Mishkin ch.4: Interest Rates

Summary

1. Three key concepts:

   Present Value     Yield to Maturity     Total Return

2. Know how to work with the key concepts:

   Task in Exams: Problem solving

3. Applications:

   Real versus nominal rates   PV of stocks: Discounted dividends
   Inflation-indexed bonds     Tracking total returns
Fundamentals

• Present Value:

\[ PV_t = \frac{\text{Payment}_{t+1}}{(1+i)} + \frac{\text{Payment}_{t+2}}{(1+i)^2} + \ldots + \frac{\text{Payment}_{t+N}}{(1+i)^N} \]

\( i \) = discount rate = interest rate used to discount future payments

• Yield to Maturity (or simply: Yield) = Particular discount rate \( i \) that solves

\[ PB = PV_t = \frac{\text{Payment}_{t+1}}{(1+i)} + \frac{\text{Payment}_{t+2}}{(1+i)^2} + \ldots + \frac{\text{Payment}_{t+N}}{(1+i)^N} \]

One-period example (Treasury Bill promises $10,000 at maturity)

\[ PB = \frac{10000}{(1+i)} = \frac{10000}{PB} \Rightarrow (1+i) = \frac{10000}{PB} \Rightarrow i = \frac{10000-P_B}{PB} \]

[Notes on Mishkin Ch.4 - P.2]
Application: Coupon Bonds

- **Defined by:**
  - Maturity date $\Rightarrow$ Years to maturity $= N$
  - Coupon $= C$
  - Face Value $= F$ $\Rightarrow$ Coupon rate $= C/F$

- **Market data:**
  - Price $= P_B$ or $P_{Bt}$ $\Rightarrow$ Yield to maturity $= i$ or $i_t$

  We use subscript $t$ (for time) when multiple dates are involved.
  Yield or “interest rate” without modifiers refers to yield to maturity.

- **Common problems:** Compute yield from price. Compute price from yield.

**Example**

- **Bond quote from WSJ:**

<table>
<thead>
<tr>
<th>Date: 4/9/2018</th>
</tr>
</thead>
<tbody>
<tr>
<td>Security</td>
</tr>
<tr>
<td>Maturity</td>
</tr>
<tr>
<td>Coupon Rate</td>
</tr>
<tr>
<td>Price</td>
</tr>
<tr>
<td>Yield</td>
</tr>
<tr>
<td>Treasury note</td>
</tr>
<tr>
<td>3/31/2021</td>
</tr>
<tr>
<td>1.25%</td>
</tr>
<tr>
<td>96.63</td>
</tr>
<tr>
<td>2.431%</td>
</tr>
</tbody>
</table>

  *Question: How is the yield calculated?*
Example

Date: 4/9/2018

<table>
<thead>
<tr>
<th>Security</th>
<th>Maturity</th>
<th>Coupon Rate</th>
<th>Price</th>
<th>Yield</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treasury note</td>
<td>3/31/2021</td>
<td>1.25%</td>
<td>96.63</td>
<td>2.431%</td>
</tr>
</tbody>
</table>

- Task #1: Set up the present value equation
  - About 3 years to maturity => N = 3
  - Price is per F=100 face value => C = (Coupon Rate) * 100 = 1.25
  - Payment at dates t+1,…,t+N-1: Coupon = C
  - Payment at maturity date t+N: Face value + final coupon = F+C

\[
PV = \frac{C}{(1+i)^1} + \frac{C}{(1+i)^2} + \frac{C+F}{(1+i)^3} = \frac{1.25}{(1+i)} + \frac{1.25}{(1+i)^2} + \frac{101.25}{(1+i)^3}
\]

- Task #2: Solve the present value equation
  - Cubic equation – solve numerically by exploiting that PV is declining in i.

Knowing how this works is essential to understanding yield quotes.

[Notes on Mishkin Ch.4 - P.4]
### Worksheet

<table>
<thead>
<tr>
<th>Discount</th>
<th>PV</th>
<th>Graph each pair</th>
</tr>
</thead>
<tbody>
<tr>
<td>try coupon: 1.25%</td>
<td>100.00</td>
<td>-&gt; par value</td>
</tr>
<tr>
<td>try low: 0.00%</td>
<td>103.75</td>
<td>-&gt; high PV =&gt; yield too low</td>
</tr>
<tr>
<td>try high: 3.00%</td>
<td>95.05</td>
<td>-&gt; low PV =&gt; yield too high</td>
</tr>
<tr>
<td>try between: 2.50%</td>
<td>96.43</td>
<td>-&gt; yield too high</td>
</tr>
<tr>
<td>try again: 2.43%</td>
<td>96.63</td>
<td>-&gt; close enough</td>
</tr>
</tbody>
</table>

![Discount PV Graph each pair](image-url)
The Total Return
(a.k.a: Return, Rate of Return)

• Definition:

\[
RET = \frac{\text{Payment} + P_{t+1} - P_t}{P_t}
\]

• Measured over a specific time period t to t+1:
  
  \( P_t = \) Price at the start; known.
  
  \( P_{t+1} = \) Price at the end; often unknown at time t.
  
  Payment = current yield or other payout during the time period t to t+1; assumed known

• Can be computed for ANY financial asset – not only bonds

• Components:

  \[
  \text{Current Yield} = \frac{\text{Payment}}{P_t}
  \]

  \[
  \text{Capital Gain} = \frac{P_{t+1} - P_t}{P_t} = \frac{\text{Change in Price}}{\text{Initial Price}}
  \]
Application in Mishkin:

Total return on a coupon bond for one year:

\[ RET = \frac{C + P_{Bt+1} - P_{Bt}}{P_{Bt}} \]

Mishkin’s formula is a special case: Payment = C. Period = one year

Remember the general principle:

\[ RET = \frac{Payment}{P_t} + \frac{P_{t+1} - P_t}{P_t} \]

e.g. if period = D days, then: Payment = (Annualized current yield) * D/365.
if asset = real estate, then: Payment = Rental income minus expenses.

• Caution: Yields are usually annualized. But current yield in RET refers to the period over which RET is calculated – may require conversion.
Illustrations in Mishkin

1. Price – Yield Relation

Example: 10-year coupon bond

<table>
<thead>
<tr>
<th>Price of Bond ($)</th>
<th>Yield to Maturity (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,200</td>
<td>7.13</td>
</tr>
<tr>
<td>1,100</td>
<td>8.48</td>
</tr>
<tr>
<td>1,000</td>
<td>10.00</td>
</tr>
<tr>
<td>900</td>
<td>11.75</td>
</tr>
<tr>
<td>800</td>
<td>13.81</td>
</tr>
</tbody>
</table>
2. Key Linkages: Yield Change - Price Change - Return

Lesson: Price responses increase with maturity

**TABLE 2** One-Year Returns on Different-Maturity 10%-Coupon-Rate Bonds When Interest Rates Rise from 10% to 20%

<table>
<thead>
<tr>
<th>(1) Years to Maturity When Bond Is Purchased</th>
<th>(2) Initial Current Yield (%)</th>
<th>(3) Initial Price ($)</th>
<th>(4) Price Next Year* ($)</th>
<th>(5) Rate of Capital Gain (%)</th>
<th>(6) Rate of Return [col (2) + col (5)] (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>10</td>
<td>1,000</td>
<td>503</td>
<td>-49.7</td>
<td>-39.7</td>
</tr>
<tr>
<td>20</td>
<td>10</td>
<td>1,000</td>
<td>516</td>
<td>-48.4</td>
<td>-38.4</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>1,000</td>
<td>597</td>
<td>-40.3</td>
<td>-30.3</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td>1,000</td>
<td>741</td>
<td>-25.9</td>
<td>-15.9</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>1,000</td>
<td>917</td>
<td>-8.3</td>
<td>+1.7</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
<td>1,000</td>
<td>1,000</td>
<td>0.0</td>
<td>+10.0</td>
</tr>
</tbody>
</table>

*Calculated with a financial calculator, using Equation 3.

Common measure of price-sensitivity: \( \%\Delta P = - \text{Duration} \times \Delta i/(1+i) \)

Pure discount bonds have duration = maturity; for coupon bonds, duration < maturity.

Optional reading: Online Appendix to Ch.4
Real Returns and Real Yields

• **Real interest rate** = Nominal interest rate minus expected inflation

\[ r = i - \pi^e \]

- Traditional measurement: Find nominal interest rates & estimate expected inflation
  
  Problem: expectations are not directly observable.

- New approach: use yields on inflation-protected securities (in U.S. since 1997)

• **Treasury Inflation-Protected Securities (TIPS):**
  
  - Face value is fixed in real terms: \( F_T = $100 \)
  
  - Nominal face value varies with CPI: \( F = F_T \times (\text{CPI}) \)
  
  - Nominal coupon varies with CPI: \( C = (\text{Coupon rate}) \times F \)
  
  - If Price = Face value, then: nominal return \( \approx (\text{Coupon rate}) + (\text{Change in CPI}) \)

\[ => \text{Real yield to maturity} = \text{real return} \approx \text{Coupon rate} \]

\[ => \text{Interpret coupon rate as real interest rate.} \]

- If Price \( \neq \) Face value, real return differs from coupon rate & not easy to compute.

\[ => \text{Rely on published sources for real yields to maturity (e.g. WSJ)} \]

• Remember: **Quoted yields on TIPS are direct measures of REAL interest rates.**
Other Applications

- **Present values of corporate stocks:**
  - Payment = Dividend. Example of infinitely lived asset
  \[
  PV_t = \frac{\text{Dividend}_{t+1}}{1+i} + \frac{\text{Dividend}_{t+2}}{(1+i)^2} + \ldots
  \]
  - Return to stocks in Mishkin ch.7.

- **British consols:**
  - Coupon bonds without repayment date.
  \[
  P_B = \frac{C}{1+i} + \frac{C}{(1+i)^2} + \ldots = \frac{C}{i}
  \]
  - Nice illustration of negative price-to-yield relationship.
More on Bond Quotes: Review and Caveats

- Most of the time, economists use published or online quotes to obtain yields.
- Traders sometimes work with yields without mentioning prices. Then $P_B = PV$ is implied. Everyone knows.

- Bond quotes have two parts:
  1. Identify the security: issuer, maturity date, coupon = fixed data.
  2. Market information: price, yield, time of quote = changes over time.

- Implied items that also change: current yield, remaining maturity.

- Assumptions to check:
  Prices are usually per $100 face value; but sometimes per $1000.
  Prices are may be decimal or fractional (e.g. "100 : 8" = $100 \frac{8}{32} = 100.25$)

  Good sources should have legends or footnotes to confirm interpretation

- Simplifications for this class:
  Disregard discounting over fractional periods; usually maturity = whole years
  Disregard lumpiness in coupons; treat payments as smooth over the year
  Disregard “accrued interest” – use prices are as quoted “clean”