Mishkin ch.4: Interest Rates

Summary

1. Three key concepts:

   - Present Value
   - Yield to Maturity
   - Total Return

2. Know how to work with the key concepts:

   Task in Exams: Problem solving

3. Applications:

   - Real versus nominal rates
   - PV of stocks: Discounted dividends
   - Inflation-indexed bonds
   - Tracking total returns
Fundamentals

• Present Value:

\[ PV_t = \frac{Payment_{t+1}}{(1+i)} + \frac{Payment_{t+2}}{(1+i)^2} + \ldots + \frac{Payment_{t+N}}{(1+i)^N} \]

\( i = \text{discount rate = interest rate used to discount future payments} \)

• Yield to Maturity (or simply: Yield) = Particular discount rate \( i \) that solves

\[ PB = PV_t = \frac{Payment_{t+1}}{(1+i)} + \frac{Payment_{t+2}}{(1+i)^2} + \ldots + \frac{Payment_{t+N}}{(1+i)^N} \]

One-period example (Treasury Bill promises $10,000 at maturity)

\[ PB = \frac{10000}{(1+i)} \Rightarrow (1+i) = \frac{10000}{PB} \Rightarrow i = \frac{10000 - PB}{PB} \]
Application: Coupon Bonds

- **Defined by:**
  - Maturity date  =>  Years to maturity = N
  - Coupon = C
  - Face Value = F  =>  Coupon rate  = C/F

- **Market data:**
  - Price = P_B or P_{Bt}  =>  Yield to maturity = i or i_t
  - Current yield = C/P_B

Use subscript t (time) when multiple dates are involved.
Yield or “interest rate” without modifiers refers to yield to maturity.

- **Common problems:** Compute yield from price. Compute price from yield.

**Example**

- **Bond quote from WSJ:**

<table>
<thead>
<tr>
<th>Date</th>
<th>Security</th>
<th>Maturity</th>
<th>Coupon Rate</th>
<th>Price</th>
<th>Yield</th>
</tr>
</thead>
<tbody>
<tr>
<td>4/9/2018</td>
<td>Treasury note</td>
<td>3/31/2021</td>
<td>1.25%</td>
<td>96.63</td>
<td>2.431%</td>
</tr>
</tbody>
</table>

**Question:** How is the yield calculated?
Example

Date: 4/9/2018
Security  Maturity  Coupon Rate  Price  Yield
Treasury note  3/31/2021  1.25%  96.63  2.431%

• Task #1: Set up the present value equation
  About 3 years to maturity => N = 3
  Price is per F=100 face value => C = (Coupon Rate) * 100 = 1.25
  Payment at dates t+1,…,t+N-1: Coupon = C
  Payment at maturity date t+N: Face value + final coupon = F+C

  \[ PV = \frac{C}{(1+i)} + \frac{C}{(1+i)^2} + \frac{C+F}{(1+i)^3} = \frac{1.25}{(1+i)} + \frac{1.25}{(1+i)^2} + \frac{101.25}{(1+i)^3} \]

• Task #2: Solve the present value equation
  Cubic equation – solve numerically by exploiting that PV is declining in i.

Knowing how this works is essential to understanding yield quotes.
<table>
<thead>
<tr>
<th>Worksheet</th>
<th>Discount</th>
<th>PV</th>
<th>Graph each pair</th>
</tr>
</thead>
<tbody>
<tr>
<td>try coupon:</td>
<td>1.25%</td>
<td>100.00</td>
<td>→ par value</td>
</tr>
<tr>
<td>try low:</td>
<td>0.00%</td>
<td>103.75</td>
<td>→ high PV =&gt; yield too low</td>
</tr>
<tr>
<td>try high:</td>
<td>3.00%</td>
<td>95.05</td>
<td>→ low PV =&gt; yield too high</td>
</tr>
<tr>
<td>try between:</td>
<td>2.50%</td>
<td>96.43</td>
<td>→ yield too high</td>
</tr>
<tr>
<td>try again:</td>
<td>2.43%</td>
<td>96.63</td>
<td>→ close enough</td>
</tr>
</tbody>
</table>
Notes on Bond Quotes

• Most of the time, economists use published or online quotes to obtain yields.
• Traders sometimes work with yields without mentioning prices.
  Then PB = PV is implied. Everyone knows.

• Bond quotes have two parts:
  (1) Identify the security: issuer, maturity date, coupon = fixed data.
  (2) Market information: price, yield, time of quote = changes over time.

• Implied items that also change: current yield, time to maturity.
• Assumptions to check:
  Prices are usually per $100 face value; but sometimes per $1000.
  Prices are may be decimal or fractional (e.g. "100 : 8" = $100 \( \frac{8}{32} \) = 100.25)

  Good sources should have legends or footnotes to confirm interpretation

• Simplifications for this class:
  Disregard discounting over fractional periods; usually maturity = whole years
  Disregard lumpiness in coupons; treat payments as smooth over the year
  Disregard “accrued interest” – use prices are as quoted “clean”
**The Total Return**
(a.k.a: Return, Rate of Return)

- Definition:

\[
\text{RET} = \frac{\text{Payment} + P_{t+1} - P_t}{P_t}
\]

- Measured over a specific time period t to t+1:
  
  \(P_t\) = Price at the start; known.
  
  \(P_{t+1}\) = Price at the end; often unknown at time t.
  
  Payment = current yield or other payout during the time period t to t+1; assumed known

- Can be computed for ANY financial asset – not only bonds

- Components:

  \[
  \text{Current Yield} = \frac{\text{Payment}}{P_t}
  \]

  \[
  \text{Capital Gain} = \frac{P_{t+1} - P_t}{P_t} = \frac{\text{Change in Price}}{\text{Initial Price}}
  \]
Application in Mishkin:

Total return on a coupon bond for one year:

\[ RET = \frac{C + P_{Bt+1} - P_{Bt}}{P_{Bt}} \]

Mishkin’s formula is a special case: Payment = C. Period = one year

Remember the general principle:

\[ RET = \frac{\text{Payment}}{P_t} + \frac{P_{t+1} - P_t}{P_t} \]

e.g. if period = X days, then: Payment = (Annualized current yield) * X/365.
if asset = real estate, then: Payment = Rental income minus expenses.

- Caution: Yields are usually annualized. But current yield in RET refers to the period over which RET is calculated – may require conversion.
**Illustrations in Mishkin**

1. **Price – Yield Relation**

Example: 10-year coupon bond

<table>
<thead>
<tr>
<th>Price of Bond ($)</th>
<th>Yield to Maturity (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,200</td>
<td>7.13</td>
</tr>
<tr>
<td>1,100</td>
<td>8.48</td>
</tr>
<tr>
<td>1,000</td>
<td>10.00</td>
</tr>
<tr>
<td>900</td>
<td>11.75</td>
</tr>
<tr>
<td>800</td>
<td>13.81</td>
</tr>
</tbody>
</table>

[Notes on Mishkin Ch.4 - P.9]
2. Key Linkages: Yield Change - Price Change - Return

Lesson: Price responses increase with maturity

<table>
<thead>
<tr>
<th>(1) Years to Maturity When Bond Is Purchased</th>
<th>(2) Initial Current Yield (%)</th>
<th>(3) Initial Price ($)</th>
<th>(4) Price Next Year* ($)</th>
<th>(5) Rate of Capital Gain (%)</th>
<th>(6) Rate of Return [col (2) + col (5)] (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>10</td>
<td>1,000</td>
<td>503</td>
<td>-49.7</td>
<td>-39.7</td>
</tr>
<tr>
<td>20</td>
<td>10</td>
<td>1,000</td>
<td>516</td>
<td>-48.4</td>
<td>-38.4</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>1,000</td>
<td>597</td>
<td>-40.3</td>
<td>-30.3</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td>1,000</td>
<td>741</td>
<td>-25.9</td>
<td>-15.9</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>1,000</td>
<td>917</td>
<td>-8.3</td>
<td>+1.7</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
<td>1,000</td>
<td>1,000</td>
<td>0.0</td>
<td>+10.0</td>
</tr>
</tbody>
</table>

*Calculated with a financial calculator, using Equation 3.

Common measure of price-sensitivity: \( \%\Delta P = - \text{Duration} \times \Delta i/(1+i) \)

Pure discount bonds have duration = maturity; for coupon bonds, duration < maturity.

Optional reading: Online Appendix to Ch.4
Real Returns and Real Yields

• **Real interest rate** = Nominal interest rate minus expected inflation
  \[ r = i - \pi^e \]

  - Traditional measurement: Find nominal interest rates & estimate expected inflation
  
    Problem: expectations are not directly observable.

  - New approach: use yields on inflation-protected securities (in U.S. since 1997)

• **Treasury Inflation-Protected Securities (TIPS):**
  
  - Face value is fixed in real terms: \( F_T = \$100 \)
  
  - Nominal face value varies with CPI: \( F = F_T \times (\text{CPI}) \)
  
  - Nominal coupon varies with CPI: \( C = (\text{Coupon rate}) \times F \)
  
  - If Price = Face value, then: nominal return \( \approx (\text{Coupon rate}) + (\text{Change in CPI}) \)

  => **Real yield to maturity = real return \( \approx \) Coupon rate**

  => Interpret coupon rate as real interest rate.

  - If Price \( \neq \) Face value, real return differs from coupon rate & not easy to compute.

  => Rely on published sources for real yields to maturity (e.g. WSJ)

• Remember: **Quoted yields on TIPS are direct measures of REAL interest rates.**

[Notes on Mishkin Ch.4 - P.11]
Other Applications

- **Present values of corporate stocks:**
  - Payment = Dividend. Example of infinitely lived asset
  \[ PV_t = \frac{Dividend_{t+1}}{1+i} + \frac{Dividend_{t+2}}{(1+i)^2} + \ldots \]
  - Return to stocks in Mishkin ch.7.

- **British consols:**
  - Coupon bonds without repayment date.
  \[ P_B = \frac{C}{1+i} + \frac{C}{(1+i)^2} + \ldots = \frac{C}{i} \]
  - Nice illustration of negative price-to-yield relationship.