Note on the Money Multiplier and the Fed Funds Market
Econ 135 - Henning Bohn

This note explains how standard money multiplier analysis and Fed funds market analysis can be modified to account for the introduction of interest on reserves. See Mishkin ch.14 for the standard money multiplier. See Mishkin ch.15 for standard Fed funds market analysis. This note assumes that you have read both chapters.

1. Assumptions

When the money multiplier and the Fed funds market are presented separately, it may not be obvious how they fit together and where they differ. Here is a unified set of assumptions:

1. The Fed imposes a reserve ratio $r$, uses open market operations to increase or reduce banks’ reserve holdings at the Fed, extends discount loans at rate $i_d$, and pays interest on reserves (IOR) at rate $i_o$. The discount rate is always greater than the interest rate on reserves.
2. The non-bank public (households, firms) has a demand for money ($M$) that depends on the price level ($P$), aggregate real activity ($Y$), and on the opportunity cost of holding money ($i$). To be specific, let $M$ refer to $M_1$ and let $i$ represent the yield on 3-month T-bills
3. The demand for money divides proportionally into a demand for currency ($C$) and a demand for checkable deposits ($D$) with proportionality factor $c = C/D$. Consequently, $C = c/(1+c)*M$ and $D = 1/(1+c)*M$ have the same determinants ($Y$, $P$, $i$) as money demand.
4. Banks effectively determine the volume of discount loans (a.k.a. borrowed reserves, $BR$) because the Fed routinely grants bank requests for discount loans. Banks have an incentive to request discount loans only if the Fed funds rate exceeds the discount rate. Assume $BR = 0$ for $i_{ff} \leq i_d$ and $BR(i_{ff})$ is increasing in $i_{ff}$ for $i_{ff} > i_d$.

Mishkin ch.15 considers the limiting case that borrowed reserves $BR(i_{ff})$ are infinitely interest-elastic at $i_{ff} = i_d$. This makes total reserve supply (meaning NBR through open market operations plus “supply” provided by banks demanding BR) horizontal at $i_{ff} = i_d$. The limiting case is a good simplification for the U.S. since 2003, but not before.

Money multiplier and Fed funds market analysis make somewhat different assumptions about excess reserves. In general, banks have two main motives for holding excess reserves: (a) as precaution, to be prepared for deposit outflows; (b) for investment purposes when the interest rate on reserves is sufficiently high. The precautionary motive is related to the size of the bank’s deposit-taking business, suggesting a ratio $ER/D$ that is small and constant under normal conditions. This is assumed in money multiplier analysis. Because money multiplier analysis was developed before the Fed started paying interest on reserves (since 2008), it abstracts from the investment motive. Fed funds market analysis,
in contrast, explicitly invokes the investment motive to explain why the demand for reserves has a horizontal segment at $i_{ff} = i_{or}$ that imposes a lower bound on the Fed funds rate.

Here is a unified set of assumptions about reserves that works for the post-2008 period; it is somewhat more precise about the determinants of reserve demand than Mishkin ch.15:

5. Assume banks have a deposit-related demand for excess reserves, $ER$, which is proportional to demand deposit: $ER_D = e_D \cdot D$, where $e_D \geq 0$ is a constant and small under normal, non-crisis conditions.

6. Assume banks hold excess reserves for investment purposes, $ER_I$, whenever the Fed funds rate would otherwise fall below the interest rate on excess reserves: $ER_I(i_{ff})$ is a declining function of $i_{ff}$ for $i_{ff} \leq i_{or}$, whereas assume $ER_I = 0$ for $i_{ff} > i_{or}$. Following Mishkin ch.15, we only consider the limiting case that excess reserves are infinitely interest-elastic at $i_{ff} = i_{or}$. Then reserve demand has a horizontal segment at $i_{or}$ and this places a lower bound on $i_{ff}$.

7. Assume required reserves are $RR = rr \cdot D$, where $rr$ is the required reserve ratio.

8. Assume the T-bill rate is linked to current and expected Fed funds rates through the term structure of interest rates, $i=f(i_{ff},...)$. This makes deposit holdings a declining function of the Fed Funds rate, so one may write $D = D(i_{ff},...)$, where he dots serve as reminder the money demand moves around in response to macroeconomic changes.

Assumption 7 requires a big caveat. U.S. reserve requirements specify average daily reserve holdings over a biweekly reserve maintenance period. Money multiplier analysis typically considers a time horizon of several weeks or months, so $RR = rr \cdot D$ applies in general. The Fed funds market is an overnight market, however. Overnight reserves may differ from $rr \cdot D$ provided the gap is made up on another day during the same reserve maintenance period. Indeed banks have an incentive to hold less in reserves on days when $i_{ff}$ is unusually high (i.e. meeting reserves costly); and they have an incentive to hold more reserves on days when $i_{ff}$ is unusually low. Overnight reserve demand should therefore be more interest-elastic than the volume of deposits, though the difference should be small when the Fed funds rate is fairly smooth—as in recent years. Thus $RR = rr \cdot D$ is best interpreted as reasonable simplification for periods without much short-term volatility in the Fed funds rate.

2. Equilibrium in the Fed Funds Market: Normal Case vs. Ample Reserves

Putting the assumptions together, the overall demand for reserves is

$$R^d = RR + ER = RR + ER_D + ER_I = rr \cdot D + ER_D + ER_I$$

Consider first the case of a Fed funds rate greater than the interest rate on excess reserves. Since there is no investment motive for excess reserves for $i_{ff} > i_{or}$,

$$R^d = rr \cdot D + ER_D = (rr + e_D) \cdot D(i_{ff},...)$$
is proportional to checkable deposits. The function is declining in the Fed funds rate because the public’s demand for checkable deposits is interest-elastic. This is the downward-sloping part of Mishkin’s reserve demand funding in ch.15. For the same range of interest rates, the ratio of excess reserves ratio to deposits is \( e = \frac{ER}{D} = \frac{ER_D}{D} = e_D \), which means it is constant as assumed in money multiplier analysis.

Let \( \hat{R}^d = (rr + e_D) \cdot D(i_{or},...) \) denote the reserve demand without the investment component when Fed funds rate just equal \( i_{or} \). The there are two types of market equilibrium:

**Case A (Normal):** If reserve supply is less or equal \( \hat{R}^d \), the reserve market has an equilibrium with \( i_{ff} \geq i_{or} \) and \( ER_I = 0 \). In the Fed funds market diagram, reserve demand and supply intersect on the downward sloping segment of reserve demand.

In this type of equilibrium, the ratio of excess reserves ratio to deposits is \( e = \frac{ER}{D} = \frac{ER_D}{D} = e_D \), which means \( e \) is constant, as assumed in money-multiplier analysis.

**Case B (Ample reserves):** If reserve supply is greater than \( \hat{R}^d \), market equilibrium requires \( ER_I = R^* - \hat{R}^d > 0 \) and \( i_{ff} = i_{or} \). In the Fed funds market diagram, reserve demand and supply intersect on the flat segment of reserve demand. In the special case that reserve supply exactly equals \( \hat{R}^d \), market equilibrium is at \( (\hat{R}^d, i_{ff}) \), the point at which the flat and downward-sloping segments of reserve demand intersect.

In this type of equilibrium, the ratio of excess reserves ratio to deposits is \( e = \frac{ER}{D} = \frac{e_D}{D} + \frac{ER_I}{D} \), which is greater than \( e_D \) and varies in response to changes in \( R^* \) and \( \hat{R}^d \). Recall that money-multiplier analysis assumed a constant excess reserve ratio. This assumption is likely not satisfied in the Ample Reserves case.

### 3. The Effects of Open Market Operations and Changes in Interest on Reserves

**Case A. Normal conditions – Experiment 1. Open market purchase**

- Fed funds market: NBR increases => reserve supply shifts left => Fed funds rate decreases, bank reserves increase.
- Market for money: MB increases while \( m \) stays constant => M1 increases, shown the Liquidity Preference diagram as money supply shift to the right => interest rate \( i \) declines until the quantity of money demanded matches the increased supply
- Overall: Proportional increase in MB, C, D, and M1.
- Note that a reduced market interest rate \( i \) increases the quantity of currency demanded by the public. This change is conceptually a nuisance (central bankers don’t worry about currency), but to deal with it correctly, one must distinguish two versions of this experiment:
(a) Open market purchase with a target for MB: then currency withdrawal causes loss of bank reserves
=> net increase in NBR is (slightly) less than change in MB.
(b) Open market purchase with a target for NBR: then currency withdrawal triggers a defensive,
additional open market purchase to keep NBR on target => All changes slightly greater than in (a).

Case A. Normal conditions – Experiment 2. Decrease in the Interest Rate on Reserves:
• Fed funds market: Horizontal segment of reserve demand shifts down, but no change in equilibrium.
• Market for money: no change.

Case B. Ample reserves – Experiment 1. Open market purchase:
• Fed funds market: NBR increases => reserve supply shifts left => Fed funds rate unchanged, excess
reserved for investment purposes increase.
• Market for money: MB increases, but ER/D also increases => m declines. Unchanged Fed funds rate
=> no change in interest rates i => no change in quantity of money demanded => no change in M1.

Case B. Ample reserves –– Experiment 2. Decrease in the Interest Rate on Reserves:
Distinguish two versions, depending how changes in currency are handled.
(a) Assuming no open market operations:
• Fed funds market: Horizontal segment of reserve demand shifts down => Fed funds rate decreases,
bank reserves remain unchanged.
• Market for money: Reduced Fed funds rate reduces i, increases demand for deposits and for
currency. Excess reserves decline because higher D implies higher RR and because of currency
withdrawal => MB increases due to higher C, m increases because of reduced ER/D => M1
increases to match increase demand for money.
(b) Assuming defensive open market operations to maintain MB target:
• Fed funds market: Open market sale to offset higher C => Reserve supply shifts left, but no impact
on the Fed funds rate => Same decrease in the Fed funds rate as in (a), less total reserves and less
excess reserves than in (a).
• Market for money: No change in MB but greater reduction in m than in (a) => Same increase in M1
as in (a).

Experiments with changes in the opposite directions would work analogously. Note that large open
market operations or large change in IOR may cause a transition between the Normal and Ample
Reserves; then the initial case applies until \( \hat{R}^d \) is reached, and other case applies for changes beyond.