

# A Tale of Two Cities and a Giffen Good\*

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## Abstract

A scenario is provided in which a house in Eden Mills, Ontario is a Giffen good. The conditions derived in the example apply to other indivisible goods as well. *Journal of Economic Literature* classification number: D11.

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## 1 Introduction

This paper describes how indivisible goods can be Giffen goods.<sup>1</sup> The ideas are explicated by using the invented, but conceivably true story of Oliver. Oliver lives an exciting, expensive life-style in the big city of Toronto. He would gladly continue to do so, but realizes he cannot afford it given his current wealth. So he contacts his stock broker and asks him to construct a gamble that, if it pays off, will make him rich enough to stay in Toronto. If he loses the gamble he will abandon big city life and buy a house in his hometown, the village of Eden Mills. Oliver believes that living in Eden

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<sup>1</sup>This complements a previous paper (Garratt, 1997) that explains how the presence of an indivisible good can make a divisible good a Giffen good.

Mills is preferred when poor to living in Toronto. He is just about to tell his broker to invest his money when he hears that housing prices in Eden Mills have gone up. Oliver realizes that he will need more money to live in Eden Mills than he initially thought, so he asks his broker to change the terms of the gamble. In the new gamble, Oliver accepts a smaller probability of winning the money he needs to stay in Toronto in exchange for being richer if he loses.

The surprising aspect of Oliver's story is that when the price of housing in Eden Mills rises, the probability that Oliver buys a house there increases. Oliver's expected demand curve for housing in Eden Mills is upward sloping. If many people like Oliver undertake similar gambles, then an increase in the price of houses in Eden Mills leads to an observed increase in the number of houses purchased.

The possibility that housing sales in some small town could increase with price is not permitted by standard theory, in which each consumer's demand for an indivisible good is defined by his reservation price for the good. The logic is simple. For prices below his reservation price he demands the good and for prices above his reservation price he does not. There is no sense in which quantity demanded of the indivisible good can vary directly with price.

When considering the purchase of an indivisible good, however, consumers can sometimes increase utility *ex ante* by randomizing over final consumption bundles. They do so by undertaking gambles in money before consumption decisions are made. A feature of the optimum gamble is that the indivisible good is always purchased for one outcome of the gamble and not purchased for the other outcome. Thus a consumer's expected demand for the indivisible good is equal to the probability of one outcome of the gamble. The following analysis provides the conditions under which there will be a Giffen effect in a consumer's expected demand for an indivisible good.

## 2 Toronto or Eden Mills?

Oliver can live with his uncle in Toronto rent free, but to live in Eden Mills he must purchase a house at price  $p$ .<sup>2</sup> Oliver's utility over money and cities

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<sup>2</sup>The price  $p$  can be interpreted as the net cost of purchasing a house in Eden Mills if Oliver cannot live rent free in Toronto.

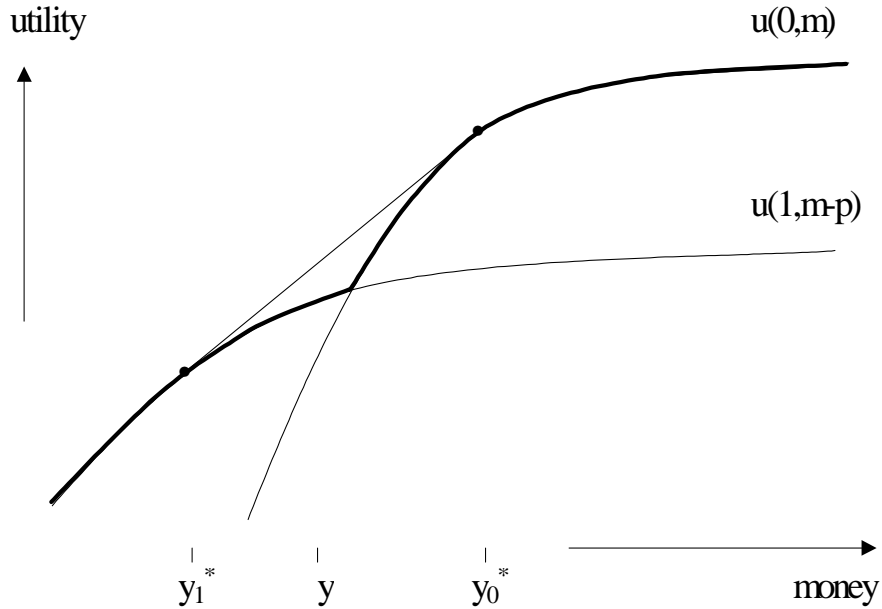


Figure 1: Indirect utility with and without gambles.

is given by  $u(e, m)$ , where  $m$  is money and  $e = 0$  or  $1$  denotes Toronto or Eden Mills (or equivalently, 0 houses in Eden Mills or 1 house in Eden Mills), respectively. Oliver's utility function  $u(e, \cdot)$  is continuous, strictly increasing and strictly concave in  $m$  for  $e = 0, 1$ . Without gambles Oliver chooses between living in Toronto with endowed income  $y$  and living in Eden Mills with income  $y - p$ , picking the alternative with the highest utility. Under the assumption that Oliver prefers living in Eden Mills when poor and Toronto when rich his indirect utility function looks like the thick curve in Figure 1.

It is easy to establish the existence of a reservation price for the house in Eden Mills,  $\hat{p}$ , such that below  $\hat{p}$  the house is purchased and above  $\hat{p}$  it is not. In cases where Oliver chooses to purchase the house at any affordable price, the reservation price is  $\hat{p} = y$ . Otherwise, it is defined as the price that

satisfies the equation<sup>3</sup>

$$u(0, y) = u(1, y - \hat{p}). \quad (1)$$

The existence of a reservation price implies the house in Eden Mills cannot be a Giffen good in the usual sense. However, having endowed income  $y$  shown in Figure 1, Oliver finds it advantageous to undertake a gamble in money prior to making his consumption decision.<sup>4</sup> Next, it is shown that the probability Oliver purchases a house in Eden Mills can vary directly with own price.

### 3 Expected demand for a house in Eden Mills

Oliver is a von Neumann-Morgenstern expected utility maximizer and thus chooses a gamble by selecting an income  $y_0$  that will allow him to stay in Toronto, an income  $y_1$  that will cause him to move to Eden Mills and buy a house, and a probability  $\pi$  of receiving the income  $y_1$ . The income  $y_0$  is received with probability  $(1 - \pi)$ . Actuarial fairness implies that

$$\pi = \frac{y - y_0}{y_1 - y_0}. \quad (2)$$

The optimization problem involves three choice variables and is written formally as

$$\max_{\pi, y_0, y_1} (1 - \pi)u(0, y_0) + \pi u(1, y_1 - p) \quad (3)$$

$$\text{subject to } (1 - \pi)y_0 + \pi y_1 = y \quad (4)$$

$$0 \leq \pi \leq 1 \quad (5)$$

$$y_0 \geq 0, y_1 \geq p. \quad (6)$$

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<sup>3</sup>The assumptions made so far on utility do not rule out a negative reservation price, i.e., a wealthy Oliver would have to be compensated to move to Eden Mills. This does not affect the claims made here. It is possible to construct examples similar to the one described here under the added assumption that at any income the consumer is always better off with the indivisible good than without it. This ensures a non-negative reservation price.

<sup>4</sup>Ng (1965) and Garratt and Marshall (1994) demonstrate the desire for gambles when the indivisible good is college education. See Bergstrom (1986) for the role of gambles in occupational choice.

The first order conditions for an interior solution are given by (4), together with

$$u'(0, y_0^*) = u'(1, y_1^* - p), \quad (7)$$

$$u(1, y_1^* - p) = u(0, y_0^*) + u'(0, y_0^*)[y_1^* - y_0^*]. \quad (8)$$

Despite the non-concavity of the objective function and the nonlinearity of the constraint (4), the values of  $y_0^*$ ,  $y_1^*$  and  $\pi^*$  that solve equations (4), (7) and (8) represent a global maximum.<sup>5</sup>

It is now possible to analyze how the optimum  $\pi$  responds to changes in  $p$ . Let  $R(0, y) = -u''(0, y)/u'(0, y)$  and  $R(1, y) = -u''(1, (y - p))/u'(1, y - p)$  denote the Arrow-Pratt measures of absolute risk aversion at points on each of the curves,  $u(0, y)$  and  $u(1, y - p)$  respectively. The implicit function theorem applied to (4), (7) and (8) yields,

$$\frac{\partial \pi^*}{\partial p} = - \left( \frac{1 - \pi^*}{R(0, y_0^*)(y_1^* - y_0^*)^2} + \frac{\pi^*}{R(1, y_1^*)(y_1^* - y_0^*)^2} \right) - \pi^* \frac{1}{(y_1^* - y_0^*)}. \quad (9)$$

Equation (9) shows the Slutsky decomposition of the effect of a price change.<sup>6</sup> For an interior solution  $\partial y_0^*/\partial y = \partial y_1^*/\partial y = 0$ . That is, a change in income within the nonconcave portion of the indirect utility curve causes Oliver to change the probabilities of the gamble but not the prizes. This is apparent from Figure 1 where it can be seen that the optimum gamble for different values of  $y$  involve the same incomes  $y_0^*$  and  $y_1^*$ . Only the probabilities of each outcome change, according to (2), as  $y$  changes. Thus, the term on the far right-hand-side of (9) is equal to  $\pi^*$  times  $\partial \pi^*/\partial y$ , which is the part of the Slutsky equation that identifies the income effect. The remaining term on the right-hand-side of (9) identifies the substitution effect.<sup>7</sup>

The substitution effect is always negative, and hence, for a Giffen effect in the probability of consuming a house in Eden Mills to be possible, the income

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<sup>5</sup>Verification of the second order conditions for similar problems is found in Marshall (1984) and Bergstrom (1986). Also see Garratt and Marshall (1994) for details.

<sup>6</sup>Hoy and Robson (1981) provide the Slutsky equation for the effect of a change in price on the demand for insurance. Upward sloping demand for insurance occurs when absolute risk aversion decreases rapidly enough to make the (negative) income effect larger than the substitution effect. In (9) the income effect does not explicitly depend on the degrees of absolute risk aversion. Moreover, a Giffen effect in the demand for an indivisible good can occur with constant absolute risk aversion (see the example below).

<sup>7</sup>This can also be verified by applying the implicit function theorem to the first-order conditions of the expenditure minimization problem.

effect on the probability of consuming the house must be negative. This is true if and only if the payoffs in the optimum gamble satisfy  $y_1^* < y_0^*$ . That is, the house in Eden Mills is consumed in the low income state. This happens in Figure 1 because we assumed Oliver preferred to live in Eden Mills when poor. But it can also happen if the indivisible good under consideration has the property that  $u(1, m) > u(0, m)$  for all  $m \geq 0$ .<sup>8</sup>

Cook and Graham (1977) define an indivisible good to be inferior (without gambles) if the consumer's reservation price (or *ransom*) is a decreasing function of income.<sup>9</sup> This requires that the marginal utility of money with wealth  $y - p$  and the indivisible good be less than the marginal utility of money with wealth  $y$  and no indivisible good. This is true for Oliver because apart from the annual writer's festival there is very little to do in Eden Mills. It is necessary to get the type of crossing shown in Figure 1, where the curve  $u(1, y - p)$  cuts the curve  $u(0, y)$  from above. Hence, inferiority of the indivisible good in the sense of Cook and Graham is necessary for there to be a Giffen effect in the expected demand for an indivisible good.

The section concludes by specifying differentiable utility functions for Oliver that produce an upward sloping expected demand curve for a house in Eden Mills. Let  $u(0, m) = -Ae^{-\alpha m} + B$  and  $u(1, m) = -e^{-\alpha m}$  where  $\alpha > 0$ ,  $A > 0$ ,  $B > 0$ , and  $\ln(A - B) > \alpha p$ . The conditions on the parameters ensure that the utility curves will intersect in the manner shown in Figure 1, and thus a gamble is desirable for some levels of endowed income. Moreover,  $R(0, m) = R(1, m) = \alpha$  for all  $m \geq 0$ , and hence by (2) and (9) it follows that for an interior solution we have

$$\frac{\partial \pi^*}{\partial p} = -\frac{1}{\alpha(y_1^* - y_0^*)^2} - \frac{y - y_0^*}{(y_1^* - y_0^*)^2}. \quad (10)$$

From the first-order equations,  $y_0^* = \frac{1}{\alpha} \ln \frac{A(\ln A - \alpha p)}{B}$  and  $y_1^* = \frac{1}{\alpha} \ln \left( \frac{\ln A - \alpha p}{B} \right) + p$ . Hence, for an interior solution  $\frac{\partial \pi^*}{\partial p} > 0$  if and only if

$$\ln \frac{A(\ln A - \alpha p)}{B} - \alpha y - 1 > 0. \quad (11)$$

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<sup>8</sup>I leave it to the reader to draw this case. Start with a utility curve  $u(1, m)$  that lies everywhere above the utility curve  $u(0, m)$ . A positive price  $p$  for the indivisible good shifts the upper curve to the right by the amount  $p$ . If  $u(1, m)$  exhibits sufficiently decreasing marginal utility of money, shifting it to the right will cause it to intersect the curve  $u(0, m)$  from above.

<sup>9</sup>Formally, an indivisible good is inferior if  $\partial \hat{p}(y) / \partial y < 0$  in equation (1).

For the purpose of illustration let  $A = 10$ ,  $\alpha = .91$ ,  $B = .1$ , and  $y = 4$ . These parameter values yield an interior solution and satisfy equation (11) for prices  $p \in [0, 1.39]$ . The expected demand over the range of prices that permits an interior solution is shown in Figure 2. At first,  $\pi^*$  increases with price (we observe a Giffen effect in Oliver's expected demand for a house in Eden Mills). But when the price gets larger  $\pi^*$  starts to fall towards zero. Eventually, the price gets large enough that Oliver desires zero probability on a house in Eden Mills.<sup>10</sup>

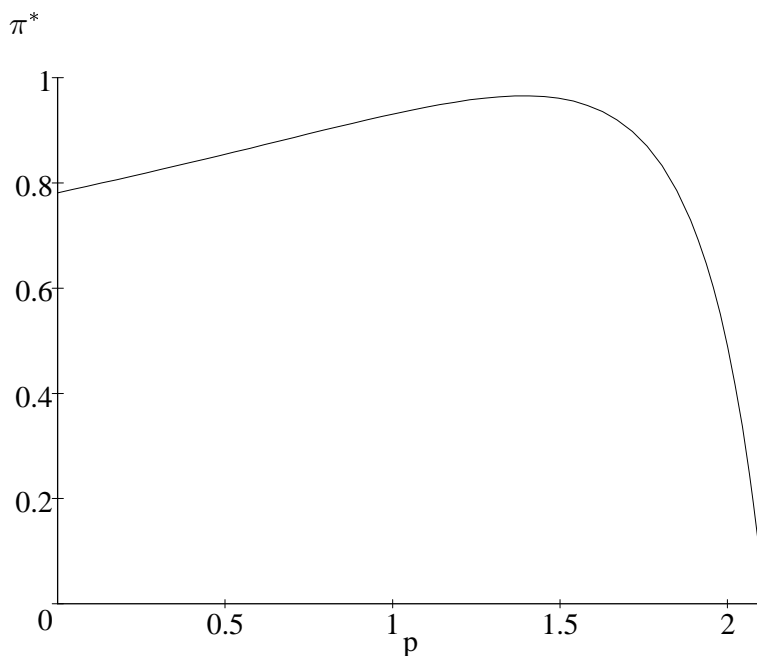


Figure 2: Expected demand for a house in Eden Mills.

## 4 Concluding Remarks

In the story that motivates this analysis Oliver lives in Toronto and has an upward sloping expected demand curve for a house in Eden Mills. Can

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<sup>10</sup>Recall an increase in price causes the curve  $u(1, m - p)$  in Figure 1 to slide to the right and  $y_0^*$  to move to the left. At the price  $p \simeq 2.1$  in this example  $y_0^* = y$  and the probability demanded on a house in Eden Mills is zero. For  $p > 2.1$  the consumer's problem has a corner solution at  $y_0^* = y$ ,  $y_1^* = 0$ , and  $\pi^* = 0$ .

things work in the opposite direction? Oliver's sister Olivia has preferences that are quite similar to Oliver's, but she is currently living a sleepy, slow-paced life in Eden Mills. She dreams of moving to Toronto and living a more exciting life-style but knows that she would need to be richer to make the move worthwhile. Is it possible that an increase in the price of apartments in Toronto would cause her to undertake a gamble with a greater chance of moving there? The answer is no. Since, like Oliver, she prefers to live in Eden Mills when poor and Toronto when rich she views housing in Toronto as a normal good (her reservation price is an increasing function of wealth), and hence her expected demand is downward sloping.

What sort of indivisible goods are likely to be Giffen goods? The paper identifies the key features. First, the good must be an important enough expenditure for consumers to arrange state contingent plans for its consumption. This suggests a large, expensive indivisible good. Second, possession of the good must lower the marginal utility of wealth over some range. Third, consumers must be willing to buy the good at low levels of wealth. All these factors contribute to making the good something the consumer wants when he loses the gamble, essentially a consolation prize. Interestingly, these features are not very different from what we know from standard price-theory analysis. Namely, we should look for Giffen goods among inferior goods (bought at a low income level) with a high income share (large goods). However, the standard 'divisible-good' discussion depends on little substitution (a small substitution effect). In contrast, the allocations considered here involve either the indivisible good and little money or no indivisible good and a lot of money, which suggests good substitutability. Examples meeting these criteria depend on individual preferences. Oliver's story is one possibility.

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