The Term Structure of Interest Rates
Mishkin ch.6

- Concept of the Yield Curve: plot bond yields against maturity
- Three theories with different assumptions about risk and return
  1. Expectations hypothesis
  2. Segmented markets theory
  3. Liquidity premium theory; also called Preferred Habitat theory.
- Question for theory: How do investors trade-off risk against expected return?
  Answer: Focus on liquidity premium theory (others for illustration)
- Question for application: What do investors expect about future interest rates?
  Remember:
  - Bond market = efficient market: Investors form rational expectations
  - Returns on long-term bonds depend on future interest rates
- Key application: Interpreting the yield curve = Extracting information about future interest rates from observed current rates at different maturities.
The Main Term-Structure Equation

\[ i_{nt} = \frac{1}{n} \left(i_t + i_{t+1}^e + \ldots + i_{t+n-1}^e\right) + l_{nt} \]

- \( i_{nt} \) = yield on n-period bonds; \( i_t \) = yield on 1-period bonds
- \( i_{t+x}^e \) = expected yield on 1-period bonds in year \( t+x \)
- \( l_{nt} \) = liquidity premium on n-period bonds. Common period = year.

Example: \( n=3 \) year investment horizon. Formula: \( i_{3t} = \frac{1}{3} \left(i_t + i_{t+1}^e + i_{t+2}^e\right) + l_{3t} \)

- Strategy 1: Buy a three-year bond with yield \( i_{3t} \)
  Cash flow: \( i_{3t}, i_{3t}, i_{3t} \). Average return = \( i_{3t} \)

- Strategy 2: Buy series of 1-year bonds.
  Cash flow: \( i_t, i_{t+1}, i_{t+2} \). Expect average return = \( \frac{1}{3} \left(i_t + i_{t+1}^e + i_{t+2}^e\right) \)

- Strategy 3: Buy a six-year bond with yield \( i_{6t} \) and sell after three years.
  Current yield \( i_{6t} \). Capital gain/loss depends on PV with discount rate \( i_{3,t+3} \).

RISK considerations:

1. Reinvestment Risk: low yield if a bond matures too early (low \( i_{t+1}, i_{t+2} \))
2. Price Risk: low price if a bond is sold before maturity (high \( i_{3,t+3} \))

• Empirical finding: premiums \( l_{nt} \geq 0 \) are paid for taking price risk.
Evidence: Yields move together
(LT yields follow ST yields, but smoothed = consistent with taking averages)
Evidence: Yield curves are typically upward sloping

Theory: Average of \( i_{nt} \) over time = Average of \( i_t \) + Average Premium

\[ \implies \text{Estimate: Average Premium} = \text{Average slope} (i_{nt} - i_t) \]

Liquidity Premium (Preferred Habitat) Theory

Expectations Theory

Liquidity Premium, \( l_{nt} \)

[Notes on Mishkin Ch.6 - Term Structure - P.4]
Interpreting Unusual Shapes of the Yield Curve

- Long rates $i_{nt}$ will be far above the short rate $i_t$, if investors expect rising interest rates $=>$ Steep yield curve signals rising yields ($i_{t+x} > i_t$)

- Long rates $i_{nt}$ will be near or below the short rate $i_t$, if investors expect falling rates $=>$ Flat or downward sloping yield curve signals declining yields ($i_{t+x} < i_t$)

Examples – Calculations for n=2 and n=3

- For n=2:  
  $$i_{2t} = \frac{1}{2} \left( i_t + i_{t+1}^e \right) + l_{2t}$$

- For n=3:  
  $$i_{3t} = \frac{1}{3} \left( i_t + i_{t+1}^e + i_{t+2}^e \right) + l_{3t}$$

- Note: Formulas apply for any base period (day, month, year, decade).
(a) Future short-term interest rates expected to rise
(b) Future short-term interest rates expected to stay the same
(c) Future short-term interest rates expected to fall moderately
(d) Future short-term interest rates expected to fall sharply
Application: Monetary Policy and the Yield Curve

\[ i_{nt} = \frac{1}{n} \left( i_t + i_{t+1}^{e} + \ldots + i_{t+n-1}^{e} \right) + l_{nt} \]

• Short-term rates are set by the central bank = policy choices.
  - Current value \( i_t \) is observed.
  - Future values \( i_{t+1}^{e}, i_{t+2}^{e}, \ldots, i_{t+n-1}^{e} \) reflect expectations about future policy.

• Application of rational expectations:
  1. Apply MP function: \( i_{t+x}^{e} = r_{t+x}^{e} + \pi_{t+x}^{e} = \bar{r} + (1 + \lambda)\pi_{t+x}^{e} \)
     - Central banks tend to raise interest rates when inflation threatens.
     => Steep yield curve signals high inflation (high \( \pi^{e} \))
  2. Apply classical theory: \( i_{t+x}^{e} = r + \pi_{t+x}^{e} \). Real economy determines natural \( r \).
     - Money growth determines \( \pi \). Recall that low \( \bar{r} \) => high \%\( \Delta M \) => high \( \pi \).
     => Steep yield curve signals market assessment that current policy (\( \bar{r} \)) will result in high money growth and inflation.

• Yield curve signals help central banks: policy is inflationary if the central bank overestimates \( Y^{P} \) – steep yield curve provides a warning signal.
• Analogous reasoning for inverted yield curve: signal of declining inflation, of tight monetary policy, of a central bank underestimating \( Y^{P} \).
**Unexpected Changes in Monetary Policy and the Yield Curve**

- Apply general logic to specific scenarios:
  1. Suppose the central bank shifts MP down temporarily: no impact on long run inflation => Reduces $i_t$ more than $i_{nt}$ => Yield curve becomes steeper.
  2. Suppose the central bank shifts MP down permanently => Higher inflation in the long-run => Higher interest rates in the far future.
     => Yield curve shifts down for short maturities, but UP at very long maturities.

- General result: Lower $i_t$ and increased slope $i_{nt} - i_t$ whenever MP shifts down

- Distinctive implications of a permanent downshift in MP: $i_{nt}$ for jumps up at long maturities if the policy change is viewed as permanent (inflationary).
  - New insight about bond prices: Easy money tends to raise most bond prices, but very long term bond prices may FALL if the policy change is permanent.

- Similar results for MP shifts up: Always higher $i_t$ and reduced slope $i_{nt} - i_t$; at long maturities, decline in $i_{nt}$ signals that the shift is viewed as permanent, i.e., likely to succeed in reducing inflation.
  - Famous historical example: U.S. monetary contraction in 1979-82