On Testing the Sustainability of Government Deficits in a Stochastic Environment

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Abstract

In recent years, a number of empirical studies have examined the long-run sustainability of U.S. debt policy, with mixed results. Bohn (1995) has shown that the existing sustainability tests are explicitly or implicitly based on a transversality condition that is invalid in a stochastic environment. This paper derives and implements a new test for sustainability that is valid under uncertainty. I find that U.S. fiscal policy has historically satisfied been on a path that satisfies the transversality condition.

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1. Introduction

In recent years, a number of empirical studies have examined the question whether U.S. fiscal policy has historically been on a sustainable path, i.e., whether it is consistent with the government’s intertemporal budget constraint. Some conclude that U.S. fiscal policy has been sustainable, others disagree. (See Hamilton and Flavin 1986, Hakkio and Rush 1986, Kremers 1989, Trehan and Walsh 1988 and 1991, and Wilcox 1989.)

This paper reexamines the sustainability issue and implements a new test for the sustainability of U.S. fiscal policy. A reexamination is needed because the existing tests are based on theoretical models that explicitly or implicitly assume an interest rate on government bonds above the average growth rate of the economy. Interest rates on U.S. government bonds have in fact been below the average growth rate of GDP for long periods, including the periods that were studied in the literature. Given such low interest rates, even a policy with constant debt-GDP ratio—which is clearly sustainable—would display primary deficits most of the time, and it would show an average growth rate of government debt above the safe interest rate. The “standard” transversality condition would be violated.

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1 Corsetti and Roubini (1991) examine the same question for a cross-section of countries.
2 The average real return on U.S. Treasury bills, for example, has been 0.23% for 1929-88 and 1.11% for 1954-1988. Both values are well below the average U.S. growth rate. Additional evidence is in Table 2 below.
3 Bohn (1995) has shown that a zero limit of future government debt discounted by the safe interest rate—the “standard” transversality condition—is not a valid necessary condition for sustainability in a stochastic setting. The literature on the sustainability of budget deficits is unfortunately based almost entirely on this condition and the associated intertemporal budget constraint (see, e.g., Hamilton and Flavin 1986, Hakkio and Rush 1986, Kremers 1989, Trehan and Walsh 1988, 1991, Wilcox 1989, Corsetti and Roubini 1991). The safe interest rate is usually proxied by the actual or the average return on government debt in this context. Kremers and Corsetti-Roubini also examine time series of debt-income (GNP or GDP) ratios, supposedly to test a stronger constraint. But if the safe interest rate is below the average growth rate, a stationary debt-income ratio is in fact a weaker constraint than the transversality condition.
The test for sustainability in this paper is motivated by the fact that even if the government can run primary budget deficits most of the time and in most states of nature, permanent primary deficits will lead to an excessive debt accumulation in at least some “bad” states of nature. If debt keeps growing relative to GNP, a sustainable policy must eventually respond by imposing a primary surplus. In contrast, a government running a Ponzi-scheme will be oblivious to an accumulation of debt in all states of nature.

Using the correct transversality condition for stochastic economies from Bohn (1995), I show that a positive response of primary surpluses to marginal changes in the debt-GDP ratio is a sufficient condition for a sustainable policy. This positive response is empirically confirmed for U.S. fiscal policy, which suggests that U.S. government policy has historically (1916-89) been on a path that satisfies the transversality condition.

The empirical analysis also shows that the link between debt and primary surpluses can easily be obscured by war-time spending, cyclical fluctuations, and the near unit-root behavior of the debt-GDP ratio. An atheoretical time-series approach fails to show a link between debt and primary surplus. But a structural equation for primary surpluses motivated by Barro’s (1979) tax smoothing model shows a significant impact of initial debt on primary surpluses.

It should be acknowledged that fiscal policy may be subject to additional constraints (other than the transversality condition) in certain economic environments, e.g., if there is an upper bound on the debt-income ratio or on tax revenues. This paper focuses on the transversality condition, because even this

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4 Bohn (1995) derives the sustainability conditions for a class of stochastic, dynamically efficient economies with complete markets. The complete markets assumption precludes problems of the type examined by Blanchard and Weil (1992).

5 See Blanchard (1984), Kremers (1989), Bohn (1991b). As noted by a referee, such additional constraints may arise naturally in overlapping generations economies.
rather weak restriction on fiscal policy has been empirically controversial. An rigorous examination of additional sustainability conditions is left for future research.\(^6\) Note, however, that the evidence for mean reversion in the debt-GDP ratio is promising, because it suggests that the debt-GDP ratio may bounded under reasonable conditions.

The paper is organized as follows. Section 2 reviews the theoretical issues and derives the sufficient condition for sustainability. Section 3 examines the empirical evidence. Section 4 concludes.

2. Conditions for Sustainability

This section will derive a testable set of sufficient conditions for policies that satisfy the intertemporal budget constraint and the transversality condition in a stochastic setting.

The economic environment for government policy is defined as follows. Denote aggregate income (GNP) by \(Y_t\), the “tax rate” by \(\tau_t = T_t/Y_t\), the ratio of government spending to income by \(g_t=G_t/Y_t\), the debt-income ratio at the beginning of a period by \(d_t = D_t/Y_t\), and the primary surplus relative to income by \(s_t = \tau_t-g_t\). Let \(R_t\) be the return on government debt and \(y_t\) be the growth rate of income. Then the usual budget equation in levels is

\[
D_{t+1} = (1+R_{t+1}) \cdot [G_t - T_t + D_t]
\]

It implies a budget equation in ratio form,

\[
d_{t+1} = x_{t+1} \cdot [g_t - \tau_t + d_t] = x_{t+1} \cdot [d_t - s_t], \tag{1}
\]

where \(x_{t+1} = (1+R_{t+1})/(1+y_{t+1})\).\(^7\) With this notation, one can show:

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\(^6\) The main complication is that if one takes the notion of an upper bound on tax revenues serious, it implies an upper bound on the debt-income ratio for all states of nature, not just in expectation (Bohn 1991b; cf. Corsetti and Roubini 1991). It would be a non-trivial task to define plausible bounds on fiscal policy that assure a bounded debt-GDP ratio with probability one.

\(^7\) Note that the return on debt has to satisfy the Euler condition \(E_t[\mathbb{u}_{t+1}(1+R_{t+1})]=1\), where \(\mathbb{u}_{t+1}\) is the marginal rate of substitution between periods \(t\) and \(t+1\).
PROPOSITION 1:

If the stream of aggregate income $Y_t$ has a finite present value and if the primary surplus can be written as

$$s_t = \mu_t + \rho \cdot d_t,$$  \hspace{1cm} (2)

where $\mu_t$ is a bounded stochastic process and $\rho > 0$, then the government policy satisfies the appropriate transversality condition and the associated intertemporal budget constraint.

PROOF: See the appendix.♦

The key element in this proposition is the requirement that the government responds to higher initial debt by increasing the primary surplus. The other assumptions are largely technical.\(^8\)

Intuitively, the central question for testing sustainability is what a time series of fiscal variables reveals about government behavior at high levels of the debt-income ratio. Given the bounds on the “other” determinants of the primary surplus that are captured by $\mu_t$, a positive $\rho$-coefficient in (2) implies that the primary surplus will turn positive if $d_t$ is sufficiently high. Even if $\mu_t$ is a large negative number so that the primary surplus is negative for a wide range of $d_t$-

\(^8\) The assumption of a finite present value of income seems reasonable, because otherwise the economy would have infinite wealth. Boundedness is a rather weak assumption because the bound may be as wide as one wishes—more complicated conditions would only be distracting. The linearity assumption is a useful simplification for the empirical analysis. The proposition could be generalized to cover all non-linear functions that can be bounded below by a linear function that satisfies (2). Overall, Proposition 1 is far from being necessary for sustainability, but it will be sufficiently general for the empirical analysis. From a theoretical perspective, no restriction on the correlation between $\mu_t$ and $d_t$ are needed; $\mu_t$ may even include functions of $d_t$ (provided they are bounded). From a practical perspective, a high correlation or a functional dependence between $\mu_t$ and $d_t$ might of course complicate the statistical inferences; this is an empirical issue to be addressed below. The key to the proof of Proposition 1 is that the $\mu_t$ part in (2) is inessential because it is dominated by the linear term if, for whatever reason, a high value of the debt-income ratio is realized.
values, the condition $\rho > 0$ is sufficient to satisfy the relevant transversality constraint.

Condition (2) is sufficient but not necessary for sustainability. Since a sustainable policies under uncertainty may display primary budget deficits in “most” states of nature, but not in all, it is difficult to rule out a policy that looks like a Ponzi-scheme for some range of debt-GDP ratios observed empirically may be part of a policy that calls for primary surpluses at higher (but unobserved) debt levels. Thus, a search for necessary conditions would not be promising. In contrast, a fiscal policy that satisfies condition (2) should yield clear in-sample evidence that the government is concerned about its debt.

It is important that condition (2) is written in terms of the primary surplus and neither involves interest rates nor the debt-GDP ratio. In a stochastic environment, movements in the debt-GDP ratio are significantly driven by random fluctuations in GDP growth and by variations in the realized return on government debt, which would complicate inferences based on the time series of government debt. The rate of return on government debt and the with-interest surplus also depend on the structure of government debt (e.g., long versus short term, nominal versus indexed). Here, no assumptions about interest rates and the debt structure need to be imposed. Condition (2) is valid whether or not interest rates are above or below the rate of economic growth.

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9 For example, consider the policy $s_t = \rho \cdot \max(0, d_t - d^*)$, where $\rho > 0$ and where $d^* > 0$ is a “large” positive constant. If the available sample has $d_t$-values below $d^*$, one would not be able to identify $d^*$ and estimate $\rho = 0$. This policy is theoretically sustainable but empirically indistinguishable from a Ponzi-scheme. See Bohn (1994) for more details on such inference problems.

10 If the primary deficit satisfies (2), the debt-income ratio will have the law of motion

$$d_{t+1} = x_{t+1} \cdot (1-\rho) \cdot d_t + x_{t+1} \cdot \mu_t$$

which clearly depends on the level of interest rates relative to the growth rate (on $x_{t+1}$).
3. Empirical Evidence

An empirical test of condition (2) will have to involve regressions of the primary surplus on initial debt. This raises questions about the time series properties of the debt and surplus series and about how one should estimate the $\mu_t$ process.

These two issues are interdependent. The literature suggests that the time series of government debt and the primary surplus may be non-stationary (see Trehan and Walsh, 1991). If debt and the primary surplus were both non-stationary, one might be able to interpret simple regression of $s_t$ on $d_t$ as a co-integrating regression without having to explicitly model the $\mu_t$ process. But if $s_t$ and $d_t$ do not have unit roots, a regression of $s_t$ on $d_t$ that omits other plausible determinants of the primary surplus is likely to yield inconsistent estimates due to a classical omitted variables problem.

3.1. Time Series Analysis

In the literature, inferences about unit roots are usually based on augmented Dickey-Fuller or Phillips-Perron tests of the null hypothesis that the series are integrated of order one (Dickey and Fuller 1981, Fuller 1976, Phillips and Perron, 1988). Table 1 shows such unit root tests for the debt-GDP ratio $d_t$ and for the surplus-GDP ratio $s_t$ for the sample period 1916-1989, for a longer sample period 1800-1988, and for subperiods that exclude the major wars. The main sample period for the analysis below will be 1916-89. The 1800-1988 sample is provided because the unit roots issue concerns the long-run properties of the data, about which a long sample may be more informative. The non-war results will be
discussed below. To save space, only the augmented Dickey-Fuller “t-statistics” and the Phillips-Perron “Zτ-statistics” are displayed.\textsuperscript{11}

Using the full 1916-89 and the full 1800-1988 samples, one cannot reject the null hypotheses that government debt is I(1) and that the deficit is I(0), where I(·) denotes the order of integration. Similar results have been obtained in the literature (e.g., Trehan and Walsh 1988, 1991).\textsuperscript{12}

The rejection of a unit root for the surplus-GDP ratio clarifies that a regression involving this variable cannot be interpreted as co-integrating regression. The non-rejection of a unit root for the debt-GDP ratio is somewhat disturbing, because a unit root in \( d_t \) is difficult to reconcile with the sustainability condition (2)—unless the \( \mu_t \) process is non-stationary but still bounded. This non-rejection may, however, reflect a power problem rather than an actual unit root.

Equation (1) describes the mapping from the current debt and primary surplus to the future debt-GDP ratio. For the 1916-1989 sample, the average real return on government debt is -0.17% while the average real growth rate of GDP is +3.46% (see Table 3 below). The average growth factor \( x_{t+1} \) in (1) is therefore about 0.97, i.e., clearly below one. Combined with the clear stationarity of \( s_t \), this equation suggests that the debt-GDP ratio does not have a unit root.\textsuperscript{13} The equation also explains why the debt-GDP ratio is so highly autocorrelated that a unit root is difficult to reject.

\textsuperscript{11} Similar results were obtained with non-augmented Dickey-Fuller tests and with the Phillips-Perron “Zp-statistics.” The Phillips-Perron values are based on a lag window of 3, but the choice did not matter much.

\textsuperscript{12} A difference to the cointegration literature is that the variables are GNP-shares, not levels. This is done partly for econometric reasons—to eliminate growth trends and potential heteroskedasticity—and partly for plausibility. If fiscal variables were defined as stationary stochastic processes in levels while GNP has a trend or unit root component, the ratio might become implausibly small or large.

\textsuperscript{13} This conclusion is reinforced below by the finding that \( p>0 \). Equation (1) and (2) imply \( d_{t+1} = x_{t+1} \cdot (1-p) \cdot d_t + x_{t+1} \cdot \mu_t \). If \( p>0 \), \( x_{t+1} \cdot (1-p) \) is even further away from 1 than \( x_{t+1} \) itself.
Overall, I conclude that it is legitimate to treat the surplus-GDP and debt-GDP ratios as stationary variables. At the very least, the GDP-ratios are sufficiently stationary that a regression of the surplus-GDP on the debt-GDP ratio does not pose spurious regression problem.

3.2. The Determinants of the Primary Surplus

If the surplus-GDP and debt-GDP ratios are stationary, a model for $\mu_t$ is needed to estimate the reaction coefficient $\rho$ in the sustainability condition (2). This section Barro’s (1979, 1986a, 1986b) tax-smoothing model as a structural model for the primary surplus.\textsuperscript{14} The model considers an optimizing government that minimizes the the cost of tax collection by smoothing marginal tax rates over time. Key features of the optimal policy are that tax rates should only depend on permanent government spending and on initial debt, i.e., not vary over the business cycle or with temporary fluctuations in spending.

The tax-smoothing model yields an equation for the primary surplus if one subtracts current non-interest outlays from taxes (all relative to GNP). The model implies that the level of temporary government spending $GVAR$ and a business cycle variable $YVAR$ are the determinants of the non-debt components of primary surplus, $\mu_t$. Including for an approximation error $\varepsilon_t$, one obtains

$$\mu_t = \alpha_0 + \alpha_G \cdot GVAR_t + \alpha_Y \cdot YVAR_t + \varepsilon_t$$

as model for the non-debt determinants of the surplus-GDP ratio and

$$s_t = \alpha_0 + \alpha_G \cdot GVAR_t + \alpha_Y \cdot YVAR_t + \rho \cdot d_t + \varepsilon_t$$  \hspace{1cm} (3)

\textsuperscript{14} This model was also used in Kremer’s (1989) sustainability paper. It was originally derived as a partial equilibrium model (Barro 1979), but it can easily be embedded in a general equilibrium setting. Regarding alternative approaches, note that a model of government behavior would also be needed for estimating discounted debt (along the lines of Hamilton-Flavin 1986, or Wilcox 1989), because the correct discount rate in the stochastic transversality depends on the stochastic process of debt. The discounted-debt tests in the literature appear to be model-independent only because they do not address the question of discount rates.
as model for the surplus-GDP ratio. The $\alpha$’s are coefficients. The variables GVAR and YVAR are taken from Barro (1986a) for 1916-83 and updated for 1984-89. The empirical analysis will focus on the question whether the estimate for $\rho$ is positive.\(^{15}\)

Estimates of equation (3) are in Table 2. All regressions use Ordinary Least Squares (OLS) estimation. White’s (1980) robust standard errors are provided to address potential heteroskedasticity problems.\(^{16}\) Line 1 shows the results for the full sample period 1916-1989. The regression in line 2 excludes the two world wars. Lines 3 and 4 show results for the sample period 1916-83, for which Barro’s original regressors are available. Line 3 has results for the entire period, while line 4 focuses on the sample period 1920-82 excluding 1941-47, which is the period Barro (1986a) used. Line 5 shows results for the post-war period 1948-89 and line 6 shows results for the period 1960-84, which is the sample period used by Hamilton and Flavin (1986) and by Wilcox (1989).

The coefficients on $d_t$ are significantly positive in all regressions and they are quantitatively reasonable. For example, the $\rho$-value of 0.056 in line 1 means that a marginal increase in initial government debt by $100 increases the primary surplus in the following year by $5.60. The estimates for $\rho$ are all between +3.0% and +5.6%. These results provide strong evidence that the U.S. government has indeed satisfied the sustainability condition of Proposition 1.

\(^{15}\) I will not examine the boundedness of $\mu_t$ in any formal way. The variable GVAR, the ratio of temporary spending to GNP, is by definition bounded in the interval [-1,1]. Boundedness of YVAR is plausible, because it is just the ratio of actual to potential GNP.

\(^{16}\) Additional corrections for serial correlation do not significantly change the results. Note that OLS yields unbiased estimates without requiring assumptions about ergodicity, provided the error $\epsilon_t$ is independent of the explanatory variables. This robustness with respect to ergodicity is reassuring, because one can construct examples in which some fiscal variables are not ergodic (see Bohn, 1994). Ergodicity is however implicitly assumed for all asymptotic inferences in this section.
The variables GVAR and YVAR enter negatively in all regressions, as the Barro model predicts. As in Barro (1986a), the YVAR coefficients are above one in absolute value, which suggests that tax revenues fall more than GNP in a recession instead of falling at the same rate. This is not quite consistent with the basic tax smoothing model, but it might be consistent with optimal policy if distortions (e.g., labor supply elasticities) vary over the business cycle.

To illustrate the contrast between the equation (3) and a simple, non-structural regression of the primary surplus on initial debt, Figures 1 and 2 show two scatterplots of the primary surplus against initial debt. Figure 1 displays the raw data. The war years are clearly visible (and labeled) as negative outliers. Apart from the outliers, there appears to be a positive relation between $s_t$ and $d_t$, as postulated under Proposition 1. But because of the outliers, a simple univariate regression of $s_t$ and $d_t$ would yield an insignificant slope coefficient of -0.011 ($t=-0.31$). In Figure 2, the primary surplus is adjusted for the estimated “other” systematic determinants of the primary surplus, GVAR and YVAR. The graph plots the non-cyclical, non-war related component of the primary surplus against $d_t$. In contrast to Figure 1, the World War II observations do not show up as outliers (except in that debt was high). After adjusting for war-related and cyclical factors, there is a clear positive relation between debt and primary surplus.

Table 3 displays sample averages of interest rates, growth rates, and of the primary surplus. The return on government bonds $r=E[R_t]$ is below the average growth rate $y=E[y_t]$ for all sample periods. The fact that $r<y$ holds in the data underscores the theoretical concerns about sustainability tests that implicitly assume $r>y$. The average primary surplus is negative for the full sample and only

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17 The return estimates in the table are annual net federal interest payments divided by debt at the start of the fiscal year, minus inflation measured by the GNP-deflator. I also computed a series for $x_{t+1}$ by dividing $d_{t+1}$ by $(d_t-s_t)$ and obtained a mean of 0.987, which implies $r-y\approx-1.2\%$. 
slightly positive in those subperiods that do not include major wars. As noted in Bohn (1995), average primary deficits are difficult to reconcile with sustainability under risk-neutrality or in a deterministic setting, but they are consistent with sustainability in a model with risk-aversion. The average level of the primary surplus does not seem to be a good indicator for sustainability; and a moderately low level of the primary surplus does not necessarily lead to a rising debt-GDP ratio.\(^{18}\)

As noted above, the debt-income ratio should display mean reversion if the interest rate is below the growth rate. The mean reversion effect should be reinforced by the estimated positive coefficients for \(\rho\) in Table 2. But the unit root tests in Section 3.1 did not find evidence for mean reversion.

The tax-smoothing model allows one to reexamine this issue. Combining equations (1) and (3), the change in the debt-GDP ratio, \(\Delta d_{t+1}\), should be a declining function of initial debt and an increasing function of YVAR and GVAR. Table 4 reports regressions of \(\Delta d_{t+1}\) on these variables. The coefficient on the lagged debt-GDP ratio \(d_t\) is significantly negative in all cases. The YVAR and the GVAR variables enter positively in all regressions, as predicted by the model, though not always significantly.

Table 4 sheds new light on the unit root tests for government debt. Under the theoretically sound “null hypothesis” that the debt-GDP ratio is a highly autocorrelated but stationary time series, the standard unit root regression of \(\Delta d_{t+1}\) on \(d_t\) is misspecified and yields inconsistent estimates, because it omits the explanatory variables YVAR and GVAR. Once YVAR and GVAR are included,

\(^{18}\) In the U.S., the debt-GNP ratio for 1989 is above the pre-war ratio but below the ratios of 1948 and 1960. The ratio in 1983 is above the ratio in 1916, but below the ratios of 1920, 1948, and 1960. The frequently used Hamilton-Flavin debt series (covering 1960-84) also implies a declining debt-income ratio. Both Hamilton-Flavin (1986) and Wilcox (1989) work with interest rates below the U.S. growth rate.
one obtains significant evidence in favor of mean reversion in the debt-GDP ratio.\footnote{Even if one argues that because of the near-unit root behavior of $d_t$, one should apply Dickey-Fuller’s critical values to evaluate the significance of $\rho$ in Table 4, most of the $t$-values on $\rho$ are significantly negative. Except for the short 1948-89 sample, all $t$-values exceed the critical value of 3.51 (for 1 percent significance).}

The omitted-variables interpretation is consistent with the finding in Table 1 that a unit root in the debt-GNP ratio can be rejected strongly when the two world wars are excluded from the 1916-89 sample and when the three major wars (Civil War, World War I and World War II) are excluded from the 1800-1988 sample.

Figure 1 illustrates the special importance of the war periods. The debt-GNP ratio tends to move towards zero during peacetime periods (with some exceptions), but these movements are interrupted by upward jumps during wars. The wartime jumps are in line with the tax-smoothing model, which predicts that temporary military spending should be largely debt-financed. But the war periods create problems in unit root tests, because high deficits often occur in the later years of a war, i.e., right after debt has already been driven up by the deficits in the early years of the war.

To illustrate this effect, Figure 3 plots the change in the debt-income ratio, $\Delta d_{t+1}$, against initial debt, $d_t$. This is the relation fitted by the unit root tests. If one abstracted from these “outliers” (labeled with years), there appears to be a negative relation. But the outliers—the values for 1942-45 in particular—are so large that no significant mean reversion is found in Dickey-Fuller test that include the war years.

From a theoretical perspective, the mean reversion result for the debt-GDP ratio is significant in light of Barro’s (1979) prediction that the debt-income ratio
should be non-stationary, i.e. that $\rho \approx r-y$.\textsuperscript{20} Given that $r < y$, one would have to be seriously concerned about sustainability if the equality $\rho \approx r-y$ were indeed true, because that would imply $\rho < 0$. Still, one may ask whether $\rho \neq r-y$ implies a rejection of the tax-smoothing model. Fortunately (for the model), the prediction $\rho \approx r-y$ does not extend to a stochastic setting with risk aversion. Though a general analysis of optimal taxation in stochastic models is clearly beyond the scope of this paper, the appendix shows for a special case that the optimal policy will satisfy $\rho > 0$ irrespective of the sign of $r-y$.

From an empirical perspective stationarity of the debt-GDP is interesting in comparison to the literature. Kremers (1989) finds a unit root in the logarithm of the debt-income ratio.\textsuperscript{21} This suggests that the choice of levels versus logarithms matters for small sample inferences. But note that if one substitutes (2) into (1) and takes logarithms and if $\mu_t$ is relatively small, the product $x_{t+1} \cdot (1-\rho) \cdot d_t$ is approximately transformed into a sum with a unit coefficient on $\log(d_t)$. A regression of $\log(d_{t+1})$ on $\log(d_t)$ may therefore produce a coefficient near one even if $\rho$ is substantially above zero.

The stationarity of the surplus-GDP and the debt-GDP ratios also raises some questions about the interpretation of sustainability tests based on co-integration. Most of these tests—e.g., Trehan and Walsh (1988,1991), Hakkio and Rush (1986)—use real levels of fiscal variables and find unit roots in government spending, debt, and taxes. From the results here, it appears that the unit root in

\textsuperscript{20} In contrast to this paper, Barro finds no empirical evidence for mean reversion. He uses (scaled) changes in nominal government debt as left hand side variable and adds a proxy for expected inflation on the right. One may wonder whether his inability to find mean reversion is due to problems associated with estimating inflation. (See also Kremers 1989.) Note that all data used in lines 3 and 4 are taken directly from Barro (1986a) so that data problems cannot explain the differences.

\textsuperscript{21} In the structural analysis, Kremers also uses Barro’s model as theoretical framework, and—like Barro—focuses on the growth of nominal debt, not on the primary surplus or the debt-income ratio as this paper. The above comments on Barro’s approach apply analogously.
real debt is either not really there or due to a unit root in GDP. For long sample
periods (say, 1800-1988), an analysis in GDP-shares is clearly preferable to an
analysis in levels, because there would be extreme heteroskedasticity if levels of
fiscal variables were used. For shorter sample periods, this heteroskedasticity
problem may not be as obvious, but it remains troubling. If the analysis is done
terms of GDP-shares, the stationarity of $s_t$ and $d_t$ implies that regressions
involving debt and deficits cannot be interpreted as co-integrating regressions.\textsuperscript{22}

The results in Tables 2 and 4 are remarkably uniform across different
sample periods. In contrast, the policy discussion about the recent “deficit-
problem” suggests that U.S. fiscal policy of the 1980s might differ from the
previous pattern. To explore this issue, Figure 5 compares actual and predicted
primary surpluses. The predictions are based on the regression in Table 2, line 1,
which uses the full sample period.\textsuperscript{23} The graph confirms that the 1980s are
somewhat unusual: Actual primary surpluses have been below the predicted
values since 1984. The primary surplus for 1983 is almost exactly as predicted. But
from 1983 to 1986, the primary surplus failed to increase as much as the model
predicts; it even declined in 1985 and 1986. Since 1986, however, the actual
surplus increased has started to catch up, so that the prediction error declined
from 1.5% of GNP in 1986 to 0.7% in 1989. Given that the underlying regression
has a standard error of 1.4%, these prediction errors are not particularly large. It is

\textsuperscript{22} Trehan and Walsh (1991) also question the relevance of cointegration restrictions, but because of
the sensitivity of these restrictions to assumptions about interest rates. Given the stationarity of the
GDP-shares, I am not convinced that the stationarity of debt and/or deficit levels by itself can
provide decisive evidence on sustainability (cf. Trehan and Walsh 1991). Regarding Bohn (1991a),
the results here provide a new justification for estimating error-corrections models with $(\tau_t, g_t)$
without including $d_t$. The non-stationarity of $\tau_t$ and $g_t$ together with the stationarity of $s_t$ implies
co-integration between $\tau_t$ and $g_t$. This can be exploited to obtain insights about the behavior of
governments in response to high deficits (Bohn 1991a), but it does not provide information about
sustainability.

\textsuperscript{23} Predictions based on the other regressions yield similar results.
therefore not surprising that tests for structural stability do not reveal significant differences between pre- and post-1983 fiscal policy.24

Another unusual feature of the 1980s is that real interest rates were much higher than in previous decades. This observation does not affect the main results in Table 2, because the sustainability test of this paper was specifically designed not to rely on assumptions about interest rates.25 The high interest rates of the early 1980s are of course included in the average returns data in Table 3; the average returns are nonetheless below the growth rate in all cases.

Overall, the estimation results provide substantial evidence that the sufficient condition for sustainability has historically been satisfied for the United States. The main evidence is that the primary deficit has responded positively to increased debt. In addition, the debt-GDP ratio shows mean reversion.

### 3.3. Non-linearities

Propositions 1 can be generalized for non-linear relationships between the surplus-GDP ratio and the debt-GDP ratio. If \( s_t = \mu_1 t + f(d_t, \mu_2 t) \) is characterized by a non-linear function \( f(\cdot) \) and bounded processes \( (\mu_1 t, \mu_2 t) \), fiscal policy still satisfies the intertemporal budget constraint, provided \( f(\cdot) \) can be bounded below by some linear function \( \eta + \rho \cdot d_t \) with \( \rho > 0 \). (The proof is a simple dominance argument.) To explore potential non-linearities, I extended equation (3) by adding up to fourth-order polynomials of \( d_t \), functions of the form \( \min(d_t - \overline{d}, 0) \) that pick out high values for debt, and separate terms for war and non-war years. Because of the obvious problems of over-fitting (e.g., the war-time observations can be perfectly

---

24 I tested for changes in the intercept, in the intercept and the \( \rho \)-coefficient, and in all four coefficients. Note that the increase in the predicted surplus from 1983-89 is mainly due to cyclical factors; only 0.6% of the predicted 4.2% increase is due to higher debt.

25 High real interest rates have an indirect influence on the regressions in Tables 2 and 4 in that \( d_t \) increases faster than it would have otherwise. But volatility in interest rates and output growth is actually helpful, because it tends to increase the in-sample variations in \( d_t \).
matched with a few “min” functions that pick out sample points with high debt) and multicollinearity, such regressions should not be taken too seriously. But they may be worth reporting to rebut concerns that the relation between the surplus-GDP ratio and the debt-GDP ratio might “flatten out” at high debt-GDP ratios. I find that the coefficients on quadratic terms in $d_t$ and $\min(d_t-d-,0)$ terms yield consistently positive, and in some cases significantly positive, point estimates, while cubic and higher order terms are never close to being significant.26

4. Conclusions

The paper derives and empirically tests a condition for the sustainability of government policy that is applicable in a stochastic setting and even if the interest rate on government bonds is below the average rate of economic growth. The paper finds that U.S. fiscal policy has historically been on a path that satisfies the transversality condition.

---

26 For example, if the quadratic term $\alpha^2(d_t-d-)^2$ is added to equation (3), the estimates for the full sample are $\rho=0.030$ (t=2.70) and $\alpha^2=0.097$ (t=3.58) and the estimates for the non-war sample are $\rho=0.020$ (t=1.43) and $\alpha^2=0.076$ (t=1.28). The coefficients on GVAR and YBAR remain almost unchanged. Compared to the first two regressions in Table 4, the $\rho$ coefficients are reduced, but the positive quadratic terms strengthen the conclusion that the transversality condition is satisfied. The coefficients in the non-war sample are individually insignificant because of multicollinearity, but they are jointly significant at the 1.2% level.
References


Appendix

A1. Proof of Proposition 1

Bohn (1995) has shown that a stochastic general equilibrium model implies the transversality condition

$$\lim_{N \to \infty} E_t[u_{t,N} \cdot D_{t+N}] = 0,$$

(A1)

where $u_{t,N}$ is the marginal rate of substitution between periods $t$ and $t+N$ and $D_{t+N} = d_{t+N} \cdot Y_{t+N}$ the level of government debt in period $N$ periods ahead. The expectation in (A1) can be interpreted as product of debt times state-contingent claims prices summed over all states of nature. Equation (A1) reduces to the standard deterministic or expected value condition

$$\lim_{N \to \infty} \frac{1}{(1+r)^N} \cdot E_t[D_{t+N}] = 0$$

(A2)

if individuals are risk-neutral or if there is no uncertainty, but not otherwise.

The rate of return $R_{t+1}$ on any security (including government debt securities) must satisfy the Euler equation $E_t[u_{t,1} \cdot (1+R_{t+1})] = 1$. But since $u_{t,1} \cdot (1+R_{t+1})$ may differ from one for any or all states of nature, the marginal rate of substitution in (A1) is cannot be measured by the discount rate on government debt. The appropriate discount rate in (A1) depends on how the overall level of government debt is distributed across state of nature and not on the promised return on any particular debt security. (See Bohn, 1995 for more details.)

To show that the policy defined by (2) satisfies (A1), we have to prove that $z_n = E_t[u_{t,n} \cdot D_{t+n}]$ converges to zero as $n \to \infty$. That is, given any $\varepsilon > 0$, there must be a value $N^*$ such that $|z_n| < \varepsilon$ for all $n \geq N^*$. By iterating on (1) and (2), one obtains

$$d_{t+n} = (1-p)^n \left( \prod_{j=1}^{n} x_{t+j} \right) \cdot d_t + \sum_{i=1}^{n} (1-p)^{n-i} \cdot \left( \prod_{j=i}^{n} x_{t+j} \right) \cdot \mu_{t+i-1}$$

(A3)
Using the relations $z_n = Y_t \cdot E_t u_{t,n} \prod_{j=1}^{n} (1+y_{t+j}) \cdot d_{t+n}$ and $E_t[u_{t,i} \cdot (1+R_{t+i+1})] = 1$, taking expectations, (A3) implies

$$z_n/Y_t = (1-\rho)^n \cdot d_t + \sum_{i=1}^{n} (1-\rho)^{n-i} \cdot E_t[a_{i-1}]$$

where $a_k = u_{t,k} \prod_{j=1}^{k} (1+y_{t+j}) \cdot \mu_{t+k}$.

By assumption, the present value of future income, $V_t = Y_t \cdot \sum_{k \geq 1} E_t u_{t,k} \prod_{j=1}^{k} (1+y_{t+j})$ is finite. Finiteness of this sum implies that the elements in the sum must converge to zero, i.e., $E_t u_{t,k} \prod_{j=1}^{k} (1+y_{t+j}) \to 0$ as $k \to \infty$. Combined with a bound on $\mu_t$, this implies $E_t a_k \to 0$ as $k \to \infty$. That is, for any $\delta > 0$ there is an $N$ such that $|E_t a_k| \leq \delta$ for all $k \geq N$. Let $\sum_{i=1}^{N} (1-\rho)^{n-i} \cdot E_t[a_{i-1}] = \Omega$, then for $n \geq N$ we have

$$|z_n/Y_t - [ (1-\rho)^n \cdot d_t + (1-\rho)^{n-N} \Omega] | = \sum_{i=N+1}^{n} (1-\rho)^{n-i} \cdot |E_t[a_{i-1}]| \leq \delta/\rho$$

Since $(1-\rho)^n \to 0$ as $n \to \infty$, the absolute value of $z_n$ will be less than $\varepsilon$ for high enough $n$, provided one picks $\delta < \varepsilon \cdot \rho / Y_t$. Q.E.D.

**A2. Tax Smoothing under Uncertainty**

Barro (1979) argues that, under certain assumption, tax-smoothing implies random walk behavior for the debt-GDP ratio. This section provides an example demonstrating that tax-smoothing in a stochastic model does not imply a non-stationary path for the debt-income ratio. Moreover, optimal taxes respond positively to unexpected jumps in government debt.

Consider the following simple Lucas (1978) exchange economy with government and with cost of tax collection (as in Barro 1979). Consumers maximize $\sum_{t \geq 0} \beta^t u(c_t)$ subject to $A_{t+1} = (1+R_{t+1}) \cdot [A_t + Y_t \cdot (1-\tau_t) - h(\tau_t) - c_t]$, where $c=$consumption, $A=$assets = claims on the government, $R =$ return on government debt, $Y =$ exogenous income, $\tau =$ tax rate, and $h(\tau) = h/2 \tau^2 =$ cost of tax collection. Government debt satisfies $D_{t+1} = (1+R_{t+1}) \cdot [D_t + Y_t \cdot \tau_t]$. To simplify, assume that utility is CRRA with risk aversion $\alpha$, that $G_t = g \cdot Y_t$ for all $t$ and that
income growth $y_t$ is lognormal i.i.d. Then the individual first order condition for $R_t$ is

$$E_t[(1+R_t)\cdot\beta\cdot(1+y_{t+1})^{-\alpha}\cdot[(1-g-h(\tau_{t+1}))/\{(1-g-h(\tau_t))\}^{\alpha}]} = 1.$$  \tag{A1}

If the government were able to borrow on complete markets, it would be straightforward to show that the welfare maximizing policy for “small” values of $h$ would be to issue income-indexed debt and to stabilize the tax rate at a fixed value at all times and for all states of nature. Therefore, to explain why there are any movements in tax rates, one has to impose restrictions on debt management. Specifically, I will impose the same assumption that Barro (1979) apparently imposes implicitly, namely that the government has to use safe debt with return $R_{t+1} = r_t$. Then the government’s first order condition is

$$E_t[[1+r_t + [D_t+G_t\cdot Y_t\cdot \tau_t] \cdot (1+r_{t+1})^{-\alpha}\cdot \beta\cdot(1+y_{t+1})^{-\alpha}\cdot(1-g-h(\tau_{t+1}))/\{(1-g-h(\tau_t))\}^{\alpha}\cdot\tau_{t+1})] = \tau_t,$$

using the the linearity of $h'(\cdot)$. It is straightforward to show by differentiating (A1) that $dr_t/dd_t$ will converge to zero in the limit as $h$ becomes small. In the limit, optimal tax policy is characterized by

$$E_t[(1+r)\cdot\beta\cdot(1+y_{t+1})^{-\alpha}\cdot\tau_{t+1}] = \tau_t$$  \tag{A2}

where $r_t = r$ is determined by $E_t[(1+r)\cdot\beta\cdot(1+y_{t+1})^{-\alpha}] = 1$. To examine what this condition implies for debt service, consider the class of linear polices $\tau_t = g + \rho\cdot d_t$. Since this class of policies implies $d_{t+1} = (1+r)/(1+y_{t+1} \cdot (1-\rho)\cdot d_t$, substitution into (A2) yields

$$(1-\rho)\cdot E_t[(1+r)^2\cdot\beta\cdot(1+y_{t+1})^{-\alpha-1}] = 1,$$  \tag{A3}

For comparison, note that the optimal policy in a hypothetical economy with complete markets will also fall into this linear class (suggesting that the linearity restriction is not unreasonable) with a parameter $\rho_v = v/(1+v)>0$, where $v$ is defined by $E_t[(1+v)\cdot\beta\cdot(1+y_{t+1})^{1-\alpha}] = 1$. (The value $v$ can be interpreted as ratio of income to the present value of income, which is positive because of dynamic efficiency.) Using the lognormality assumption, one has
\[ \log(1+r) = -\log(\beta) + \alpha x - \alpha^2 / 2 \sigma^2 \]
\[ \log(1+v) = -\log(\beta) + (\alpha+1)x - (\alpha+1)^2 / 2 \sigma^2 \]
\[ \log(1-\rho) + 2 \log(1+r) = -\log(\beta) + (\alpha-1)x - (\alpha-1)^2 / 2 \sigma^2 \]

where \((x, \sigma^2)\) are the mean and variance of \(\log(1+y_t)\). After some algebra, this implies

\[ \log(1-\rho) + \log(1+v) = -\sigma^2 > 0, \]

hence \(\rho > \rho_v > 0\). That is, regardless of the interest rate \(r\), the optimizing government will run a primary budget surplus that is at least as large as the surplus it would have run under the optimal policy with complete markets. In particular, the government will not try to exploit an interest rate on safe debt below the average growth rate to run primary deficits.

**A3. Description of the Data**

Except for the budget surplus \(s_t\), the data in Section 3 are based on Barro (1986a). For 1916-83, the series YVAR and GVAR were taken directly from Barro (1986a), except that an adjustment was made for YVAR in 1925 and 1930 where the values did not match those in Barro (1986b). The variables were updated for 1984-89 using the methods explained in Barro (1981, equation 14; 1986a, p. 204) and Sahasakul (1986; equations 20, 21). The resulting values for 1984-89 are 0.0042, 0.0037, 0.0033, 0.0015, 0.0, and -0.0004 for YVAR, and -0.0028, -0.0032, -0.0043, -0.0087, -0.0137, and -0.0159 for GVAR, respectively. The debt series \(d_t\) is the ratio of privately held public debt (from the WEFA database and Federal Reserve Banking and Monetary Statistics) to GNP (from the WEFA database and pre-1929 Commerce Department data). It differs lightly from Barro’s series on the private debt-GNP ratio because of data revisions (mainly in GNP) and because of the inclusion of minor amounts of non-interest bearing debt. In lines 3 and 4 of Tables 2 and 4, Barro’s original debt-GNP series was used for comparison.
The primary budget surplus $s_t$ for calendar years 1929-89 was constructed by dividing the difference of federal receipts and non-interest outlays by nominal GNP, where all series were taken from the National Income and Product Accounts (WEFA database). The surplus for 1916-28 was obtained by interpolating a fiscal year surplus series (with sources described below).

In Table 1, the fiscal-year debt series for 1800-1988 refers to public debt. All fiscal year series were taken from Bohn (1991a). They are largely based on the Historical Statistics of the United States and The Budget of the United States. A description of this data set is available from the author. In Table 2, $y$ is average real GNP growth and $r$ is the average ratio of federal interest outlays divided by initial debt on a fiscal year basis, minus the growth rate of the GNP deflator. The GNP deflator was taken as inflation measure to maintain comparability to $y$. 
Table 1: Unit Root Tests

Panel A: Augmented Dickey-Fuller Tests

<table>
<thead>
<tr>
<th>Samples:</th>
<th>Calendar Years 1916-89</th>
<th>Fiscal Years 1800-1988</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>All Years</td>
<td>Excl. Wars†</td>
</tr>
<tr>
<td>Debt $d_t$</td>
<td>2.20</td>
<td>3.07*</td>
</tr>
<tr>
<td>Surplus $s_t$</td>
<td>4.34**</td>
<td>5.12**</td>
</tr>
</tbody>
</table>

Panel B: Phillips-Perron Tests

<table>
<thead>
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<th>Samples:</th>
<th>Calendar Years 1916-89</th>
<th>Fiscal Years 1800-1988</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>All Years</td>
<td>Excl. Wars†</td>
</tr>
<tr>
<td>Debt $d_t$</td>
<td>2.24</td>
<td>4.58**</td>
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<tr>
<td>Surplus $s_t$</td>
<td>3.56**</td>
<td>6.24**</td>
</tr>
</tbody>
</table>

**Legend:** The test statistics in Panel A are absolute t-values from augmented Dickey-Fuller regressions estimated with two lags. Phillips-Perron “Zτ-statistics” are in Panel B. Both test the null hypothesis of a unit root. The critical values for both statistics are 2.89 at the 5% level and 3.45 at the 1% level (based on 100 observations). The variables $s_t$ and $d_t$ are the primary U.S. budget surplus and U.S. government debt (private debt for 1916-89, public debt for 1800-1988), respectively, each divided by GNP. The 1916-89 sample has calendar year variables (74 observations), the 1800-1988 sample has fiscal year values (189 observations). Details and data sources are described in the appendix.

† = sample period 1920-40 and 1948-89; 63 observations.
Table 2: Determinants of the Budget Surplus

Dependent Variable: Budget Surplus $s_t$

<table>
<thead>
<tr>
<th>Sample</th>
<th>Constant</th>
<th>GVAR</th>
<th>YVAR</th>
<th>$d_t$</th>
<th>$R^2$</th>
<th>$\sigma$</th>
<th>DW</th>
</tr>
</thead>
<tbody>
<tr>
<td>1916-89</td>
<td>-0.019</td>
<td>-0.798</td>
<td>-1.503</td>
<td>0.056</td>
<td>0.939</td>
<td>0.014</td>
<td>1.47</td>
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<tr>
<td></td>
<td>(-5.255)</td>
<td>(-32.348)</td>
<td>(-3.600)</td>
<td>(6.090)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1920-89 excl. 40-47</td>
<td>-0.009</td>
<td>-0.532</td>
<td>-2.034</td>
<td>0.030</td>
<td>0.635</td>
<td>0.011</td>
<td>1.45</td>
</tr>
<tr>
<td></td>
<td>(-1.882)</td>
<td>(-3.705)</td>
<td>(-4.758)</td>
<td>(2.806)</td>
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<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[-1.548]</td>
<td>[-2.886]</td>
<td>[-4.081]</td>
<td>[2.151]</td>
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<td></td>
</tr>
<tr>
<td>1916-83</td>
<td>-0.018</td>
<td>-0.800</td>
<td>-1.446</td>
<td>0.055</td>
<td>0.942</td>
<td>0.015</td>
<td>1.54</td>
</tr>
<tr>
<td></td>
<td>(-4.906)</td>
<td>(-31.672)</td>
<td>(-3.360)</td>
<td>(5.998)</td>
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</tr>
<tr>
<td>1920-82 excl. 40-47</td>
<td>-0.008</td>
<td>-0.532</td>
<td>-1.954</td>
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<td>0.630</td>
<td>0.012</td>
<td>1.56</td>
</tr>
<tr>
<td></td>
<td>(-1.713)</td>
<td>(-3.613)</td>
<td>(-4.439)</td>
<td>(2.817)</td>
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</tr>
<tr>
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<td>[-3.857]</td>
<td>[2.203]</td>
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<tr>
<td>1948-89</td>
<td>-0.015</td>
<td>-0.570</td>
<td>-2.135</td>
<td>0.040</td>
<td>0.663</td>
<td>0.010</td>
<td>1.63</td>
</tr>
<tr>
<td></td>
<td>(-3.294)</td>
<td>(-3.741)</td>
<td>(-4.087)</td>
<td>(3.599)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1960-84</td>
<td>-0.012</td>
<td>-0.393</td>
<td>-2.158</td>
<td>0.044</td>
<td>0.730</td>
<td>0.007</td>
<td>1.44</td>
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<tr>
<td></td>
<td>(-2.057)</td>
<td>(-2.017)</td>
<td>(-4.381)</td>
<td>(2.036)</td>
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</tr>
<tr>
<td></td>
<td>[-2.139]</td>
<td>[-2.097]</td>
<td>[-3.702]</td>
<td>[2.623]</td>
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</tr>
</tbody>
</table>

**Legend:** () = t-statistics, [] = White’s (1980) robust t-statistics, $\sigma$ = standard error, DW = Durbin-Watson statistic. The variable $s_t$ is the primary U.S. budget surplus divided by GNP, $d_t$ is privately-held debt/GNP at the start of the year, GVAR and YBAR are measures of temporary government spending and of cyclical variations in output, respectively, which are based on Barro (1986a). All estimates are OLS with annual data. Details and data sources are described in the appendix.
Table 3: Sample Averages

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Surplus $s_t$</td>
<td>-1.25%</td>
<td>0.27%</td>
<td>-1.30%</td>
<td>0.42%</td>
<td>0.40%</td>
<td>0.02%</td>
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<tr>
<td>GNP-growth $y$</td>
<td>3.29%</td>
<td>2.84%</td>
<td>3.23%</td>
<td>2.71%</td>
<td>3.30%</td>
<td>3.14%</td>
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<tr>
<td>Real Returns $r$</td>
<td>-0.17%</td>
<td>1.39%</td>
<td>-0.57%</td>
<td>1.02%</td>
<td>0.26%</td>
<td>-0.14%</td>
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<tr>
<td>Difference $r-y$</td>
<td>-3.46%</td>
<td>-1.44%</td>
<td>-3.81%</td>
<td>-1.69%</td>
<td>-3.05%</td>
<td>-3.28%</td>
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<tr>
<td>t-value on $r-y$</td>
<td>-3.22</td>
<td>-1.45</td>
<td>-3.29</td>
<td>-1.51</td>
<td>-5.59</td>
<td>-6.26</td>
</tr>
</tbody>
</table>

Legend:
Averages are taken over annual observations. The variable $s_t$ is the primary U.S. budget surplus divided by GNP, GNP-growth is the growth rate of real GNP. Real returns are net interest payments on the public debt divided by initial debt, minus inflation. Data sources are described in the appendix.
**Table 4: Determinants of Changes in the Debt-GNP Ratio**

Dependent Variable: Change in the Debt-GNP ratio $\Delta d_{t+1}$

<table>
<thead>
<tr>
<th>Sample</th>
<th>Constant</th>
<th>GVAR</th>
<th>YVAR</th>
<th>$d_t$</th>
<th>$R^2$</th>
<th>$\sigma$</th>
<th>DW</th>
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<tbody>
<tr>
<td>1916-89</td>
<td>0.038</td>
<td>0.753</td>
<td>1.443</td>
<td>-0.131</td>
<td>0.779</td>
<td>0.029</td>
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</tr>
<tr>
<td></td>
<td>(5.160)</td>
<td>(15.060)</td>
<td>(1.705)</td>
<td>(-7.060)</td>
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<td></td>
</tr>
<tr>
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<td>[3.305]</td>
<td>[15.668]</td>
<td>[2.189]</td>
<td>[-3.507]</td>
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</tr>
<tr>
<td>1920-89 excl. 40-47</td>
<td>0.019</td>
<td>0.071</td>
<td>2.506</td>
<td>-0.083</td>
<td>0.385</td>
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<td>[-3.755]</td>
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<tr>
<td>1916-82</td>
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<td>0.779</td>
<td>1.230</td>
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<td>0.794</td>
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<tr>
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<td>(4.767)</td>
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</tr>
<tr>
<td>1920-82 excl. 40-47</td>
<td>0.017</td>
<td>0.085</td>
<td>2.349</td>
<td>-0.085</td>
<td>0.371</td>
<td>0.025</td>
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<td></td>
<td>(1.759)</td>
<td>(0.272)</td>
<td>(2.514)</td>
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<td>[1.840]</td>
<td>[0.247]</td>
<td>[2.596]</td>
<td>[-3.318]</td>
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<tr>
<td>1948-89</td>
<td>0.009</td>
<td>0.161</td>
<td>3.786</td>
<td>-0.050</td>
<td>0.662</td>
<td>0.012</td>
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<td>(1.203)</td>
<td>(0.584)</td>
<td>(4.967)</td>
<td>(-2.462)</td>
<td></td>
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</tr>
<tr>
<td></td>
<td>[2.030]</td>
<td>[0.656]</td>
<td>[8.165]</td>
<td>[-2.634]</td>
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<tr>
<td>1960-84</td>
<td>0.016</td>
<td>0.518</td>
<td>3.201</td>
<td>-0.076</td>
<td>0.861</td>
<td>0.007</td>
<td>2.19</td>
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<tr>
<td></td>
<td>(2.766)</td>
<td>(2.715)</td>
<td>(6.641)</td>
<td>(-3.568)</td>
<td></td>
<td></td>
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</tr>
<tr>
<td></td>
<td>[3.649]</td>
<td>[4.143]</td>
<td>[9.877]</td>
<td>[-3.442]</td>
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</tbody>
</table>

**Legend:** () = t-statistics, [] = White’s (1980) robust t-statistics, $\sigma$ = standard error, DW = Durbin-Watson statistic. The variable $d_t$ is publicly-held debt/GNP at the start of the year, GVAR and YBAR are measures of temporary government spending and of cyclical variations in output, respectively. All estimates are OLS with annual data. Details and data sources are described in the appendix.
Figure 1: Primary Surplus versus Initial Debt

Legend:
The figure plots privately-held government debt/GNP at the start of a period on the horizontal axis against the primary budget surplus/GNP on the vertical axis. The plot is for 1916-89, with the labels referring to specific years. Data sources are described in the appendix.
Figure 2: Adjusted Primary Surplus versus Initial Debt

Legend:
The figure plots privately-held government debt/GNP at the start of a period on the horizontal axis against the difference of primary surplus/GNP and its “other” determinants $\mu_t$, on the vertical axis. The other determinants, $\mu_t$, are estimated from the regression in Table 2, line 1. The plot is for 1916-89, with the labels referring to specific years. Data sources are described in the appendix.
Legend:
The figure shows the publicly-held debt of the federal government divided by GNP at the start of each fiscal year (in percent). Data sources are described in the appendix.
Figure 4: Change in Debt versus Initial Debt

Legend:
The figure plots privately-held government debt/GNP at the start of a period on the horizontal axis against the change in this variable over the subsequent year on the vertical axis. The plot is for 1916-89, with the labels referring to specific years. Data sources are described in the appendix.
Figure 5: Actual and Predicted Primary Surpluses

Legend:
The figure shows the primary surplus of the U.S government as ratio to GNP and it shows the primary surpluses that are predicted by the statistical model of Table 2, line 1.