Econ 204A: Organization

- Class Page: [www.econ.ucsb.edu/~bohn/204A/204Aindex.html](http://www.econ.ucsb.edu/~bohn/204A/204Aindex.html)
  - Information is updated throughout the quarter.
  - Check for announcements. Class page announcements are assumed known.

- Open door policy for graduate students. NH3016.
  - Official office hours posted on the class page.

- E-mail: [henning.bohn@ucsb.edu](mailto:henning.bohn@ucsb.edu). Put “Econ 204A” in the subject line.

- Grading: Weekly problem sets, midterm (in class), final exam.
Introduction to Macroeconomics

- Objectives of macroeconomics:
  To analyze the economy as a whole, to explain economic growth and economic fluctuations, and to assess economic policy.

- Outline of Econ 204A:
  - **Introduction**: Intertemporal choice problems.
    - Tools: Constrained optimization. Graphical analysis.
  - **The Solow Model**: The Mechanics of Capital, Production, and Economic Growth.
    - If time: Introduction to New Growth.
  - **The Ramsey Model**: Optimal Consumption and Savings/Investment over Time.
    - Applications to Fiscal Policy and to Money. Digression to discrete time.
  - **The Diamond model**: Modeling Overlapping Generations.
Intertemporal Choice: Consumption

- Individual decision problem:
  - Given a series of wage incomes $w_t$. Given real interest rate $r_t=r$ (constant).
  - Individuals choose consumption $c_t$ and asset holdings $a_t$ subject to
    Budget equations: $a_t = (1 + r)a_{t-1} + w_t - c_t$

- Building intuition: Two-period version with graphical analysis. Then generalize.
  - Consumption now ($c_1$) vs. consumption later ($c_2$).
  - Assume given initial wealth $A = (1 + r)a_0$.

- Budget equations imply an intertemporal budget constraint (IBC):
  - use $a_1 = (1 + r)a_0 + w_1 - c_1 = A + w_1 - c_1$ and
    $a_2 = (1 + r)a_1 + w_2 - c_2$
  - impose the terminal condition $a_2 = 0$:

    $\Rightarrow a_1 = -\frac{1}{1+r} [w_2 - c_2] \Rightarrow 0 = A + w_1 - c_1 + \frac{1}{1+r} [w_2 - c_2]$

    $\Rightarrow$ IBC: $c_1 + \frac{1}{1+r} c_2 = w_1 + \frac{1}{1+r} w_2 + A$.

- Means: Present value of consumption = Present value of income plus initial wealth.
Two Periods: Graphical Analysis

- Budget line has slope \(-(1+r)\). Increase in r \(\Rightarrow\) steeper slope.

  Feasible set: Area under the budget line.

- Endowment point is \((A+w_1, w_2)\). Higher A, \(w_1\), \(w_2\) \(\Rightarrow\) budget line shifts “out”.
Two Periods: Math

- Optimization problem: maximize $U = u(c_1) + \beta u(c_2)$
  - subject to IBC: $c_1 + \frac{1}{1+r} c_2 = A + w_1 + \frac{1}{1+r} w_2$
- Approach #1: substitute constraint into objective. Problem is:
  
  Max $U = u(A + w_1 + \frac{1}{1+r} w_2 - \frac{1}{1+r} c_2) + \beta u(c_2)$.
  
  FOC for $c_2$: $-\frac{1}{1+r} u'(c_1) + \beta u'(c_2) = 0$ $\Rightarrow$ $\frac{1}{1+r} u'(c_1) = \beta u'(c_2)$

- Approach #2: use Lagrangian. Define shadow value $\lambda$. Problem is:
  
  Max $L = u(c_1) + \beta u(c_2) + \lambda \cdot \left( A + w_1 + \frac{1}{1+r} w_2 - c_1 - \frac{1}{1+r} c_2 \right)$

  $\Rightarrow$ FOC for $c_1$ and $c_2$: $u'(c_1) = \lambda$ and $\beta u'(c_2) = \lambda \cdot \frac{1}{1+r} \Rightarrow \frac{1}{1+r} u'(c_1) = \beta u'(c_2)$

- Same conditions. If utility is strictly concave, the solution $(c_1, c_2)$ is unique.
Interpretation 1: Consumption Smoothing

- **Consumption Smoothing Intuition:**
  - Suppose time preference factor is approximately equal to the discount factor:
    \[ \beta \approx \frac{1}{1+r} \Rightarrow u'(c_1) \approx u'(c_2) \Rightarrow c_1 \approx c_2 \]
  - Insight: *Consumption is a “smooth” series.* True even if the income series varies.

- **Benchmark:** suppose \( c_1 \approx c_2 \). Then \( c_1 + \frac{1}{1+r} c_2 = c_1 \cdot (1 + \frac{1}{1+r}) = A + w_1 + \frac{1}{1+r} w_2 \)
  \[ \Rightarrow c_1 = c_2 = \frac{1}{1+(1+r)^{-1}} \cdot (A + w_1 + \frac{1}{1+r} w_2) \]
  - Find: *Consumption is a fraction of total lifetime resources.*

- **Permanent Income** = Annuity equivalent of the income actual stream (Friedman 1957)
  \[ y^P = \frac{w_1 + \frac{1}{1+r} w_2}{1+(1+r)^{-1}} \] Then \( c_1 = y^P + \frac{1}{1+(1+r)^{-1}} \cdot A \).
  - Find: *Consumption \( \approx \) Permanent labor income plus a fraction of initial wealth.*
Permanent Income Model with many periods

• Generalize to n periods: maximize \( U = \sum_{t=1}^{n} \beta^{t-1} u(c_t) \)

  - Budget equations \( a_t = (1 + r)a_{t-1} + w_t - c_t \) with terminal condition \( a_n = 0 \).

  \( \Rightarrow \) Intertemporal budget constraint: \( \sum_{t=1}^{n} \frac{1}{(1+r)^{t-1}} c_t = A + \sum_{t=1}^{n} \frac{1}{(1+r)^{t-1}} w_t \)

  - Benchmark: If \( \beta \approx 1/(1+r) \), optimal consumption is constant. Then:

    \[
    c_1 = \frac{1}{\sum_{t=1}^{n} 1/(1+r)^{t-1}} (A + \sum_{t=1}^{n} \frac{w_t}{(1+r)^{t-1}})
    \]

    Interpretation: \( \sum_{t=1}^{n} 1/(1+r)^{t-1} = \frac{1+r}{r} [1 - (1+r)^{-n}] \) = present value of a fixed annuity.

  - Simple approximation for large \( n \) and small \( r \): \( \frac{1}{\sum_{t=1}^{n} 1/(1+r)^{t-1}} \approx \frac{r}{1+r} \approx r \)

    \textit{Consumption} \sim \textit{Permanent labor income} + \textit{Interest on initial assets}.

• Lessons from permanent income theory:

  1. **Distinguish changes in current income** from changes in **permanent income**: Permanent changes have a much greater impact than temporary changes.

  2. **Future income matters**; under uncertainty, expectations matter.
Interpretation 2: Incentives to Save

- High interest rates provide incentives to consume less and save more:
  \[ 1 + r > \frac{1}{\beta} \text{ in } \frac{1}{1+r} u'(c_1) = \beta u'(c_2), \text{ then } u'(c_1) > u'(c_2) \implies c_2 > c_1. \]

1. High interest rates “tilt” the consumption path upwards. Consumption grows over time.
2. For any given present value, growing consumption must start at a lower level.

   => Initial consumption tends to be less than permanent income intuition would suggest.

- Caveat: High r has income effects. Positive for savers, negative for borrowers.

- Example (Power utility): \( u(c) = \frac{1}{1-\theta} c^{1-\theta} \) with \( \theta > 0, \theta \neq 1 \)

  - Then: \( u'(c) = c^{-\theta} \). FOC is \( \frac{1}{1+r} c_1^{-\theta} = \beta c_2^{-\theta} \implies \frac{c_2}{c_1} = [(1+r)\beta]^{1/\theta} \) increasing in \( r \).

  - In logs: \( \ln(\frac{c_2}{c_1}) = \frac{1}{\theta} [\ln(1+r) + \ln(\beta)] \)

- Definition: \( \frac{\partial \ln(c_2/c_1)}{\partial \ln(1+r)} = \frac{(1+r)}{(c_2/c_1)} \frac{\partial (c_2/c_1)}{\partial (1+r)} \) = Elasticity of intertemporal substitution (EIS)

  - Convenient property of power utility: EIS = 1/\( \theta \), constant

  - Note: \( u(c) = \ln(c) \) implies \( u'(c) = c^{-1} \) has constant EIS=1.
Decision Problems with Production

- Output is produced with capital and labor: \( Y_t = F(K_t, L_t) \)
  - Properties: Increasing; concave; constant returns to scale. (More later.)
  - Firm profits = Output – wage cost – cost of capital. (All in real terms.)
  - Real wage = Marginal product of labor: \( w_t = F_L(K_t, L_t) \)
  - Interest rate = Marginal product of capital – depreciation rate.

- Static model as warm-up: take \( K \) as given. No capital investment.
  - Time constraint: available hours = \( h \). Time for work (\( l \)) plus leisure (\( h-l \))

- Individual problem: Households maximize utility from consumption and leisure:
  maximize \( U = u(c, h-l) \) subject to \( c = w \cdot l + \pi \) \([\pi = \text{other income}]\)
  FOC: \( u_c(c, h-l) \cdot w = u_{h-l}(c, h-l) \)
  \( \Rightarrow \) Consumption-leisure graph with indifference curves and budget line (slope \(-w\)).

- Market equilibrium: assume \( N \) identical households, firms are owned by households
  - Firms operating profits (given \( K \)): \( \Pi = F(K, L) - w \cdot L \).
  - Allocation is symmetric: \( l = \frac{1}{N} L, \pi = \frac{1}{N} \Pi \)

  \( \Rightarrow \ c = w \cdot \frac{L}{N} + \frac{1}{N} \Pi = \frac{1}{N} F(K, L) = F(k, l) \) where \( k = \frac{K}{N} \).

  Aggregate tradeoff between per-capita consumption and leisure is concave.
  - Firms maximize profits \( \Rightarrow F_l(k, l) = w \). FOCs determine equilibrium \((c, l)\).
Social Planning Perspective

- What if a central authority (government) made all economic decisions?
  - Social planner maximizes household utilities subject to the production constraint.
    
    Maximize $U = u(c,h - l)$ s.t. $c = F(k,l) \implies$ FOC: $u_c(c,h - l) \cdot f_l(k,l) = u_{h-l}(c,h - l)$.
  - Find: Same conditions as in the market equilibrium – same solution.

- What if there was a single household owning and operating a firm?
  Household would maximize $U = u(c,h - l)$ s.t. $c = F(k,l)$. Same solution.

- Welfare Theory (digression/intro only)
  - Pareto Optimality as benchmark. An allocation is Pareto Optimal if no one can be made better off without making someone else worse off. Here special case with symmetry.
  - First fundamental welfare theorem: Competitive equilibrium is Pareto optimal.
    Assuming competitive, price-taking behavior, increasing utility.
  - Second fundamental welfare theorem: under some conditions, Pareto optimal solutions can be implemented as market equilibrium
    In general requires lump sum transfers, a concave production function, a concave utility function. Here no transfers needed because of symmetry.
• Application of welfare theory in macroeconomics:
  - Set up the social planning problem to find Pareto optimal allocations.
  - Interpret the social planner as a representative agent who maximizes U subject to a per-capita version of aggregate constraints (e.g. agent operating a representative firm).
  - If the second welfare theorem applies, the social planning/representative agent problem provides a convenient and practical way to find market solutions.

• Common approach:
  1. Solve the representative agent problem to obtain equilibrium quantities, say \((c^*, l^*)\)
  2. Given the optimal quantities, find market clearing “prices” from the FOC:
     In the example: \((c^*, l^*)\) implies a unique factor price
     \[
     w = u_{h-l}(c^*, h-l^*) / u_c(c^*, h-l^*). 
     \]
  3. Interpret the implied prices as equilibrium prices in a market economy with many agents.
     Works for most problems studied in this class. (Exception: distortionary taxes.)
Dynamic Model: Production Economy with many discrete time periods

- Resource constraints: \( Y_t = F(K_t, L_t) = I_t + C_t \) and \( K_{t+1} = I_t + (1 - \delta) \cdot K_t \).
- Household utility: \( U = u(c_1, h - l_1) + \beta \cdot u(c_2, h - l_2) + \ldots \)
- Alternative interpretations:
  1. Market allocation with firms and households; markets for labor, goods, capital.
  2. Social planner maximizes \( U \) subject to production constraints.
  3. Representative household maximizes \( U \) s.t. per-capita resource constraints.

- Observations:
  - Marginal increase in \( l_t \) allows a marginal increase in \( c_t \) by \( F_L \).
    
    \[ \frac{\partial u}{\partial c_t} \cdot F_L() = \frac{\partial u}{\partial l_t} \text{ and } F_L = w_t \text{ apply, same as in the static production model.} \]
  
  - Marginal increase in \( K/N \) reduces \( c_t \) by same amount and increases \( c_{t+1} \) by \( F_K + (1 - \delta) \).

    \[ \frac{\partial u}{\partial c_t} = \beta \cdot \frac{\partial u}{\partial c_{t+1}} \cdot [F_K(K_{t+1}, L_{t+1}) + 1 - \delta]. \text{ Also: } F_K + 1 - \delta = 1 + r \Rightarrow F_K = r + \delta \]

  *Interpretation: Marginal product of capital = interest + depreciation = cost of capital*

  *Note: Concave production => aggregate tradeoff between \( c_t \) and \( c_{t+1} \) is concave.*

- Challenge: track dynamics of capital and output: \( K_t \rightarrow Y_t \rightarrow I_t \rightarrow K_{t+1} \rightarrow Y_{t+1} \ldots \)

  *How does such an economy evolve over time? How does economic growth occur?*

- To start: examine dynamics with fixed saving rate = Solow growth model.
Context: A Brief History of Macroeconomics

• Pre-Keynesian ("Classical") Analysis
  - Extension of microeconomics. Quantity theory of money.

• John Maynard Keynes. ISLM interpretation by John Hicks.
  - Missing theory of inflation => The Phillips curve => Neoclassical Synthesis (~1960s)

• Rational expectations and the Lucas Critique (1975).

• Real Business Cycles: Fluctuations in a stochastic growth model.

• Current consensus: DSGE = Dynamic stochastic general equilibrium models
  - With infinite-lived dynasties or with overlapping generations of finite-lived agents.
  - With or without informational frictions that yield ‘Keynesian’ features.

• Central issues: Population; preferences; technology
  For this class: omit uncertainty and imperfect information.
Learning Objectives

• Conceptual:
  1. Macroeconomics is based on microeconomic principles – optimal choices subject to constraints, equilibrium reasoning, welfare theory.
  2. Macroeconomic insights from simple models:
     - Permanent income; intertemporal substitution, consumption-leisure tradeoff

• Technical skills:
  1. Optimization – with and without constraints.
  2. Solving and interpreting dynamic and static choice problems.

Problem sets for practice.