, the initial price will be higher under monopoly and will rise at a slower rate relative to perfect competition. The reason why the price of the monopolist lies above that of the competitive market, and grows more slowly, may not be apparent. Remember that with zero extraction costs under mo-
monopoly, user costs equal marginal revenue, not price, and thus marginal revenues must grow at rate $r$ as in equation (3.12). With a linear demand schedule, an $r$ percent increase in marginal revenue yields a less than $r$ percent increase in price. Thus if prices are to increase at a slower rate than under competition, they must be initially higher. Figure 3.3 contrasts the competitive price path with the monopolist’s price path when facing a linear demand schedule. Thus, the monopolist’s higher initial price will promote more conservation in earlier periods, allowing for relatively greater production and lower prices in the later periods, extending the economic life of the resource. This example supports the claim, “the monopolist is the conservationist’s best friend.” While this may be true, we simply note that the monopolist exacts a huge fee for performing this rationing function and that the monopolist’s price path distorts the optimal intertemporal resource allocation of perfect competition. From society’s perspective, the present value of lower prices over period 0 to $t_0$ in Figure 3.3 far outweighs the present value of the monopolist’s lower prices after time $t_0$. But of course this explains precisely why the monopolist chose an initially higher but more slowly increasing price path.

Still another intriguing aspect of Hotelling’s 1931 paper was that he explicitly considered the case of the monopolist facing a demand curve that had a constant price elasticity at every price. Hotelling demonstrated that under such conditions, the monopolist’s price path will be identical to that of the competitive market! Some scholars have accepted Hotelling’s result as if it were a prophecy. To them, the question of whether OPEC is a cartel is moot—the price of oil would be the same under either regime!

To reach this conclusion based on Hotelling’s simplistic model would be to commit an egregious error, for two reasons. The demand for oil is not a static function with a constant price elasticity. As dis-

---

**FIGURE 3.3** Possible monopolistic versus competitive price paths.
cussed in Chapter 7, the long-run demand schedule is much more price elastic than its short-run counterpart. Since it may take twenty years to achieve the full long-run price adjustment, a monopolist can exploit the short-run inelasticity of oil demand by initially charging a higher price. Thereafter, prices may even decline for a substantial period as consumers react to the price jump. Thus, the nature of the demand for oil suggests that the monopolist's price path is likely to be more like Figure 3.3, with a higher price initially and a slower rate of price increase thereafter than would occur under competition.\(^5\)

Notwithstanding these objections, some may argue that even if the two price paths are not identical, the period of time \(t_0\) in Figure 3.3 during which the monopolist's price exceeds the competitive level is relatively short and the period thereafter of lower prices under monopoly is relatively long. Applying this logic to today's situation, one might argue that OPEC's prices may exceed that of a competitive market, but within ten years we will reach a time when the price of oil will be actually lower, thanks to OPEC. Obviously, it matters greatly whether \(t_0\) is 10 years or 50 years. Unfortunately, our ability to pinpoint \(t_0\) is poor. Furthermore, it matters greatly whether some "backstop" fuel will be present to place a ceiling on future price increases. (A backstop fuel is a fuel available in effectively unlimited quantities at a constant extraction cost per unit.) If a backstop fuel exists, the high future prices that would occur under competition would not materialize. We will explore in more detail later the price implications of a backstop fuel.

**The Size of the Resource Base**

Irrespective of whether one posits monopoly or competition, user costs play an influential role in oil price determination. Since user costs are based on expectations of present and future supply and demand conditions, it is instructive to look in greater detail at factors that influence producers' perceptions of user costs.

In order to attribute a scarcity premium to oil, oil producers must first form expectations about the magnitude of the underlying oil reserve base. The size of existing oil reserves may be a poor indicator, since these are only the reserves found to date and future exploration will surely result in new discoveries. If new discoveries occur as expected, producers need not revise their estimates of user costs. Figure 3.4 illustrates a Hotelling-type competitive market with zero production costs. Over the period that producers' expectations of user costs remain unchanged (period \(0\) to \(t_0\)), the oil price rises at the rate of interest. Suppose that at time \(t_0\), geologists sharply increase their estimate of the reserve base. The scarcity value of oil being thus reduced, user costs will be revised downward sharply. Following the revision, as long as expectations are unchanged (period \(t_0\) to \(t_1\)), prices
again rise at the rate of interest. Now assume that in period $t_1$, oil producers become convinced that the ultimate resource base is much smaller than they had ever thought. Prices will immediately shoot upward and thereafter rise with the rate of interest until expectations are revised again. Figure 3.4 illustrates the interesting point that prices need not follow a smooth price path since changing expectations will cause switching to alternate price paths. Thus Hotelling's model may be correct, but have only limited predictive power because prices are continually jumping from one price path to another due to changing expectations.

The Presence of a Backstop Fuel

In view of the vast potential supplies of unconventional crude oil from tar sands, oil shales, and coal, economists have been prompted to consider the impact their development will have on the price of petroleum. For simplicity, let us assume that the reserves of these oil substitutes become infinitely elastic at some price $P^*$. Obviously, these resources are also nonrenewable, but the reserve base may be so large that their user costs are effectively zero. Alternatively, the backstop fuel may be a renewable energy source such as solar energy. In either case, at price $P^*$, virtually unlimited supplies will be available. As Figure 3.5 indicates, the time path of oil price is substantially altered. No longer does the price continue to rise indefinitely at the rate of interest. The solid line price path depicts a world of perfect foresight. The price increases at rate $r$ until it reaches $P^*$, at which time the backstop fuel would enter the market to meet all demand at the price $P^*$. Presumably, backstop fuel producers watch the price rise and correctly anticipate that the backstop fuel plants should be ready in year $t_0$ with sufficient capacity to meet expected demand. Also, according to this view, oil producers would want to dispose of all of their oil before $t_0$. 

CRITERIA FOR EFFICIENT DYNAMIC RESOURCE ALLOCATION
since after \( t_0 \) the user cost would become constant. In reality, conventional oil production will continue past \( t_0 \) because oil fields cannot be exhausted instantaneously. Also, it seems plausible to conjecture that oil prices might even overshoot the price of the backstop fuel, \( P^* \), if at \( t_0 \) the introduction of the backstop fuel is initially insufficient to meet market demand. The dotted line occurring after \( t_0 \) shows how prices might temporarily overshoot while the backstop fuel industry is adjusting to meet demand (period \( t_0 \) to \( t_1 \)). After \( t_1 \), oil prices would be constrained to \( P^* \).

What implications do backstop fuels have for the pricing of oil today? Assume for argument’s sake that shale oil and coal liquefaction exist as backstop fuels for crude oil at a constant cost of $70 per barrel (i.e., suppose \( P^* = 70 \)). Let us further assume that existing crude oil reserves are sufficient for 30 years (\( t_0 = 30 \)). What is the appropriate price of oil today? Applying Hotelling’s model with \( P_{30} = 70 \), we arrive at a current price in period 0 in Figure 3.5 of $4.01 for a 10 percent discount rate.

\[
P_0 = \cdots = \frac{P_{30}}{(1.1)^{30}}
\]

(3.13)

\[
P_0 = \frac{70}{17.45} = 4.01
\]

For a discount rate of 5 percent, the current price would be $16.20. Calculations such as these are obviously highly subjective and inexact, but even with highly conservative assumptions about the backstop fuel price and years of available oil reserves, world oil prices appear to be well above the levels implied by a competitive market.
To Hotelling, the choice of the appropriate discount rate was obvious—use the market rate of interest. Since inflation was not a problem in 1931, Hotelling was content to state his results in nominal terms, not adjusted for inflation. Today, future inflation rates are perhaps more uncertain than oil prices, causing practitioners to favor oil price forecasts expressed in dollars of constant purchasing power. Thus, the preference is to forecast the real price of oil, that is, the future price of oil deflated by the future general price index. The real rate of interest \( r^* \) is simply the nominal rate of interest \( r \) less the expected inflation rate \( i \), as shown below:

\[
r^* = r - i
\]

Hotelling’s framework, nevertheless, remains valid. We simply substitute the real rate of interest for the nominal rate of interest and interpret oil prices in dollars of constant purchasing power.

Having resolved the inflation confusion, one must ask whether the real rate of interest, usually estimated at 2 to 3 percent per annum, is the appropriate real discount rate. If oil reserves were a riskless asset, private investors would utilize the real rate of interest since it reflects the real, long-run return on a risk-free asset. Oil producers usually argue that geological and political risks of oil exploitation require using a much higher real discount rate to reflect the elements of uncertainty and guarantee a normal real return on the average. Professor Adelman points out that the political risks are so large in the Middle East that a real discount rate of 9 percent may be conservative.\(^6\)

To illustrate the effects of changes in the real discount rate, Figure 3.6 depicts price paths under three alternative discount rates. From

\[\text{FIGURE 3.6 Price paths under alternative discount rates.}\]
period 0 to \( t_0 \), producers employ a very high real discount rate of 25 percent in anticipation of nationalization. Now suppose the risk of nationalization abates and the real discount rate is reduced to 5 percent. Finally, in period \( t_1 \) the state obtains control of production and adopts a zero real discount rate. With a zero real discount rate, the user costs are constant over time. If a backstop fuel were available at \( P^* \), the competitive price would jump to \( P^* \) and remain forever at that real price level. After all, a zero discount rate implies indifference between a dollar today and a dollar in 30 years, when the backstop fuel would be available.

The Magnitude of the Long-Run Price Elasticity of Oil Demand

Since demand conditions affect marginal revenues and thereby the user costs, the price path even in a competitive market depends on the price elasticity of demand. In the short run, the price elasticity for crude oil is generally known to be quite inelastic. The magnitude of the long-run elasticity is known with much less certainty. The problem is exacerbated by the fact that a substantial adjustment period is required to alter the energy efficiency of the existing capital stock. Long lags, combined with unprecedentedly high price levels, make for a great deal of uncertainty regarding the value of the long-run price elasticity. Figure 3.7 contrasts two price paths, one under high elasticity expectations and one under low price elasticity expectations. With expectations of a low price elasticity, the price path begins at price \( P_0 \) and rises thereafter at the discount rate as usual. Suppose that in period \( t_0 \) producers come to believe that long-run demand is much more price elastic than previously believed. Clearly, at the prices projected

\[ \text{FIGURE 3.7 The importance of the price elasticity of demand.} \]

\[ 84 \text{ ENERGY ECONOMICS AND POLICY} \]
for the future, consumers will not demand the previously projected production. This revision in expectations about future price elasticities of demand forces an abrupt downward adjustment in the user cost. Even though the price rises at the discount rate after $t_0$, the base from which it rises is much lower and the price path lies below the original price path.

The Rate of World Economic Growth

Still another factor influencing the calculation of user costs is the growth in oil demand resulting from world economic growth. Besides price, the major determinant of oil consumption is the level of economic activity. As discussed in Chapter 7, there is a strong positive correlation between the standard of living and energy consumption. The point is that if producers expect rapid long-run growth rates, ceteris paribus, user costs will be higher. Although Figure 3.7 illustrated the case of low and high price elasticity expectations, it could have just as easily represented the effects of different expectations about economic growth rates. The period from 0 to $t_0$ would be representative of expectations assuming high economic growth rates, while the period from $t_0$ into the future would signal a period in which economic growth projections had been scaled downward.

Increasing Marginal Production Costs

Even though Hotelling’s assumption of zero marginal production cost seems quite applicable to oil produced in the Persian Gulf, a close look at oil production elsewhere reveals that marginal production costs increase over a broad output range. In a given oil province, the lower cost deposits are exploited first, leaving additional reserves to be found at higher prices. As the production from stripper wells across Illinois and Kansas proves, at higher prices it is always possible to wring additional oil from the earth’s coffers.

A confusing aspect of existing oil reserve estimates is that they are classified as “known” reserves capable of being extracted at present prices and technology. They provide no estimate of the reserve base available from additional exploration at existing prices and technology, nor any measure of the size of uneconomic reserves that may become economic with higher prices or improved technology.

Suppose we assume that additional reserves and production will be forthcoming at higher prices. What effect will this have on the price path? Figure 3.8a assumes for pedagogical purposes that reserves are of three types. Type 1 reserves are limited to quantity $Q_1^*$, and, like Persian Gulf reserves, are available at a negligible extraction cost. Type 2 reserves are available in quantity $Q_2^* - Q_1^*$ at a marginal extraction cost of $C_2^*$ per unit. Finally, we assume a renewable re-
source (type 3 reserves) that offers a backstop fuel available at cost $C_3^*$ per unit.

As in Figure 3.5, will the price path rise at $r$ percent until it hits the backstop? No. As shown in Figure 3.8b, prices rise at a relatively much faster rate during the period from 0 to $t_1$ (when the lowest cost reserves are depleted). Over this period extraction costs are zero, user costs are equivalent to price, and therefore price will rise relatively quickly. The new price path beginning at $t_1$ and ending at $t_2$ rises at a much slower rate than the price path over the period 0 to $t_0$. The backstop fuel price is reached in period $t_2$ just as type 2 reserves are ultimately depleted, and backstop fuel production commences. Prices escalate at a slower rate in each subsequent era because extraction costs become a bigger part of marginal costs. For the period from $t_1$ to $t_2$, user costs are only a portion of total marginal costs, and prices rise only to reflect the increase in user costs. Even though the user cost for type 2 reserves will escalate at $r$ percent per year, prices will rise only at a fraction of that rate because extraction costs, which are constant at $C_2^*$, make up such a big fraction of overall marginal costs and hence, price in a competitive market. Of course, after $t_2$ the backstop is reached and user cost for reserve type 3 is zero. Thus without any user costs, prices will be constant over time at $P_3 = C_3^*$.

How can we generalize the results of Figure 3.8b? Suppose that instead of three types of reserves, there were numerous reserve types. Prices would be expected to rise at a rate well below the rate of interest as, over time, extraction costs make up increasing fractions of the price. Furthermore, if technological change reduces the marginal production costs over time, it is even possible that the reduction in $MC^p$ could exceed the rise in $U$, so that total marginal costs and price might actually decline. Thus user costs may not be a large portion of overall marginal costs and thereby not greatly affect resource price patterns. Many energy economists feel that it was this zero extraction cost assumption that led Hotelling so far astray.7

86  **ENERGY ECONOMICS AND POLICY**
1. Why will prices jump to the backstop fuel cost if \( r = .0 \)?

2. Explain why \( P_0 = 16.20 \) if \( r = .05 \) in equation (3.13). If production was sufficient for 30 years along the price path beginning at \( 4.01 \) per barrel, why will it take longer to reach the backstop moving along a higher price path? Supposing that at a 5 percent discount rate it takes 40 years, show that \( P_0 = 9.94 \).

3. Using Figure 3.8b, explain why the lowest cost reserves will always be exploited first. Also explain why no one owning type 1 reserves would choose to hold them past period \( t_1 \).

**Market Failures in a Dynamic Context**

The previous discussion sets forth the criteria for efficient resource allocation over time, regardless of whether this goal is to be achieved by market forces or by central planning. Given the primary reliance on market forces in Western countries, it is appropriate to outline the conditions under which intertemporal market failures could occur, opening the door for possible inefficient or nonoptimal intertemporal resource allocation. Once having isolated these conditions, policymakers have a basis from which to argue for public intervention to correct any such failures.

The standard types of static market failures outlined earlier, such as monopoly and externalities, may also occur in a dynamic framework. Since their resource allocation effects are similar, there is no reason to restate them here and provide dynamic illustrations. The question is whether the dynamic framework of analysis provides additional possibilities for market failures. These possibilities center on the question of the social rate of discount. Here there are two possibilities for market failure. First, market rates of interest, which guide firms' calculation of values, may differ from the social discount rate, leading to an inefficient allocation of resources over time. The second possibility is that social and market interest rates may be equivalent and yet the firm may select its own private discount rate that exceeds the social rate of discount.

**If Market Interest Rates Differ from the Social Rate of Discount**

One potential source of dynamic market failure arises when firms discount future user costs at market interest rates that differ from the social rate of discount. Typically, the assertion is that market rates of interest exceed the social rate of discount, implying that future user costs are lower than they would otherwise be, thus causing overpro-

**Criteria for Efficient Dynamic Resource Allocation** 87
duction of resources in the current period, and correspondingly, underproduction in future periods.

On the face of it, it would seem that the market rate of interest must reflect the social discount rate because the market rate of interest reflects society's rate of time preference. A common result from most principles textbooks is that the rate of interest is determined by the intersection of the savings schedule and the marginal efficiency of investment schedule. The marginal efficiency of investment schedule is obtained by ranking the investment possibilities according to the rate of interest that yields a zero present value. On the savings side, consumers compare current consumption with future consumption, concluding that the ratio of the marginal utility of current consumption to the marginal utility of future consumption is \(1 + r\), where \(r\) is the social rate of time preference. The problem is that while the rate of interest will reflect the existing society's social rate of discount, it makes no allowance for future generations, who are not present to express their social rate of time preferences.

Noneconomists tend to be particularly critical of the discounting of future consumption because it places a greater value on present consumption than future consumption. To illustrate the small weight given future generations by the discount process, the present value of one dollar 50 years from today is worth less than 1 cent today, assuming a 10 percent discount rate. Examples such as these prompted the well-known British economist A.C. Pigou to conclude that the whole process of discounting is morally indefensible. To Professor Pigou, the appropriate rate of social discount is zero. The value of one unit of a nonrenewable resource to persons living in a future age when the resource has been physically exhausted may well be infinite, if no substitutes are then available; even if its value is finite (and possibly quite large), it should not be discounted by current consumers at any rate.

Basically, three separate arguments have been set forth in defense of using market interest rates as the appropriate rate of social discount. First, the example of a dollar in 50 years being worth less than 1 cent today overstates the degree to which future generations are disregarded. In the usual case, the appropriate discount rate is a real discount rate, which for a riskless asset is around 3 percent. Thus, applying a 3 percent rate instead of a 10 percent discount rate, the value of $1 in 50 years is $0.23 rather than $.01. Thus, when performed properly, discounting may be a bit nearsighted, but it is not blind.

The second argument in favor of utilizing market rates of interest is based on past experience. Presumably, one consequence of using the market rate of interest to discount future consumption is that future generations will eventually be left with a world of increasing resource scarcity and associated falling per capita incomes. If discounting is morally indefensible, it must be because of its deleterious effects on
future generations. Is the present generation the inheritor of an empty resource vault caused by the profligate ways of past generations?

It is likely that the highest grade ores, the largest oil fields, and the lowest cost coal deposits have already been exploited, as theory would predict. In terms of productive factors other than raw materials, previous generations have bequeathed to us more knowledge, better educational facilities, and high-technology capital equipment. Paradoxically, even in terms of nonrenewable resources, technology seems to have provided economic abundance in the face of increasing physical depletion. Even though the quality of the in-situ resources exploited today is far poorer than those of a century ago, the cost of producing the associated resource-using final goods has declined over time. Barnett and Morse\textsuperscript{10} conducted a study of 29 important nonrenewable resources over the period 1870–1957. They found that, when corrected for inflation, the prices of all but a few of these resources have actually fallen over this period. Data for more recent periods analyzed by V. Kerry Smith indicate there is no strong evidence supporting either secularly rising or declining relative resource prices.\textsuperscript{11} Margaret Slade found that for 11 nonrenewable commodities the former tendency for prices to decline seems to hold no longer as prices have stabilized and in some cases increased.\textsuperscript{12} It seems clear that technology has for some time run depletion a strong race. In sum, as long as each generation provides future generations with the technology to produce energy-intensive goods at current or even moderately higher real prices, there would appear to be no compelling argument against the current practice of discounting future consumption by the market rate of interest.

The third argument is more a defense of the status quo method of allocating resources than one which argues that the social rate of discount equals the market rate. In most countries, the rights to subsurface minerals belong to the government rather than to private interests. As a consequence, a part of the resource base is not necessarily subject to the use of market rates of discount. To the extent that the rate of discount applied by the governments in developing their natural resources is less than the market interest rate, production is deferred for future generations. There is the danger that the government may select such a low social rate of discount that when the resources are ultimately developed, technology will have provided a cheaper substitute. However, if the government rate of discount is only slightly below the market rate of interest, this may indicate a desirable negative risk premium, particularly if society is risk-averse with respect to future unanticipated shortages. Since firms maximize the expected present value of a resource, the market rate of interest may not reflect this risk aversion. A system of private ownership of part of the resource base together with government ownership of the remainder

CRITERIA FOR EFFICIENT DYNAMIC RESOURCE ALLOCATION
may lead to some average rate of discount between the market rate of interest and the government rate of discount. The resulting average rate may be more appropriate, especially to the degree that it more properly reflects the risk aversion phenomenon.

Perhaps the greatest threat to efficient resource allocation is governmental decisions based neither on market rates of interest nor on a legitimate measure of the social rate of discount. Political exigencies may dictate either premature development of resources or underexploitation of resources. History suggests that bad decisions are not restricted to dictators and ill-informed bureaucrats. For example, in the late nineteenth century, there was much concern in Britain that the continued exploitation of British coal would leave the country without any coal reserves, facing much higher future energy costs. The famous British economist William Stanley Jevons had this to say on the coal question in 1866.13

It is shown that we owe almost all our arts to continental nations, except those great arts which have been called into use here by the cheapness and excellence of our coal. It is shown that the constant tendency of discovery is to render coal a more and more efficient agent, while there is no probability that when our coal is used up any more powerful substitute will be forthcoming . . . If we lavishly and boldly push forward in the creation and distribution of our riches, it is hard to overestimate the pitch of beneficial influence to which we may attain in the present. But the maintenance of such a position is physically impossible. We have to make the momentous choice between brief greatness and longer continued mediocrity.

Had Britain opted to conserve its coal at that time, would it have been better off? History suggests quite the opposite. Those in Jevons’ generation would have faced much higher prices for imported coal and/or a slower rate of economic development. Even future generations would probably not have benefitted. It is likely that even today the cost of delivered coal from the technically progressive U.S. mines would be less than that of the “low” cost British reserves of the nineteenth century. In addition, British oil from the North Sea probably dominates both of these alternatives.

**If Private Discount Rates Exceed the Market Rate**

Assuming that the social rate of discount is approximated by the market rate of interest, let us examine the possibility that the firm’s private rate of discount exceeds the market rate of interest. One might conclude this is impossible, since another firm would acquire the resource because it would be willing to pay a higher price. Because the acquiring firm is using the market rate of interest to discount future profits rather than the higher discount rate of the firm owning the
resource, the present value of the resource is higher to the acquiring firm. Therefore, can we dismiss this situation as an impossibility? Yes, assuming the costs of transferring the resource do not exceed the value differential.

The question is, under what circumstances would all firms be unwilling to apply the market rate of interest as a discount factor? The answer pertains to any risk premium that may be inherent in the peculiar nature of the resource. To the extent that the private and social risk premiums are equivalent, it is entirely appropriate to adopt a discount rate in excess of the market interest rate. Such a risk premium might arise if future technological change might make the resource worthless. Thus, whether the resource is owned privately or publicly, the same risk premium would be appropriate.

There are situations where there is a private risk premium, but no corresponding social risk premium. Consider the case of an oil company operating in a country subject to revolutions. Recognizing that a change in government could mean nationalization, the firm is likely to adopt a private rate of discount well above the market rate of interest. From that firm's perspective, or any other firm's perspective, not to make some allowance for such eventualities would display very poor business acumen. Nevertheless, from the perspective of the inhabitants of the country in question, the effect is to accelerate current production in excess of the socially optimal level. Over the longer term, the total amount of foreign and/or private investment is reduced, so that there is an underexploitation in the long run. Curiously enough, the dilemma would not necessarily be solved by transferring the development of the resource base to the government in power. To the extent that each successive and typically short-lived government maximizes its own wealth, similar short-run overproduction and long-run underinvestment occurs. Regardless of the solution, it is clear that the citizens of countries subject to such political instabilities pay a high price in the short-run overexploitation of the country's resource base and long-term underdevelopment of its resources.

Myopic exploitation of resources in which user costs are effectively disregarded is not limited to politically unstable countries. The institutions determining property rights are vital. In the United States, subsurface mineral rights are generally privately owned. Therefore, it is common to find many oil producers operating in one oil field. Furthermore, with a legal system determining ownership by the rule of capture, producers are entitled to all the oil their wells can produce. Thus, unrestrained competitive behavior in the 1930s led producers to disregard user costs. After all, a barrel not produced today might not be available for capture by the same producer tomorrow. The results were massive overdrilling and overproduction, often permanently damaging the oil reservoir. Later, in Chapter 7, we return to this fascinating form of market failure.

CRITERIA FOR EFFICIENT DYNAMIC RESOURCE ALLOCATION
AN ECONOMIST'S PRESCRIPTION FOR EFFICIENT ENERGY RESOURCE ALLOCATION: A PRICING PROBLEM

The previous discussions of static and dynamic criteria for efficient resource allocation emphasize that the key to efficient resource allocation is the way in which energy is priced over time. At any point in time, the price of the energy resource should reflect its marginal social costs. This implies that any divergence between social and private costs arising as a consequence of public goods or other externalities should be corrected by internalizing such external costs. In terms of market structure, the equivalence of price, reflecting marginal social benefits, and marginal social costs requires a competitive market framework. Monopoly elements must be eliminated if prices are to equal marginal social costs.

In order for resources to be allocated efficiently over time, it is furthermore necessary that the user cost or opportunity cost of selling a resource in some future period must also be included in the marginal social cost. Even though this user cost does not involve an out-of-pocket payment such as production costs, it does reflect a real resource cost in that the resource could be used alternatively in the future.

We saw that the economic definition of conservation, rather than implying a simple reduction in the current rate of resource exploitation, implies that the user costs of a resource should rise over time at the social rate of discount. Therefore, over time, prices should rise to reflect these user cost changes. If they do not, resource allocation is not efficient over time. Based on currently available evidence, the market rate of interest offers a sufficiently close approximation to the social rate of discount. To the extent that nondiversifiable risks are present, the rate of discount should be adjusted to reflect these differences.

At first it might seem strange to tell the student to examine whether energy prices are efficient. There is, after all, a tendency to focus on the quantity of energy production, arguing that some fuels are being overproduced while others are underproduced. By focusing on quantities, we too often forget that we are trying to achieve that output combination where the marginal social benefit of the last unit produced equals its marginal social cost. By first looking at prices we are obliged to measure marginal social costs and benefits. Therefore, we view efficient energy allocation as a pricing problem. If energy is priced such that the marginal social benefits equal the marginal social costs both statically and dynamically, we need not be concerned with
actual consumption and production levels. The socially optimal quantities of energy production and consumption are the byproducts of efficient prices. This interpretation is followed subsequently throughout this book.

1. One can assume that the salvage value of the firm is included in profits in period $n$.

2. A more elegant proof involves the solution to a constrained optimization problem assuming that there are only 60 barrels available:

$$\max_{Q_0, Q_1} \Lambda = P_0 Q_0 - C(Q_0) + \frac{1}{1 + r} [P_1 Q_1 - C(Q_1)]$$

subject to $Q_0 + Q_1 = 60$, or equivalently, maximize the Lagrangian expression,

$$L = P_0 Q_0 - C(Q_0) + \frac{1}{1 + r} [P_1 Q_1 - C(Q_1)] - \lambda (Q_0 + Q_1 - 60)$$

The first-order conditions are:

(i) $\frac{\partial L}{\partial Q_0} = U_0 - \lambda = 0$ [where $U_0 = \frac{\partial (P_0 Q_0 - C(Q_0))}{\partial Q_0}$]

(ii) $\frac{\partial L}{\partial Q_1} = \frac{1}{1 + r} U_1 - \lambda = 0$

(iii) $\frac{\partial L}{\partial \lambda} = -Q_0 - Q_1 + 60 = 0$

By setting (i) equal to (ii), we can prove that

$$U_0 = \frac{1}{1 + r} U_1$$

Note also that this formula combined with equation (iii) gives a two-equation system with two unknowns, $Q_0$ and $Q_1$, that can be solved to obtain optimal production rates and prices. Another interesting point is that the Lagrange multiplier $\lambda$ represents the present value of the user cost in each period (i) and (ii).

4. In solving mathematically for the optimal output levels, we use the following two facts: (1) The present value of user costs must be equal in both periods, and (2) only 60 barrels will be produced in the two periods. (See Note 2, above.) These facts yield two equations with two unknowns:

\[ U_0 = \frac{1}{1 + r} U_1 \]
\[ MR_0 - MC_0 = \frac{1}{1.1} (MR_1 - MC_1) \]
\[ 50 - .5Q_0 = \frac{1}{1.1} (50 - .33Q_1) \]

(i) \[ -.5Q_0 + .33Q_1 = -5 \]

(ii) \[ Q_0 + Q_1 = 60 \]

Solving (i) and (ii) simultaneously, one obtains the optimal output levels \( Q_0^* \) and \( Q_1^* \) as follows:

\[ Q_0^* = 28.2 \quad \text{and} \quad Q_1^* = 31.8 \]

Then substituting \( Q_0^* \) and \( Q_1^* \) into the demand equations, optimal prices \( P_0^* \) and \( P_1^* \) can be solved for:

\[ P_0^* = 50 - .5(Q_0^* = 28.2) = 35.90 \]
\[ P_1^* = 5 - .33(Q_1^* = 31.8) = 39.50 \]


