THE NORMAL FORM

For each player, we can define a function \( u_i : S \Rightarrow \mathbb{R} \) so that, for each strategy profile \( s \) in \( S \) that the players choose, \( u_i(s) \) is player \( i \)'s utility (payoff) in the game; this is called the payoff function.

Consider the game:

\[
\begin{array}{cccc}
I & I & 2 & 1 \\
O & O & B & \\
2, 2 & 1, 3 & 3, 4 & \\
\end{array}
\]

\( S_1 = \{OA, OB, IA, IB\} \)
\( S_2 = \{O, I\} \)

There are eight (4*2) strategy profiles in this game. A convenient way to describe the strategy spaces and payoff functions is to draw a matrix (this is harder with \( n \)-player games, of course), with each row corresponding to a strategy of player 1 and each column corresponding to a strategy of player 2.

<table>
<thead>
<tr>
<th>Player 1</th>
<th>( O )</th>
<th>( I )</th>
</tr>
</thead>
<tbody>
<tr>
<td>OA</td>
<td>2, 2</td>
<td>2, 2</td>
</tr>
<tr>
<td>OB</td>
<td>2, 2</td>
<td>2, 2</td>
</tr>
<tr>
<td>IA</td>
<td>4, 2</td>
<td>1, 3</td>
</tr>
<tr>
<td>IB</td>
<td>3, 4</td>
<td>1, 3</td>
</tr>
</tbody>
</table>

Each cell corresponds to a strategy profile. Strategies and payoffs can be taken to be a fundamental representation of a game. A formal definition:

A game in normal form (also called strategic form) consists of a set of players, \( \{1, 2, \ldots, n\} \), strategy spaces for the players, \( S_1, S_2, \ldots, S_n \), and payoff functions for the players, \( u_1, u_2, \ldots, u_n \).

Thus, we can convert an extensive-form game into a normal-form game. Can we do the reverse? Consider the following two extensive-form games:
Both of these games are equivalent to the same normal-form game shown below, but have a very different strategic feel.

Game theorists have been debating whether the normal form contains all of the relevant information about strategic settings. However, when players make all of their decisions before observing what other players do (as in one-shot or static games, for example), there is no discrepancy between the extensive form and the normal form.