Ethical discounting in incomplete markets

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Abstract

A large literature draws a stark distinction between ‘positive’ (i.e., based on prices) and ‘normative’ (i.e., based on ethics) approaches to choosing discount rates for project appraisal. This paper unifies these approaches in a model of a sophisticated ethical agent who may trade part of her endowment on incomplete markets. Such an agent should use market interest rates to discount payoffs in marketed states. Discount rates in non-marketed states, however, depend on both ethics and prices. Normative discount rates that do not explicitly account for those trading opportunities that do exist exhibit significant biases, even if trade is highly constrained.

How should ethical agents – benevolent governments, philanthropic foundations, or altruistic investors – discount the future when evaluating marginal projects? The literature on social discounting offers two materially different approaches to this question, one ‘positive’, the other ‘normative’ (see e.g. Gollier & Hammitt, 2014, for a review). In the positive approach markets are paramount; discount rates are taken to reflect the opportunity costs of investment, which are captured by market rates of return. The normative approach, on the other hand, emphasises ethics; discount rates reflect the marginal rate of substitution between consumption in the future and consumption in the present, as calculated by a benevolent ethical planner. Prices play no explicit role in the normative approach.

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– the key issues relate to specifications of planner preferences, and exogenous forecasts of consumption growth.\footnote{One might argue that consumption growth depends implicitly on markets. Examining the consequence of that observation in an explicit model of an ethical agent’s consumption distribution problem is one of the points of this paper.}

If markets are complete these two approaches coincide: marginal rates of substitution at optimal allocations are related one-for-one to state prices. However, proponents of the normative approach have suggested that real-world market imperfections often render this relationship suspect. Market imperfections generically imply that equilibria are inefficient, and there is therefore no reason to believe that observed prices are relevant for social welfare computations (so the argument goes). Moreover, pervasive market incompleteness (e.g., the absence of markets for risk-free payoffs in the distant future) implies that the positive approach is silent on how payoffs in non-marketed states should be evaluated. Such states are critical for the evaluation of some projects (e.g., investments that affect the distant future, such as climate change mitigation).\footnote{This view is well summarised by Gollier (2013): “The problem of incomplete and inefficient financial markets is particularly acute when considering longer time horizons. In particular, future generations cannot trade on these markets, thereby raising more concerns about [intertemporal efficiency]. Moreover, one does not observe liquid safe assets with maturities longer than 30 years. This implies that the positivist arbitrage argument cannot be used to determine the efficient [long run discount rate]. The determination of the...long run discount rate should then rely on the...models used by ethicists.”}

This paper adds two nuances to the debate on ‘ethical’ discounting in imperfect economies. The first, elementary, point is that the presence of market imperfections in general, and incompleteness in particular, does not imply that prices are irrelevant for the cost-benefit analysis of marginal projects. A sophisticated ethical agent should use those markets that do exist to reallocate her endowment across time and states of the world so as to satisfy her normative preferences. Such an agent should use market interest rates to discount marginal payoffs in \textit{marketed} states, even though the economy may be at an inefficient (or ethically undesirable) equilibrium. This observation holds regardless of whether the planner seeks to maximise an arbitrary ethically motivated objective function, or whether she seeks to satisfy consumers’ preferences (provided that consumers are utility maximising).

The second, more subtle, point is that market trading opportunities alter the appropriate discount rate in \textit{non-marketed} states. The value of a small change in consumption in a non-marketed state should be computed at the post-trade consumption bundle; this changes the relative values of non-marketed states even though consumption in those states is unaffected by trade. Numerators of discount factors in non-marketed states will differ from their pre-trade values if preferences are not additively separable. In addition, since
initial consumption is tradable the appropriate denominator for non-marketed discount factors is the marginal utility of market income (which depends on prices), and not the marginal utility of pre-trade initial consumption. The latter change occurs even if preferences are additively separable. These effects can have quantitatively important consequences for discount rates.

I demonstrate these effects in a simple Arrow-Debreu style model applied to an incomplete markets setting. The model shows that a normative approach to social discounting that does not account for market trading opportunities explicitly gives rise to biased cost-benefit rules. By calibrating the model to existing applications of normative social discounting we can estimate the magnitude of the bias, and analyse how it depends on preferences, prices, and the form of market incompleteness.

**Related literature**

The literature on discounting in project evaluation is vast, see e.g. Lind (1982); Portney & Weyant (1999); Groom et al. (2005); Gollier (2012); Arrow et al. (2013, 2014); Gollier & Hammitt (2014); Cropper et al. (2014); Groom & Hepburn (2017) for surveys. Discussion of the relative merits of the positive and normative approaches to discounting in imperfect economies extend back to e.g. Marglin (1963); Feldstein (1964); Baumol (1968); Bradford (1975). These authors focus on the differential effects of public projects on consumption and investment – see Goulder & Williams (2012); Gollier (2013) for more recent contributions in this vein. These issues can be handled parsimoniously by converting a project’s impacts on investment into consumption equivalents using the shadow price of capital, and discounting these using the consumption (i.e., normative) discount rate. By contrast, this paper focusses exclusively on the consumption impacts of projects. The paper integrates positive and normative approaches in a simple unified framework that takes ethics as primitive, but explicitly accounts for trading opportunities on incomplete markets.

A great many papers have derived discount rates for project evaluation by comput-

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3Incomplete markets are often studied in models of financial market equilibrium in which agents transfer wealth through investments in portfolios of securities. As our focus is on the value of marginal payoffs, and not the properties of equilibria (we take prices as exogenous), the Arrow-Debreu contingent claims formulation is more direct for our purposes.

4Like the rest of the literature cited here, this paper focusses on the appraisal of marginal projects. See e.g. Arrow & Kurz (1970); Sandmo & Dreze (1971) for general equilibrium models of distorted economies where effective discount rates on public investment are determined by equating marginal rates of return in the public and private sectors. Barrage (2018) studies a general equilibrium framework in which planners and consumers have different rates of pure time preference.
ing intertemporal marginal rates of substitution from ethically motivated social welfare functions. Gollier (2012) provides a comprehensive overview; see e.g. Fleurbaey & Zuber (2015); Millner (2019) for more recent contributions. This paper shows that these discount rates are generically biased if ethical agents can trade on the market, and suggests that this bias is quantitatively important in existing applications of normative social discounting.

Similarly, several governments, including those of the United Kingdom and France, use a normative approach to motivate discount rate schedules for public project appraisal. This approach makes no reference to current market prices and their implications for discount rates in both marketed and non-marketed states. I show that even if governments can trade only small fractions of aggregate consumption, discount rates in marketed states are strongly influenced by prices, and normative discount rates in non-marketed states require substantial adjustment to account for trading opportunities.

While government discounting policy is a natural application of the results in this paper, they apply just as well to institutional and private investors who pursue ethical objectives (see e.g. the effective altruism movement (MacAskill, 2016)). Roth Tran (2019) provides a discussion of investment policy for foundations that emphasises the importance of accounting for correlations between investment payoffs and normative marginal utilities, in analogy with the consumption-based capital asset pricing model. These issues are complementary to the framework I develop, which deals with the problem of ethical project evaluation in incomplete markets in general.

1 The model

Time is indexed by $t \in \{0, \ldots, t_{\text{max}}\}$ and states of the world are indexed by $s \in \{1, \ldots, N\}$. The state of the world at $t = 0$ is known with certainty. We consider an ethical agent (‘the planner’) who has normative preferences over time- and state-dependent consumption $c_{t,s}$. Bundles of consumption goods will be denoted by bold-faced letters, e.g. $\mathbf{c} = (c_0, c_{1,1}, \ldots, c_{1,N}, c_{2,1}, \ldots, c_{2,N}, \ldots, c_{t_{\text{max}},1}, \ldots, c_{t_{\text{max}},N})$. Planner preferences are represented by a monotonic, differentiable, and strictly quasi-concave utility function $U(\mathbf{c})$ that satisfies Inada-like conditions (i.e., $\lim_{\omega_{t,s} \to 0} \frac{\partial U}{\partial c_{t,s}}(\omega) = \infty$ for all $t, s$). The planner has an initial claim to a consumption bundle $\omega$.

There are several possible interpretations of the model – the planner could represent an individual ethical investor, a foundation, or a benevolent government. In the case of an ethical investor or foundation the endowment $\omega$ may be thought of as the dividend
stream from the portfolio of assets the planner holds *ab initio*. This portfolio could include financial and real assets. In the interpretation where the planner is a government $\omega$ can be understood as a baseline bundle of aggregate consumption for the economy as a whole. In this interpretation $\omega$ should be understood to already reflect the trading activities of private agents. Note however that this does not imply that $\omega$ will be optimal from the perspective of the planner. The planner’s preferences may differ from those of private agents, for example because she aims to represent the interests of unborn generations, or to account for externalities that agents impose on one another. In principle the planner can tax consumption arbitrarily and redistribute income so as to further these broader social objectives. However, large scale taxation and redistribution will of course impose its own social costs. To account for this I consider the case where only a small part of the endowment $\omega$ is tradable in Section 2.2 below. The trading activity I consider in that section may be considered inframarginal with respect to the level of taxation – only resources that are *already* under the control of the government, given status quo levels of taxation and redistribution, are traded. I abstract from these issues in the first instance in order to build intuition for the results, and to speak to the other interpretations of the model where assuming the endowment is fully tradable is more natural.

1.1 Cost-benefit analysis in complete markets

Assume that markets are complete, and that the price of a unit of consumption in state $(t, s)$ is $p_{t,s}$. The planner’s indirect utility from her endowment is:

$$V(\omega) = \max_c U(c) \text{ s.t. } \sum_{t,s} p_{t,s} c_{t,s} = \sum_{t,s} p_{t,s} \omega_{t,s}. \quad (1)$$

Consider a marginal project that yields a bundle of payoffs $\pi = (\pi_{t,s})_{t=0,\ldots,t_{max},s=1,\ldots,N}$. From the envelope theorem, the effect of this project on the planner’s indirect utility in state $(t, s)$ is:

$$\frac{\partial V}{\partial \omega_{t,s}} \pi_{t,s} = \lambda p_{t,s} \pi_{t,s},$$

where $\lambda > 0$ is the planner’s marginal utility of income. Since $\pi$ is assumed to be small relative to $\omega$ (i.e., the project is marginal),

$$V(\omega + \pi) - V(\omega) \approx \lambda \sum_{t,s} p_{t,s} \pi_{t,s}. $$
Writing the prices $p_{t,s}$ in terms of the interest rates $r_{t,s}$ that would prevail on credit markets (i.e., $p_{t,s} = (1 + r_{t,s})^{-t}$), we find that the project is welfare improving iff

$$
\sum_{t,s} (1 + r_{t,s})^{-t} \pi_{t,s} > 0.
$$

This is the market net present value rule, originally proposed by Fisher (1930). The fact that planner preferences play no role in this criterion is sometimes known as Fisher’s separation theorem. When markets are complete project evaluation is entirely independent from consumption distribution.\(^5\)

### 1.2 ‘Naïve’ normative cost-benefit analysis

The opposite extreme from the above analysis of complete markets is the case of an ethical agent who makes no use of the markets to reallocate her endowment, even if some markets do exist. Such a planner’s consumption is entirely determined by her initial endowment $\omega$, which is taken as an exogenous input to the cost-benefit exercise. Although extreme, this case mirrors the normative approach to social discounting (see e.g. Gollier, 2012; Gollier & Hammitt, 2014). In that approach consumption paths are exogenously specified, and do not depend explicitly on prevailing market conditions. The cost-benefit rule is derived by considering the direct effect of marginal projects on the planner’s utility function, evaluated at the exogenously given reference consumption bundle $\omega$:

$$
U(\omega + \pi) - U(\omega) \approx \sum_{t,s} \left. \frac{\partial U}{\partial c_{t,s}} \right|_{\omega} \pi_{t,s}.
$$

Define a consumption discount rate $\rho_{t,s}$ at time $t$ and in state $s$ as:

$$
(1 + \rho_{t,s})^{-t} = \left( \frac{\partial U}{\partial c_{t,s}} \right|_{\omega} \left( \frac{\partial U}{\partial c_{0}} \right|_{\omega} \right).
$$

\(^5\)Fisher (1930, p. 139) emphasizes the strong market completeness assumptions that underlie this result: ‘All this is true under the assumption...that...you can borrow and lend or buy and sell ad libitum and without risk. If this assumption is not true...the choice among [investments] might or might not fall upon that one having the maximum present [market] value, depending on the other circumstances involved, particularly his preferences as regards time shape.’
Then the cost-benefit rule in this case can be written as:

$$\sum_{t,s} (1 + \rho_{t,s})^{-t} \pi_{t,s} > 0.$$  

We will refer to this rule as the ‘naïve’ cost-benefit rule, and to $\rho_{t,s}$ as the ‘naïve’ consumption discount rate, as they do not take explicit account of the trading opportunities that the market, even though incomplete, affords.

### 1.3 ‘Sophisticated’ cost-benefit analysis in incomplete markets

In reality markets are invariably incomplete, but not entirely absent. Given this, an approach that is intermediate between the two extremes outlined above seems most relevant for real-world applications. A sophisticated ethical agent should still use those markets that do exist to redistribute the marketed part of her endowment to satisfy her preferences. In general, this redistribution will alter marginal rates of substitution, even in non-marketed states. Once trade is accounted for the ‘naïve’ formula for $\rho_{t,s}$ in (3) will no longer be appropriate. In addition the market based discount rates in (2) will also no longer apply to non-marketed states. We demonstrate this initially assuming that the planner’s entire marketed endowment is tradable, but examine the case where only a portion of the endowment is tradable in Section 2.2 below.

Assume that a subset $\Omega$ of states are marketed, i.e. $(t, s) \in \Omega$ iff there is a market price for consumption in state $(t, s)$. We assume that $|\Omega| \geq 2$, and that there is at least one non-marketed state. We also assume that the state at $t = 0$ is marketed. This assumption is not essential, but helps with the interpretation of the model (it clearly holds in all practical applications). The planner’s indirect utility from her endowment $\omega$ is then:

$$W(\omega) = \max_{c} U(c) \text{ s.t. } \sum_{(t,s) \in \Omega} p_{t,s} c_{t,s} = \sum_{(t,s) \in \Omega} p_{t,s} \omega_{t,s}$$

$$\forall (t, s) \notin \Omega, \quad c_{t,s} = \omega_{t,s}.$$  

Let $\mu > 0$ be the planner’s marginal utility of market income in this incomplete markets setting, and let $c^* = c^*(\omega)$ be the planner’s optimal consumption bundle after trading $\omega$ on the incomplete markets. Since $\partial U(c)/\partial \omega_{t,s} = 0$ for $(t, s) \in \Omega$, and the budget constraint is
independent of $\omega_{t,s}$ for $(t, s) \notin \Omega$, the envelope theorem yields

$$
\frac{\partial W}{\partial \omega_{t,s}} = \mu_{t,s} \quad (t, s) \in \Omega \\
\frac{\partial W}{\partial \omega_{t,s}} = \left. \frac{\partial U}{\partial c_{t,s}} \right|_{c^*} \quad (t, s) \notin \Omega.
$$

(4)

Note that for $(t, s) \notin \Omega$, $c^*_{t,s} = \omega_{t,s}$.

We can thus define the appropriate cost benefit rule in an incomplete market setting as

$$
W(\omega + \pi) > W(\omega) \iff \sum_{(t,s)\in\Omega} p_{t,s} \pi_{t,s} + \frac{1}{\mu} \sum_{(t,s)\notin\Omega} \left. \frac{\partial U}{\partial c_{t,s}} \right|_{c^*} \pi_{t,s} > 0,
$$

(5)

where $\pi$ is assumed small relative to $\omega$.

We can write this condition in terms of the market interest rates $r_{t,s}$, and an incomplete markets consumption discount rate $\delta_{t,s}$ defined for $(t, s) \notin \Omega$ through:

$$
(1 + \delta_{t,s})^{-t} = \frac{1}{\mu} \left. \frac{\partial U}{\partial c_{t,s}} \right|_{c^*}.
$$

(6)

The project improves welfare iff:

$$
\sum_{(t,s)\in\Omega} (1 + r_{t,s})^{-t} \pi_{t,s} + \sum_{(t,s)\notin\Omega} (1 + \delta_{t,s})^{-t} \pi_{t,s} > 0.
$$

(7)

For those states where a market exists, the appropriate discount rate is still the associated market rate. This simply reflects the fact that consumption in a marketed state sustains an opportunity cost that is priced by the market (captured by the planner’s budget constraint). Notice that this finding did not require us to assume that the market is at an efficient or ethically optimal equilibrium. Provided that the planner is concerned with marginal projects, and that a competitive equilibrium of the economy exists, she should still use market prices to discount marginal payoffs in marketed states. Although competitive equilibria are not (constrained) Pareto efficient when markets are incomplete, they do generically exist (Geanakoplos & Polemarchakis, 1986). That is all that is required for the cost-benefit rule in (7) to apply. Although we have derived this result using the preferences of a single ethical planner, it applies just as well in the more classical setting where the planner evaluates the effects of the project on all currently living consumers. This follows since in marketed states all (utility maximising) consumers’ marginal rates of substitution

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6Note that the restriction to marginal projects defines cost-benefit analysis.
will be equal to market prices in equilibrium.\footnote{The case where the planner places non-zero weight on consumers who are not yet born is just a special case of the general model with arbitrary planner preferences \( U(c) \).}

For those states that are not marketed the opportunity cost of consumption is not priced by the market. Nevertheless, the planner’s trading opportunities still affect discount rates in these states, as they determine the optimal consumption bundle \( c^* \) and the marginal utility of market income \( \mu \). These quantities depend on both preferences and market conditions.

By comparing equation (3) to equation (6) we can define a measure of the fractional bias in the naïve consumption discount factor. For \((t, s) \notin \Omega\), define

\[
\Delta_{t,s} = \frac{\frac{\partial U}{\partial c_{t,s}} \bigg| \omega}{\frac{1}{\mu} \frac{\partial U}{\partial c_{t,s}} \bigg| c^*}.
\]

This quantity – the ratio of naïve to sophisticated discount factors – is greater (smaller) than 1 when the naïve cost-benefit rule over (under) estimates the value of the non-marketed state \((t, s)\). Crucially, the naïve discount factor (the numerator of \(\Delta_{t,s}\)) is computed at the pre-trade consumption bundle \(\omega\), while the sophisticated discount factor (the denominator of \(\Delta_{t,s}\)) is computed at the post-trade bundle \(c^*\), and also accounts for the tradability of initial consumption through the marginal utility of market income \(\mu\).

Similarly, the absolute bias in naïve discount rates is defined as

\[
\Delta^r_{t,s} = \left( \frac{\frac{\partial U}{\partial c_{t,s}} \bigg| \omega}{\frac{1}{\mu} \frac{\partial U}{\partial c_{t,s}} \bigg| c^*} \right)^{-\frac{1}{t}} - \left( \frac{1}{\mu} \frac{\partial U}{\partial c_{t,s}} \bigg| c^* \right)^{-\frac{1}{t}}.
\]

2 Applications and extensions

In this section I apply and extend the analysis above. I start by assuming that the planner’s preferences are additively separable – this is by far the most common assumption in applications of normative social discounting. I compute indicative estimates of the bias in naïve discount rates in some simple numerical examples. I then examine how the results change if the planner cannot trade her entire marketed endowment – such constraints are especially relevant for governments who control only a part of aggregate consumption. Finally, I examine an example that demonstrates that non-separability can have non-trivial
additional effects on discount rates in non-marketed states.

2.1 Additively separable preferences

To begin to estimate the bias factors $\Delta_{t,s}$ we first assume that preferences are additively separable. This significantly simplifies the analysis, and allows us to derive some non-parametric results, as well as some analytical results in special cases. The crucial simplification that additive separability delivers is that the marginal rate of substitution between current consumption and consumption in state $(t,s)$ does not depend on any intermediate state. This leads to the following simple observation:

**Proposition 1.** Suppose that $U(c)$ is additively separable across time and states, and that

$$\lim_{t \to \infty} \left( \frac{\partial U}{\partial c_{t,s}} \bigg|_{\omega} \right)^{-\frac{1}{\mu}}$$

exists for all $s$. Then for all $(t,s) \notin \Omega$,

(a) $\Delta_{t,s} = \frac{\mu}{\frac{\partial U}{\partial c_{0}} \bigg|_{\omega}}$

(b) $\lim_{t \to \infty} \Delta_{t,s} = 0$.

Part (a) of this observation is intuitive. If the marginal utility of income $\mu$ is greater than the marginal utility of consumption at the initial allocation, a small project that provides a marginal increase in marketed income is more valuable than a small change in initial consumption $\omega_0$. Thus the denominator of the pure consumption discount factor underestimates the marketed benefits of the project, and over estimates its non-marketed benefits (i.e., discount factors in non-marketed states are biased upwards). The opposite case obtains when $\mu < \frac{\partial U}{\partial c_{0}} \bigg|_{\omega}$.

Part (b) shows that when preferences are additively separable naïve discount rates are asymptotically unbiased. This is an immediate consequence of the fact that naïve discount factors are biased by a constant factor in each non-marketed state. A partial converse of part (b) of this result also holds: if $U(c)$ is not additively separable naïve consumption discount rates are biased in all states and at all maturities. This is generically true, except in some artificial cases where coincidentally it is the case that $\Delta_{t,s} = 0$ for a specific pair $(t,s)$.

To illustrate the quantitative implications of neglecting market trading opportunities we can work with an analytically tractable model in which the planner’s preferences are
also homothetic:  
\[ U(c) = \sum_{t,s} m_{t,s} \frac{c_{t,s}^{1-\eta}}{1-\eta}, \]  
(10)

where \( \eta \geq 0 \) is the elasticity of marginal utility. In this case we have the following result:

\textbf{Proposition 2.} Assume that preferences are given by (10), and that \( \Omega \) is the set of states that are priced by the market. Then for any \((t, s) \notin \Omega\),

\[ \Delta_{t,s} = \left[ \frac{\sum_{(t,s) \in \Omega} p_{t,s} \left( \frac{m_{t,s}}{p_{t,s}} \right)^{1/\eta}}{\sum_{(t,s) \in \Omega} p_{t,s} \omega_{t,s}} \right]^\eta. \]  
(11)

We can get some intuition for \( \Delta_{t,s} \) in this case by considering a deterministic model in which there is only one state of the world.  

Assume in addition that  
\[ p_t = (1 + r)^{-t}, \]
\[ m_t = (1 + \rho)^{-t}, \]
\[ \omega_t = (1 + g)^t, \]

and that markets exist up to time \( T - 1 < t_{\text{max}} \), i.e., \( \Omega = \{0, \ldots, T-1\} \). Then some simple calculations show that for all \( t \notin \Omega \),

\[ \Delta_t = \Delta(T) = \left[ \frac{\left(1 + \rho\right)^{1/\eta}(1 + r)^{\frac{\eta-1}{\eta}}\left[1 - 1 \right]^{-T} - 1}{\left[1 + r\right](1 + g)^{-T} - 1} \right]^\eta, \]  
(12)

where we have dropped the redundant time index, and made the dependence on \( T \), the time horizon of the market, explicit. This expression becomes more intuitive in the continuous time limit.  

\[ \Delta(T) = \left[ \frac{\eta(r-g)}{\rho + (\eta-1)r} \frac{e^{-(\rho+(\eta-1)r)T}}{e^{-(r-g)T} - 1} \right]^\eta. \]  
(13)

\footnote{A special case of this model is the discounted expected utility model with an iso-elastic utility function, i.e. \( m_{t,s} = (1+\rho)^{-t}q_{t,s} \), where \( \rho \geq 0 \) is the pure rate of time preference and \( q_{t,s} \) is the subjective probability of state \( s \) at time \( t \).}

\footnote{Although this is an extreme assumption, note that government discounting policy in the United Kingdom and elsewhere is often based on deterministic versions of the Ramsey rule – see the discussion below.}

\footnote{In this limit we send \( \rho \rightarrow \rho\Delta t, r \rightarrow r\Delta t, g \rightarrow g\Delta t, T \rightarrow T/\Delta t, \) and take the limit as \( \Delta t \to 0 \).}
Observe that

\begin{align*}
\lim_{T \to 0} \Delta(T) &= 1 \\
\frac{d\Delta(T)}{dT} &\geq 0 \text{ if } r \geq \rho + \eta g. \\
r = \rho + \eta g &\Rightarrow \forall T, \Delta(T) = 1.
\end{align*}

Thus the naïve cost-benefit rule is unbiased if

\[ \rho + \eta g = r. \]  \tag{14}

This relationship is very familiar; it is the condition for an intertemporal optimum in a Ramsey-style growth model. The left hand side is the Ramsey rule for the consumption discount rate at the initial allocation. The right hand side is the market interest rate. Equality of these two quantities is equivalent to saying that the initial endowment is allocated efficiently, given market interest rates. If this is the case there is no bias in the ‘naïve’ cost-benefit rule, regardless of the extent of market incompleteness. Deviations from (14) indicate that bias will be present in the naïve rule. The magnitude of bias is controlled by the quantity

\[ \frac{1}{\eta} |\rho + \eta g - r|. \]

We can illustrate the magnitude of \( \Delta(T) \) by calibrating the model to existing studies of normative discounting. Consider the baseline specification of Gollier & Hammitt (2014). These authors state that ‘the positive approach cannot be applied for time horizons exceeding 20 or 30 years, because there are no safe assets traded on markets with such large maturities.’ For large maturities they advocate a normative approach based on the Ramsey rule, in which the (deterministic) consumption discount rate is given by \( \rho + \eta g \). In their preferred specification, \( \rho = 0, \eta = 2, g = 2\%/yr \). Substituting these parameter values, \( T = 30 \) yrs for the time horizon of the market, and \( r = 3\%/yr \) for the market risk free rate, into (13), we find that using the naïve consumption discount rate would underestimate the value of payoffs in non-marketed states (i.e., \( t > 30 \) yrs) by approximately 15% in this case. Figure 1 plots the naïve and sophisticated discount rates as a function of maturity for several values of \( r \), holding the other parameters in this example fixed. The figure demonstrates that an ethical investor or foundation who uses naïve normative discount
Figure 1: ‘Naive’ and ‘sophisticated’ discount rates for the parameter values in Gollier & Hammitt (2014): \((\rho, \eta, g, T) = (0, 2, 2\%/yr, 30\ yrs)\), and \(r \in \{1.5, 3, 5\}\%/yr\).

rates to evaluated projects could make highly inefficient investment choices.\(^{11}\)

2.2 Constraints on trade

The model we have studied thus far assumed that the planner is free to trade the marketed part of her endowment in an unrestricted manner. This is a fair assumption for an ethically minded investor or foundation, but perhaps less so for a government (due to debt ceilings, the distortionary effects of taxation, etc.). Governments may face economic, legal, or political constraints on their ability to buy and sell assets, either public or private, on the market. In this section I show that such concerns may change some of the quantitative

\(^{11}\)Although Gollier & Hammitt (2014) focus on applications to government discounting policy, we may still apply their calibration to the case of an ethical investor or foundation whose endowment is fully tradable. In effect, by calibrating the investor’s consumption endowment to these authors’ estimates of aggregate consumption growth, we assume that the investor’s consumption endowment is perfectly correlated with economic growth.
details of the preceding analysis, but do not change the core insights.

Suppose that the planner may only trade a fraction \( \sigma_{t,s} \in [0,1] \) of her endowment in state \((t,s)\). We assume that \( \sigma_{t,s} \) is sufficiently small that any trades the government makes may be considered inframarginal relative to the level of taxation in the economy, i.e., only assets (real or financial) that are already under public control are traded. For non-marketed states \((t,s) \notin \Omega\), \( \sigma_{t,s} = 0 \). Let \( \circ \) denote the Hadamard (i.e., element-wise) product operator. Then the planner’s optimisation problem is

\[
\max_{c \geq 0} U((1 - \sigma) \circ \omega + c) \quad \text{s.t.} \quad \sum_{(t,s) \in \Omega} p_{t,s}(\sigma_{t,s} \omega_{t,s}) = \sum_{(t,s) \in \Omega} p_{t,s}c_{t,s}, \quad (t,s) \notin \Omega \Rightarrow c_{t,s} = 0.
\]

For \((t,s) \in \Omega\), define

\[
x_{t,s} = (1 - \sigma_{t,s})\omega_{t,s} + c_{t,s}.
\]

Then this problem can be seen to be equivalent to

\[
Z(\omega) = \max_{x} U(x) \quad \text{s.t.} \quad \sum_{(t,s) \in \Omega} p_{t,s}x_{t,s} = \sum_{(t,s) \in \Omega} p_{t,s}\omega_{t,s},
\]

\[
\forall (t,s) \in \Omega, (1 - \sigma_{t,s})\omega_{t,s} - x_{t,s} \leq 0
\]

\[
\forall (t,s) \notin \Omega, x_{t,s} = \omega_{t,s}.
\]

There is thus one additional set of constraints in this optimization problem. For \((t,s) \in \Omega\) we have

\[
\frac{\partial Z}{\partial \omega_{t,s}} = \mu p_{t,s} + \nu_{t,s}(1 - \sigma_{t,s})
\]

where \( \nu_{t,s} \geq 0 \) is the Karush-Kuhn-Tucker multiplier associated with the inequality constraint in state \((t,s)\), and the complimentary slackness conditions are:

\[
\nu_{t,s}[(1 - \sigma_{t,s})\omega_{t,s} - x_{t,s}] = 0.
\]

If the \((t,s)\) inequality constraint binds then we have \( \nu_{t,s} > 0 \). In this case the effect of a small change in \( \omega_{t,s} \) on indirect utility can be evaluated as if the state \((t,s)\) were a combination of a marketed and non-marketed state. The formula for \( \frac{1}{\mu} \frac{\partial Z}{\partial \omega_{t,s}} \) in this case contains one term that depends on price \( (p_{t,s}) \), and another that depends on the value
of changing the constrained value of consumption in state \((t, s)\) \((\nu_{t,s}(1 - \sigma_{t,s})/\mu)\). The latter term is analogous to the equivalent term that arises in a non-marketed state. On the other hand, if the \((t, s)\) constraint does not bind, we have \(\nu_{t,s} = 0\), and state \((t, s)\) is evaluated as if it were fully tradable. Thus, constraints on trade can in effect be handled by taking linear combinations of market and non-market discount factors in states with binding constraints, and ignoring non-binding constraints.

Exactly which states’ constraints are binding depends on the empirical details. The results in Figure 1 are robust to trading constraints (i.e., none of the constraints is binding), provided that at least 10% of the endowment in each marketed state is tradable. Appendix C demonstrates in detail how the results in that figure change with tighter constraints on trade. For a more policy relevant application, consider the discounting schedule for public project evaluation in the United Kingdom’s ‘Green Book’ (HM Treasury, 2018).\(^{12}\) The UK uses the normative Ramsey rule to calibrate its social discount rate, with the preference parameters given by \(\rho = 1.5\%/yr\), \(\eta = 1\). The Green Book’s forward discount rates are piecewise constant, with an initial consumption growth rate of \(g = 2\%/yr\) that declines with maturity. The dashed blue curve in Figure 2 displays the equivalent spot discount rates. The dash-dotted red curve in Figure 2 displays the real yield curve for UK government debt on 1st November 2019.\(^{13}\) The Bank of England computes yields up to maturities of 40 years, so we take \(T = 40\) yrs.\(^{14}\) We can use the formalism above, the yield curve, and the values of \((\rho, \eta)\) from the Green Book to compute the ‘sophisticated’ normative discount rates, accounting for trading constraints. These are displayed in the solid curves in Figure 2 for values of \(\sigma\) ranging from 0.01% to 10%. This figure suggests that, as of 1st November 2019, project appraisal based on the Green Book’s discounting schedule is potentially highly myopic (i.e. discount rates are too high) relative to what would be optimal if even limited trading opportunities were exploited.

\(^{12}\) We focus on the UK for two reasons. First, the Green Book takes an explicitly normative approach based on the Ramsey rule. Second, index-linked government bonds with fairly long maturities are liquidly traded in the UK, and the Bank of England makes estimates of the associated real yield curve available to the public.

\(^{13}\) Yield curve data are obtained from https://www.bankofengland.co.uk/statistics/yield-curves.

\(^{14}\) While some longer indexed-link government bonds are traded in the UK, they are less liquid, and default risk becomes non-negligible for very long maturities so these bonds cannot be considered risk free. Note that the bias factor \(|\Delta(T) - 1|\) is increasing in \(T\), so this analysis may be considered a lower bound on the correction to discount factors at non-marketed maturities.
2.3 Non-separable preferences

Thus far our analysis has focussed on the case where the planner’s preferences are additively separable. This is by far the most common case in existing applications of normative social discounting. The non-separable case is however still of interest. When preferences are non-separable market prices influence both the numerator and the denominator of discount factors in non-marketed states. A familiar case where non-separability arises is when the planner’s preferences are nonlinear in continuation values, for example if she has Epstein-Zin preferences (Epstein & Zin, 1989) or is ambiguity averse (Klibanoff et al., 2009). While it is in principle straightforward to extend the numerical examples above to account for
this kind of non-separability, calibrating such models poses some challenges. To calibrate
the model in the presence of uncertainty we need to be able to infer Arrow-Debreu state
prices for payoffs in different states of the world from observed asset prices. Although this
is a simple exercise in complete markets, state prices are non-unique when markets are
incomplete (see e.g. Cochrane, 2005). While detailed models of the asset classes in the
economy could provide bounds on state prices, such an analysis would take us a long way
from existing applications of normative social discounting.¹⁵

Instead of attempting such an exercise, I will simply demonstrate that non-separability
can have non-trivial effects in an example. Consider a two period Epstein-Zin model where
there are two states \((1, H)\) and \((1, L)\) in the second period, one of which, say \((1, L)\), is
non-marketed. Specifically, suppose that the planner’s indirect utility function is

\[
V(\omega_0, \omega_1, H, \omega_1, L) = \max_{c_0, c_1, H} \frac{1}{1 - \eta} \left[ c_0^{1-\eta} + \frac{1}{1 + \rho} (q c_1^{1-\alpha} H + (1 - q) c_1^{1-\alpha} L)^{1-\alpha} \right] \\
\text{s.t. } c_0 + \frac{1}{1 + r_H} c_1, H = \omega_0 + \frac{1}{1 + r_H} \omega_1, H, \\
c_1, L = \omega_1, L, \tag{16}
\]

where \(\eta\) is the inverse of the elasticity of intertemporal substitution, \(\alpha\) is the coefficient of
relative risk aversion, \(q\) is the subjective probability of state \((1, H)\), and \(r_H\) is the market
rate of return in state \((1, H)\). If \(\eta = \alpha\) the preferences in (16) are additively separable; in
all other cases they are not. We can decompose the ‘sophisticated’ discount factor in state
\((1, L)\) as follows:

\[
\frac{1}{\mu} \frac{\partial U}{\partial c_{1,L}} \bigg|_{c^*} = \frac{\partial U}{\partial c_{1,L}} \bigg|_{\omega} \times \frac{\partial U}{\partial c_0} \bigg|_{\omega} \times \frac{\partial U}{\partial c_{1,L}} \bigg|_{c^*} \\
\text{Naïve discount factor} \quad \text{Separable adjustment} \quad \text{Non-separable adjustment}
\]

The second factor in this expression is the adjustment to the denominator of the naïve
discount factor that occurs even if preferences are separable. The third factor is a new
term that only differs from 1 if preferences are non-separable; it captures adjustments

¹⁵Almost all applications of social discounting I am aware of deal exclusively with risk free discount rates
and payoffs. France is something of an exception (Quinet, 2013). Giglio et al. (2015) estimate long run
discount rates for real estate by exploiting the structure of the housing market in the UK and Singapore.
Their work is however quite far from delivering Arrow-Debreu state prices. See Gollier (2019) for an
analysis of the welfare costs of ignoring the correlations between project payoffs and macroeconomic risks.
Figure 3: Illustration of the effect of non-separability on sophisticated discount rates in non-marketed states for the example in (16). Preference parameters are given by \((\rho, \eta) = (0, 2)\), and \(\alpha\), the coefficient of relative risk aversion, varies on the \(x\) axis. The period length is 40 years, and \(\omega_0 = 1\). In the marketed ‘high growth’ state the agent’s endowment is \(\omega_{1,H} = (1.02)^{40}\), while in the non-marketed ‘low growth’ state her endowment is \(\omega_{1,L} = (0.99)^{40}\). State \((1,H)\) occurs with probability \(q = 0.95\). The market rate of return in state \((1,H)\) is \(r_H = (1.03)^{40} - 1\).

to the numerator of the naïve discount factor. I plot the associated adjustments to the naïve discount rate in state \((1,L)\) in a calibrated example in Figure 3. The figure shows that for plausible parameter values the non-separable adjustment to the discount rate is of comparable magnitude to the separable adjustment.

3 Conclusion

This paper presented a simple unified model of positive and normative discounting in which an ethical agent trades part of her endowment on incomplete markets. The model shows that such an agent should use market rates to discount payoffs in marketed states, and that discount rates in non-marketed states depend on both ethics and prevailing market prices. Accounting for trading opportunities leads to quantitatively significant adjustments.
to normative discount rates in non-marketed states. Neglecting such adjustments leads to cost-benefit rules that are ethically inefficient, in the sense that they do not accurately reflect ethical agents’ stated objectives, and the means they have to pursue them through the markets.
Appendix

A Proof of Proposition 1

Since $U(c)$ is additively separable, we have

$$U(c) = \sum_{t,s} w_{t,s} u_{t,s}(c_{t,s})$$

and hence for $(t, s) \notin \Omega$,

$$\frac{\partial U}{\partial c_{t,s}} \bigg|_{\omega} = w_{t,s} u'_{t,s}(\omega_{t,s}) = \frac{\partial U}{\partial c_{t,s}} \bigg|_{c^*}.$$

Thus,

$$\Delta_{t,s} = \frac{w_{t,s} u'_{t,s}(\omega_{t,s})}{u'_{t,s}(\omega_{t,s})} \cdot \frac{1}{\mu} \mu w_{t,s} u'_{t,s}(\omega_{t,s})$$

For part (b) notice that

$$\lim_{t \to \infty} \Delta_{t,s}^r = \lim_{t \to \infty} \left( w_{t,s} u_{t,s}(\omega_{t,s}) \right)^{-1/t} \left[ \left( \frac{\partial U}{\partial c_0} \bigg|_{\omega} \right)^{\frac{1}{t}} - \left( \frac{\partial U}{\partial c_0} \bigg|_{c^*} \right)^{\frac{1}{t}} \right].$$

Since $\lim_{t \to \infty} \left( w_{t,s} u_{t,s}(\omega_{t,s}) \right)^{-1/t} = \lim_{t \to \infty} \left( \frac{\partial U}{\partial c_{t,s}} \bigg|_{\omega} \right)^{-\frac{1}{t}}$ exists by assumption, and

$$\lim_{t \to \infty} \left( \frac{\partial U}{\partial c_0} \bigg|_{\omega} \right)^{\frac{1}{t}} - \left( \frac{\partial U}{\partial c_0} \bigg|_{c^*} \right)^{\frac{1}{t}} = 0,$$

we find

$$\lim_{t \to \infty} \Delta_{t,s}^r = 0$$

for all $s$. 

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B Proof of Proposition 2

The optimisation problem is
\[
\max_{c_{t,s}(t,s)\in\Omega} \sum_{t,s} m_{t,s} \frac{c_{t,s}^{1-\eta}}{1-\eta} \quad \text{s.t.} \quad \sum_{(t,s)\in\Omega} p_{t,s}(\omega_{t,s} - c_{t,s}) = 0.
\]

Defining \(\mu\) to be the Lagrange multiplier for the constraint, the first order conditions for \((t, s) \in \Omega\) yield
\[
c_{t,s}^* = \left( \frac{\mu p_{t,s}}{m_{t,s}} \right)^{-1/\eta}.
\]

We can solve for \(\mu\) using the constraint \(\sum_{(t,s)\in\Omega} p_{t,s} c_{t,s}^* = \sum_{(t,s)\in\Omega} p_{t,s} \omega_{t,s}\) to find
\[
\mu = \left( \frac{\sum_{(t,s)\in\Omega} p_{t,s} \left( \frac{m_{t,s}}{p_{t,s}} \right)^{1/\eta}}{\sum_{t,s} p_{t,s} \omega_{t,s}} \right)^{\eta}.
\]

Thus for \((t, s) \not\in \Omega\) the fractional bias in discount factors is
\[
\Delta_{t,s} = \omega_0^{\eta} \left( \frac{\sum_{(t,s)\in\Omega} p_{t,s} \left( \frac{m_{t,s}}{p_{t,s}} \right)^{1/\eta}}{\sum_{t,s} p_{t,s} \omega_{t,s}} \right)^{\eta}.
\]

C Constraints on trade

In this section we demonstrate how the results in Figure 1 change when the agent’s ability to trade her endowment is constrained. In order to do this we solve the consumption allocation problem (15) numerically accounting for constraints on trade. Specifically, we assume that \(\sigma_{t,s}\), the share of the endowment in state \((t, s)\) that is tradable, is independent of \((t, s)\) and equal to \(\sigma\). We solve the problem repeatedly for several values of \(\sigma\). The results for \(r = 1.5\%/\text{yr}\) and \(r = 3\%/\text{yr}\) are qualitatively similar, so we just present results for \(r = 1.5\%/\text{yr}\) and \(r = 5\%/\text{yr}\). In all cases, none of the trading constraints is binding if \(\sigma > 10\%\).
Figure 4: Discount rates with trading constraints for the example in Figure 1.
References


