Mishkin ch.4: Interest Rates

Summary

1. Three key concepts:

   Present Value       Yield to Maturity       Total Return

2. Know how to work with the key concepts:

   Task in Exams: Problem solving

3. Applications:

   Real versus nominal rates       PV of stocks: Discounted dividends
   Inflation-indexed bonds          Tracking total returns
Fundamentals

• Present Value:

\[ PV_t = \frac{Payment_{t+1}}{(1+i)} + \frac{Payment_{t+2}}{(1+i)^2} + \ldots + \frac{Payment_{t+N}}{(1+i)^N} \]

\[ i = \text{discount rate} = \text{interest rate used to discount future payments} \]

• Yield to Maturity (or simply: Yield) = Particular discount rate \( i \) that solves

\[ PB = PV_t = \frac{Payment_{t+1}}{(1+i)} + \frac{Payment_{t+2}}{(1+i)^2} + \ldots + \frac{Payment_{t+N}}{(1+i)^N} \]

One-period example (Treasury Bill promises $10,000 at maturity)

\[ PB = \frac{10000}{(1+i)} \Rightarrow (1+i) = \frac{10000}{PB} \Rightarrow i = \frac{10000 - PB}{PB} \]
**Application: Coupon Bonds**

- **Defined by:**
  - Maturity date \( \Rightarrow \) Years to maturity = \( N \)
  - Coupon = \( C \)
  - Face Value = \( F \) \( \Rightarrow \) Coupon rate = \( C/F \)

- **Market data:**
  - Price = \( P_B \) or \( P_{Bt} \) \( \Rightarrow \) Current Yield = \( C/P_B \)
  - Yield to maturity = \( i \) or \( i_t \)

  *(Time subscript \( t \) used when timing matters.)*

- **Common problems:** Compute yield from price. Compute price from yield.

**Example**

- **Bond quote from WSJ:**

<table>
<thead>
<tr>
<th>Date</th>
<th>Security</th>
<th>Maturity</th>
<th>Coupon Rate</th>
<th>Price</th>
<th>Yield</th>
</tr>
</thead>
<tbody>
<tr>
<td>7/31/2017</td>
<td>Treasury note</td>
<td>7/31/2020</td>
<td>2.00%</td>
<td>101.4141</td>
<td>1.516%</td>
</tr>
</tbody>
</table>

*Question: How is the yield calculated?*
## Example

<table>
<thead>
<tr>
<th>Date:</th>
<th>7/31/2017</th>
</tr>
</thead>
<tbody>
<tr>
<td>Security</td>
<td>Maturity</td>
</tr>
<tr>
<td>Treasury note</td>
<td>7/31/2020</td>
</tr>
</tbody>
</table>

- **Task #1: Set up the present value equation**
  - Three years to maturity $\Rightarrow N = 3$
  - Price is per $F=100$ face value $\Rightarrow C = (\text{Coupon Rate}) \times 100 = 2.00$
  - Payment at dates $t+1,\ldots,t+N-1$: $\text{Coupon} = C$
  - Payment at maturity date $t+N$: $\text{Face value} + \text{final coupon} = F+C$

\[
PV = \frac{C}{(1+i)} + \frac{C}{(1+i)^2} + \frac{C+F}{(1+i)^3} = \frac{2}{(1+i)} + \frac{2}{(1+i)^2} + \frac{102}{(1+i)^3}
\]

- **Task #2: Solve the present value equation**
  - Cubic equation – solve numerically by exploiting that $PV$ is declining in $i$.

\textit{Knowing how this works is essential to understanding yield quotes.}
Worksheet | Discount | PV       | Graph each pair
--- | --- | --- | ---
try coupon: | 2.00% | 100.00 | = Par value
try lower: | 1.00% | 102.94 | $P$ too high => $i$ too low
try higher: | 1.60% | 101.16 | $P$ too low => $i$ too high
try again: | 1.50% | 101.46 | Better
and again: | 1.52% | 101.40 | Close enough
WSJ quote: | 1.516% | 101.4141 |
Notes on Bond Quotes

- Most of the time, economists use published or online quotes to obtain yields.
- Traders sometimes work with yields without mentioning prices.
  Then $P_B = PV$ is implied. Everyone knows.

- Bond quotes have two parts:
  1. Identify the security: issuer, maturity date, coupon = fixed data.
  2. Market information: price, yield, time of quote = changes over time.

- Implied items that also change: current yield, time to maturity.
- Assumptions to check:

  Prices are usually per $100 face value; but sometimes per $1000.
  Prices are may be decimal or fractional (e.g. "100 : 8" = $100 \frac{8}{32} = 100.25$)

  Good sources should have legends or footnotes to confirm interpretation

- Simplifications for this class:
  Disregard discounting over fractional periods; usually maturity = whole years
  Disregard lumpiness in coupons; treat payments as smooth over the year
  Disregard “accrued interest” – use prices are as quoted “clean”
The Total Return
(a.k.a: Return, Rate of Return)

• Definition:

\[ \text{RET} = \frac{\text{Payment} + P_{t+1} - P_t}{P_t} \]

• Measured over a specific time period \( t \) to \( t+1 \):
  - \( P_t \) = Price at the start; known.
  - \( P_{t+1} \) = Price at the end; often unknown at time \( t \).
  - Payment = current yield or other payout during the time period \( t \) to \( t+1 \); assumed known

• Can be computed for ANY financial asset – not only bonds

• Components:

  \[ \text{Current Yield} = \frac{\text{Payment}}{P_t} \]

  \[ \text{Capital Gain} = \frac{P_{t+1} - P_t}{P_t} = \frac{\text{Change in Price}}{\text{Initial Price}} \]
Application in Mishkin:

Total return on a coupon bond for one year:

\[ RET = \frac{C + P_{B_{t+1}} - P_{B_t}}{P_{B_t}} \]

Mishkin’s formula is a special case: Payment = C. Period = one year

Remember the general principle:

\[ RET = \frac{\text{Payment}}{P_t} + \frac{P_{t+1} - P_t}{P_t} \]

- Applies to all kinds of financial investments
  - e.g. Real Estate, then: Payment = Rental income minus expenses.

- Caution: Yields are usually annualized. But “current yield” in RET refers to the period over which RET is calculated – may require conversion.
  - e.g. if period = X days, then: Payment = (Annualized current yield) * X/365.
Illustrations in Mishkin

1. Price – Yield Relation

Example: 10-year coupon bond

<table>
<thead>
<tr>
<th>Price of Bond ($)</th>
<th>Yield to Maturity (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,200</td>
<td>7.13</td>
</tr>
<tr>
<td>1,100</td>
<td>8.48</td>
</tr>
<tr>
<td>1,000</td>
<td>10.00</td>
</tr>
<tr>
<td>900</td>
<td>11.75</td>
</tr>
<tr>
<td>800</td>
<td>13.81</td>
</tr>
</tbody>
</table>
2. Key Linkages: Yield Change - Price Change - Return

Lesson: Price responses increase with maturity

<table>
<thead>
<tr>
<th>(1) Years to Maturity When Bond Is Purchased</th>
<th>(2) Initial Current Yield (%)</th>
<th>(3) Initial Price ($)</th>
<th>(4) Price Next Year* ($)</th>
<th>(5) Rate of Capital Gain (%)</th>
<th>(6) Rate of Return [col (2) + col (5)] (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>10</td>
<td>1,000</td>
<td>503</td>
<td>−49.7</td>
<td>−39.7</td>
</tr>
<tr>
<td>20</td>
<td>10</td>
<td>1,000</td>
<td>516</td>
<td>−48.4</td>
<td>−38.4</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>1,000</td>
<td>597</td>
<td>−40.3</td>
<td>−30.3</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td>1,000</td>
<td>741</td>
<td>−25.9</td>
<td>−15.9</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>1,000</td>
<td>917</td>
<td>−8.3</td>
<td>+1.7</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
<td>1,000</td>
<td>1,000</td>
<td>0.0</td>
<td>+10.0</td>
</tr>
</tbody>
</table>

*Calculated with a financial calculator, using Equation 3.

Common measure of price-sensitivity: \( \%\Delta P = - \text{Duration} \times \Delta i/(1+i) \)

Pure discount bonds have duration = maturity; for coupon bonds, duration < maturity.

Optional reading: Online Appendix to Ch.4
Real Returns and Real Yields

- **Real interest rate** = Nominal interest rate minus expected inflation
  \[ r = i - \pi^e \]
  
  - Traditional measurement: Find nominal interest rates & estimate expected inflation
  
  Problem: expectations are not directly observable.

  - New approach: use yields on inflation-protected securities (in U.S. since 1997)

- **Treasury Inflation-Protected Securities (TIPS):**
  
  - Face value is fixed in real terms: \( F_r = 100 \)
  
  - Nominal face value varies with CPI: \( F = F_r \times (\text{CPI}) \)

  - Nominal coupon varies with CPI: \( C = (\text{Coupon rate}) \times F \)

  - If Price = Face value, then: nominal return \( \approx (\text{Coupon rate}) + (\text{Change in CPI}) \)

  \[ \Rightarrow \quad \text{Real yield to maturity} = \text{real return} \approx \text{Coupon rate} \]

  \[ \Rightarrow \quad \text{Interpret coupon rate as real interest rate}. \]

  - If Price \( \neq \) Face value, real return differs from coupon rate & not easy to compute.

  \[ \Rightarrow \quad \text{Rely on published sources for real yields to maturity (e.g. WSJ)} \]

- **Remember:** *Quoted yields on TIPS are direct measures of REAL interest rates.*
Other Applications

- **Present values of corporate stocks:**
  - Payment = Dividend. Example of infinitely lived asset
  
  \[ PV_t = \frac{\text{Dividend}_{t+1}}{1+i} + \frac{\text{Dividend}_{t+2}}{(1+i)^2} + \ldots \]

  - Return to stocks in Mishkin ch.7.

- **British consols:**
  - Coupon bonds without repayment date.
  
  \[ P_B = \frac{C}{1+i} + \frac{C}{(1+i)^2} + \ldots = \frac{C}{i} \]

  - Nice illustration of negative price-to-yield relationship.