Notes on the Keynesian Model
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This note presents the Keynesian model as a system of equations that can be solved algebraically. For the tests, knowing the main equations and doing graphical analysis should suffice. Hence it’s not crucial for you to understand every line in this note. However, you should work through the math at least once to gain a better understanding of the graphs. Students who like algebra better than graphs may also the information here to solve applied questions algebraically.

This is not a substitute for the class or for the required readings. Sections 1-3 assume that you have read Mishkin ch.21-22. Section 4 explains the simple IS curve used in class and how it differs from the IS curve in Mishkin ch.20. (The latter is for students who read ch.20—not required). Section 5 explains how the analysis must be modified if the real interest rate is constrained by lower bound on nominal interest rates; best do not read the (more complicated) Zero Lower Bound section in Mishkin ch.23. Section 6 presents the Taylor Rule as generalization of the MP-curve. Sections 4-6 are recommended readings, meant to supplement the class presentation.

In class, I focus on explaining the Keynesian model in words and with diagrams. I don’t walk through the underlying math, mainly because most students find diagrams easier to follow. However, going through the math at least once – at your own pace – will help you better understand the diagrams and how to use them correctly. Some students may find the math easier than the diagrams, or useful to double-check answers from graphs.

1. Main Equations
The Keynesian model has three main equations:

(1) AS-curve: \( \pi = \gamma (Y - Y^p) + \pi^e + \rho \)
Here \( \gamma > 0 \) (gamma) is a slope coefficient that indicates how strongly inflation responds to the gap between output and potential; \( \rho \) symbolizes a price shock: normally \( \rho = 0 \), but it can be positive or negative.

(2) IS-curve: \( Y = Y_d(r, \ldots) = Y - d \cdot r \).
Mishkin’s algebra for the IS curve is quite cumbersome; to simplify, I write IS as linear downward-sloping function with intercept \( Y \) and negative slope \( d > 0 \). Section 4 below explains how this relates to Mishkin ch.20.

(3) MP-curve: \( r = \bar{r} + \lambda \cdot \pi \)
Here \( \lambda > 0 \) (lambda) is a slope coefficient that indicates how strongly the central banks responds to the inflation; \( \bar{r} \) is the intercept, i.e., the interest rate the central bank would set at zero inflation.
Overall, the model has three equations for the three endogenous variables (π, Y, r). Potential output, expected inflation, and the various intercepts and shocks are exogenous factors. For any given combination of exogenous determinants, the model delivers a unique solution for (π, Y, r). Most applied questions are about the impact of various changes. To solve problems, one must determine how the solution changes when one (or more) of the exogenous factors change.

2. Algebraic Solution
To solve the model, it helps to note that each of the three equations includes only two of the three endogenous variables. This suggests a solution by successive elimination of variables is convenient. In terms of pure math, one could do the substitution in any sequence. Mishkin’s strategy, which I follow here, is to eliminate r first. The main step is to substitute the MP equation into the IS equation to obtain

\[ Y = \bar{Y} - \bar{d} \cdot (r + \lambda \cdot \pi) = (\bar{Y} - \bar{d} \cdot r) - \bar{d} \cdot \lambda \cdot \pi \]

The result is an equation that expresses the demand for goods as function of inflation, called AD-curve. Since \( \lambda > 0 \) and \( \bar{d} > 0 \), the AD curve has a negative slope. The intercept \( \bar{Y} - \bar{d} \cdot \bar{r} \) is increasing in \( \bar{Y} \) and decreasing in \( \bar{r} \).

After eliminating r, the AS- and AD-curves are system of two linear equations in the unknowns (π, Y). Because AS has positive slope whereas AD has negative slope, there is always a unique intersection, which provides the solution for (π, Y). To obtain a final solution for output, one may substitute AS into AD:

\[ Y = \bar{Y} - \bar{d} \cdot \bar{r} - \bar{d} \cdot \bar{r} \cdot \lambda \cdot \pi = \{ \bar{Y} - \bar{d} \cdot \bar{r} + \bar{d} \cdot \bar{r} \cdot \gamma \cdot Y^p - \bar{d} \cdot \bar{r} \cdot \lambda \cdot \pi^c - \bar{d} \cdot \bar{r} \cdot \lambda \cdot \rho \} - \bar{d} \cdot \bar{r} \cdot \gamma \cdot Y \]

Moving all Y-terms to the left and dividing yields a solution for output:

\[ Y = \frac{1}{\bar{d} \cdot \bar{r} \cdot \gamma \cdot \lambda} \{ \bar{Y} - \bar{d} \cdot \bar{r} + \bar{d} \cdot \bar{r} \cdot \gamma \cdot Y^p - \bar{d} \cdot \bar{r} \cdot \lambda \cdot \pi^c - \bar{d} \cdot \bar{r} \cdot \lambda \cdot \rho \} \]

Thus Y increases when \( \bar{Y} \) increases, when \( \bar{r} \) decreases, when potential output rises, when expected inflation declines, and when \( \rho \) is negative. Knowing the solution for Y, one can substitute Y into AS to obtain a solution for \( \pi \) and then substitute \( \pi \) into MP to obtain a solution for r (both are too long and messy to be insightful, hence the math is omitted).

The point of this derivation is to show you (a) that algebraic solutions exist – inviting you to work with them if you are mathematically inclined; and (b) that the math produces “big” formulas; the latter suggests that most students are probably better off using graphs to solve problems that trying to memorize the formulas. However, understanding the AD-curve is important, so everyone should understand the math leading to (*).
3. AD-AS Analysis

The idea of AD-AS analysis is to graph the AS curve (equation 1) and the AD-curve (equation *) into and output-inflation diagram to solve problems graphically. The AS equation describes inflation (firm’s decisions about pricing) as function of the aggregate demand for goods, so it can drawn as is into \((\pi, Y)\)-diagram. If \(\rho = 0\), the AS curve runs through the point \((Y^p, \pi^e)\); if \(\rho \neq 0\), the AS curve is shifted up or down accordingly. The AD-curve economically describes how aggregate demand depends on inflation; to draw it accurately into a diagram with \(\pi\) on the vertical axis, the equation needs to be inverted to read:

\[
\pi = \frac{\pi - \bar{\pi}}{d \cdot \lambda} - \frac{1}{d \cdot \lambda} Y
\]

You may think of (**) as specifying the inflation rate that would be needed to induce the central bank to set interest rates in a way that produces a given aggregate demand \(Y\). The main point of (**) is to point out that in a \((\pi, Y)\)-diagram, the slope of AD is \(-\frac{1}{d \cdot \lambda}\). That is, the AD-curve is steep when \(\bar{d}\) and \(\lambda\) are small, and it is relative flat when \(\bar{d}\) and \(\lambda\) are large. The intercept is proportional to \(\bar{Y} - \bar{d} \cdot \bar{\pi}\), which means the curve shifts to the right (or equivalently, up) when \(\bar{Y}\) increases, and to the left (down) \(\bar{\pi}\) increases.

Now we have the tools to solve problems. We start with an equilibrium position of the economy – some initial solution for \((\pi, Y, r)\). Then a shock hits. The key question is always which of the basic equations are affected. To review (details in Mishkin ch.21-23):

(1) AS shifts when potential output changes, when expected inflation changes exogenously, or when firms’ cost of production changes (so \(\rho\) is non-zero). If AS shifts up, so in the \((\pi, Y)\)-diagram, \(Y\) declines and \(\pi\) increases. Given higher \(\pi\), the MP curve implies higher \(r\).

(2) MP shifts when monetary policy changes; this can be expressed in terms of interest rates (e.g., “Fed decides to raise the interest rates” means shift up in \(\bar{r}\)) or in terms of money supply and liquidity effect (e.g., “Fed increases money growth” means the interest rate drops via the liquidity effect, so it’s shift down in \(\bar{r}\)). If MP shifts up, AD shifts left, so in the \((\pi, Y)\)-diagram, \(Y\) and \(\pi\) decrease. Note that the IS curve is unchanged, so lower \(Y\) requires higher \(r\).

(3) IS shifts when a component of \(C+I+G+NX\) changes exogenously, either autonomously (\(C, I, NX\)) or because of a change in fiscal policy; and increase/decrease implies an increase/decrease in \(\bar{Y}\). If \(\bar{Y}\) increases, IS shifts right, AD shifts right, so in the \((\pi, Y)\)-diagram, \(Y\) and \(\pi\) increase. Here the MP curve is unchanged, so higher \(\pi\) implies higher \(r\).
Here are three items that require care (or produce wrong answers):

1. Monetary vs. fiscal policy: Monetary policy is about Federal Reserve policy with respect to money or interest rate targets, and it affects MP. Fiscal policy is about government spending or taxes, and it affects IS.

2. The direction of supply shocks: Economic changes that force firms to raise prices will shift the AS curve up, or equivalently, to the left. Algebraically, such shifts are represented by positive values for ρ (meaning: higher inflation). In the press, such shocks are sometimes described as “negative” supply shocks; this is intuitively appealing because the impact on output is negative, but ρ is positive. Best think of ρ as shock to inflation.

3. Expected inflation: The relevant expectation in the Keynesian model is a measure summarizing the beliefs about inflation held by firms and workers when they set prices and nominal wages. The model assumes that \( \pi^e \) is given in the short run and changes slowly in response to changes in actual inflation. In the context of Mishkin ch.7 (which discusses different types of expectations), these are adaptive expectations. Importantly, the \( \pi^e \) in the AS curve may differ from rational expectations held by economists or investors in financial markets. (It’s almost worth having different symbols, but Mishkin does not, so you will have to remember the distinction when we turn to financial markets.)

4. A Streamlined IS curve

Mishkin provides a detailed presentation of the IS curve in ch.20. For our purposes, the simple IS relationship \( Y = Y^d - d \cdot r \) suffices. This section explains how this relationship is derived and has some comments on how to interpret it.

Starting point for the IS curve is that aggregate demand has four components, consumption C, capital investment I, government spending G, and net exports NX.

For consumption, Mishkin assumes that C depends on disposable income Y-T and on exogenous (“autonomous”) component \( \bar{C} \), so \( C = \bar{C} + mpc \cdot (Y - T) \), where mpc stands for marginal propensity to consume out of current disposable income. If you remember intermediate macro, this might strike you are overly simplistic. Notable omissions are that incentives to save (rather than consume) depend on real interest rates, that permanent and temporary changes in income and taxes have different effects, and that expectations about future income and taxes should influence decisions about consumption versus savings. Implicitly, the omitted elements are reflected in \( \bar{C} \); this means \( \bar{C} \) shifts when expectations about future income and taxes change. Also, the effect of a tax cut on consumption may differ from the mpc from current income. This suggests writing
\[ C = \overline{C} + mpc \cdot Y - mpcT \cdot T - z \cdot r \]

with separate coefficients for \( Y \) and \( T \) and with an interest rate response (\( z \)). [This is more elaborate than in Mishkin, worth thinking through, but I will simplify below.]

For capital investment, it’s uncontroversial that capital investment depends negatively on the cost of borrowing, which Miskkin writes as linear function

\[ I = \overline{I} - d \cdot (r + \overline{f}) \]

with slope \( d \) and indicator \( \overline{f} \) for financial frictions that raise corporate borrowing costs. If you remember intermediate macro, there are again omission that are implicitly subsumed into the autonomous component \( \overline{I} \), notably expectations about the profitability of investment, which depends on expectations about future sales, marginal tax rates, business regulations, etc. News about any of these factors would trigger shifts in \( \overline{I} \).

For net exports, Mishkin assumes that \( NX = \overline{NX} - x \cdot r \) has an autonomous component and depends negatively on the interest rate.

Combining all four components, one obtains

\[ Y = C + I + G + NX \]

or equivalently

\[ Y = \frac{1}{1 - mpc} [\overline{C} - mpcT \cdot T + \overline{I} + G + \overline{NX} - d \cdot (r + \overline{f})] - \frac{d + z + x}{1 - mpc} \cdot r, \]

which is the IS curve. This is equation (12) in Mishkin ch.20, except I allow for \( mpcT \neq mpc \) and \( z \neq 0 \). To summarize all the autonomous components, define

\[ \overline{Y} = \frac{1}{1 - mpc} [\overline{C} - mpcT \cdot T + \overline{I} + G + \overline{NX} - d \cdot (r + \overline{f})] \]

and let \( \overline{d} = \frac{d + z + x}{1 - mpc} \) denote the slope, then the IS curve reduces to \( Y = \overline{Y} - \overline{d} \cdot r \). To remember for solving problems: (1) The IS curve is negatively sloped; (2) IS shifts when \( \overline{Y} \) changes; (3) \( \overline{Y} \) depends on fiscal policy, the autonomous components of demand, and financial frictions.

One reason I do not require Mishkin ch.20 is that Mishkin tries to describe some complicated issues with oversimplified formulas. Addressing them adequately would require even more math, a bigger investment in math than warranted for this class. One issue is the response of NX to interest rate changes. The argument for \( x > 0 \) involves linkages between interest rates and exchange rates, and between exchange rates and the competitiveness of exports that are complicated and not necessarily effective in the short run. Hence I will interpret most changes in NX simply as shifts in \( \overline{NX} \) and disregard the interest rate channel (i.e., approximate \( x \approx 0 \)). A second issue is response of consumption to various changes in disposable income. Mishkin’s assumption of a single marginal propensity to consume (\( mpc \)) is unfortunate because consumption responses differ depending on the source of the
change. (This motivates my distinction between mpc and mpc\_T above.) There is good evidence that the marginal propensity to consume from temporary changes in Y is small (less than \( \frac{1}{4} \)). Moreover, the marginal propensity to consume from taxes (mpc\_T) depends significantly on expectations whether future government will close the resulting fiscal deficits/surpluses created through adjustments in future taxes or in spending. If consumers expect that a tax change is reversed later, the marginal propensity to consume is small. (Previous macro classes may have discussed the logic of ‘Ricardian neutrality’, which says the effect should be zero.). The assumption that higher taxes reduce consumption is implicitly assuming that tax changes are generally not reversed within the planning horizon of current consumers. Finally, \( \tilde{f} \) has a broader interpretation: Economically, this variable captures everything that makes the required return on capital investment greater than the benchmark real interest rate. This includes financial frictions, but also applies to risk premiums and to policy issues such as taxes or concerns about business regulations.

Bottom line: Instead trying to describe details of aggregate demand algebraically, I find it more useful to write IS as simple equation: \( Y = \bar{Y} - d\cdot r \). You should know that the IS curve has negative slope and you should know which disturbances would shift the curve. Moreover, to memorize what goes into \( \bar{Y} \), it’s reasonable for short-run analysis to approximate mpc\~0; then
\[
\bar{Y} = \bar{C} - mpc\_T\cdot T + \bar{I} - d\tilde{f} + G + NX
\]
This provides a concise list of forces that shift IS:

- IS shifts right when the G increases or when the autonomous components of C, I, or NX increase.
- IS shifts left when taxes increase or when financial frictions increase.

In economic applications, you should assume \( \bar{C} \) changes whenever households have an economic reason to change their consumption (at given disposable income); assume \( \bar{I} - d\tilde{f} \) changes whenever firms have a reason to change their capital investment (at given safe rate r); and assume \( NX \) changes whenever importers or exporters have a reason to change their behavior.

5. Modifications needed to account for the ZLB
ZLB is shorthand for the idea that short-term nominal interest rates cannot be negative, because if banks tried to charge negative interest rates, customers would withdraw their balances in cash. (In practice, central banks such as the ECB have successfully imposed slightly negative interest rates. This has worked because holding cash balances is costly for large institutions. Zero is nonetheless a good approximation for macroeconomic purposes.)

Mathematically, the ZLB the inequality constraint \( i \geq 0 \). Since \( r = i - \pi^e \), this implies \( r \geq -\pi^e \). Around the world, for most of economic history, the MP curve \( r = \bar{r} + \lambda \cdot \pi \) typically implies real interest rates that a positive or at most slightly negative, whereas expected inflation is 2\% or higher, so
$r >> -\pi^c$. This explains why textbooks written before 2008 rarely mentioned the ZLB and assumed that the linear MP curve would always apply. From 2008-2015, U.S. short-term interest rates were essentially zero, and Federal Reserve officials made clear that they would have like to reduce real interest rates further. That is, the desired real rate $\bar{r} + \lambda^1 \cdot \pi$ was less than $-\pi^c$, or equivalently, the desired nominal interest rate $\bar{r} + \lambda^1 \cdot \pi - \pi^c$ was negative. The ZLB implies that the desired rates were not feasible. Actual interest rates were at the lower bounds $i = 0$ and $r = -\pi^c$.

With the ZLB, feasible monetary is described by $r = \max\{\bar{r} + \lambda^1 \cdot \pi, -\pi^c\}$. This is the standard MP curve except that it has a horizontal segment at $r = -\pi^c$ that applies for inflation rates low enough that $\bar{r} + \lambda^1 \cdot \pi < -\pi^c$. The two segments meet at an inflation rate of $\pi = -(\bar{r} + \pi^c)/\lambda^1$, which is the inflation rate for which the standard MP curve implies a zero nominal interest rates. This segment is practically relevant if the central bank want to pursue an expansionary policy (negative $\bar{r}$) and expected inflation is low.

Recall from Section 2, that the AD curve is constructed by mapping alternative inflation rates to quantities of output demanded, using the IS and MP curved and taking expected inflation as given. For low inflation rates (if $\pi < -(\bar{r} + \pi^c)/\lambda^1$) the ZLB implies $r = -\pi^c$ and hence $Y = \bar{Y} - d \cdot (-\pi^c)$. This value does not depend on inflation and puts an upper bound on the quantities of output demanded. In the AD-AS diagram, the implied AD curve has a vertical segment for low inflation rates. Most of the time, aggregate demand at $\pi = -(\bar{r} + \pi^c)/\lambda^1$ is so far above potential output that this segment is irrelevant. However, if there is large negative demand shock (low $\bar{Y}$) that shifts the aggregate demand curve sharply to the left, the vertical segment also shifts left. Then $Y = \bar{Y} - d \cdot (-\pi^c)$ may fall below potential output. Ramification of this scenario are discussed in class.

6. A more general MP-curve: The Taylor Rule

Mishkin’s MP-curve is simplified version of a more complete description of monetary policy known as the Taylor Rule:

$$r = \bar{r} + \lambda^1 \cdot \pi + \lambda^2 \cdot (Y - Y^p)$$

The term $\lambda^1 \cdot \pi$ has the same role as in the MP-curve and indicates how strongly the central bank responds to inflation at any given level of output. The Taylor principle again calls for $\lambda^1 > 0$. The term $\lambda^2 \cdot (Y - Y^p)$ models a central bank response to the output gap. This is motivated by the insight that expected and actual inflation are likely to rise when the output gap is positive (or fall if the gap is negative). Responding to the output gap with $\lambda^2 > 0$ serves to preempt inflation before inflationary expectations get established. Economist John Taylor famously estimated that the Federal Reserve has follow such a rule with coefficients $\lambda^1 = 0.5$ and $\lambda^2 = 0.5/Y^p$. 

The Keynesian model works similarly with Taylor rule instead of MP-curve. Combining IS-curve and Taylor rule imply \( Y = \bar{Y} - \bar{d} \cdot (\bar{r} + \lambda_1 \cdot \pi + \lambda_2 \cdot (Y - Y^p)) \). Like (*), this is a relationship between output and inflation that can be drawn as AD-curve. A new element is that monetary policy will change when there is new information about potential output; otherwise, AD-AS analysis is unchanged. Put differently, the Taylor rule suggests that in the model with MP-curve, new information about potential output may induce the central bank to change \( \bar{r} \).