Price Controls and Rent Dissipation with Endogenous Transaction Costs

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When the price of a good is controlled and the allocation rule is first-come, first-served, consumers compete to be first by arriving before the market opens and forming a queue. The waiting time rises until, in equilibrium, the money-plus-time price clears the market.¹ This characterization of rationing by waiting, together with the observation that waiting dissipates the rent that a price ceiling extracts from suppliers, is familiar from such textbook treatments of the subject as Donald McCloskey (1982 pp. 338–46). This is not the end of the story, however. When confronted by queues, consumers will engage in activities that economize on waiting costs. While such rational behavior would seem to mitigate the dissipation of rent, we find that the opposite can occur. Pursuit of such activities can add to the welfare cost of rationing by waiting.

Our principal results can be illustrated by a simple example with 100 identical consumers and a good that is fixed in supply at 100 units per day. Without a price control, the equilibrium price is $10 per unit, and each person consumes one unit per day. Consumers purchase the good at a retail store, and each purchase takes time. This time cost can be reduced by shopping infrequently and buying a large amount per trip, but infrequent trips imply a storage cost. The consumer's optimal purchase size minimizes the sum of shopping and storage costs. Assume that the optimal purchase size is two units per trip, so each consumer shops every other day. Consequently, the store has 50 customers per day, each buys two units, and the total demand is 100 units per day.

A price ceiling is now placed on the good, lowering its price to $5 and increasing each consumer's demand. If purchase size is held constant at two units, each consumer must shop more frequently, and the store will have more than 50 customers per day. Since supply remains fixed, some demands cannot be met. Suppose the allocation rule is first-come, first-served, so that only the demands of the first 50 customers are met. Consumers will now compete to be first by arriving at the store earlier than usual.

How early must a consumer arrive to be among those served? As discussed in Charles Holt and Roger Sherman (1982), arrival times are like bids for the rent created by the price ceiling. Each consumer will choose an optimal arrival time, given the arrival times of all other consumers. The optimal time must be early enough to guarantee that the consumer is served before the supply is exhausted, but not so early as to require unnecessary waiting.

If consumers are indifferent regarding the time of day when they wait, then the waiting time to be served will remain constant throughout the day. For this to occur, consumers must enter the queue at the same rate at which they are served. Suppose it takes five minutes for the store to serve a single customer. Suppose, further, that the equilibrium waiting plus shopping time is 55 minutes. Then, the first customer to be served when the store opens must arrive 50 minutes before opening, the second must arrive 45 minutes before opening, and so forth. The last customer to be served must arrive 55 minutes before the supply is exhausted and the store closes.

In equilibrium, waiting time must be long enough to equate demand for the good to

¹The evolution of queues under first-come, first-served allocations is discussed by Yoram Barzel (1974) and Harold Alderman (1987).
the fixed supply. If all consumers have the same value of time, the good's equilibrium full price (money plus time cost) with the ceiling will equal the full price without it.\(^2\)

The equality between full prices with and without the ceiling determines the equilibrium waiting time. Let the value of time spent shopping and waiting be $0.20 per minute. Without the ceiling, the price of the good is $10.00, and the only time cost is the five minutes it takes to serve a customer. Since each customer buys two units per purchase, the time cost per unit is $0.50, and the full price is $10.50. With the ceiling, the price is $5.00, and the time cost entails the wait to be served as well as the service time. Let \(w\) be the waiting time in minutes, so the time cost per unit is $0.50 + $0.10w, and the full price is $5.50 + $0.10w. For this full price to equal $10.50 (the full price without the ceiling), waiting time must rise to 50 minutes, making the cost of waiting $5 per unit.

Consumers are no better off with the ceiling than without it, because the full price is the same in either situation. Suppliers are worse off by $5 per unit, and this represents a deadweight loss. In effect, the price ceiling transfers $5 per unit from suppliers to consumers, but this rent is dissipated as consumers compete for it. This is the standard analysis of the deadweight loss of rationing by waiting.\(^3\)

The analysis is incomplete, however, because it does not allow for adjustments consumers are likely to make in response to rationing by waiting. Consider, for example, purchasing four units of the good per shopping trip instead of two. This cuts the per-unit cost of waiting and service in half, from $5.50 per unit to $2.75 per unit, and the full price falls from $10.50 to $7.75. The larger purchase size does increase storage cost, but assume that the value of time saved more than offsets this cost.

If all consumers attempt to realize this gain by doubling purchase size, the gain will quickly evaporate. The resulting full price, $7.75, is lower than the market-clearing price, so demand exceeds supply and the waiting time increases. To reestablish the equilibrium full price, $10.50, the waiting time per purchase must increase until the sum of waiting and service time has doubled. Each consumer makes half as many shopping trips as before but spends twice as much time per trip. The cost of storage is therefore a deadweight loss which must be added to the deadweight loss of waiting.\(^4\)

Intuitively, the additional rent dissipation occurs because the cost of increased storage does not help ration the price-controlled good. In the example, we implicitly assume that the cost of storage does not vary with per-period consumption. Storage cost generally will vary with the size of each purchase (e.g., the cost of a larger refrigerator varies with its capacity), but this does not add to the marginal full price of the good. The consumer who invests in additional storage in order to make larger purchases incurs a fixed cost per period in return for a lower marginal cost for the price-controlled good. A lower marginal cost is not sustainable, however; it results in excess demand, which causes the waiting time per purchase to rise to a new, higher, market-clearing level.

This example illustrates how individually rational adjustments to rationing by waiting...

\(^2\)If the full price with the ceiling were less than the full price without it, each consumer’s demand would exceed one unit per day. The number of customers, each buying two units per trip, would exceed 50 per day, and the stock would be exhausted before all customers in the queue were served. This is not an equilibrium, because those not served would choose to arrive earlier, given the arrival times of others. A symmetric argument demonstrates that the equilibrium full price with the ceiling cannot exceed the equilibrium full price without it.

\(^3\)The central result, that some individuals lose and no one gains from the price ceiling, is a consequence of assuming that all individuals are identical. This simplifying assumption clearly is inappropriate if one seeks to understand why a price ceiling is imposed.

\(^4\)Note that in the new equilibrium, the consumer has no incentive to return to the old purchase pattern of two units per trip and eliminate the added storage cost. In fact, since the time cost per purchase has doubled, the individual payoff to making larger purchases is greater than it was in the original equilibrium.
can be self-defeating. It also suggests that rules prohibiting such adjustments, such as maximum purchase sizes, may be socially beneficial. However, it is only an example. There are other margins customers may adjust in response to increased waiting time. For example, if the wait is sufficiently long, customers may read in line or listen to music. They may even hire others to wait in line for them. Each of these activities is designed to lower the cost of time spent waiting, so the effect of pursuing these activities is to reduce the full price of the good.

Are all such activities self-defeating? To address this question, we develop a general model of transaction cost under rationing by waiting. The model is general enough to include the activities of increasing purchase size and decreasing the opportunity cost of waiting time. The deadweight loss of rationing by waiting was thoroughly analyzed by Yoram Barzel (1974). Our analysis generalizes his by allowing buyers to engage in activities to reduce the cost of waiting; such activities figure in the empirical work of Harold Alderman (1987) and Deacon and Sonstelie (1985, 1989). We abstract from individual differences in demand, a subject treated by H. E. Frech and William Lee (1987) and Wing Suen (1989). Furthermore, we do not consider the use of queues to solve peak-load problems, as in Robert Barro and Paul Romer (1987), nor do we consider the distributional impacts of rationing by waiting, as in Donald Nichols et al. (1971). Finally, we ignore the supply response to a price ceiling by assuming that the quantity to be sold is fixed.

I. Individual Behavior Under Rationing by Waiting

Consider an economy with two goods, a numeraire \( x \) and a good subject to a price ceiling \( y \). There is a single representative agent whose utility function is \( U(x,y) = x + F(y) \), where \( F' > 0 \) and \( F'' < 0 \). The economy's endowment is \( \bar{x}, \bar{y} \) per period.

Purchasing \( y \) involves a shopping cost, which we represent in terms of the time needed to make a purchase. Depending on the nature of the commodity, this might include time spent getting to and from the store, time spent selecting the item from the shelves, and time spent verifying payment. In what follows, shopping time per purchase is assumed to be fixed. Commodity \( y \) is subject to an effective price ceiling, and the allocation rule is first-come, first-served. Competition among consumers to be first results in a queue and, hence, a waiting cost per purchase, as in the preceding example. The cost of time spent shopping and waiting, a cost per purchase, is represented as a claim on the numeraire.\(^5\)

The per-period magnitude of this time cost equals the product of the opportunity cost of time, the shopping plus waiting time per purchase, and the number of purchases per period. The consumer will rationally take actions to mitigate these costs. We refer to these actions as "activities," denoted \( z = (z_1, \ldots, z_n) \), and assume that each involves a per-period cost. The term "transaction cost" is used to denote the value of time spent purchasing the price-controlled good plus the cost of any activities undertaken. The transaction-cost function is denoted \( C(t, y, z) \).

This is our general formulation of transaction cost; specific examples imply particular functional forms. Here are two that we have already discussed.

Example 1: Shopping Less Frequently.—This is the case analyzed in the Introduction. Let \( z \) be a scalar representing the number of units bought per purchase. The number of purchases per period is \( y/z \), and the time cost per period is \( vty/z \), where \( v \) is the value of time. Storage cost per period increases with storage capacity, which equals \( z \); let storage cost equal \( z^a \). Then, the total

\(^5\)If our representative-consumer model is taken literally, no queues would exist. In equilibrium, the consumer always demands the endowment \( \bar{y} \), net transactions are zero, and he need never enter a queue to purchase \( y \). To resolve this seeming inconsistency, assume that the endowment of \( y \) is owned by a representative firm. The representative consumer earns the profit of that firm, \( p \bar{y} \), but must buy its product in the market.
transaction cost is

\[ C(t, y, z) = (vty / z) + z^\alpha. \]

In what follows, it proves useful to assume \( \alpha > 1 \).

Example 2: Reducing the Opportunity Cost of Time.—Waiting in line for any length of time can be irritating, but if the wait is sufficiently long, customers typically find diversions to make the time pass more easily. They may read, listen to music, or even hire someone else to stand in line for them. Each of these examples can be thought of as an activity intended to lower the opportunity cost of time spent waiting. To represent this, let \( v - z \) be the opportunity cost of a unit of time spent waiting. To simplify, assume that customers can buy just one unit per purchase, so the waiting cost per period is \((v - z)(v)\). Increasing \( z \) clearly lowers the cost of time spent waiting. Assume that the cost of activity \( z \) is \( y^\beta z^\gamma \), where \( \beta \) and \( \gamma \) are both positive. The total transaction cost per period is, therefore,

\[ C(t, y, z) = (v - z)ty + y^\beta z^\gamma. \]

In what follows, it proves useful to assume \( \gamma > 1 \).

These two examples illustrate the variety of activities that fall under our general formulation.

Since activities do not enter the utility function, the consumer chooses \( z \) to minimize transaction cost. We represent the solution to this problem with the transaction-cost function:

\[ C^*(t, y) = \min_z C(t, y, z). \]

The cost-minimizing value of \( z \) is denoted by \( z^*(t, y) \).

The total cost per period of purchasing \( y \) equals the transaction cost plus the amount paid to the seller, \( py + C^*(t, y) \), where \( p \) is the price of \( y \). This total, plus the consumer's purchase of the numeraire, \( x \), must be financed from endowed income, which is \( \bar{x} + p\bar{y} \). The consumer's choice problem is, therefore,

\[
\begin{align*}
(2) & \quad \max_{x, y} x + F(y) \\
\text{subject to} & \quad x + py + C^*(t, y) = \bar{x} + p\bar{y}.
\end{align*}
\]

The first- and second-order conditions for an interior maximum are:

\[
\begin{align*}
(3) & \quad F'(y) = p + C^*_y(t, y) \\
(4) & \quad F''(y) - C^*_y(t, y) < 0
\end{align*}
\]

where \( C^*_y \) and \( C^*_y \) are first and second partial derivatives of \( C^* \) with respect to \( y \). The first-order condition requires the marginal benefit of \( y \) to equal the sum of its price and marginal transaction cost.

II. Equilibrium Transaction Time

Equilibrium is established when demand for the controlled good, given its price and transaction time, equals supply. Since price is controlled, it is the transaction time that adjusts to clear the market. Equilibrium transaction time is thus a function of the price of the good. This section discusses the properties of that function.

If \( p \) and \( t \) clear the market, the demand for \( y \) at these values must equal \( \bar{y} \). This implies that the first-order condition for

\[ C^*(t, y) = \min_z C(t, y, z). \]

The second-order condition requires the slope of the demand schedule to be less than the slope of the marginal-transaction-cost schedule, and this is not guaranteed by the assumptions adopted so far. In fact, in Example 1, \( C^*_y < 0 \), so both demand and marginal-cost schedules are downward sloping, and they may cross in the wrong direction. If so, the first-order condition will identify a minimum, and the true maximum may occur at a boundary, where \( x = 0 \) or \( y = 0 \). We ignore this possibility in what follows, however, and assume that the first-order condition identifies an interior maximum.

\[ ^6 \text{It is logical to constrain } z \text{ to be less than } v. \text{ In what follows, we assume that this constraint never binds.} \]

\[ ^7 \text{The second-order condition requires the slope of the demand schedule to be less than the slope of the marginal-transaction-cost schedule, and this is not guaranteed by the assumptions adopted so far. In fact, in Example 1, } C^*_y < 0 \text{, so both demand and marginal-cost schedules are downward sloping, and they may cross in the wrong direction. If so, the first-order condition will identify a minimum, and the true maximum may occur at a boundary, where } x = 0 \text{ or } y = 0. \text{ We ignore this possibility in what follows, however, and assume that the first-order condition identifies an interior maximum.} \]
utility-maximization must be satisfied for y = \bar{y}; that is

\begin{equation}
F'(\bar{y}) = p + C_{y}\left(t, \bar{y}\right).
\end{equation}

This is the equilibrium condition for the price-controlled market. Assume the existence of some initial equilibrium with price \(\bar{p}\) and transaction time \(\bar{t}\). This initial equilibrium could either be controlled or uncontrolled. In the latter case, there would be no queue, so \(\bar{t}\) would equal the time required for shopping.

Assuming \(C_{yt}(\bar{t}, \bar{y}) \neq 0\), equation (5) implicitly defines \(t\) as a function of \(p\). Let \(t^*(p)\) be this function, and note that

\begin{equation}
t^*(\bar{p}) = -1/C_{yt}(\bar{t}, \bar{y}).
\end{equation}

It is natural to presume that \(t^*(\bar{p}) < 0\) (i.e., that a decrease in price increases waiting time). However, in view of equation (6), this is true only if \(C_{yt} > 0\) (i.e., only if an increase in waiting time increases marginal transaction cost).

It is sensible to assume that an increase in the length of the queue raises the marginal cost of the controlled good if the consumer cannot take mitigating actions. This condition is written as \(C_{yt} > 0\) in our notation, and it is satisfied in standard models of rationing by waiting. In the present case, however, the consumer can take actions to mitigate higher waiting costs, and those actions can affect the marginal transaction cost. In Example 1, for instance, an increase in transaction time leads the consumer to buy more per purchase, which tends to decrease marginal transaction cost. Thus, \(C_{yt}^*\) might be negative, even if \(C_{yt} > 0\). If \(C_{yt}^*\) were negative, however, a longer queue would reduce marginal transaction cost and cause excess demand to increase rather than decrease. Rationing by waiting is a stable process only if \(C_{yt}^* > 0\), therefore, and this condition is assumed to hold in what follows.\(^8\) From (6), this condition implies \(t^*(\bar{p}) < 0\), so an increase in the ceiling price reduces the length of the queue.

To establish a benchmark for analyzing the effect of adjustments in transaction activities, let us first fix the vector \(z\) at \(\bar{z}\), its cost-minimizing level in the initial equilibrium. With \(z\) fixed, the transaction-cost function becomes \(C(t, y, \bar{z})\) instead of \(C^*(t, y)\), and we can now repeat the steps above using \(C(t, y, \bar{z})\) instead of \(C^*(t, y)\). The result is an equilibrium waiting-time function, denoted \(\hat{t}(p)\), that applies if activities are fixed. By analogy to equation (6), the derivative of this function is

\begin{equation}
\hat{t}'(\bar{p}) = -1/C_{yt}(\bar{t}, \bar{y}, \bar{z}).
\end{equation}

Now compare the effect on waiting time of a marginal decrease in price in the two situations. When the consumer is free to adjust \(z\), the marginal increase in waiting time is \(1/C_{yt}(\bar{t}, \bar{y}, \bar{z})\); when \(z\) is fixed, it is \(1/C_{yt}(\bar{t}, \bar{y}, \bar{z})\). Allowing \(z\) to adjust acts to increase waiting time if and only if

\begin{equation}
C_{yt}(\bar{t}, \bar{y}) < C_{yt}(\bar{t}, \bar{y}, \bar{z}).
\end{equation}

To evaluate this inequality, recall that \(C^*(\bar{t}, \bar{y})\) was found by minimizing \(C(\bar{t}, \bar{y}, z)\) with respect to \(z\). Thus, in the initial equilibrium

\begin{equation}
C^*(\bar{t}, \bar{y}) = C(\bar{t}, \bar{y}, z^*(\bar{t}, \bar{y}))
\end{equation}

and, by the envelope theorem,

\begin{equation}
C_{yt}(\bar{t}, \bar{y}) = C_y(\bar{t}, \bar{y}, z^*(\bar{t}, \bar{y})).
\end{equation}

Differentiating (10) with respect to \(t\) yields

\begin{equation}
C_{yt}(\bar{t}, \bar{y}) = C_{yt}(\bar{t}, \bar{y}, \bar{z}) + C_{yz}(\bar{t}, \bar{y}, \bar{z}) \cdot z(t)(\bar{t}, \bar{y})
\end{equation}

\(^8\)In the unstable case, equilibrium occurs at one of two possible corner solutions, where either (i) all of the consumer's endowment of numeraire is spent on waiting costs or (ii) the transaction cost \(C^*\) equals \(F(y)\).
where $C_{yz}$ is the $1 \times n$ vector of partial derivatives of $C_y$ with respect to $z_i$, $i = 1, \ldots, n$, and $z^*_i$ is the $n \times 1$ vector of partial derivatives of $z^*_{i_t}$, $i = 1, \ldots, n$, with respect to $t$. Inequality (8) is satisfied, and the consumer’s adjustment in $z$ increases waiting time, if and only if

$$C_{yz}(i, \bar{y}, \bar{z}) \cdot z^*_i(i, \bar{y}) < 0.$$  

The left-hand side of (12) is the effect of adjusting $z$ on the marginal transaction cost of $y$. If the consumer pursues actions, $z$, that decrease marginal transaction cost, the result is an increased waiting time. This finding is summarized as the following proposition.

PROPOSITION 1: Suppose that the ceiling price of $y$ is reduced, causing the time required per transaction to increase. An individually rational adjustment in transaction activities results in a longer equilibrium waiting time if and only if that adjustment reduces the marginal transaction cost of $y$, given the initial waiting time, that is,

$$t^*(\bar{p}) / \hat{t}(\bar{p}) > 1$$

if and only if

$$C_{yz}(i, \bar{y}, \bar{z}) \cdot z^*_i(i, \bar{y}) < 0.$$  

The intuition behind Proposition 1 is illustrated in Figure 1. Marginal transaction cost, $C^*_y(t, \bar{y})$ is graphed against transaction time. Marginal transaction cost is increasing in $t$, as required for stability. The initial equilibrium is at point $e$, where marginal transaction cost equals $F'(\bar{y}) - \bar{p}$. If the ceiling price were lowered to $\bar{p}$, the equilibrium would move to $\bar{e}$, and the waiting time would rise to $\hat{t}$.

Now consider the change in waiting time if the activity vector is fixed at $\bar{z}$. Again, marginal transaction cost is graphed against transaction time, but this time the relevant function is $C_y(t, \bar{y}, \bar{z})$ instead of $C^*_y(t, \bar{y})$. From equation (10), the two functions are equal when $t = \hat{t}$; from equation (11), $C_y(t, \bar{y}, \bar{z})$ crosses $C^*_y(t, \bar{y})$ from below if

$$C_{yz}(i, \bar{y}, \bar{z}) \cdot z^*_i(i, \bar{y}) < 0.$$  

If the latter condition holds, as assumed in Figure 1, a given increase in $t$ will cause a greater increase in marginal transaction cost if activities are fixed than if the consumer can vary them in an individually rational way. Consequently, a given increase in $t$ is more effective at rationing demand if activities are fixed, as opposed to variable, and the waiting time required to restore equilibrium is accordingly smaller.

These two curves can be used to separate the effect of a decrease in the ceiling price into two, distinct steps. In the first step $z$ is held constant at its initial level, and $t$ increases from $\hat{t}$ to $\hat{t}$ to ration excess demand. At waiting time $\hat{t}$, however, the consumer wishes to change $z$, which is the second step. Since $C_{yz}(i, \bar{y}, \bar{z}) \cdot z^*_i(i, \bar{y}) < 0$, the change in $z$ decreases marginal transaction cost; this increases demand, causing excess demand to reappear and making a further increase in waiting time necessary. Thus, the adjustment in $z$ leads to a longer equilibrium waiting time.

If $C_{yz}(i, \bar{y}, \bar{z}) \cdot z^*_i(i, \bar{y}) > 0$, the marginal cost with $z$ fixed crosses $C^*_y$ from above. In the second step, the adjustment in $z$ would increase marginal transaction cost, causing excess supply and a decrease in waiting time. In this case, the adjustment in $z$ causes a shorter waiting time.
The two examples described earlier illustrate these two cases. In Example 1, the consumer mitigates higher waiting cost by increasing the amount bought per purchase. The transaction-cost function is

\[ C(t, y, z) = \left( \frac{vy}{z} \right) + z^a \]

where \( z \) is the amount bought per purchase, \( v \) is the value of time, and storage cost equals \( z^a \). The cost-minimizing value of \( z \) is

\[ z^* = \left( \frac{vy}{\alpha} \right)^{1/(1+a)}. \]

If \( z \) is fixed at \( \bar{z} \), the effect of a change in transaction time on marginal transaction cost is

\[ C_{yt} = v / \bar{z}. \]

If the consumer is allowed to make cost-minimizing adjustments in \( z \) when transaction time increases, the effect on marginal transaction cost is \( C^*_{yt} = C_{yt} + C_{yz} z^* \), from equation (11). Since \( C_{yz} = -v t z^2 < 0 \) and \( z^* = z^*/(1 + \alpha) t > 0 \), the adjustment in \( z \) decreases marginal transaction cost; specifically,

\[ C^*_{yt} = \frac{\alpha}{1 + \alpha} \left( \frac{u}{\bar{z}} \right), \]

which is less than the effect of \( t \) on marginal cost when \( z \) is held fixed.

The adjustment in \( z \) decreases marginal transaction costs, so the increase in equilibrium transaction time must be greater when \( z \) is allowed to vary than when it is held fixed. The magnitude of the difference can be determined from equations (6) and (7), which imply

\[ \frac{t^*(\bar{p})}{t(\bar{p})} = \frac{C_{yt}}{C^*_{yt}} = \frac{1 + \alpha}{\alpha} \]

in this example. Given any reduction in the ceiling price, the increase in waiting time when the consumer adjusts \( z \) in a privately rational way is \( (1 + \alpha)/\alpha \) times as high as it would be if \( z \) were fixed.

In Example 2, the consumer engages in an activity that reduces the opportunity cost of time. The transaction-cost function in this example is

\[ C(t, y, z) = (v - z) ty + y^\beta z^\gamma \]

where \( z \) is the level of the activity, \( v - z \) is the resulting opportunity cost of time, and \( y^\beta z^\gamma \) is the cost of the activity. The cost-minimizing value of \( z \) is

\[ z^* = (ty^{1-\beta} z^{-1})^{1/(\gamma - 1)}. \]

If \( z \) is held constant, the effect of a change in transaction time on marginal transaction cost is

\[ C_{yt} = v - \bar{z} \]

which is just the opportunity cost of time. Alternatively, if \( z \) can be varied, then the consumer will increase it when \( t \) increases. The effect of this increase in \( z \) on marginal transaction cost is determined by \( C_{yz} \). Using the first-order condition from cost-minimization, \( C_{yz} \) can be reduced to \( t(\beta - 1) \).

Since \( C_{yz} = t(\beta - 1) \) and \( z^* > 0 \), Proposition 1 implies that the net effect of adjusting \( z \) on the equilibrium transaction time depends on whether \( \beta \) is greater or less than unity. To see this, suppose that the ceiling price were reduced with \( z \) initially fixed and that the waiting time were increased to clear the market. Next, the consumer adjusts \( z \), and the market moves to a new equilibrium. If \( \beta < 1 \), so that \( C_{yz} z^* < 0 \), the adjustment in \( z \) will lower marginal transaction cost. This creates excess demand and causes a further increase in \( t \), as in Example 1. Alternatively, if \( \beta > 1 \) so that \( C_{yz} z^* > 0 \), adjusting \( z \) raises the marginal cost of \( y \), reducing demand and causing \( t \) to decrease.

The second case seems puzzling; why would the consumer take an action that raises the marginal cost of \( y \)? The answer is that increasing activity \( z \) can lower total transaction cost while raising the marginal transaction cost of \( y \). The effect of changing \( z \) on total transaction cost is

\[ C_z = -ty + y y^\beta z^\gamma \]

\[ = yC_{yz} + (1 - \beta) y y^\beta z^\gamma. \]

If \( \beta > 1 \), it is quite possible for an increase
in $z$ to raise marginal transaction cost ($C_{yz} > 0$) while reducing total transaction cost ($C_z < 0$).

III. Transaction Activities and Social Welfare

The rational consumer chooses activities to minimize transaction costs for a given waiting time. Given $t$, therefore, adjusting $z$ raises consumer welfare. The preceding section showed that adjusting $z$ may lead to a longer waiting time, however, which reduces consumer welfare. The present section sorts out these opposing forces by establishing a condition that determines whether or not individually rational adjustments are socially self-defeating.

Recall that the representative consumer's utility function is $x + F(y)$. In equilibrium, this individual consumes the entire endowment of the price-controlled good, $\bar{y}$, and consumes the initial endowment of numeraire less the amount dissipated by transaction costs, $\bar{x} - C(t, \bar{y}, \bar{z})$. When the consumer is allowed to choose activities, consumer welfare, as a function of the ceiling price, is

$$ (13) \quad W^*(p) = \bar{x} + F(\bar{y}) - C^*(t^*(p), \bar{y}). $$

The rent transfer from sellers to buyers due to the price ceiling is netted out in this expression, because the representative consumer is both a buyer and a seller of $y$. Thus, a reduction in $p$ can only affect welfare by changing the transaction cost.

The derivative of the welfare function evaluated at the initial equilibrium is

$$ (14) \quad W'^*(\bar{p}) = -C^*_i(\bar{i}, \bar{y})t'^*(\bar{p}). $$

Assuming the equilibrium to be stable, a decrease in the ceiling price must increase transaction time, which implies that $t'^*(\bar{p})$ is negative. Since we assume $C_i$ to be positive, the right-hand side of (14) is positive. Thus, a decrease in the ceiling price necessarily decreases welfare.

If transaction activities are held fixed at $\bar{z}$, consumer welfare as a function of $p$ is

$$ (15) \quad \tilde{W}(p) = \bar{x} + F(\bar{y}) - C(\hat{t}(p), \bar{y}, \bar{z}) $$

and the derivative of (15) evaluated at the initial equilibrium is

$$ (16) \quad \tilde{W}'(\bar{p}) = -C_i(\bar{i}, \bar{y}, \bar{z})\hat{t}'(\bar{p}). $$

This derivative must also be positive.

Comparing (14) to (16) allows us to compare the change in welfare when activities are free to vary with the change when activities are fixed. The envelope theorem implies

$$ (17) \quad C^*_i(\bar{i}, \bar{y}) = C_i(\bar{i}, \bar{y}, \bar{z}). $$

Incorporating (17) with (14) and (16), the ratio of the two changes in welfare can be reduced to

$$ (18) \quad \frac{W^*(\bar{p})}{\tilde{W}'(\bar{p})} = \frac{t^*(\bar{p})}{\hat{t}'(\bar{p})}. $$

If this ratio is greater than unity, lowering the ceiling price leads to a larger decline in welfare if the consumer adjusts transactions activities than if activities are held fixed. In this case, the individually rational adjustment in transaction activities is socially wasteful. The implications of (18) are summarized in the following proposition.

**PROPOSITION 2:** Suppose that the ceiling price of $y$ is reduced, causing the time required per transaction to increase. An individually rational adjustment in transaction activities is socially wasteful if and only if it leads to a longer equilibrium waiting time; that is,

$$ W^*(\bar{p})/\tilde{W}'(\bar{p}) > 1 $$

if and only if $t^*(\bar{p})/\hat{t}'(\bar{p}) > 1$.

Proposition 1 in the preceding section gives a condition that determines when an
adjustment in \( z \) increases transaction time. Combining that result with Proposition 2 yields the following.

**PROPOSITION 3:** Suppose that the ceiling price of \( y \) is reduced, causing the time required per purchase to increase. An individually rational adjustment in transaction activities is socially wasteful if and only if it reduces the marginal transaction cost of \( y \), given the initial waiting time; that is,

\[
W^*(\bar{p}) / \hat{W}'(\bar{p}) > 1
\]

if and only if \( C_{yz}(\bar{i}, \bar{y}, \bar{z}) \cdot z^*(\bar{i}, \bar{y}) < 0 \).

Proposition 3 is our main result. It gives a condition that determines when an individually rational adjustment to rationing by waiting is socially self-defeating. Example 1 above, in which the consumer shops less frequently in response to longer queues, illustrates this outcome. Not all adjustments are self-defeating, however. In Example 2, the consumer takes an action that reduces the opportunity cost of waiting time. If this action raises the marginal cost of \( y \), it improves social welfare. The difference between the two examples lies in the effect of \( z \) on the marginal cost of the controlled good. In the first example, the consumer undertakes a costly activity that lowers its marginal cost. Although individually rational, this worsens the nonprice rationing problem and leads to a longer queue and lower equilibrium utility. In the second case, the consumer undertakes an activity that, while lowering total transaction cost, actually raises the marginal cost of the price-controlled good. This adjustment allows the market to clear with a smaller waiting time and leads to increased equilibrium utility.

**IV. How Much Rent Is Dissipated?**

In the standard account of a price control with identical consumers and rationing by waiting, the price ceiling transfers rent from sellers to buyers, but this rent is exactly dissipated in waiting cost. Exact dissipation depends on the explicit assumption of identical consumers and on the implicit assumption that consumers cannot adjust behavior on other margins when confronted by a queue. The preceding section showed that such adjustments may cause a further reduction in welfare, so the loss due to non-price rationing may exceed the rent transferred from suppliers. We term this phenomenon “overdissipation” and examine it further in the present section.

The rent transferred from a unit decrease in \( p \) is just \( \bar{y} \), the amount of the controlled good consumed. We use the term “exact dissipation” to denote the case in which a marginal reduction in the ceiling price lowers welfare by exactly \( y \), so \( W^*(\bar{p}) - \bar{y} = 0 \). Overdissipation occurs if \( W^*(\bar{p}) - \bar{y} > 0 \), and underdissipation occurs if \( W^*(\bar{p}) - \bar{y} < 0 \).

This expression, \( W^*(\bar{p}) - \bar{y} \), can be reduced to a more transparent form by combining equations (6) and (14):

\[
W^* - \bar{y} = \frac{\bar{y}}{C_{yt}^*} \left( \frac{C^*}{\bar{y}} - C_{yt}^* \right).
\]

All derivatives in this expression are evaluated at the initial equilibrium. Since \( C_{yt}^* \) must be positive for stability, overdissipation occurs if \( (C^*/\bar{y}) - C_{yt}^* \) is greater than zero, underdissipation occurs if this expression is less than zero, and exact dissipation occurs if it equals zero. The fraction \( C^*/\bar{y} \) is the derivative of average transaction cost with respect to transaction time, and \( C_{yt}^* \) is the derivative of marginal transaction cost with respect to transaction time. This leads immediately to the following proposition.

**PROPOSITION 4:** Overdissipation occurs when a marginal increase in transaction time increases average transaction cost more than marginal transaction cost (\( (C^*/\bar{y}) - C_{yt}^* > 0 \)). Underdissipation occurs in the reverse situation; exact dissipation occurs when the effects on average and marginal cost are equal.

This proposition reflects the different roles played by average and marginal transaction cost. When \( t \) increases, it increases both the marginal and average transaction cost of acquiring \( y \). The increase in marginal
cost serves to ration demand and helps accomplish the transfer of rent from seller to buyer. The increase in average transaction cost, however, serves only to reduce welfare. The distinction between marginal transaction cost and average transaction cost does not arise in the standard analysis of rationing by waiting, because this analysis ignores the possibility of varying the amount per purchase or the opportunity cost of waiting time. The transaction cost per unit consumed is the product of three constants in this case: the value of time, the waiting time per purchase, and the inverse of the amount bought per purchase. Hence, marginal transaction cost equals average transaction cost in this case, which implies exact dissipation.

When looking for examples of over- and underdissipation, it is useful to consider the special case in which exact dissipation would occur if transaction activities were fixed. In this special case, Proposition 3 leads immediately to examples of over- and underdissipation. If exact dissipation occurs when activities are fixed, then

\[(20) \quad C_{y_I}(t, y, z) = C_r(t, y, z) / y.\]

It can be shown that this condition holds if and only if the transaction-cost function takes the following form:

\[(21) \quad C(t, y, z) = yg(t, z) + h(y, z).\]

The crucial feature of (21) is that a function

\[9\]Trien T. Nguyen and John Whalley (1986) have a more general transaction-cost function, but they also assume that marginal transaction cost equals average transaction cost.

\[10\] Rewrite (21) as

\[C_{y_I}(t, y, z) = \frac{1}{C_r(t, y, z)} = \frac{1}{y}.\]

Integrate both sides of this equation with respect to \( y \) yielding \( \ln(C(t, y, z)) = \ln(y) + \ln(k(t, z)) \), where \( \ln(k(t, z)) \) is a constant of integration. Take the antilog of both sides of this equality and then integrate with respect to \( t \) yielding \( C(t, y, z) = yg(t, z) + h(y, z) \), where \( g(t, z) \) is the integral of \( k \) with respect to \( t \) and \( h \) is a constant of integration.

of transaction time, \( g \), is multiplied by the quantity consumed, \( y \). This linearity implies that the derivative of marginal transaction cost with respect to time is equal to the derivative of average transaction cost with respect to time. That derivative is just \( g_r \).

If the transaction technology satisfies (22), Proposition 3 implies that overdissipation occurs if the consumer changes activities in a way that lowers the marginal cost of \( y \) \( (C_{y_I}^* \cdot z_I^* < 0) \), and underdissipation occurs if these changes raise the marginal cost of \( y \). Both of the examples presented earlier display exact dissipation when activities are fixed. Hence, when activities are variable, Example 1 exhibits overdissipation, and Example 2 exhibits underdissipation if \( \beta > 1 \).

More generally, over- and underdissipation can occur even with transaction activities fixed, if transaction time interacts with a nonlinear function of the quantity consumed. Here is one plausible story leading to such an outcome. Waiting in line is frustrating, and it becomes increasingly so as the length of the wait increases. As a consequence, the marginal waiting cost rises with the amount of time spent waiting per period. Suppose, for example, that the transaction cost is a sum of waiting and storage costs, \( \frac{1}{2}(y / z)^2 + \frac{1}{3}z^3 \), where \( z \) is the amount per purchase. Given any \( z \), marginal transaction cost rises more rapidly than average transaction cost as \( t \) increases. Thus, underdissipation results with \( z \) fixed, and the consumer gains from the price ceiling. In effect, the consumer captures some rent because the transaction cost for inframarginal units of \( y \) is relatively low. The consumer will wish to increase \( z \) when \( t \) increases, however, and this lowers welfare because \( C_{y_I} < 0 \). Yet the loss that results from increasing \( z \) is not large enough to eliminate the rent obtained from inframarginal units purchased. Hence, underdissipation occurs in this example, even though the adjustment in \( z \) is self-defeating.\[11\]

\[11\] In this example, \( z^* = (ty)^{2/3} \) and \( C_{y_I} < 0 \), so adjusting \( z \) is self-defeating by Proposition 3. \( C^* = \frac{2}{3}(ty)^{2/3} \), which implies \( (C^* / y) = C_{y_I}^* < 0 \), however; hence, underdissipation occurs by Proposition 4.
V. Discussion

The phenomenon addressed here is a form of rent-seeking and is therefore related to the contributions of Gordon Tullock (1967), Anne Krueger (1974), and Richard Posner (1975). In our model, a rent is created when the price suppliers can charge is held below its market-clearing level. Given the first-come, first-served allocation rule, consumers compete by waiting, and with identical customers, waiting exactly dissipates the rent created by the ceiling. This is reminiscent of results found in the rent-seeking literature cited above.

Consumers in our model also find it individually rational to compete by investing in activities to mitigate waiting costs, and our emphasis on these additional margins for competition is a departure from this literature. If adjustments along these margins decrease the marginal cost of buying the good, waiting time must increase even more. These adjustments are self-defeating in the sense that the welfare cost of the ceiling would be lower if they had not occurred. In fact, more rent can be dissipated through this competition than is lost by sellers.\(^{12}\)

Our central welfare implication is similar to a result derived by Tullock (1980) in a much different context. In his model, individuals expend effort competing for a fixed prize to be allocated to a single player. Each player takes the effort of others as given, and by expending effort, each can increase his probability of winning and, hence, reduce the probability for his rivals. The resulting Nash equilibrium admits overdissipation of rent if the marginal cost of effort is declining. His policy conclusion, that “it would be desirable to establish institutions so that the marginal cost (for effort) is very steeply rising” (Tullock, 1980 p. 104) is equivalent to our prescription that consumers be constrained from taking actions that reduce marginal transaction cost.

An allocation rule that restricts the ways by which consumers compete for supplies must also specify penalties for noncompliance (e.g., fines for hiring others to wait). The model we developed is sufficiently general to address this consideration. To the individual, such penalties add to the cost of pursuing activities and hence reduce the extent to which they are undertaken. If the penalty is denoted \(\Pi(z)\), the total cost from competing on these margins becomes \(C(t, y, z) + \Pi(z)\). However, if the penalties take the form of fines and receipts are re-distributed, \(\Pi(z)\) does not represent a social cost. This suggests an enforcement policy that levies transferrable penalties for engaging in self-defeating activities.

Although our analysis was developed in terms of consumer adjustments to rationing by waiting, the general approach has broader application. In an uncontrolled market, sellers find it rational to take actions that reduce the transaction cost faced by buyers. Examples are pleasant surroundings and courteous clerks. Such amenities are costly, but sellers provide them because they reduce the consumer’s total shopping cost by making time spent shopping more pleasant. They also typically reduce the consumer’s \textit{marginal} transaction cost for a given transaction time, however. If a price ceiling is imposed, such amenities will, by Propositions 1 and 3, result in a longer queue and lower welfare. When a good is price-controlled and rationed by waiting, therefore, unpleasant stores and discourteous clerks are optimal.

Some of the activities pursued to mitigate time costs may be undertaken collectively. Developing nations often sell price-controlled goods in government-operated ration shops and allocate available supplies on a first-come, first-served basis (Alderman, 1987). When deciding where such shops

\(^{12}\)Our analysis prescribes allocation rules that prohibit this self-defeating competition. Steven N. S. Cheung (1974) also stresses that individuals will compete on a variety of margins for the rent caused by a price ceiling. When a good is rationed by waiting, he notes that allowing consumers to buy more than one unit per “wait” simply lengthens the queue, leaving the waiting time per unit unchanged (Cheung, 1974 p. 68). He goes on to claim, however, that actions taken to minimize individual transaction cost will reduce rent dissipation, and consequently he argues for policies that allow consumers to pursue such actions. Our analysis shows that this argument is not generally correct.
should be located, a seemingly important consideration is the transport cost consumers face getting to and from these price-controlled outlets. To motivate the discussion, suppose that the typical consumer initially walks five miles to a centralized store and then waits in line for the price-controlled good. Both walking and waiting help ration demand. To reduce transport cost, the government might decide to incur a fixed cost and disperse distribution to smaller shops located nearer consumers. This would lower the marginal cost of the good, given the initial waiting time, and the length of the queue would necessarily increase to restore equilibrium. Welfare is reduced in this case, because the cost of dispersing the system is wasted.

Common-property resources share two key features with the price-controlled goods examined in this paper; they are offered at a money price below equilibrium (usually zero) and often are allocated by rules that approximate first-come, first-served. For example, rights to fish stocks and underground water are unpriced and often are secured by the first to appropriate them. Thus, our approach to price-controlled goods may offer insights into the allocation of common property as well. Consider, for example, a group of farmers who withdraw water from a common aquifer. The aquifer is recharged naturally at a constant rate, and no price is charged for water withdrawn. Although water is unpriced, the cost of pumping is a transaction cost, and the marginal cost of pumping limits demand. Equilibrium is established when the depth to water yields a marginal pumping cost that equates the demand for water to the recharge rate. Suppose a new pump becomes available, one that is more expensive to purchase but has a lower variable cost. By lowering the marginal pumping cost, the new pump would cause the rate of withdrawal to increase beyond the rate of natural recharge. The depth to water would increase until a new equilibrium is established where, once again, the marginal cost of water just equates demand to natural recharge. Equilibrium welfare is lower, in this case, because the increased purchase price for the new pump yielded no reduction in equilibrium pumping cost.

Our final, more speculative, extension concerns government provision of public campgrounds at prices that are below the market-clearing price. In such cases, it is often a waiting list or a reservation system, rather than a queue, that ration demand. The waiting list rations demand by postponing consumption or by requiring consumers to fix plans too far in advance; in this sense, it raises the cost per transaction in a way that is similar to the queue (Cotton Lindsay and Bernard Feigenbaum, 1984). In an effort to reduce this cost, the public provider may try to make reservations easier to arrange (e.g., by installing a computerized booking system or by allowing reservations to be made at more convenient locations). Yet, if any such "improvement" lowers the cost of booking a site, the length of the waiting list and the degree of advance planning will grow and eliminate at least part of the expected gain.

REFERENCES


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