Econ 204A: Organization

• Class Page: [www.econ.ucsb.edu/~bohn/204A/204AIndex.html](http://www.econ.ucsb.edu/~bohn/204A/204AIndex.html)
  - Information is updated throughout the quarter.
  - Check for announcements. Class page announcements are assumed known.

• Open door policy for graduate students. NH3016.
  - Official office hours posted on the class page.

• E-mail: [henning.bohn@ucsb.edu](mailto:henning.bohn@ucsb.edu). Put “Econ 204A” in the subject line.

• Grading: Weekly problem sets, midterm (in class), final exam.
Introduction to Macroeconomics

• Objectives of macroeconomics:
  To analyze the economy as a whole, to explain economic growth and economic fluctuations, and to assess economic policy.

• Outline of Econ 204A:
  1. Introduction: Intertemporal choice problems.
     - Tools: Constrained optimization. Graphical analysis.
     - Applications to growth accounting, open economy. Introduction to New Growth.
  3. The Ramsey Model: Optimal Consumption and Savings/Investment over Time.
     - Applications to Fiscal Policy and to Money. Digression to discrete time.
A. Intertemporal Choice: Consumption and Savings

- Individual decision problem:
  - Given a series of wage incomes $w_t$. Given real interest rate $r_t = r$ (constant).
  - Individuals choose consumption $c_t$ and asset holdings $a_t$ subject to
    Budget equations: $a_t = (1 + r)a_{t-1} + w_t - c_t$

- Building intuition: Two-period version with graphical analysis. Then generalize.
  - Consumption now ($c_1$) vs. consumption later ($c_2$).
  - Assume given initial wealth $A = (1 + r)a_0$.

- Budget equations imply an intertemporal budget constraint (IBC):
  - use $a_1 = (1 + r)a_0 + w_1 - c_1 = A + w_1 - c_1$ and
    $a_2 = (1 + r)a_1 + w_2 - c_2$
  - impose the terminal condition $a_2 = 0$:
    => $a_1 = -\frac{1}{1+r}[w_2 - c_2]$ => $0 = A + w_1 - c_1 + \frac{1}{1+r}[w_2 - c_2]$
    => IBC: $c_1 + \frac{1}{1+r}c_2 = w_1 + \frac{1}{1+r}w_2 + A$.
  - Means: Present value of consumption = Present value of income plus initial wealth.
Two Periods: Graphical Analysis

- Budget line has slope \(-(1+r)\). Increase in \(r\) \= steeper slope.

  Feasible set: Area under the budget line.

- Endowment point is \((A+w_1, w_2)\). Higher \(A, w_1, w_2\) \= budget line shifts “out”.
Two Periods: Math

- Optimization problem: maximize $U = u(c_1) + \beta u(c_2)$
  - subject to IBC: $c_1 + \frac{1}{1+r} c_2 = A + w_1 + \frac{1}{1+r} w_2$

- Approach #1: substitute constraint into objective. Problem is:
  
  Max $U = u\left(A + w_1 + \frac{1}{1+r} w_2 - \frac{1}{1+r} c_2\right) + \beta u(c_2).$

  FOC for $c_2$: $-\frac{1}{1+r} u'(c_1) + \beta u'(c_2) = 0 \quad \Rightarrow \quad \frac{1}{1+r} u'(c_1) = \beta u'(c_2)$

- Approach #2: use Lagrangian. Define shadow value $\lambda$. Problem is:
  
  Max $L = u(c_1) + \beta u(c_2) + \lambda \cdot \left(A + w_1 + \frac{1}{1+r} w_2 - c_1 - \frac{1}{1+r} c_2\right)$

  $\Rightarrow$ FOC for $c_1$ and $c_2$: $u'(c_1) = \lambda$ and $\beta u'(c_2) = \lambda \cdot \frac{1}{1+r} \Rightarrow \frac{1}{1+r} u'(c_1) = \beta u'(c_2)$

- Same conditions. If utility is strictly concave, the solution $(c_1, c_2)$ is unique.
Interpretation 1: Consumption Smoothing

• Consumption Smoothing Intuition:
  - Suppose time preference factor is approximately equal to the discount factor:
    \[ \beta \approx \frac{1}{1+r} \Rightarrow u'(c_1) \approx u'(c_2) \Rightarrow c_1 \approx c_2 \]
  - Insight: Consumption is a “smooth” series. True even if the income series varies.

• Benchmark: suppose \( c_1 \approx c_2 \). Then \( c_1 + \frac{1}{1+r} c_2 = c_1 \cdot (1 + \frac{1}{1+r}) = A + w_1 + \frac{1}{1+r} w_2 \)

\[ \Rightarrow c_1 = c_2 = \frac{1}{1+1/(1+r)} \cdot (A + w_1 + \frac{1}{1+r} w_2) \]

  - Find: Consumption is a fraction of total lifetime resources.

  - Note: If wage income was a constant \( y \), then \( \frac{w_1 + \frac{1}{1+r} w_2}{1+1/(1+r)} = \frac{y + \frac{1}{1+r} y}{1+1/(1+r)} = y \).

• Permanent Income = Annuity equivalent of the income actual stream (Friedman 1957)

Here \( y^P = \frac{w_1 + \frac{1}{1+r} w_2}{1+1/(1+r)} \). Then \( c_1 = y^P + \frac{1}{1+1/(1+r)} \cdot A \).

  - Find: Consumption \( \approx \) Permanent labor income plus a fraction of initial wealth.
Permanent Income Model with many periods

- Generalize to n periods: maximize $U = \sum_{t=1}^{n} \beta^{t-1} u(c_t)$
  
  - Budget equations $a_t = (1 + r)a_{t-1} + w_t - c_t$ with terminal condition $a_n = 0$.
  
  =>$\text{Intertemporal budget constraint: } \sum_{t=1}^{n} \frac{1}{(1+r)^{t-1}} c_t = A + \sum_{t=1}^{n} \frac{1}{(1+r)^{t-1}} w_t$

- Benchmark: If $\beta \approx 1/(1+r)$, optimal consumption is constant. Then:

  $$c_1 = \frac{1}{\sum_{t=1}^{n} 1/(1+r)^{t-1}} (A + \sum_{t=1}^{n} \frac{w_t}{(1+r)^{t-1}})$$

  Interpretation: $\sum_{t=1}^{n} 1/(1+r)^{t-1} = \frac{1+r}{r} [1 - (1 + r)^{-n}]$ is present value of a fixed annuity.

- Simple approximation for large n and small r: $\frac{1}{\sum_{t=1}^{n} 1/(1+r)^{t-1}} \approx \frac{r}{1+r} \approx r$

  Consumption $\sim$ Permanent labor income + Interest on initial assets.

- Implications of permanent income theory:

  1. **Distinguish changes in current income** from changes in permanent income:
     
         Permanent changes have a much greater impact than temporary changes.
  
  2. **Future income matters**: Current consumption depends on expectations about future income.
**Interpretation 2: Incentives to Save**

- High interest rates provide incentives to consume less and save more:
  
  \[ 1 + r > \frac{1}{\beta} \text{ in } \frac{1}{1+r} u'(c_1) = \beta u'(c_2), \text{ then } u'(c_1) > u'(c_2) \Rightarrow c_2 > c_1. \]

1. High interest rates “tilt” the consumption path upwards. Consumption grows over time.
2. For any given present value, growing consumption must start at a lower level.

  \[ \Rightarrow \text{ Initial consumption tends to be less than permanent income intuition would suggest.} \]

- **Caveat: High } r \text{ has income effects.} \text{ Positive for savers, negative for borrowers.} \]

- Example (Power utility): \( u(c) = \frac{1}{1-\theta} c^{1-\theta} \text{ with } \theta > 0, \theta \neq 1 \)
  
  - Then: \( u'(c) = c^{-\theta} \). FOC is \( \frac{1}{1+r} c_1^{-\theta} = \beta c_2^{-\theta} \Rightarrow \frac{c_2}{c_1} = [(1 + r)\beta]^{1/\theta} \) increasing in \( r \).
  
  - In logs: \( \ln(\frac{c_2}{c_1}) = \frac{1}{\theta} [\ln(1 + r) + \ln(\beta)] \]

- Definition: \( \frac{\partial \ln(c_2/c_1)}{\partial \ln(1+r)} = \frac{(1+r)}{(c_2/c_1)} \frac{\partial (c_2/c_1)}{\partial (1+r)} = \text{Elasticity of intertemporal substitution (EIS)} \)
  
  - Convenient property of power utility: EIS = 1/\theta, constant

  - Note: \( u(c) = \ln(c) \text{ implies } u'(c) = c^{-1} \) has constant EIS=1.
B. Decision Problems with Production

- Output is produced with capital and labor: \( Y_t = F(K_t, L_t) \)
  - Properties: Increasing; concave; constant returns to scale. (More later.)
  - Firm profits = Output – wage cost – cost of capital. (All in real terms.)
  - Real wage = Marginal product of labor: \( w_t = F_L(K_t, L_t) \)
  - Interest rate = Marginal product of capital – depreciation rate.

- Static model as warm-up: One period. Take \( K \) as given. No capital investment.
  - Time constraint: available hours = \( h \). Time for work (\( l \)) plus leisure (\( h-l \))

- Individual problem: Households maximize utility from consumption and leisure:
  - maximize \( U = u(c, h-l) \) subject to \( c = w \cdot l + \pi \) \( [\pi = \text{other income}] \)
  - FOC: \( u_c(c, h-l) \cdot w = u_{h-l}(c, h-l) \)
  - \( \Rightarrow \) Consumption-leisure graph with indifference curves and budget line (slope \(-w\)).

- Market equilibrium: assume \( N \) identical households, firms are owned by households
  - Firms operating profits (given \( K \)): \( \Pi = F(K, L) - w \cdot L. \)
  - Allocation is symmetric: \( l = \frac{1}{N} L, \pi = \frac{1}{N} \Pi \)
  - \( \Rightarrow \) \( c = w \cdot \frac{L}{N} + \frac{1}{N} \Pi = \frac{1}{N} F(K, L) = F(k, l) \) where \( k = \frac{K}{N} \).
  - Aggregate tradeoff between per-capita consumption and leisure is concave.
  - Firms maximize profits \( \Rightarrow F_l(k, l) = w \). FOCs determine equilibrium (\( c, l \)).
Social Planning Perspective

• What if a central authority (government) made all economic decisions?
  - Social planner maximizes household utilities subject to the production constraint.
    Maximize $U = u(c,h - l)$ s.t. $c = F(k,l)$ => FOC: $u_c(c,h - l) \cdot F_l(k,l) = u_{h-l}(c,h - l)$.
  - Find: Same conditions as in the market equilibrium – same solution.

• What if there was a single household owning and operating a firm?
  Household would maximize $U = u(c,h - l)$ s.t. $c = F(k,l)$. Same solution.

• Welfare Theory (digression/intro only)
  - Pareto Optimality as benchmark. An allocation is Pareto Optimal if no one can be made better off without making someone else worse off. Here special case with symmetry.
  - First fundamental welfare theorem: Competitive equilibrium is Pareto optimal.
    Assuming competitive, price-taking behavior, increasing utility.
  - Second fundamental welfare theorem: under some conditions, Pareto optimal solutions can be implemented as market equilibrium
    In general requires lump sum transfers, a concave production function, a concave utility function. Here no transfers needed because of symmetry.
• Application of welfare theory in macroeconomics:
  - Set up the social planning problem to find Pareto optimal allocations.
  - Interpret the social planner as a representative agent who maximizes U subject to a per-capita version of aggregate constraints (e.g. agent operating a representative firm).
  - If the second welfare theorem applies, the social planning/representative agent problem provides a convenient and practical way to find market solutions.

• Common approach:
  1. Solve the representative agent problem to obtain equilibrium quantities, say \((c^*, l^*)\)
  2. Given the optimal quantities, find market clearing “prices” from the FOCs.
     In the example: \((c^*, l^*)\) implies a unique factor price
     \[ w = u_{h-l}(c^*, h-l)/u_c(c^*, h-l^*). \]
  3. Interpret the implied prices as equilibrium prices in a market economy with many agents.
     Works for most problems studied in this class. (Exception: distortionary taxes.)
C. Dynamic Models: Production Economy with many periods

- Resource constraints: \( Y_t = F(K_t, L_t) = I_t + C_t \) and \( K_{t+1} = I_t + (1 - \delta) \cdot K_t \).

- Household utility: \( U = u(c_1, h - l_1) + \beta \cdot u(c_2, h - l_2) + \ldots \)

- Alternative interpretations:
  1. Market allocation with firms and households; markets for labor, goods, capital.
  2. Social planner maximizes U subject to production constraints.
  3. Representative household maximizes U s.t. per-capita resource constraints.

- Observations:
  - Marginal increase in \( I_t \) allows a marginal increase in \( c_t \) by \( F_L \).
    \[ \frac{\partial u}{\partial l_t} \cdot F_L() = \frac{\partial u}{\partial c_t} \text{ and } F_L = w_t \text{ apply, same as in the static production model.} \]

  - Marginal increase in \( K/N \) reduces \( c_t \) by same amount and increases \( c_{t+1} \) by \( F_K + (1 - \delta) \).
    \[ \frac{\partial u}{\partial c_t} = \beta \cdot \frac{\partial u}{\partial c_{t+1}} \cdot [F_K(K_{t+1}, L_{t+1}) + 1 - \delta]. \text{ Also: } F_K + 1 - \delta = 1 + r \Rightarrow F_K = r + \delta \]

  Interpretation: Marginal product of capital = interest + depreciation = cost of capital

  Note: Concave production \( \Rightarrow \) aggregate tradeoff between \( c_t \) and \( c_{t+1} \) is concave.

- Challenge: track dynamics of capital and output: \( K_t \rightarrow Y_t \rightarrow I_t \rightarrow K_{t+1} \rightarrow Y_{t+1} \ldots \)

  How does such an economy evolve over time? How does economic growth occur?

- Dynamics of capital accumulation are conveniently presented in continuous time.
- Starting point: Solow growth model – assumes fixed saving rate to simplify.
Context: A Brief History of Macroeconomics

- Pre-Keynesian (“Classical”) Analysis
  - Extension of microeconomics. Quantity theory of money.

- John Maynard Keynes. ISLM interpretation by John Hicks.
  - Missing theory of inflation => The Phillips curve => Neoclassical Synthesis (~1960s)


- Real Business Cycles: Fluctuations in a stochastic growth model.

- Current consensus: DSGE = Dynamic stochastic general equilibrium models
  - With infinitely-lived dynasties or with overlapping generations.
  - With or without informational frictions that yield ‘Keynesian’ features.

- Central issues: Population; preferences; technology
  For this class: omit uncertainty and imperfect information.
Learning Objectives

• Conceptual:
  1. Macroeconomics is based on microeconomic principles – optimal choices subject to constraints, equilibrium reasoning, welfare theory.
  2. Macroeconomic insights from simple models:
     - Permanent income; intertemporal substitution, consumption-leisure tradeoff

• Technical skills:
  1. Optimization – with and without constraints.
  2. Solving and interpreting dynamic and static choice problems.

Problem sets for practice.