Fiscal Policy

- Many issues in practice – not all suitable for representative agent modeling
  
- Suitable issues:
  - Effects of government spending – real spending on good & services
  - Optimal fiscal policy with regard to government spending
  - Benchmark neutrality results on financing: Ricardian neutrality/equivalence theorem
  - Effects of tax distortions – incentive effects

- More elaborate models needed for:
  - Redistributional taxation – requires model with heterogonous population
  - Stabilization policy – requires model with stochastic disturbances
  - Government debt and social security – better with generational turnover

- Outline:
  I. Effects of government spending assuming lump-sum taxes; with note on optimal policy
  II. Ricardian neutrality
  III. Examples with tax distortions – examples only.
Fiscal Policy I: Government Spending

• Assumption: Public spending $G$ per efficiency unit of labor.
  - Assume spending is tax-financed: $T = G$, lump-sum.
  - Here abstract from productivity and population growth (could be added)
  - Best interpret as government consumption. [Public capital would require different model.]

• Resource constraint per-capita: $\dot{k} = f(k) - \delta k - c - G$
  - Phase diagram with $c(k)$-line: $c(k) = f(k) - \delta k - G$. Higher $G$ shifts the $c(k)$-line down.

• Preferences: utility over private consumption $U = \int_0^\infty e^{-\rho t} u(c) dt$.
  - Treat public spending as exogenous. (Standard macro view. Endogenous in public finance.)

  $\Rightarrow$ Hamiltonian: $H(c, k, \lambda, t) = e^{-\rho t} u(c) + \lambda \cdot [f(k) - \delta k - c - G]$
  - Note: $G$ enters only into the constraint $\Rightarrow$ Same Euler equations as without government.

  $\frac{\dot{c}}{c} = \frac{1}{\theta(C)} \left( f'(k) - \delta - \rho \right)$

  - Steady state conditions:
    $\dot{k} = 0 \Rightarrow c^* = f(k^*) - \delta k^* - G^*$
    $\dot{c} = 0 \Rightarrow f'(k^*) = \delta + \rho$

  - Implied phase diagrams: Fixed $k^*$-line. Shifts in $c(k)$-line whenever $G(t)$ changes.
Application #1: Permanent Increase

![Diagram showing the effects of a permanent increase in government purchases.](image)

**FIGURE 2.8 The effects of a permanent increase in government purchases**

- Find: Instant jump to the new steady state. Government spending has no impact on capital stock, no impact on the interest rate, crowds out private consumption one-for-one.
Application #2: Temporary Increase

- **Graphs:** See Romer Fig.2.9 (next slide)

- **Key idea:** Follow different “arrows” phase diagrams as time passes.
  - Until \( t_0 \): Economy at the initial steady state with normal.
  - Interval \([t_0, t_1]\): Movements in the phase diagram follow \( c(k) \) function with higher \( G \).
  - After \( t_1 \): Movements in the phase diagram follow \( c(k) \) function with normal \( G \).

- **Solve backwards:**
  - Return to the steady state in the long run \( \Rightarrow \) Must be on the ‘normal’ saddle path at \( t_1 \).
  - Determine \( c(0) \) such that movements during \([t_0, t_1]\) end on the saddle path at \( t_1 \).

- **Find:** Temporary increase government spending has
  - a temporary negative effect on capital accumulation: crowding out;
  - a temporary negative impact on consumption.
  - a temporary positive impact on interest rates.

Optional Exercise: Suppose at time \( t_0 \), the government announces that spending will increase permanently, starting at time \( t_1 \). Determine the effects.
Romer Fig. 2.9

(a)

(c)

\[ c = 0 \]

\[ \dot{k} = 0 \]

\[ k^* \]

\[ \rho + \theta g \]

\[ t_0 \]

\[ t_1 \]

Time
Note on Preferences and Optimal Policy

• Suppose individuals have preferences over private and public consumption:

$$U = \int_0^\infty e^{-\rho t} u(c,G) dt$$

- Standard approach in public finance: Social planner optimizes over \(G\)
- Hamiltonian: \(H(c,G,k,\lambda,t) = e^{-\rho t} u(c,G) + \lambda \cdot [f(k) - \delta k - c - G]\)

- Apply the Maximum Principle with two choice variables:

$$\frac{\partial H}{\partial c} = e^{-\rho t} \frac{\partial u}{\partial c} - \lambda = 0 \text{ and } \frac{\partial H}{\partial G} = e^{-\rho t} \frac{\partial u}{\partial G} - \lambda = 0$$

=> Optimality requires \(\frac{\partial u}{\partial c} / \frac{\partial u}{\partial G} = 1\) = unit marginal rate of substitution.

• Result: Optimal to divide to total consumption \(\tilde{c} = c + G\) into private and public components such that each provides the same utility on the margin.

• Express the model in terms of total consumption:

- write \(c = c(\tilde{c}), \ G = G(\tilde{c}), \) and \(\tilde{u}(\tilde{c}) = u[c(\tilde{c}),G(\tilde{c})]\)

=> Welfare problem: maximize

$$U = \int_0^\infty e^{-\rho t} \tilde{u}(\tilde{c}) dt \text{ s.t. } \dot{k} = f(k) - \delta k - \tilde{c}$$

• Interpretation: The optimal growth model without government sector can be interpreted as model in which public spending is subsumed into private spending.
Fiscal Policy II: Government debt and deficits

• Assumption: Given path of spending can be financed with taxes T or debt D.
  - Government budget equation:
    \[ \dot{D} = G(t) + r(t) \cdot D(t) - T(t) \]
  - Integrate over time:
    \[ D(t) = D(0) \cdot e^{\int_0^t r(v) dv} + \int_0^t (G(s) - T(s)) \cdot e^{\int_s^t r(v) dv} ds \]
  - Apply present value factors \( p_{0,t} = e^{-\int_0^t r(v) dv} \),
    \[ p_{0,t}D(t) = D(0) + \int_0^t p_{0,s} G(s) ds - \int_0^t p_{0,s} T(s) ds \]
  - Impose the transversality condition \( \lim_{t \to \infty} p_{0,t}D(t) = 0 \). Obtain the

• Government’s intertemporal budget constraint:
  \[ D(0) + \int_0^\infty p_{0,s} G(s) ds = \int_0^\infty p_{0,s} T(s) ds \]
  - Interpretation: Initial debt and the present value of spending must be backed by the present value of taxes.

• Claim (Ricardian neutrality): Financing of public spending does not matter.
Proof of Ricardian neutrality

• Step 1: Individual budget equation with taxes:
  \[ \dot{a} = w(t) + r(t) \cdot a(t) - c(t) - T(t) \]
  
  => Intertemporal budget constraint:
  \[ a(0) + \int_0^\infty p_{0, s} w(s) ds = \int_0^\infty p_{0, s} c(s) ds + \int_0^\infty p_{0, s} T(s) ds \]

• Step 2: Combine with the government budget constraint:
  \[ a(0) + \int_0^\infty p_{0, s} w(s) ds = \int_0^\infty p_{0, s} c(s) ds + D(0) + \int_0^\infty p_{0, s} G(s) ds \]

• Step 3: Individual assets = capital + government bonds. Write K = a – D, assuming L=1
  \[ K(0) + \int_0^\infty p_{0, s} w(s) ds = \int_0^\infty p_{0, s} c(s) ds + \int_0^\infty p_{0, s} G(s) ds \]

• Find: Taxes and government bonds cancel out.

  PV of consumption = Private resources – PV of public spending

• Interpretation:
  1. Ricardian neutrality: Given the spending path, taxes and deficits have no impact on individual decisions.
  2. Ricardian equivalence: Tax-financed and deficit-financed government spending have equivalent effects on individual decisions.
  3. Government bonds should NOT be considered net wealth (Barro)
Practical relevance: Are budget deficits really irrelevant?

1. Good intuition – taxpayers own the government debt.

   Modifications needed with tax distortions, finite lives, credit market imperfections, uncertainty about incidence of future taxes, etc. (later) – all best understood as departures from the benchmark.

2. Key insight for more elaborate models: if the income effects of tax policy cancel out, the substitution effects remain => Shift of emphasis since 1970s to tax distortions / incentive / “supply side” effects.

3. Property of representative agent models: accept or use different model.

Fiscal Policy III: Tax Incentives

Example from Romer: Suppose taxes are imposed on capital income.

- Individual savings: \[ \dot{a} = w(t) + (1 - \tau) \cdot r(t) \cdot a(t) - c(t) \]

  => Optimality condition: \[ \frac{\dot{c}}{c} = \frac{1}{\delta(c)} \left[ r(1 - \tau) - \rho \right] \]

- Steady state conditions:
  \[ \dot{k} = 0 \quad \Rightarrow \quad c^* = f(k^*) - \delta k^* - G^* \quad \text{where} \quad G^* = \tau[f'(k^*) - \delta]k^* \]
  \[ \dot{c}/c = 0 \quad \Rightarrow \quad (1 - \tau)[f'(k^*) - \delta] = \rho \]

- Phase diagrams: k*-line shifts to the left; c(k) as before => Taxes with incentive effects do matter.

Note on decentralization: When taxes are distortionary, market equilibrium is not Pareto-optimal

=> Cannot use planning problem to compute equilibrium (Advanced: add “implementability constraints”)

Example with labor-leisure choice

- Example not in Romer: setting with labor-leisure choice and \( \tau(t) = \) labor income tax rate

Maximize: \( U = \int_0^\infty e^{-\rho t} u(c, 1 - l) dt \) with unit time allocated to \( l = \) labor, \( (1-l) = \) leisure

subject to: \( \dot{a} = (1 - \tau(t)) \cdot w(t) \cdot l(t) + r(t) \cdot a(t) - c(t) \)

- Hamiltonian: \( H(c, l, a, \lambda, t) = e^{-\rho t} u(c, 1 - l) + \lambda \cdot [(1 - \tau)wl + ra - c] \)

- Apply the Maximum Principle with two choice variables:
  \[
  \frac{\partial H}{\partial c} = e^{-\rho t} \frac{\partial u(c, 1-l)}{\partial c} - \lambda = 0 \quad \text{and} \quad \frac{\partial H}{\partial l} = e^{-\rho t} \left( -\frac{\partial u(c, 1-l)}{\partial l} \right) + (1 - \tau)w\lambda = 0
  \]

  \( \Rightarrow \) Individual optimality condition: \( \frac{\partial u(c, 1-l)}{\partial c} \) depends on the tax rates.

- Government: spends \( G \), has revenues \( T(t) = \tau(t) \cdot w(t) \cdot l(t) \) \( \Rightarrow \) \( \dot{D} = G + rD - \tau \cdot w \cdot l \) and

  \( \dot{K} = \dot{a} - \dot{D} = w \cdot l + r \cdot (a - D) - c - G = F(K, l) - \delta K - c - G = l \cdot [f(k) - \delta k] - c - G \)

  \( \Rightarrow \) Taxes cancel out for capital accumulation, but affect labor supply.

- Suppose \( u() \) is separable \( \Rightarrow \) Euler equation \( \frac{\dot{c}}{c} = \frac{1}{\theta(C)} [f'(k) - \delta - \rho] \)

- Suppose \( (G, \tau) \) are constant \( \Rightarrow (k, c, l) \) converge to a steady state with: \( f'(k^*) - \delta = \rho \)

  \[
  \frac{\partial u(c^*, 1-l^*)}{\partial l} = (1 - \tau) \cdot F_L(K^*, l^*) = (1 - \tau) \cdot w(k^*), \quad c^* = l^* \cdot [f(k) - \delta k^*] - G.
  \]

  - Strictly concave utility \( \Rightarrow \) \( \frac{\partial c^*}{\partial \tau} < 0 \), \( \frac{\partial l^*}{\partial \tau} < 0 \)

- Contrast to welfare problem: max \( U \) s.t. \( \dot{K} = F(K, l) - \delta K - c - G \) \( \Rightarrow \) \( \frac{\partial u(c^*, 1-l^*)}{\partial l} = w(k^*) \)

  \( \Rightarrow \) Tax distortions reduce welfare as compared to an allocation with lump-sum taxes.
Learning Objectives for Fiscal Policy

• Applied: Ability to do fiscal policy applications
  - Determine the impact of government spending in the optimal growth model.
    
    Problem sets for practice.

• Conceptual:
  - Know the impact of government spending in the optimal growth model.
  - Know when Ricardian neutrality applies.
  - Know that distortionary taxes create complications.