Econ 594SA: Organization

  - Information is updated throughout the quarter.
  - Check for announcements. Class page announcements are assumed known.

• Open door policy for graduate students.
  - Official office hours: MW 2:30-3:30. NH3016.

• E-mail: [bohn@econ.ucsb.edu](mailto:bohn@econ.ucsb.edu). Put “Econ 594” in the subject line.

• Grading for Part I: two problem sets, plus “midterm” exam.
Macroeconomics

• Objectives of macroeconomics:
  To analyze the economy as a whole, to explain economic growth and economic fluctuations, and to assess economic policy.

• Outline of 594SA – Part I
  • Introduction: Intertemporal choice problems.
    - Tools: Constrained optimization. Graphical analysis. (Known already?)
  • The Solow Model: The Mechanics of Economic Growth.
  • Optimal Growth: Consumption and savings/investment choices over time.
    - Applications to Fiscal Policy.
    - If time: Application to money.
Intertemporal Choice: Consumption

• Individual decision problem:
  - Given a series of wage incomes $w_t$. Given real interest rate $r$ (assume constant).
  - Individuals choose consumption $c_t$ and asset holdings $a_t$
    Subject to budget equations: $a_t = (1 + r)a_{t-1} + w_t - c_t$

• Building intuition: Two-period setting with graphical analysis. Then generalize.
  - Consumption now ($c_1$) vs. consumption later ($c_2$).
  - Assume given initial wealth $A = (1 + r)a_0$.

• Budget equations imply an intertemporal budget constraint (IBC):
  - use $a_1 = (1 + r)a_0 + w_1 - c_1 = A + w_1 - c_1$ and
    $a_2 = (1 + r)a_1 + w_2 - c_2$
  - impose the terminal condition $a_2 = 0$:
    $a_1 = -\frac{1}{1+r}[w_2 - c_2] \implies 0 = A + w_1 - c_1 + \frac{1}{1+r}[w_2 - c_2]$
    $IBC: c_1 + \frac{1}{1+r}c_2 = w_1 + \frac{1}{1+r}w_2 + A$.
  - Means: Present value of consumption = Present value of income plus initial wealth.
Two Periods: Graphical Analysis

- Budget line has slope $-(1+r)$. Increase in $r \Rightarrow$ steeper slope.

  Feasible set: Area under the budget line.

- Endowment point is $(A+w_1, w_2)$. Higher $A$, $w_1$, $w_2 \Rightarrow$ budget line shifts “out”.
Two Periods: Math

• Optimization problem: maximize $U = u(c_1) + \beta u(c_2)$
  
  - subject to IBC: $c_1 + \frac{1}{1+r} c_2 = A + w_1 + \frac{1}{1+r} w_2$

• Approach #1: substitute constraint into objective. Problem is:
  
  $$\text{Max } U = u\left(A + w_1 + \frac{1}{1+r} w_2 - \frac{1}{1+r} c_2\right) + \beta u(c_2).$$

  $=>$ FOC for $c_2$: $-\frac{1}{1+r} u'(c_1) + \beta u'(c_2) = 0$  
  $=>$  

• Approach #2: use Lagrangian; define shadow value = $\lambda$. Problem is:
  
  $$\text{Max } L = u(c_1) + \beta u(c_2) + \lambda \cdot \left(A + w_1 + \frac{1}{1+r} w_2 - c_1 - \frac{1}{1+r} c_2\right)$$

  $=>$ FOC for $c_1$ & $c_2$: $u'(c_1) = \lambda$ and $\beta u'(c_2) = \lambda \cdot \frac{1}{1+r}$  

• Same conditions. If utility is strictly concave, the solution $(c_1,c_2)$ is unique.
Interpretation: Consumption Smoothing

- **Intuition:** Suppose time preference is approximately equal to the interest rate:

\[ \beta \approx \frac{1}{1+r} \Rightarrow u'(c_1) \approx u'(c_2) \Rightarrow c_1 \approx c_2 \]

- Insight: *Consumption is a “smooth” series.* True even if the income series varies.

- **Benchmark case:** suppose \( c_1 \approx c_2 \). Then

\[ c_1 + \frac{1}{1+r} c_2 = c_1 \cdot (1 + \frac{1}{1+r}) = A + w_1 + \frac{1}{1+r} w_2 \]

\[ \Rightarrow c_1 = c_2 = \frac{1}{1/(1+r)-1} \cdot (A + w_1 + \frac{1}{1+r} w_2) \]

- Find: *Consumption is approximately a fraction of total lifetime resources.*

- Note: If disposable income were a constant \( y \), then

\[ \frac{w_1 + \frac{1}{1+r} w_2}{1/(1+r)-1} = \frac{y + \frac{1}{1+r} y}{1/(1+r)-1} = y. \]

- **Permanent Income** = Annuity equivalent of the income actual stream (Friedman 1957)

\[ y^P = \frac{w_1 + \frac{1}{1+r} w_2}{1/(1+r)-1}. \]

Then \( c_1 = y^P + \frac{1}{1/(1+r)-1} \cdot A. \)

- Find: *Consumption \approx Permanent Income plus a fraction of initial wealth.*
**Interpretation: Savings Incentives**

- **Savings incentives:** High interest rates provide incentives to consume less & save more:
  - If $1+r > 1/\beta$, then $u'(c_1) > u'(c_2) \Rightarrow c_1 < c_2$. (Caveat: High $r$ also reduces $y^P$)
  - High interest rates “tilt” the consumption path upwards. Consumption grows over time.
  - Growing consumption must start at a lower level to satisfy the IBC.
    $\Rightarrow$ Initial consumption is (slightly) less than permanent income intuition would suggest.

- **Example (Power utility):** $u(c) = \frac{1}{1-\theta} c^{1-\theta}$ with $\theta > 0, \theta \neq 1$
  - Then $\frac{1}{1+r} u'(c_1) = \beta u'(c_2) \iff \frac{1}{1+r} c_1^{-\theta} = \beta c_2^{-\theta} \iff c_2 = [(1+r)\beta]^{1/\theta} c_1$
  - Combine with $A + w_1 + \frac{1}{1+r} w_2 = c_1 + \frac{1}{1+r} c_2 = c_1 + \frac{1}{1+r} [(1+r)\beta]^{1/\theta} c_1$
  - Obtain $c_1 = \frac{1}{1+\frac{1}{1+r}[(1+r)\beta]^{1/\theta}} (A + w_1 + \frac{1}{1+r} w_2)$
Many Periods: Permanent Income Model

- Generalize to arbitrary number of periods n. Terminal condition \( a_n = 0 \).

- Intertemporal budget constraint:

\[
\sum_{t=1}^{n} \frac{1}{(1+r)^{t-1}} c_t = A + \sum_{t=1}^{n} \frac{1}{(1+r)^{t-1}} w_t
\]

- If consumption is about constant (meaning: \( \beta \approx 1/(1+r) \)), the budget constraint implies

\[
c_1 = (A + \sum_{t=1}^{n} \frac{1}{(1+r)^{t-1}} w_t) / (\sum_{t=1}^{n} \frac{1}{(1+r)^{t-1}})
\]

Interpretation: \( \sum_{t=1}^{n} \frac{1}{(1+r)^{t-1}} = \frac{1+r}{r} [1 - (1+r)^{-n}] = \text{present value of a fixed annuity.} \)

For large n and small r, \( 1/\sum_{t=1}^{n} \frac{1}{(1+r)^{t-1}} \approx \frac{r}{1+r} \approx r \)

=> Consumption is approximately a fraction r of lifetime resources.

- Implications of permanent income theory:

1. **Distinguish changes in current income** (holding future income constant) from changes in permanent income (current and future income): Permanent changes have a much greater impact than temporary changes.

2. **Future income matters** => Expectations about future income matter.

   [Aside: uncertainty would complicate matters – ignored here.]

3. **Consumption growth** depends on interest rates relative to the rate of time preference.
Intertemporal Choice with Production

• Output is produced with capital and labor: \( Y_t = F(K_t, L_t) \)
  - Properties: Increasing; concave; constant returns to scale. (More later.)
  - Firm profits = Output – wage cost – cost of capital. (All in real terms.)
  - Real wage = Marginal product of labor: \( w_t = F_L(K_t,L_t) \)
  - Interest rate = Marginal product of capital – depreciation rate.

• Warm-up: A static model. Given K. No capital investment.
  - Time constraint: available hours = h. Time for work (l) plus leisure (h-l)
  - Households maximize utility from consumption and leisure: \( U = u(c, h-l) \)
    subject to a budget constraint: \( c = w \cdot l + \pi \) [given other funds \( \pi \)]
    FOC: \( u_1(c, h-l) \cdot w = u_2(c, h-l) \)
    => Consumption-leisure graph with indifference curves and budget line (slope –w).
  - Assume firms are owned by households: \( \Pi = F(K, L) - w \cdot L \). Population = N.
  - Market equilibrium with identical households: \( l = \frac{1}{N} L, \pi = \frac{1}{N} \Pi \)
    \( c = w \cdot \frac{L}{N} + \frac{1}{N} \Pi = \frac{1}{N} [w \cdot L + F(K, L) - w \cdot L] = \frac{1}{N} F(K, L) \)
    => Aggregate tradeoff between per-capita consumption and leisure is concave.
• What if a “social planner” (government) made all the decisions?
  - Social planner maximizes household utilities subject to production constraint.
    Maximize $U = u(c, h - l)$ s.t. $c = \frac{1}{N} F(K, N \cdot l)$.
    
    FOC: $u_1(c, h - l) \cdot \frac{1}{N} F_L \cdot N = u_1(c, h - l) \cdot F_L = u_2(c, h - l)$. Same as household FOC.
  - Note: Planning solutions are Pareto Optimal. (Here simple: all households identical.)
    => Social planner is a useful “device” to compute Pareto-optimal allocations.

- First fundamental welfare theorem: Competitive equilibrium is Pareto optimal.
  (Assuming price-taking/competitive behavior, increasing utility.)

- Second fundamental welfare theorem: All Pareto optimal solutions can be implemented as market equilibrium. (In general: with lump sum transfers).
  (Requires a concave production function and a concave utility function. Here no transfers needed because households are identical.)

• Interpret the social planner as a representative household who also operates a firm.
  1. Solving a representative agent problem is a convenient way to obtain market allocations.
  2. Given the optimal allocation, market clearing “prices” follow from the FOC.
    [Here: optimal (c,l) implies unique $w = u_2(c, h - l) / u_1(c, h - l)$.]
General Case: Production Economy with discrete time periods
(Here a sketch only)

• Resource constraints: \( Y_t = F(K_t, L_t) = I_t + C_t \) and \( K_{t+1} = I_t + (1 - \delta) \cdot K_t \).
- Household utility: \( U = u(c_1, h - l_1) + \beta \cdot u(c_2, h - l_2) + \ldots \)
- Alternative interpretations:
  1. Market allocation with firms and households; markets for labor, goods, capital.
  2. Social planner maximizes U subject to production constraints.
  3. Representative household maximizes U s.t. per-capita resource constraints.
• Observations:
  - Marginal increase in \( l_t \) allows a marginal increase in \( c_t \) by \( F_L \).
    \[ \frac{\partial u}{\partial c_t} F_L(K_t, L_t) = \frac{\partial u}{\partial l_t} \]
    must apply – as in the static production model; also: \( F_L = w_t \).
  - Marginal increase in \( K/N \) reduces \( c_t \) by same amount & increases \( c_{t+1} \) by \( F_K + (1 - \delta) \).
    \[ \Rightarrow \text{Optimality condition is } \frac{\partial u}{\partial c_t} = \beta \cdot \frac{\partial u}{\partial c_{t+1}} [F_K (K_t, L_t) + 1 - \delta] \]
    [also: \( F_K + 1 - \delta = 1 + r \).]

  Interpretation: Marginal product of capital = interest + depreciation = cost of capital
  [Note: Concave production => aggregate tradeoff between \( c_t \) and \( c_{t+1} \) is concave.]
  - Key challenge: Dynamics of capital and output: \( K_t \rightarrow Y_t \rightarrow I_t \rightarrow K_{t+1} \rightarrow Y_{t+1} \ldots \)

  How does such the economy evolve over time? How does economic growth occur?
Learning Objectives

• Conceptual:
  1. Macroeconomics is based on microeconomic principles – optimal choices subject to constraints, market equilibrium, welfare theorems.
  2. Microeconomic intuition: income and substitution effects.
  3. Macroeconomic intuition from simplified models:
     - Intertemporal consumption choices and permanent income.
     - Static production models and consumption-leisure tradeoff.

• Technical skills:
  1. Optimization – with and without constraints.
  2. Solving and interpreting intertemporal and static choice problems.

Problem sets for practice.