### Mishkin ch.4: Interest Rates

#### Summary

1. Three key concepts:

   | Present Value | Yield to Maturity | Total Return |

2. Know how to work with the key concepts:

   **Task in Exams: Problem solving**

3. Applications:

   - Real versus nominal rates
   - PV of stocks: Discounted dividends
   - Inflation-indexed bonds
   - Tracking total returns
Fundamentals

• Present Value:

\[ PV_t = \frac{Payment_{t+1}}{(1+i)} + \frac{Payment_{t+2}}{(1+i)^2} + \ldots + \frac{Payment_{t+N}}{(1+i)^N} \]

\( i = \text{discount rate} = \text{interest rate used to discount future payments} \)

• Yield to Maturity (or simply: Yield) = Particular discount rate \( i \) that solves

\[ PB = PV_t = \frac{Payment_{t+1}}{(1+i)} + \frac{Payment_{t+2}}{(1+i)^2} + \ldots + \frac{Payment_{t+N}}{(1+i)^N} \]

One-period example (Treasury Bill promises $10,000 at maturity)

\[ PB = \frac{10000}{(1+i)} \Rightarrow (1+i) = \frac{10000}{PB} \Rightarrow i = \frac{10000 - PB}{PB} \]
Application: Coupon Bonds

• Defined by: Maturity date => Years to maturity = N
  Coupon = C
  Face Value = F => Coupon rate = C/F

• Market data: Price = P_B or P_Bt => Yield to maturity = i or i_t

  We use subscript t (for time) when multiple dates are involved.
  Yield or “interest rate” without modifiers refers to yield to maturity.

• Common problems: Compute yield from price. Compute price from yield.

Example

• Bond quote from WSJ:

  Date: 4/9/2018
  Security    Maturity    Coupon Rate    Price    Yield
  Treasury note  3/31/2021  1.25%     96.63     2.431%

  Question: How is the yield calculated?
Example

Date: 4/9/2018
Security Maturity Coupon Rate Price Yield
Treasury note 3/31/2021 1.25% 96.63 2.431%

• Task #1: Set up the present value equation
  
  About 3 years to maturity => \( N = 3 \)
  Price is per F=100 face value => \( C = \text{(Coupon Rate)} \times 100 = 1.25 \)
  Payment at dates t+1,…,t+N-1: Coupon = C
  Payment at maturity date t+N: Face value + final coupon = F+C

  \[
  PV = \frac{C}{(1+i)} + \frac{C}{(1+i)^2} + \frac{C+F}{(1+i)^3} = \frac{1.25}{(1+i)} + \frac{1.25}{(1+i)^2} + \frac{101.25}{(1+i)^3}
  \]

• Task #2: Solve the present value equation
  
  Cubic equation – solve numerically by exploiting that PV is declining in i.

  Knowing how this works is essential to understanding yield quotes.
Worksheet | Discount | PV | Graph each pair
--- | --- | --- | ---
try coupon: | 1.25% | 100.00 | \(\rightarrow\) par value
try low: | 0.00% | 103.75 | \(\rightarrow\) high PV \(\Rightarrow\) yield too low
try high: | 3.00% | 95.05 | \(\rightarrow\) low PV \(\Rightarrow\) yield too high
try between: | 2.50% | 96.43 | \(\rightarrow\) yield too high
try again: | 2.43% | 96.63 | \(\rightarrow\) close enough
The Total Return
(a.k.a: Return, Rate of Return)

• Definition:

\[
\text{RET} = \frac{\text{Payment} + P_{t+1} - P_t}{P_t}
\]

• Measured over a specific time period t to t+1:
  
  \( P_t \) = Price at the start; known.
  
  \( P_{t+1} \) = Price at the end; often unknown at time t.

  Payment = current yield or other payout during the time period t to t+1; assumed known

• Can be computed for ANY financial asset – not only bonds

• Components:

  \[
  \text{Current Yield} = \frac{\text{Payment}}{P_t}
  \]

  \[
  \text{Capital Gain} = \frac{P_{t+1} - P_t}{P_t} \text{ = Change in Price} \text{ Initial Price}
  \]
Application in Mishkin:

Total return on a coupon bond for one year:

\[ \text{RET} = \frac{C + P_{Bt+1} - P_{Bt}}{P_{Bt}} \]

Mishkin’s formula is a special case: Payment = C. Period = one year

Remember the general principle:

\[ \text{RET} = \frac{\text{Payment}}{P_t} + \frac{P_{t+1} - P_t}{P_t} \]

e.g. if period = D days, then: Payment = (Annualized current yield) * D/365.
   if asset = real estate, then: Payment = Rental income minus expenses.

• Caution: Yields are usually annualized. But current yield in RET refers to the
   period over which RET is calculated – may require conversion.
Illustrations in Mishkin

1. Price – Yield Relation

Table 1 Yields to Maturity on a 10%-Coupon-Rate Bond Maturing in Ten Years (Face Value = $1,000)

<table>
<thead>
<tr>
<th>Price of Bond ($)</th>
<th>Yield to Maturity (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,200</td>
<td>7.13</td>
</tr>
<tr>
<td>1,100</td>
<td>8.48</td>
</tr>
<tr>
<td>1,000</td>
<td>10.00</td>
</tr>
<tr>
<td>900</td>
<td>11.75</td>
</tr>
<tr>
<td>800</td>
<td>13.81</td>
</tr>
</tbody>
</table>
2. Key Linkages: Yield Change - Price Change - Return

Table 2 One-Year Returns on Different-Maturity 10%-Coupon-Rate Bonds When Interest Rates Rise from 10% to 20%

<table>
<thead>
<tr>
<th>(1) Years to Maturity When Bond Is Purchased</th>
<th>(2) Initial Current Yield (%)</th>
<th>(3) Initial Price ($)</th>
<th>(4) Price Next Year* ($)</th>
<th>(5) Rate of Capital Gain (%)</th>
<th>(6) Rate of Return [col (2) + col (5)] (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>10</td>
<td>1,000</td>
<td>503</td>
<td>-49.7</td>
<td>-39.7</td>
</tr>
<tr>
<td>20</td>
<td>10</td>
<td>1,000</td>
<td>516</td>
<td>-48.4</td>
<td>-38.4</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>1,000</td>
<td>597</td>
<td>-40.3</td>
<td>-30.3</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td>1,000</td>
<td>741</td>
<td>-25.9</td>
<td>-15.9</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>1,000</td>
<td>917</td>
<td>-8.3</td>
<td>+1.7</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
<td>1,000</td>
<td>1,000</td>
<td>0.0</td>
<td>+10.0</td>
</tr>
</tbody>
</table>

- Lesson: Price responses increase with maturity.
  Quantitative measure of price-sensitivity: \( \%\Delta P = - \text{Duration} \times \Delta i/(1+i) \)
  Discount bonds have duration = maturity; coupon bonds have duration < maturity.
  Optional reading: Online Appendix to Ch.4
Real Returns and Real Interest Rates

• **Real interest rate** = Nominal interest rate minus expected inflation
  \[ r = i - \pi^e \]

  - Traditional measurement: Find nominal interest rates & estimate expected inflation
    - Problem: expectations are not directly observable.
  - Better approach: exploit yields on inflation-protected securities (in U.S. since 1997)

• **Treasury Inflation-Protected Securities (TIPS):**
  - Face value is fixed in real terms: \( F_r = $100 \)
  - Nominal face value varies with CPI: \( F = F_r \times (CPI) \)
  - Nominal coupon varies with CPI: \( C = (\text{Coupon rate}) \times F \)
  - If Price = Face value, then: nominal return \( \approx (\text{Coupon rate}) + (\text{Change in CPI}) \)
    \[ \Rightarrow \text{Real yield to maturity} = \text{real return} \approx \text{Coupon rate} \]
    \[ \Rightarrow \text{Interpret coupon rate as real interest rate.} \]
  - If Price \( \neq \) Face value, real return differs from coupon rate & not easy to compute.
    \[ \Rightarrow \text{Rely on published sources for real yields to maturity (e.g. WSJ)} \]

• Remember: **Quoted yields on TIPS are direct measures of REAL interest rates.**

• Expected inflation: compute \( \pi^e = i - r \) from observed Treasury yields (i and r).


**Other Applications**

- Present values of corporate stocks:
  - Payment = Dividend. Example of infinitely-lived asset.

\[
PV_t = \frac{\text{Dividend}_{t+1}}{1+i} + \frac{\text{Dividend}_{t+2}}{(1+i)^2} + \ldots
\]

  - More on stocks in Mishkin ch.7.

- **British consols** = coupon bonds without repayment date.
  - Simple present value formula from math of geometric sums:

\[
P_B = \frac{C}{1+i} + \frac{C}{(1+i)^2} + \ldots = \frac{C}{i}
\]

  - Illustrates negative price-yield relationship.
More on Bond Quotes: Review and Caveats

• Most of the time, economists use published or online quotes to obtain yields.

• Traders sometimes work with yields without mentioning prices.
  Then \( P_B = PV \) is implied. Everyone knows.

• Bond quotes have two parts:
  (1) Identify the security: issuer, maturity date, coupon = fixed data.
  (2) Market information: price, yield, time of quote = changes over time.

• Implied items that also change: current yield, remaining maturity.

• Assumptions to check:
  Prices are usually per $100 face value; but sometimes per $1000.
  Prices are may be decimal or fractional (e.g. "100 : 8" = 100 \( \frac{8}{32} \) = 100.25)

  Good sources should have legends or footnotes to confirm interpretation

• Simplifications for this class:
  Disregard discounting over fractional periods; usually maturity = whole years
  Disregard lumpiness in coupons; treat payments as smooth over the year
  Disregard “accrued interest” – use prices are as quoted “clean”