Global Warming and the Population Externality

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Abstract

We calculate the harm a birth imposes on others when greenhouse gas emissions are a problem and a cap limits emissions damage. This negative population externality, which equals the corrective Pigovian tax on having a child, is substantial in calibrations. In our base case, the Pigovian tax is 21 percent of a parent's lifetime income in steady state and 5 percent of lifetime income immediately after imposition of a cap, per child. The optimal population in steady state, which maximizes utility taking account of the externality, is about one quarter of the population households would choose voluntarily.

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We study a simple idea: if greenhouse gas emissions are a problem, then so is population. Specifically, we quantify the harm a birth imposes on others (the *population externality*). That population matters is not novel; a novelty is to put numbers on how much it matters if total emissions are a problem.

The mechanism of the population externality depends on whether policy restricts emissions. Because the externality may be very large if emissions are a serious problem but are left unrestricted,[1] we study a cap. This avoids exaggerating the population externality. Under a binding cap, total emissions are constant and equal per-person emissions times population, so a marginal birth requires lower per-person emissions. This reduces everyone else's living standards.[2]

A consequence is that the *optimal population*, which maximizes the utility of the representative household taking account of the externality, is lower than the *natural population,* which is the population households would choose without any population policy to balance the externality.

We use a balanced-growth setting in which output is produced from labor and greenhouse gas emissions. To avoid assuming away part of the emissions problem, we assume factor productivities grow exogenously at constant rates as in Solow-

[1]Without a binding cap, a birth directly means more emissions and emissions damage borne by others. Calculating the population externality then requires taking a stand on the economic damage from marginal emissions. Marginal-damage assumptions are unnecessary under a cap.

The literature closest to the current paper is David Kelly and Charles Kolstad (2001), who study the case without a cap. They argue that the assumptions in integrated assessment models that productivity and population growth fall exogenously to zero are empirically unrealistic and reduce the emissions problem. They also calculate of the welfare cost of a marginal birth assuming productivity and population growth fall exogenously to zero.

[2]This logic is general: if the true state of the world is that greenhouse gas emissions cause no external damage and a cap is nonetheless imposed, then a population externality is still induced.

type models. Population and hence labor are determined endogenously by dynastic households with Barro-Becker (1988, 1989) preferences who choose fertility optimally.

To model the arrival of knowledge about global warming, we divide time into two stylized eras. The first is an "exponential-growth era" in which the possibility of global warming is unrecognized, emissions are unrestricted, and population and emissions grow exponentially. The second is a "cap era" in which people know emissions might do damage and government imposes a cap that eventually binds. The cap era is Malthusian in that population growth reduces incomes which in turn restrains population growth, but is non-Malthusian in that living standards rise over time because of exogenous productivity growth.

We assume the transition between eras occurs at an instant, which sidesteps learning. We calibrate the model to the exponential-growth era and study the natural and optimal populations in the cap era, calculating the sequence of Pigovian taxes on having a child (*optimal child taxes*). These taxes measure the size of the population externality as well as the policy incentives needed to get households voluntarily to choose the optimal population. Both the population externality and the difference between the natural and optimal populations are large in calibrations. Thus if emissions are a serious enough problem so a cap is warranted, then too many people is also a serious problem.

The optimal child policy in the model is at odds with current tax, welfare, and schooling policies that subsidize children. A policy of discouraging fertility is also at odds with calls to encourage population growth in order to maintain the solvency of public pension systems such as social security.

The finding of negative population externalities contrasts with findings in new growth theory (e.g. Michael Kremer, 1993; Charles Jones, 1999) that human capital may generate scale or spillover effects, which amount to positive population externalities. It is an open question whether the sum of all population externalities is positive or negative.

Our analysis follows Jon Harford (1998), who shows that when fertility is endogenous and people do things that generate negative externalities, then efficiency requires optimal child taxes in addition to taxes on the underlying externality. In our case, a cap is equivalent to a tax on emissions.

We model the essential details of a population externality under a cap in section I. Section II describes the natural population in the exponential-growth era. Section III describes the natural population in the cap era. Section IV describes the optimal population and child taxes in the cap era. In section V, we extend the model by adding time costs of children, a more general technology, and exogenously-given productivity growth, which are important for calibrations. Section VI contains calibrations. Proofs are in an unpublished appendix, to be available online.

I. SETTING

Denote the time-t adult population by N_t and aggregate labor by $L_t = l_t N_t$ where l_t is per-capita labor. (Per-capita means per-adult.) A representative firm produces output Y_t under perfect competition from labor and E_t units of greenhouse gas emissions according to $Y_t = F(L_t, E_t)(1 - \delta_t)$, where F captures the productivities of labor and emissions as inputs and δ_t is the share of output lost (damage) from global warming at t. We add exogenously-growing factor productivity in section V.

We assume F has constant returns so $F(L, E) = Lf(e)$, where $e_t \equiv E_t/L_t$ is the *emissions ratio* and f is output per unit of labor. We assume $f(0) = 0$ for now but relax this in section V. The marginal product of emissions must be driven to zero if emissions are unrestricted so we assume there is a positive value $e^+ < \infty$ at which $f'(e^+) = 0$ with $f'(e_t) > 0$ and $f''(e_t) < 0$ for $0 \leq e_t < e^+$, as in figure 1.[3] Thus the emissions ratio is constant at e^+ in the exponential-growth era and input growth is

[3]Production resembles production in the Solow model, with E replacing K.

balanced: $E = e^{+}L$ grows at the same rate as L. In the cap era with a binding cap, on the other hand, total emissions are constant so growth in population and hence labor causes e to decline, driving down output per unit of labor as indicated by the arrows in the figure. In this way, a binding cap introduces a Malthusian force.

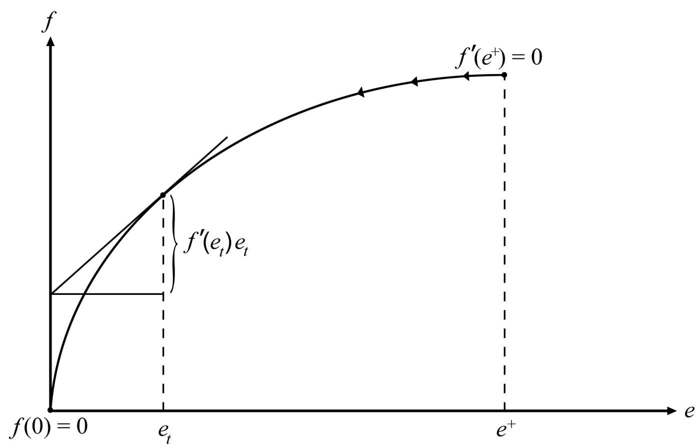

Figure 1. Input Distortion, and Geometry of the Real Population Externality

Although knowledge of how marginal damage depends on emissions is needed to determine the optimal sequence of caps, it is not needed to determine the population externality if the caps keep emissions low enough to avoid damage. We assume this: the cap $\hat{E} < \infty$ is constant over time and holds damage to zero so δ drops and

$$Y_t = F(L_t, E_t) = L_t f(e_t) \tag{1}$$

in both eras. The cap $\hat{E}$ may be optimal or suboptimal.[4]

[4]A constant cap $\hat{E}$ is optimal if damage is low up to $\hat{E}$ and rises sharply enough when emissions exceed $\hat{E}$. A constant cap eliminates interactions between changes in the cap and the population externality. The assumptions that $\hat{E}$ is constant and that $\delta_t = 0$ when a cap is in place could be generalized by adding environmental state variables such as air temperatures and specifying a process in which damage results from changes in the environmental state variables, which themselves

Starting in a period indexed $t = 0$, government implements a cap by creating and auctioning $\hat{E}$ permits each period, each permit allowing one unit of emissions in the period so $E_t \leq \hat{E}$ for $t \geq 0$. Treating permits as valid for a single period is in line with U.S. legislative proposals that state that permits are not property rights and that nothing restricts future government from terminating or limiting an emission allowance.[5]

In the market for emissions permits, the government is the supplier and the representative firm is the demander. The firm maximizes profits $L_t f(e_t) - p_t E_t - w_t L_t$, where p_t is the price of permits and w_t is the wage. The first-order conditions are $p_t = f'(e_t)$ and $w_t = w(e_t) \equiv f(e_t) - e_t f'(e_t)$.

The quantity of permits demanded at $p_t = 0$ is $e^+ L_t$. If $e^+ L_t < \hat{E}$, the cap does not bind, $p_t = 0$, and $e_t = e^+$. If $e^+ L_t > \hat{E}$, the cap binds, $p_t > 0$, and $e_t = \hat{E}/L_t < e^+$. (If $e^+ L_t = \hat{E}$, then $p_t = 0$ and $e_t = e^+$.) Compactly, the emissions ratio is $e(L_t) \equiv \min(e^+, \hat{E}/L_t)$ for any L_t.

Because $L_t = l_t N_t$, a cap means the emissions ratio depends on population:

$$e_t = e(l_t N_t) = \min(e^+, \frac{\hat{E}}{l_t N_t}). \tag{2}$$

To highlight the dependence, we assume for now that per-capita labor is fixed and normalized to one. Then labor equals population, per-capita emissions equal $e(N_t)$, and per-capita output equals $f(e(N_t))$.

depend on current and past emissions. The optimal cap in the transition to steady state would then generally change over time and the magnitude of the changes would depend on detailed properties of the damage process, about which knowledge is imperfect.

[5]We show in the appendix that the population externality would be internalized by parents if current government can and does establish iron-clad permanent *private* property rights to the *public* revenue stream from all permits to $t = \infty$. This is an interesting idea but is difficult to achieve in practice. Permanence fails if government later changes policy to expropriate the "permanent" rights. The U.S. legislative proposals acknowledge that this cannot be prevented.

Population in turn depends on fertility $n_t \geq 0$. A large number of representative dynastic households each contain a single adult who chooses n_t continuously to maximize utility. When all households choose n_t, the population growth factor is also n_t, that is, $N_{t+1} = n_t N_t$.

Adults trade off own consumption and number of children. An adult's consumption $c_t \geq 0$ is per-capita income y_t less the output cost of having and raising children to adulthood, so $c_t = y_t - \chi n_t$ where χ is the output cost of a child. Maximum feasible fertility is y_t/χ.

We follow Robert Barro and Gary Becker's (1988, 1989) specification of household preferences. A period-t adult's utility U_t is the sum of utility u from own consumption plus utility from children:

$$U_t = u(c_t) + \beta(n_t)U_{t+1}, \tag{3}$$

where children are identical and utility from children is the utility of a child times a weight β that depends on the number of children.

We assume power utility with parameter $\theta > 0$:

$$u(c) = \frac{1}{1-\theta}c^{1-\theta}; \tag{4}$$

the power form is needed later to allow for balanced growth. We also assume β is a power function with parameters $b_0 > 0$ and $b > 0$:[6]

$$\beta(n) = b_0 n^{1-b}. \tag{5}$$

In Barro and Becker's original specification, u is positive so $\theta < 1$, and β is increasing and concave. These ensure that parent's utility rises at a decreasing rate with the number of children. Larry Jones and Alice Schoonbroodt (2007) and Jones et al

[6]Power β has a sensible property: it is equivalent to assuming the utility an adult derives from grandchildren, $\beta(n_t)\beta(n_{t+1})U_{t+2}$, is independent of the number of children—see appendix.

(2008) show that parent's utility also rises at a decreasing rate with the number of children if utility is negative so $\theta > 1$, as long as β is decreasing and convex; they argue this case may better explain historical fertility trends. We therefore consider two cases: a Barro-Becker case with $\theta < 1$ and $b < 1$, and a Jones-Schoonbroodt case with $\theta > 1$ and $b > 1$.[7]

From (3) - (5), the contribution of children's consumption to parent's utility is $\beta(n_t)u(c_{t+1}) = \frac{b_0}{\omega(1-b)}(n_t c_{t+1}^{\omega})^{1-b}$ where $\omega \equiv \frac{1-\theta}{1-b}$ is the weight a parent places on per-child consumption relative to the number of children. Equal curvatures ($\theta = b$, so $\omega = 1$) mean the contribution depends on children's aggregate consumption, $n_t c_{t+1}$. We do not rule out ω greater or less than one, but $\omega \approx 1$ seems reasonable because children's aggregate consumption may be an economic resource for the parent. Values far less than one (parents care little about their children's consumption relative to the number of children), on the other hand, may be difficult to square with small families in which parents devote substantial resources to ensuring children's consumptions.

In choosing fertility, a household takes its income as well as the incomes and fertilities of future generations as given. The latter determine the utility of children. Generically (dropping time subscripts), the household maximizes

$$V(n, y, U) \equiv u(y - \chi n) + \beta(n)U$$

by choice of $n \in [0, y/\chi]$ given $y > 0$ and finite U, where $U > 0$ in the Barro-Becker case and $U < 0$ in the Jones-Schoonbroodt case.

The first-order condition balances the costs and benefits of children:[8]

$$V_n(n, y, U) = -u'\chi + \beta' U = 0. \tag{6}$$

[7]Utility (3) is infinite if discount factors given by β are too great. In the Barro-Becker case we assume $\beta < 1$ at maximum feasible fertility to ensure $\beta(n) < 1$ for all feasible n, so utility is finite. In the Jones-Schoonbroodt case, we assume $b_0 < 1$ so there is an $n^\circ < 1$ at which $\beta(n^\circ) = 1$ with $\beta(n) < 1$ for all $n > n^\circ$. This implies finite utility on paths with constant population.

[8]The second-order condition, $V_{nn} = u''\chi^2 + \beta'' U < 0$, holds by assumptions on primitives.

Because β and u are a power functions, the marginal value of children becomes infinite so $V_n \to \infty$ as $n \to 0$, and the marginal cost of children becomes infinite so $V_n(n, y, U) \to -\infty$ as $n \to y/\chi$ (that is, as $c \to 0$). Continuity of V_n then implies that for any finite $y > 0$ and finite U, there is a unique optimal fertility strictly between zero and y/χ.

Income and children's utility drive fertility. The partial elasticity of fertility with respect to income is

$$\varepsilon_{n,y} \equiv \frac{y}{n}\frac{\partial n}{\partial y} = -\frac{yV_{ny}}{nV_{nn}} = \left[\frac{b}{\theta} \cdot \frac{c}{y} + \frac{\chi n}{y}\right]^{-1}. \tag{7}$$

This elasticity is positive by assumptions on primitives. Note that child quality (Becker, 1960) is constant at this point so (7) does not capture changes in fertility associated with changes in child quality. We add an exogenous trend in child quality in section V.

The sign of β' determines how fertility changes with children's utility, because $\frac{\partial n}{\partial U} = \frac{\beta'}{-V_{nn}}$. In the Barro-Becker case, $\beta' > 0$, so fertility rises with U. In the Jones-Schoonbroodt case, $\beta' < 0$, so fertility falls with U.

We close the model by assuming government redistributes revenue from emissions auctions as equal lump sums to households. Per-capita income is therefore wages plus transfers, $TR_t = p_t E_t / N_t = p_t e_t$. From the firm's first-order conditions, per-capita income equals output per unit of labor: $y_t = w_t + p_t e_t = f(e_t)$.

The combination of a permit auction with revenue redistribution to households can represent a range of policies that restrict emissions. The combination is equivalent here to: (i) issuing and giving permits to households who then sell them at price p_t; (ii) issuing and giving permits to firms owned by households; and (iii) imposing a tax on emissions at rate f', which would just hold total emissions to $\hat{E}$, and redistributing the revenue to households.

In sum, the model describes an endogenous population-growth process. In the exponential-growth era, output per adult is $f(e^+)$, which leads to time paths of fertility and population. When a cap is imposed and population growth is positive, e_t and $f(e_t)$ eventually fall as depicted by the arrows in figure 1. This alters the time path of population. The exact path after a cap is imposed depends on how fertility responds to induced changes in income and children's utility.

II. POPULATION IN THE EXPONENTIAL-GROWTH ERA

A perfect-foresight solution in the exponential-growth era is a steady state with constant fertility and utility that solves the household's first-order condition. In any steady state, (3) implies

$$U = \frac{u(f(e) - \chi n)}{1 - \beta(n)}. \tag{8}$$

The steady-state relationship between emissions and natural fertility is found by substituting (8) into the first-order condition (6) to eliminate U:

$$S(n, e) \equiv -u'(f(e) - \chi n)\chi + \frac{\beta'(n)}{1 - \beta(n)} u(f(e) - \chi n) = 0. \tag{9}$$

With no cap, the emissions ratio is e^+. Steady-state fertility n^+ is the fertility that solves $S(n^+, e^+) = 0$. Such an n^+ exists and is unique.[9] Steady-state utility U^+ is the value of (8) at (n^+, e^+).

In general, n^+ may be greater or less than one. It is greater than one as long as child costs are not too great a fraction of output. To focus on equilibria in which

[9] As $n \to f(e^+)/\chi$, $u' \to \infty$, so $S(n, e^+) \to -\infty$. In the Barro-Becker case as $n \to 0$, $\beta' \to \infty$, so $S(n, e^+) \to \infty$. Because S is continuous, $S(n^+, e^+) = 0$ for some $n^+ \in (0, f(e^+)/\chi)$. In the Jones-Schoonbroodt case, $1/(1 - \beta(n)) \to \infty$ as $n \to n^\circ$ from above (where $\beta(n^\circ) = 1$), so $S(n, e^+) \to \infty$. Because S is continuous, $S(n^+, e^+) = 0$ for some $n^+ \in (n^\circ, f(e^+)/\chi)$. (For $n < n^\circ$, $\beta(n) > 1$, so $S < 0$.) From (9), $\frac{\partial S}{\partial n} = u''\chi^2 - \frac{\beta' u' \chi}{1-\beta} + \frac{\beta''(1-\beta) - (\beta')^2}{1-\beta} u$, which reduces to $u''\chi^2 + \beta'' u$ at n such that $S = 0$. Because $\beta'' < 0$ and $u > 0$ in the Barro-Becker case, and $\beta'' > 0$ and $u < 0$ in the Jones-Schoonbroodt case, $\frac{\partial S}{\partial n} < 0$. Hence S crosses zero only once.

population and emissions ($E_t = e^+ L_t$) grow, we assume

$$\chi < \phi f(e^+) \text{ where } \phi \equiv 1/\left(1 + \frac{(1-\theta)(1-b_0)}{(1-b)b_0}\right) < 1. \tag{10}$$

Equation (10) rearranges to $S(1, e^+) > 0$, which ensures $n^+ > 1$. Population and emissions then grow without bound at constant rate $n^+ - 1 > 0$.

III. NATURAL POPULATION IN THE CAP ERA

Population cannot grow without bound after a cap is imposed because this would eventually drive output per unit of labor $f(e)$ below χ so fertility would fall below replacement. Under a regularity condition described below, the natural population instead converges monotonically after a cap is imposed to a unique steady-state level N_{ss}. (Subscripts ss denote a variable's steady-state value.)

A perfect-foresight path in the cap era satisfies (3) and the household's first-order condition (6) for all t, which can be written as pair of first-order difference equations in $\{U_t, N_t\}_{t\geq 0}$:[10]

$$\begin{aligned} U_t &= u(f(e(N_t)) - \chi\frac{N_{t+1}}{N_t}) + \beta(\frac{N_{t+1}}{N_t}))U_{t+1}, & (11) \\ V_n(t) &\equiv \beta'\left(\frac{N_{t+1}}{N_t}\right)U_{t+1} - u'\left(f(e(N_t)) - \frac{N_{t+1}}{N_t}\chi\right)\chi = 0. & (12) \end{aligned}$$

Steady state is a pair (U_{ss}, N_{ss}) that satisfies (11) and (12) with $U_t = U_{t+1} = U_{ss}$ and $N_t = N_{t+1} = N_{ss}$. The latter implies that steady-state fertility equals replacement: $n_{ss} = 1$. In the cap era with $n_{ss} = 1$ and $e_{ss} = e(N_{ss})$, the steady-state condition is $S(1, e_{ss}) = 0$. We show in the appendix that $S(1, e)$ crosses zero exactly once on $[f^{-1}(\chi), e^+]$ so e_{ss} exists and is unique. Thus $N_{ss} = \hat{E}/e_{ss}$ exists and is unique. Because $S(1, e^+) > 0$, it must be that $e_{ss} < e^+$ so the cap binds in steady state.

[10]The system depends on $\hat{E}$ through e, but we suppress the dependence notationally except when considering how alternative values of $\hat{E}$ affect the economy.

If fertility is too sensitive to changes in population then a population increase from t to $t+1$ can reduce fertility so much that population decreases from $t+1$ to $t+2$. To rule out such "overshooting," we restrict the sensitivity of fertility to changes in population by assuming

$$\varepsilon_{n_t,y_t}\left(\frac{f'(e_t)e_t}{f(e_t)}\right) < 1, \tag{13}$$

at the steady state and at all t.[11] We show in the appendix that the system (11) and (12) is then saddle-path stable and that population converges monotonically to N_{ss} from any initial population $N_0 > 0$.

Fertility along the perfect-foresight natural path (*natural fertility,* η) is a function of population and the level of the cap, $n_t = \eta(N_t \mid \hat{E})$. Because $\hat{E}$ enters the model only through (2) as a determinant of e_t, natural fertility is homogeneous of degree zero: $\eta(N_t \mid \hat{E}) = \eta(\xi N_t \mid \xi\hat{E})$ where $\xi > 0$ is a constant. In words: natural fertility at population N_t under cap $\hat{E}$ equals fertility at population ξN_t under cap $\xi\hat{E}$, because both have the same e_t.

In the Barro-Becker case, natural fertility lies below n^+ and falls with N_t to the steady state at N_{ss}, as in figure 2a. To understand this, a cap has no effect on income as long as $N_t \leq \hat{E}/e^+$ so the cap does not bind, but reduces income once it binds. Reduced income in turn reduces fertility increasingly as N_t rises above $\hat{E}/e^+$. Because the cap eventually binds and utility is determined recursively, utility is less than U^+ as soon as a cap is imposed, which also acts to reduce fertility for all N_t in the Barro-Becker case.

In the Jones-Schoonbroodt case (figure 2b), natural fertility lies above n^+ and rises

[11] In (13), ε_{n_t,y_t} is the partial elasticity of fertility with respect to income (7) and the factor share $f'(e_t)e_t/f(e_t)$ is also the elasticity of income with respect to population. Note that (13) is not very strong. Because $f'(e)e < f(e)$, (13) holds if $\varepsilon_{n,y} \leq 1$,which holds in turn if $b \geq \theta$. On the other hand, if $\varepsilon_{n,y} > 1$ then fertility tends to fall off sharply as a declining emissions ratio reduces income so the factor share remains small, and (13) can still easily hold.

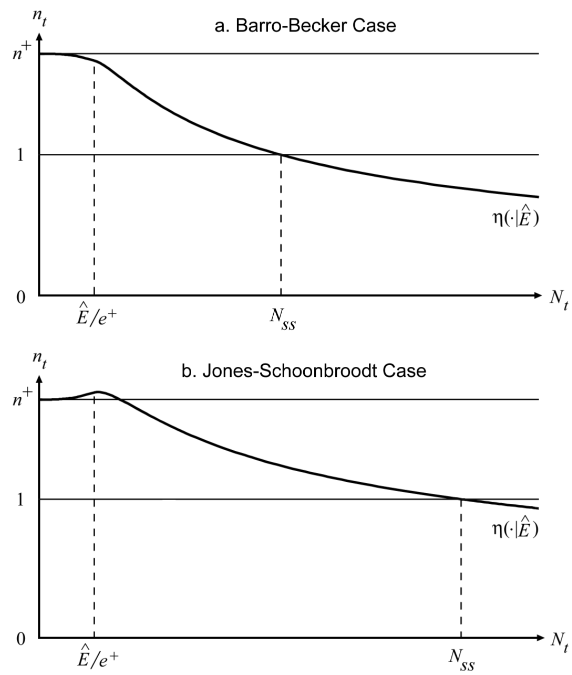

Figure 2. Natural Fertility

as long as the cap does not bind, then peaks and falls below n^+ as N_t becomes closer to N_{ss}. This reflects opposing income and utility effects. As in the Barro-Becker case, income declines as N_t rises above $\hat{E}/e^+$ and utility is less than U^+. In the Jones-Schoonbroodt case, however, reduced utility raises fertility. This lifts fertility above n^+ when a cap is imposed and causes fertility to rise with N_t for $N_t \leq \hat{E}/e^+$ and also slightly above $\hat{E}/e^+$. For N_t sufficiently close to N_{ss}, the income effect dominates and fertility lies below n^+. Because fertility first rises in the Jones-Schoonbroodt case, steady-state population tends to be greater in it than in the Barro-Becker case.

Population dynamics after a cap is imposed follow from the natural fertility function. Figure 3 illustrates in the Barro-Becker case. To minimize clutter, we choose units so $e^+ = 1$ and $N_0 = 1$. Consider a freeze, meaning a cap at the emissions level that would otherwise occur in the exponential-growth era at $t = 0$, which is $e^+ N_0 = 1$. Without a cap, fertility would be n^+ in period 0, at a. When the cap is imposed, fertility instead jumps down to $\eta(N_0 \mid 1) = \eta(1 \mid 1)$, at b. In period 1, the economy is therefore at c with population $N_1 = \eta(N_0 \mid 1)N_0 = \eta(N_0 \mid 1)$ and fertility $\eta(N_1 \mid 1)$. The economy then iterates down the fertility function and converges to population N_{ss} with fertility $\eta(N_{ss} \mid 1) = 1$.

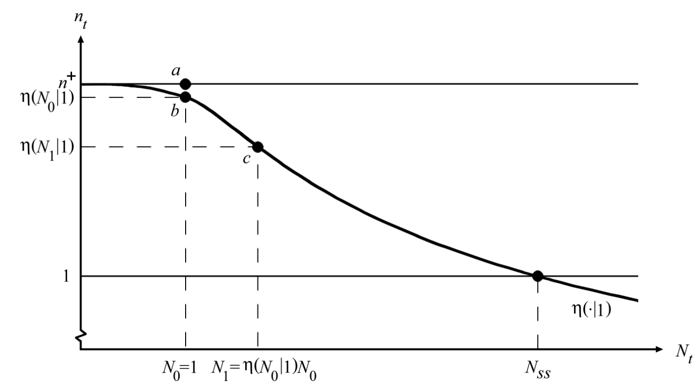

Figure 3. Population Dynamics under an Emissions Freeze

Natural fertility functions for caps other than a freeze can be derived from fertility for a freeze. A 25-percent cut, for instance, is $\hat{E} = 0.75$. Because η is homogeneous

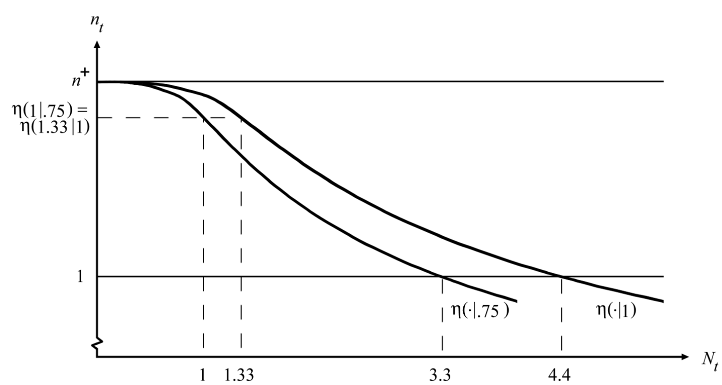

Figure 4. An Emissions Cut Versus a Freeze

of degree zero, $n_t = \eta(N_t \mid 0.75) = \eta(N_t/0.75 \mid 1)$, that is, fertility at population N_t and cap $\hat{E} = 0.75$ equals fertility at population $N_t/0.75$ and $\hat{E} = 1$. Graphically, this means the fertility function for a 25-percent cut, $\eta(\cdot \mid 0.75)$, is the function for a freeze,

$\eta(\cdot \mid 1)$, shifted 25 percent of the distance to the vertical axis, as in figure 4. (Because $N_{ss} = \hat{E}/e_{ss}$ is proportional to $\hat{E}$, steady-state population given a 25-percent cut is also 0.75 times steady-state population under a freeze.) Similarly, fertility functions for caps that do not immediately bind lie to the right of $\eta(\cdot \mid 1)$.

Any cap eventually binds and leads to the same steady-state emission ratio e_{ss}. The impact effect of imposing a binding cap is to reduce e_0 from e^+ to $\hat{E}/N_0$; subsequent dynamics take e_t the rest of the way to e_{ss}. The greater the value of $\hat{E}$ and hence the lower the impact reduction in e_0, the greater is the adjustment of e_t after period 0.

IV. OPTIMAL POPULATION IN THE CAP ERA

A household is small compared with total population so in maximizing utility, it ignores the external reduction in the emissions ratio and hence in everyone's future income caused by its having a child. We characterize the optimal population sequence, which maximizes the utility of the representative household taking account of the population externality.[12]

The optimal population problem for arbitrary t and hence given $N_t > 0$ is to maximize U_t by choice of future populations $N_{t+1}, N_{t+2}, \ldots$. The problem can be written as a dynamic programming problem with Bellman equation

$$V^*(N_t) = \max_{n_{t+1}} \{u(f(e(N_t)) - \chi n_{t+1}) + \beta(n_{t+1})V^*(N_{t+1})\}, \tag{14}$$

where $n_{t+1} = N_{t+1}/N_t$ and where the value function V^* captures the dependence of U_{t+1} on N_{t+1}. This is not a "standard" dynamic programming problem because the discount factor $\beta(n_{t+1})$ is endogenous. Alvarez (1999) shows that solutions can nonetheless be obtained by solving the transformed problem of maximizing $U_t^\circ \equiv \beta(N_t)U_t$, which has the same optimal policy.

In detail, $U_t^\circ \equiv \beta(N_t)[u(c_t) + \beta(n_{t+1})U_{t+1}] = \beta(N_t)u(c_t) + b_0\beta(N_{t+1})U_{t+1} = u^\circ(N_t, N_{t+1}) +$

[12]Parents care about offspring so this includes utility from future generations' consumptions.

$b_0 U^\circ_{t+1}$ where $u^\circ(N_t, N_{t+1}) \equiv \beta(N_t) u(f(e(N_t)) - \chi N_{t+1}/N_t)$, so the transformed problem has Bellman equation

$$V^\circ(N_t) = \max_{0 \le N_{t+1} \le \frac{1}{\chi} N_t f(e(N_t))} \{u^\circ(N_t, N_{t+1}) + b_0 V^\circ(N_{t+1})\}, \tag{15}$$

with $0 < b_0 < 1$.[13] Solutions to this problem exist for all (θ, b). (Technical claims in this section are proved in the appendix.)

To compare the optimal and natural steady states, let stars denote optimal values. For any (θ, b), steady-state optimal population N^*_{ss} satisfies the optimal steady-state condition $S^*(\hat{E}/N^*_{ss}) = 0$, where

$$S^*(e) \equiv (1 - b_0) S(1, e) - b_0 u'(f(e) - \chi) f'(e) e.$$

The optimal and natural steady-state conditions thus differ by a term that reflects the population externality. A root $e^*_{ss} = \hat{E}/N^*_{ss}$ that solves $S^*(\hat{E}/N^*_{ss}) = 0$ exists and lies strictly between e_{ss} and e^+.[14] Because $e_{ss} < e^*_{ss}$, the steady-state optimal population N^*_{ss} is less than the steady-state natural population N_{ss}.

If $\omega = \frac{1-\theta}{1-b} \le 1$, the value function V° is unique, strictly concave, and differentiable, and optimal population is a single-valued continuous function $N_{t+1} \equiv H(N_t \mid \hat{E})$ that also maximizes $V^*(N_t) = V^\circ(N_t)/\beta(N_t)$.[15] The optimal population sequence $\{N^*_t\}_{t \ge 0}$ starting at $t = 0$ from given $N_0 > 0$ is obtained by iterating on H. Moreover, e^*_{ss} is unique and the elasticity condition (13) is sufficient for the optimal population to converge monotonically to N^*_{ss} from any initial $N_0 > 0$. If $\omega > 1$, matters are more complicated but similar results hold if ω is not too great.[16]

[13] In footnote 7, we assumed $b_0 < 1$ in the Jones-Schoonbroodt case and $\beta(n) < 1$ for all feasible n in the Barro-Becker case. Using (10), the latter implies $b_0 < 1$.

[14] The details are that S^* is continuous, $S^*(e_{ss}) = -\beta u' f' e < 0$ (because $S(1, e_{ss}) = 0$ and $e_{ss} < 1$), and $S^*(e^+) > 0$ (because $S(1, e^+) > 0$ and $f'(e^+) = 0$).

[15] Applications of the Barro-Becker model commonly assume $\omega \le 1$ (e.g. Jones and Schoonbroodt, 2007). This helps ensure strict concavity of V° by ensuring that u° is concave. We allow $\omega > 1$.

[16] For instance, suppose (13) holds and $1/\omega > 1 - \varepsilon_{n_t, y_t} \kappa(e)$ for $e \in (f^{-1}(\chi), e^+)$, where $\kappa(e_t) \equiv$

Fertility along the optimal path (*optimal fertility,* η^*) follows from the optimal population as $\eta^*(N_t \mid \hat{E}) \equiv H(N_t \mid \hat{E})/N_t$. As with natural fertility, optimal fertility is: (i) homogeneous of degree zero in population and the level of the cap; (ii) jumps when a cap is imposed, from n^+ to $\eta^*(N_0 \mid \hat{E})$; and (iii) approaches $n^*_{ss} = 1$ as population converges to N^*_{ss}.

The Population Externality: Comparing Natural and Optimal Populations

We evaluate the population externality using the Bellman equation (14). Because the value function V^* in (14) is differentiable, optimal fertility satisfies:

$$\begin{aligned} V^*_n &\equiv -u'(c^*_t)\chi + \beta'(n^*_t)V^*(N^*_{t+1}) + \beta(n^*_t)N^*_t \frac{dV^*}{dN}(N^*_{t+1}) \\ &= V_n(n^*_t, f(e(N^*_t)), V^*(N^*_{t+1})) + \beta(n^*_t)N^*_t \frac{dV^*}{dN}(N^*_{t+1}) = 0, \end{aligned} \tag{16}$$

using the definition of V_n in (6). The household sets $V_n = 0$ so optimal and natural paths differ. The term $\beta(n^*_t)N^*_t \frac{dV^*}{dN}(N^*_{t+1})$ is ignored by households; it measures the population externality in units of parent's utility.

To evaluate $\frac{dV^*}{dN}(N^*_{t+1})$, the envelope theorem applied to (14) implies:

$$\frac{dV^*}{dN}(N^*_{t+1}) = -u(c^*_{t+1})f'(e(N^*_{t+1}))\frac{e(N^*_{t+1})}{N^*_{t+1}} + \ \beta(n^*_{t+1})n^*_{t+1}\frac{dV^*}{dN}(N^*_{t+2}). \tag{17}$$

Reapplying (17) iteratively to eliminate successive future derivatives of V^*, the future terms collapse into the discounted sum

$$\frac{dV^*}{dN}(N^*_{t+1}) = -\frac{1}{N^*_{t+1}} \sum_{i=1}^{\infty} \left[\prod_{j=1}^{i-1} \beta(n^*_{t+j})\right] u'(c^*_{t+i})f'(e(N^*_{t+i}))e(N^*_{t+i}). \tag{18}$$

$-\frac{f''(e)e^2}{f(e)} > 0$ measures the contribution from concavity of f. Then N^*_{ss} is unique, the optimal population sequence converges monotonically to N^*_{ss} from any $N_0 > 0$, and V° is strictly concave for $N_t \geq \hat{E}/e^+$ so H is single-valued and V° and V^* are differentiable. If the cap binds when it is imposed ($\hat{E} \leq e^+N_0$), the optimal population path is also unique. (If $\hat{E} > e^+N_0$, the optimization problem (15) may not be concave when $\omega > 1$ so it is difficult to rule out multiple optimal population paths.)

As in Harford (1998) the infinite sum reflects the fact that a birth at t creates a new dynasty whose members increase populations after $t+1$ and also generate externalities.

The terms $f'(e_{t+i})e_{t+i}$ in (18) are aggregate real population externalities measured in units of output or equivalently in units of descendants' consumption at $t+i$. Remaining terms convert the real population externalities into units of parent's (period-t) utility. The negative sign shows the externality reduces utility.

There are three interpretations of a real term $f'(e_{t+i})e_{t+i}$. First, the market value at price f' of the emissions produced by a person born at $t+i-1$, which come at the expense of emissions by everyone else under a cap. Second, the loss of per-capita output caused by a person born at $t+i-1$: output $f(e(N_{t+i}))$ is lower by $f'(e_{t+i})e_{t+i}/N_{t+i}$; summing over the population at $t+i$ gives an aggregate loss of $f'(e_{t+i})e_{t+i}$. Third, the dilution of rents from auction revenue. When government auctions $\hat{E}$ permits, it receives total revenue $p_{t+i}\hat{E} = f'(e_{t+i})\hat{E}$ that it redistributes as equal lump sums so each person indirectly receives emission revenue $f'(e_{t+i})e_{t+i}$. With an additional birth at $t+i-1$, the population at $t+i$ loses the revenue $f'(e_{t+i})e_{t+i}$ that goes to the additional person.

Because any cap eventually binds, the discounted externality sum $\frac{dV^*}{dN}(N^*_{t+1})$ is strictly negative for all N_t in the cap era, even if population is initially so low that the cap does not yet bind so $f'(e(N_t)) = 0$.[17] Thus starting from any population N_t, the optimal population at $t+1$ is always less than the natural population.[18]

[17]Formally: $e(N^*_t) \to e^*_{ss} < e^+$ implies $f'(e_t) > 0$ for some t so $\frac{dV^*}{dN}(N^*_{t+1}) < 0$.

[18]Formally: $\frac{dV^*}{dN}(N^*_{t+1}) < 0$ plus concavity of V (see footnote 8) imply that the fertility n^*_t that solves (16) is strictly less than the fertility n_t that satisfies the household first-order condition evaluated along the optimal path, $V_n(n_t, f(e(N^*_t)), V^*(N^*_{t+1})) = 0$.

Pigovian Taxes on Having Children

To compute the sequence of child taxes needed to change fertility and population from natural to optimal levels, we assume child-tax revenue is redistributed to households as equal lump sums. Let τ_t denote a tax per child and let $\bar{n}_t$ denote the average over households of n_t in t, so each household pays child taxes $\tau_t n_t$ and receives lump-sum revenue $\tau_t \bar{n}_t$.[19]

With child taxes, overall child costs include taxes and overall transfers includes lump-sum redistributions of child-tax revenue, so the household generically maximizes $u(w + TR - \chi n - \tau n) + \beta(n)U$ taking w, $TR = pE/N + \tau\bar{n}$, and τ as given. The first-order condition is

$$V_n(n, w + TR, U|\tau) \equiv -u'(w + TR - \chi n - \tau n)(\chi + \tau) + \beta' U = 0. \tag{19}$$

To implement the optimal population sequence, each optimal tax τ_t^* must be set so n_t^*, which solves (16), also solves (19). Setting V_n^* from (16) equal to V_n from (19) and noting that $w_t + TR_t - \tau_t^* n_t^* = f(e(N_t^*))$ and $U_{t+1} = V^*(N_{t+1}^*)$ along the optimal path, τ_t^* must satisfy $u'(f(e(N_t^*)) - \chi n_t^*)\tau_t^* = -\beta(n_t^*) N_t^* \frac{dV^*}{dN}(N_{t+1}^*)$. From (18),

$$\begin{aligned} \tau_t^* &= \frac{\beta(n_t^*) N_t^*}{u'(c_t^*)} \left(-\frac{dV^*}{dN}(N_{t+1}^*) \right) \\ &= \frac{\beta(n_t^*)}{n_t^*} \sum_{i=1}^{\infty} \left[\prod_{j=1}^{i-1} \beta(n_{t+j}^*) \right] \frac{u'(c_{t+i}^*)}{u'(c_t^*)} f'(e(N_{t+i}^*)) e(N_{t+i}^*). \end{aligned} \tag{20}$$

Because the externality sum $\frac{dV^*}{dN}(N_{t+1}^*)$ is strictly negative, optimal taxes are strictly positive for all $t \geq 0$. Optimal child taxes are Pigovian, as in Harford (1998): the

[19] There may be obstacles to implementing a given sequence of child taxes. Extracting taxes from parents may be difficult, for instance, and some ways of preventing population growth may be ethically unacceptable. On the other hand, a range of policies that include filing-status differences, personal exemptions, public-school spending, and welfare programs currently subsidize children. Because the issue of pre-existing child subsidies is complex, we adopt the conservative approach of assuming that $\tau_t = 0$ before a cap is imposed.

optimal tax equals the discounted present value of the externalities generated by a child and all descendants of the child. The terms in (20) other than the real externalities $f'(e(N^*_{t+1}))e(N^*_{t+1})$ can be interpreted as the number of descendants in a future period times products of single-period discount factors.[20] Overall, the τ^*_t measure population externalities in units of parent's consumption.

V. EXTENSIONS

To make calibrations more meaningful, we extend the model in three ways:

Time costs of children

Parents devote substantial time to children. To include time costs, we assume having a child requires a constant amount of parental time, ψ, in addition to output χ. Time spent having a child reduces labor supply so $l_t = 1 - \psi n_t$ depends on fertility, total labor supply $L_t = (1 - \psi n_t)N_t$ differs from population N_t, and the emissions ratio depends on fertility: $e_t = \min\left(e^+, \hat{E}/[(1-\psi n_t)N_t]\right)$.

With time costs, the cost of a child becomes $\chi + \psi w(e_t)$, the sum of the output cost and foregone wages. The household takes the wage and transfers of permit revenue $TR_t = p_t E_t / N_t$ as given in maximizing utility so these replace income as determinants of fertility. In equilibrium, household income is the sum of wage income

[20] In detail,

$$\frac{\beta(n^*_t)}{n^*_t}\left[\prod_{j=1}^{i-1}\beta(n^*_{t+j})\right]\frac{u'(c^*_{t+i})}{u'(c^*_t)} = \left(\prod_{j=1}^{i-1} n^*_{t+j}\right)\left[\prod_{j=0}^{i-1}\frac{\beta(n^*_{t+j})}{n^*_{t+j}}\frac{u'(c^*_{t+j+1})}{u'(c^*_{t+j})}\right],$$

where $\prod_{j=1}^{i-1} n^*_{t+j} = L^*_{t+i}/L^*_{t+1}$ is descendants at time $t+i$ per child born at time $t+1$. The terms $\frac{\beta(n^*_{t+j})}{n^*_{t+j}}\frac{u'(c^*_{t+j+1})}{u'(c^*_{t+j})}$ can be interpreted as single-period discount factors. Specifically, if individuals could trade consumption loans that are settled by their children, $\frac{\beta(n^*_{t+j})}{n^*_{t+j}}\frac{u'(c^*_{t+j+1})}{u'(c^*_{t+j})}$ would be the market-clearing price in period j of a loan that pays one consumption unit in period $j+1$.

and transfers, and equals labor times output per unit of labor, so $y_t = (1-\psi n_t)w(e_t) + f'(e_t)E_t/N_t = (1-\psi n_t)f(e_t)$. The generic household first-order condition becomes $V_n(n, w, TR, U) \equiv -u'\left((1-\psi n)w + TR - \chi n\right) \cdot (\chi + \psi w) + \beta' U = 0$.

These changes carry through the resulting dynamics and are easily incorporated into calibrations. Notably, the function S defined in (9) gains terms and becomes

$$S(n, e) \equiv -u'((1-\psi n)f(e) - \chi n)(\chi + \psi w(e)) + \frac{\beta'(n)}{1-\beta(n)} u((1-\psi n)f(e) - \chi n). \quad (21)$$

Roots of the resulting steady-state conditions $S(n^+, e^+) = 0$ and $S(1, e_{ss}) = 0$ exist as before. The roots are the steady-state values of fertility in the exponential-growth era, n^+, and the emissions ratio in the cap era, e_{ss}. Condition (10), which ensures $n^+ > 1$, gains a time-cost term $\psi f(e^+)$ and becomes $\chi + \psi f(e^+) < \phi f(e^+)$. The steady-state natural population is $N_{ss} = \hat{E}/[(1-\psi)e_{ss}]$. The optimal population with time costs similarly implies a steady-state optimal emissions ratio e^*_{ss} with $e_{ss} < e^*_{ss} < e^+$, and steady-state optimal population $N^*_{ss} = \hat{E}/[(1-\psi)e^*_{ss}] < N_{ss}$.[21]

Backstop Technology

A common assumption in integrated assessment models is that a "backstop" technology may permit output to be positive without emissions.[22] If there is a positive backstop output level $f(0)$, steady-state natural and optimal populations exist as above if $f(0)$ is low enough, specifically, if $f(0) < f^B \equiv \frac{\chi}{\phi - \psi}$.[23]

If $f(0) > f^B$, however, income loss from a cap is insufficient to reduce fertility

[21]The optimal steady-state condition is now $S^*(e^*_{ss}) = 0$, where $S^*(e) \equiv (1-b_0)S(1, e) - b_0(1-\psi)u'((1-\psi)f(e) - \chi)f'(e)e$. Note that uniqueness of e_{ss} and e^*_{ss} requires regularity conditions that are detailed in the appendix and are satisfied in calibrations below.

[22]e.g. William Nordhaus and Joseph Boyer (2000), Kelly and Kolstad (2001).

[23]If $f(0) < \chi/(1-\psi)$, then there is an $e = f^{-1}(\chi/(1-\psi)) > 0$ at which the marginal utility of consumption is infinite so $S(1, f^{-1}(\chi/(1-\psi))) < 0$ with S as defined in (21), and the steady-state

to replacement so the natural population does not converge to a steady-state value. Instead, fertility converges to the unique root $n_{ss} > 1$ of $S(n_{ss}, 0) = 0$, population grows without bound, $e_t \to 0$, and $f(e_t) \to f(0)$. In the limit, concavity implies $f'(e)e \to 0$ so the population externality vanishes and optimal fertility converges to the same limit n_{ss} as natural fertility.[24] For all finite periods, however, the population externality exists so the optimal child tax is positive, and natural fertility exceeds optimal fertility so the natural population exceeds the optimal population at all t and in the limit.[25]

The backstop output level $f(0)$ is key to knowing the economy's fate under a cap. As long as $f(0) < f^B$, a cap ultimately leads to a steady state with output low enough to choke off population growth. This true even if the cost of eliminating *almost all* emissions is small: fertility then would remain high so e would continue to drop, until output is low enough so $n_{ss} = 1$.

Exogenous Growth in Factor Productivity

To add exogenous productivity growth, we assume production is

$$Y_t = F(L_t \lambda^t, E_t \alpha^t), \tag{22}$$

where $\lambda \geq 1$ is an exogenously given growth factor for labor productivity and $\alpha \geq 1$ is an exogenously given growth factor for emissions productivity. Greater emissions productivity α^t means fewer emissions are needed to produce a given output from a

analysis in section III holds, mutatis mutandis. Moreover if $f(0) \geq \chi/(1-\psi)$ but $f(0) < f^B$,then $S(1, 0) < 0$, and again the reasoning of section III implies there is a unique steady-state natural emissions ratio $e_{ss} > 0$ defined by $S(1, e_{ss}) = 0$.

[24]Because $e_t l_t N_t = \hat{E}$, a value $e_t = 0$ is inconsistent with $\hat{E} > 0$. Thus there is no meaningful $e_{ss} = 0$, but allocations with $e_t > 0$ in which $e_t \to 0$ are meaningful.

[25]In the non-generic case with $f(0) = f^B$, $e_t \to 0$ and $n_t \to 1$ so all natural and optimal limit conditions reduce to $S(1, 0) = S^*(0) = 0$, where S^* was defined in footnote 21.

given amount of labor.

We assume the output cost of a child grows with labor productivity λ^t, so the household budget becomes $c_t = y_t - \lambda^t \chi n_t$. The idea is that greater productivity means more human capital, which requires that more resources be put into each child. Put differently, λ^t captures an exogenous trend in child quality that raises the cost of a child and tends to make fertility fall as productivity rises.[26]

An economy with growing productivity is equivalent to an economy with stationary values of productivity-adjusted variables, marked with tildes. The key state variable is growth-adjusted population, $\tilde{N}_t \equiv N_t \lambda^t / \alpha^t$. Also define $\tilde{n}_t \equiv n_t \lambda / \alpha$; this is the growth factor for productivity-adjusted population and for total emissions, which rise because of population and labor-productivity growth, and fall because of emissions-productivity growth.

With variables and parameters defined in growth-adjusted terms[27]:

$$\begin{aligned} & u((1-\psi n_t)w_t + TR_t - (\lambda^t \chi + \tau_t)n_t) + \beta(n_t)U_{t+1} \\ & \qquad = \lambda^{(1-\theta)t} \left[u((1-\tilde{\psi}\tilde{n}_t)\tilde{w}_t + \widetilde{TR}_t - (\tilde{\chi} + \tilde{\tau}_t)\tilde{n}_t) + \tilde{\beta}(\tilde{n})\tilde{U}_{t+1} \right], \end{aligned}$$

so choosing n to maximize $u((1-\psi n)w + TR - (\chi+\tau)n) + \beta(n)U$ with given (w, TR, U) is equivalent to choosing $\tilde{n}$ to maximize $u((1-\tilde{\psi}\tilde{n})\tilde{w} + \widetilde{TR} - (\tilde{\chi}+\tilde{\tau})\tilde{n}) + \tilde{\beta}(\tilde{n})\tilde{U}$ with given $(\tilde{w}, \widetilde{TR}, \tilde{U})$.

The equivalent problem has the same form as the problem without productivity growth except that growth-adjusted (tilde) variables replace regular variables. All analysis from previous sections goes through with growth-adjusted variables and pa-

[26]We do not model the demographic transition, however: the proportionality of the output cost of a child to λ^t simply ensures that child costs do not vanish or explode as a fraction of income merely because productivity grows, which ensures balanced growth.

[27]Specifically: $\tilde{e}_t \equiv \min(e^+, \frac{\hat{E}}{(1-\tilde{\psi}\tilde{n}_t)\tilde{N}_t})$, $\tilde{y}_t \equiv (1-\tilde{\psi}\tilde{n}_t)f(\tilde{e}_t) = y_t/\lambda^t$, $\tilde{w}_t \equiv f(\tilde{e}_t) - f'(\tilde{e}_t)\tilde{e}_t$, $\tilde{\chi} \equiv \chi\alpha/\lambda$ (so $\tilde{\chi}\tilde{n}_t = \chi n_t$), $\tilde{\psi} \equiv \psi\alpha/\lambda$ (so $\tilde{\psi}\tilde{n}_t = \psi n_t$), $\tilde{\beta}(\tilde{n}_t) \equiv \lambda^{(1-\theta)}\beta(\tilde{n}_t\alpha/\lambda) = \lambda^{(1-\theta)}\beta(n_t)$, $\tilde{U}_t \equiv U_t/\lambda^{(1-\theta)t}$, $\tilde{\tau}_t \equiv \tau_t(\alpha/\lambda)/\lambda^t$, and $\widetilde{TR}_t = f'(\tilde{e}_t)E_t/\tilde{N}_t + \tilde{n}_t\tilde{\tau}_t$. (Note that $TR_t = f'(e_t)E_t/N_t + n_t\tau_t$.)

rameters replacing regular variables and parameters:

In the exponential-growth era, the emissions ratio is $\tilde{e}_t = e^+$ and household income follows $y_t^+ \equiv (1-\tilde{\psi}\tilde{n}_t)f(e^+)\lambda^t$. A perfect-foresight solution is pair $(\tilde{n}^+, \tilde{U}^+)$ with $\tilde{U}^+ = u((1-\tilde{\psi}\tilde{n}^+)f(e^+)-\tilde{\chi}\tilde{n}^+)/(1-\tilde{\beta}(\tilde{n}^+))$, where $\tilde{n}^+$ is optimal given $\tilde{U}^+$. In any solution, growth-adjusted population grows at rate $\tilde{n}^+ - 1$. Because $E_t = \tilde{e}_t(1-\tilde{\psi}\tilde{n}^+)\tilde{N}_t$ and $\tilde{e}_t = e^+$, emissions also grow at rate $\tilde{n}^+ - 1$. We assume $\tilde{\chi} + \tilde{\psi}f(e^+) < \tilde{\phi}f(e^+)$ where $\tilde{\phi} = 1/\left(1 + \frac{(1-\theta)(1-\tilde{\beta}(1))}{(1-b)\tilde{\beta}(1)}\right) < 1$, so $\tilde{n}^+ > 1$.[28]

In the cap era, growth-adjusted population converges to steady-state value $\tilde{N}_{ss}$. Unless $\alpha = \lambda$, actual population N_t therefore changes over time. Specifically, $\tilde{n}_{ss} = n_{ss}\lambda/\alpha = 1$ implies that actual fertility is $n_{ss} = \alpha/\lambda$. This is a balanced-growth condition.[29] An intuition is that growth in labor productivity (λ) introduces an increasing trend in each person's emissions footprint and growth in emissions productivity (α) introduces a decreasing trend, so exogenous productivity growth overall introduces per-capita emissions growth with factor λ/α per period. To hold total emissions constant in steady state, this means population must grow with factor α/λ. Similarly, steady-state optimal fertility is $n_{ss}^* = \alpha/\lambda$.

Four growth factors describe steady state in the cap era. Natural and optimal populations grow with factor α/λ, as just noted. Outputs per person and living standards grow with factor λ because $\tilde{y}_{ss} = y_t/\lambda^t$ and $\tilde{y}_{ss}^* = y_t^*/\lambda^t$ are constant. Total output, the product of population and output per person, grows with factor α, the product of α/λ and λ. Finally total emissions are constant, as output grows with

[28]Emissions have increased historically, consistent with $\tilde{n}^+ > 1$. If future fertility were to fall sufficiently due to changes in tastes or if α/λ were to fall sufficiently, then $\tilde{n}^+$ could fall below one. Then emissions and the emissions problem would eventually vanish.

[29]The production function (22) implies that output growth arises from growth in the inputs $l_t N_t \lambda^t$ and $E_t\alpha^t$. In steady state with actual fertility constant at n_{ss}, effective labor $(1-\psi n_{ss})N_t\lambda^t$ has growth factor $n_{ss}\lambda$. Because emissions are capped at $\hat{E}$, the input $E_t\alpha^t$ has growth factor α. Balanced growth requires $n_{ss}\lambda = \alpha$, or $n_{ss} = \alpha/\lambda$.

the emissions-productivity growth factor α. The outcome is Malthusian modified for productivity growth: living standards continue to grow as long as $\lambda > 1$; total output continues to grow as long as $\alpha > 1$; and population grows (or shrinks) unless $\alpha = \lambda$.

Taxes in the transformed economy, $\tilde{\tau}_t \equiv \tau_t(\alpha/\lambda)/\lambda^t$, are taxes per growth-adjusted child. To express the optimal taxes $\tilde{\tau}_t^*$ as taxes per actual child (τ_t^*), it is necessary to divide out the growth-adjustment correction $(\alpha/\lambda)/\lambda^t$. The actual tax grows with factor λ, as does actual income along the optimal path, $y_t^* \equiv (1-\tilde{\psi}\tilde{n}_t^*)f(\tilde{e}_t^*)\lambda^t$. In the calibrations below we remove the growth factors by reporting optimal taxes as shares of income[30]

$$\{\tau/y\}_t^* \equiv \frac{\tau_t^*}{y_t^*} = \frac{\tilde{\tau}_t^*\lambda/\alpha}{(1-\tilde{\psi}\tilde{n}_t^*)f(\tilde{e}_t^*)}.$$

VI. CALIBRATIONS

To assess the population externality, we calibrate the model to a growing world economy with annual steady-state population growth of 1.4 percent, per-capita output growth of 1.7 percent, and aggregate emissions growth of 1.8 percent, which were actual rates over 1990-2005.[31] A period equals 30 years, so $n^+ = 1.52$, $\tilde{n}^+ = n^+\lambda/\alpha = 1.72$, $\alpha = 1.48$, and $\lambda = 1.67$.[32] The value of b_0 is chosen so the household's first-order condition holds given these growth rates. We choose units so $\tilde{e}^+ = 1$, $f(1) = 1$, and $\tilde{N}_0 = 1$. Unless noted, all variables except child taxes are growth-adjusted.

We consider two production functions. Cobb-Douglas production is $f(\tilde{e}) = f_0\tilde{e}^{f_1}(f_2-$

[30] To interpret the tax in a real-world with two-adult households, the mother and father can each be seen as paying one-half of the tax so the tax on any single child as a share of household income is one-half of $\{\tau/y\}_t^*$. For the couple to replace themselves takes two children, so each would pay $\{\tau/y\}_t^*$ for replacement. (Note that we abstract from mixing of dynasties.)

[31] see World Resources Institute (2008).

[32] Specifically $n^+ = \exp(30\cdot .014) = 1.52$. Per-capita income grows with factor λ so $\lambda = \exp(30\cdot .017) = 1.67$. Total emissions grow at the same rate as productivity-adjusted population, so $\tilde{n}^+ = n^+\lambda/\alpha = \exp(30\cdot .018) = 1.72$. This implies $\alpha = n^+\lambda/\tilde{n}^+ = 1.48$.

$\tilde{e})^{1-f_1}$, where f_0, $f_1 \in (0,1)$, and f_2 are parameters.[33] With Cobb-Douglas production, the factor share of emissions rises monotonically from zero at e^+ to f_1 as $\tilde{e}$ falls to zero. For any f_1, units choices pin down f_0 and f_2: $\tilde{e}^+ = 1$ implies $f'(1) = 0$ so $f_2 = 1/f_1$, and $f(1) = 1$ implies $f_0 = [f_1/(1-f_1)]^{1-f_1}$. To set f_1, we assume it costs 3 percent of output to reduce emissions by 25 percent, so $f(0.75) = 0.97$. This implies $f_1 = 0.483$. A 3-percent cost is in the range of estimates in Stern (2007). We also evaluate a 2-percent cost below, with $f_1 = 0.371$.

Cobb-Douglas production does not allow a positive backstop. To study a backstop and get a sense of how sensitive results are to the form of production, we also consider the abatement-cost specification used in many integrated assessment models: $f(\tilde{e}) = 1-(1-g_0)(1-\tilde{e})^{g_1}$, where g_0 and g_1 are parameters and backstop output is $f(0) = g_0$.[34] When we assume no backstop ($g_0 = 0$), we set g_1 by again assuming it costs 3 percent of output to reduce emissions by 25 percent, so $g_1 = 3.32$. With a positive backstop, we leave the curvature g_1 unchanged and simply assume a positive g_0, which proportionately reduces abatement costs at any $\tilde{e}$.

We assume children have an output cost of $\chi = 0.138$ and a time cost of $\psi =$

[33] The form may be derived from three primitive assumptions: (*i*) labor is used to produce two intermediate goods in amounts y_1 and y_2 according to the linear technology $y_1 + y_2 = f_2$; (*ii*) a unit of good 1 generates a unit of emissions so $\tilde{e} = y_1$, whereas good 2 generates no emissions; and (*iii*) output per unit of labor is a Cobb-Douglas function $f_0 y_1^{f_1} y_2^{1-f_1}$. The Cobb-Douglas form matters. If output per unit of labor were a CES function of y_1 and y_2 with an elasticity other than one, the factor share of emissions would approach either zero or one as $\tilde{e} \to 0$, which may be undesirable to impose.

[34] An interpretation is that a unit of labor gives a unit of output and a unit of emissions if no resources are devoted to abatement, and the cost of abating $1-\tilde{e}$ units of emissions is $(1-g_0)(1-\tilde{e})^{g_1}$ units of output. The factor share of emissions in abatement-cost cases has a knife-edge, which partly motivates why we use Cobb-Douglas for most calibrations. Without a backstop, the factor share rises monotonically from zero at $\tilde{e} = 1$ to *one* at $\tilde{e} = 0$, but with any positive backstop, the factor share rises from zero at $\tilde{e} = 1$ to a peak, then falls to *zero* at $\tilde{e} = 0$.

0.110. The output cost is from the sum of expenditures by families on children plus expenditures on K-12 and college education. The time cost assumes the difference between male and female labor-force participation rates is due solely to time devoted to having children so that with zero children, the average participation rate would equal the current male rate (0.76) instead of the current average of male and female rates (0.685). Details are in the appendix.

The time cost implies that per-capita labor in the exponential-growth era is $1 - \tilde{\psi}\tilde{n}^+ = 0.833$ and per-capita income is $\tilde{y}^+ = (1 - \tilde{\psi}\tilde{n}^+)f(e^+) = 0.833$.

Base Case

Our base case is a Barro-Becker case with equal utility curvatures ($\theta = b$), Cobb-Douglas production, and a cap that freezes emissions. A reasonable range of estimated

Table 1. Steady States

Case	Regime	$\tilde{N}_{ss}$	$\tilde{e}_{ss}$	$\tilde{y}_{ss}$	$f'\tilde{e}/f$	$\{\tau/y\}^*_{ss}$
Barro-Becker (base case), $\theta = b = .8$	natural	9.16	.101	.408	.457	
	optimal	2.39	.386	.721	.365	.211
Jones-Schoonbroodt, $\theta = b = 2$	natural	44.1	.021	.195	.469	
	optimal	11.1	.083	.373	.478	.957
Barro-Becker, $\theta = b = .4$	natural	4.97	.186	.529	.432	
	optimal	1.43	.644	.835	.250	.106
Barro-Becker, $\theta = .95$, $b = .8$	natural	9.90	.093	.394	.459	
	optimal	7.32	.126	.452	.450	.068
Jones-Schoonbroodt, $\theta = 1.25$, $b = 2$	natural	28.7	.032	.239	.475	
	optimal	21.7	.042	.273	.472	.155
2 percent cost of 25% emission cut	natural	15.6	.059	.418	.357	
	optimal	3.96	.233	.665	.311	.180

θ values is $0.5-5.0$ (see e.g. Masao Ogaki and Carmen Reinhardt, 1998; Ravi Bansal and Amir Yaron, 2004). For the Barro-Becker case the range is $0.5-1.0$, so we set $\theta=b=0.8$ in the base case.

Population is normalized so $\tilde{N}_0=1$, and $1-\tilde{\psi}\tilde{n}^+=0.833$, so emissions at $t=0$ would be 0.833 without a cap. Thus a freeze means $\hat{E}=0.833$.

Steady-state results are in table 1. In the base case, the growth-adjusted natural population in steady-state is 9.16 times $\tilde{N}_0$, and the steady-state emissions ratio is 0.101 times the emissions ratio without a cap. A cap substantially lowers incomes: per-capita output falls from $\tilde{y}^+=0.833$ to $\tilde{y}_{ss}=0.408$.

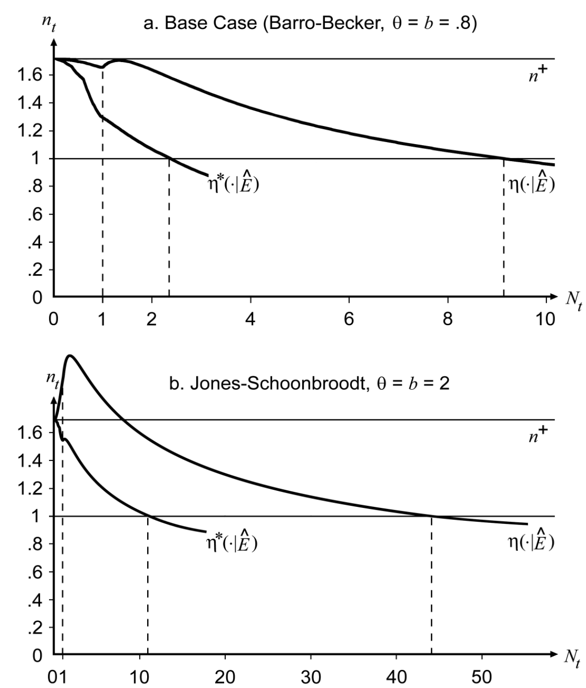

Figure 5. Base Case Fertilities

The growth-adjusted optimal population in steady state is 2.39 so the natural population is almost four (9.16/2.39) times the optimal population. The optimal emissions ratio of 0.386 is almost four times the natural emissions ratio. Per-capita optimal output, 0.721, exceeds per-capita natural output and is 13.5 percent below output at $t = 0$.

The natural fertility functions (figure 5a) imply that the natural and optimal populations converge smoothly from $\tilde{N}_0 = 1$ to steady-state values.[35] After fivegenerations, $\tilde{N}_5 = 6.84$, for instance, and after ten generations, $\tilde{N}_{10} = 8.97$, close to the steady-state value of 9.16.

Figure 6 shows the actual (not adjusted) natural and optimal populations. In the exponential-growth era, population increases exponentially. With a cap in steady state, the natural and optimal populations grow at rate $(\alpha - \lambda)/\lambda = -0.113$ so both actual populations peak after a cap is imposed and then fall. From the figure, world population under a cap would peak at about four times its current level with no population policy and would peak slightly above its current level under the optimal population policy.

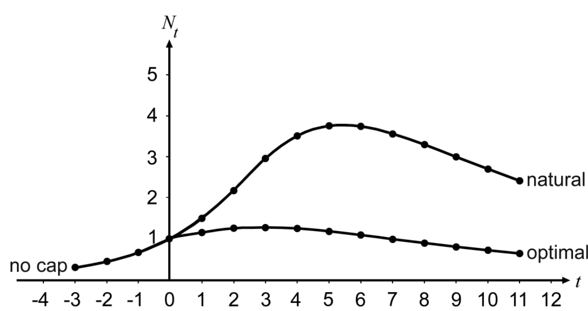

Figure 6. Populations

Permit revenue is 45.7 percent of output in the natural steady state and 36.5 percent of output in the optimal steady state. These numbers are large given that the Federal spending share over 1990-2005 averaged about 20 percent of output. Emissions revenue is small right after the cap is imposed but increases sharply as the

[35]The appendix describes the numerical procedures. In the figure, slopes of the natural and optimal fertility functions become more positive (or less negative) around $N = 1$ because the cap begins to bind so the wage and hence the time cost of children start to fall as N rises above one.

emissions ratio falls below one. With the freeze, for instance, revenue jumps from 0.7 percent of output at $t = 0$ to 27.1 percent of output at $t = 1$.

The optimal child tax in steady state is 21.1 percent of per-capita income.[36] To get a sense of this, personal income in the U.S. is (very) roughly $55,000 per adult per year, which may be interpreted as uncapped income ($\tilde{y}^+$) measured in dollars per year. Steady-state optimal income ($\tilde{y}^*_{ss}$), which is 13.5 percent less than uncapped income, would then be about $48,000. Thus a child tax of 21.1 percent is equivalent to a tax of about $10,000 each year for 30 years (the length of a generation in the model) for each child.[37] An alternative sense is that the annual cost of a child is about $13,000, of which about $6,000 is time costs. The optimal child tax therefore raises the full cost of a child in steady state by about three-quarters, from $13,000 to $23,000, to just under half ($23,000/$48,000) of income.

Optimal child taxes in steady state are independent of $\hat{E}$, but optimal child taxes along the path from $t = 0$ depend on $\hat{E}$ as shown in table 2. In the base case with a freeze (row one), the tax is 5.2 percent of income at $t = 0$, about a quarter of the steady-state value, and 11.6 percent of income at $t = 1$, a bit more than half the steady-state value. With a more restrictive cap, optimal child taxes are higher after imposition and with a less restrictive cap optimal child taxes are lower, as illustrated by a 25 percent cut in row two and a cap that is 25 percent slack in row three.

[36] Income excludes redistributions of child-tax revenue, so a tax of 21.1 percent of income is equivalent to a tax of 17.4 (= 21.1/1.211) percent of income plus redistributions of child-tax revenue.

[37] Kelly and Kolstad (2001) calculate welfare costs from a marginal child in the range $200-$800. Such costs are tiny compared with costs of $10,000 per year for 30 years. Kelly and Kolstad implicitly assume a backstop output of $f(0) = .93$ and also assume that population grows at an exogenously given rate that itself decreases at an exogenously given rate. With their production function in our model, the optimal policy would be to drive the emissions ratio to zero in steady state. This does not happen in their calculations because they assume growth slows enough so the backstop is never reached.

Table 2. Time Paths of Optimal Child Taxes

Case	$\{\tau/y\}_0^*$	$\{\tau/y\}_1^*$	$\{\tau/y\}_2^*$	$\{\tau/y\}_3^*$	$\{\tau/y\}_4^*$	$\{\tau/y\}_{ss}^*$
base case ($\hat{E} = .833$)	.052	.116	.152	.174	.188	.211
$\hat{E} = .62$	.122	.155	.176	.189	.197	.211
$\hat{E} = 1.04$	.028	.083	.133	.162	.180	.211
abatement cost, $f(0) = 0$	.055	.141	.195	.223	.237	.249
abatement cost, $f(0) = .4$	.037	.101	.142	.166	.179	.201
abatement cost, $f(0) = .6$	.026	.079	.093	.100	.098	0

Sensitivity Analysis–Utility

Table 1 also reports sensitivity analyses of individual base-case assumptions. First is a Jones-Schoonbroodt case with $\theta = b = 2$. Fertility paths are in figure 5b.[38] Steady-state population is substantially greater, income is lower, and optimal child taxes are higher than in the base case. In steady-state, the natural population is 44.1 times the transition population, income is $\tilde{y}_{ss} = 0.195$, and the child tax is 95.7 percent of income.[39] The implied ratio of the natural to optimal steady-state populations $\tilde{N}_{ss}/\tilde{N}_{ss}^*$ is still about four, however. In the transition, the optimal child tax is 10.3 percent of income at $t = 0$ and 18.1 percent of income at $t = 1$. Assuming high values of θ and b might make sense if one believes fertility is insensitive to economic incentives, but this is a pessimistic assumption here.

With low enough θ, on the other hand, fertility would respond elastically to changes in population so a cap would act like a switch that turns off fertility, and the optimal

[38]The transition from the exponential-growth to the cap era lowers children's utility, which raises fertility in the Jones-Schoonbroodt case. A greater income reduction (higher population) is then needed to reduce fertility to replacement.

[39]This is equivalent to a tax of 48.9 (= 95.7/1.957) percent of income plus redistributions of child-tax revenue.

child tax could be quite small. To see whether this happens with reasonable parameter values, we consider $\theta = b = 0.4$, at the low end of empirical estimates of θ. The resulting $\tilde{N}_{ss}$ is still 4.97 times the transition population (and $\tilde{N}^*_{ss} = 1.43$ so $\tilde{N}_{ss}/\tilde{N}^*_{ss} = 3.47$), and the optimal steady-state child tax is still 10.6 percent of income. To obtain steady-state Pigovian taxes below 10 percent, it would be necessary to assume an even lower θ.

The population externality is thus large because empirically reasonable utility assumptions imply that the desire to have children remains strong as e_t falls below e^+, so e_t ends up being driven a fair bit below e^+. The real population externality in a period $f'(e_t)e_t$, also depends on the form of f as shown in figure 1. The real externality would be small if the slope f' is always small, but this would mean the entire emissions problem could be eliminated at little cost by simply restricting emissions to zero. Similarly the population externality would be small if f were to decline only slightly below $f(e^+)$ until e_t is small, so almost all emissions could be eliminated at little cost. Sufficiently far in the future when population grows enough, however, incomes would still be low.

We also consider the equal-curvatures assumption, $\omega = 1$. Because ω measures how much parents care about per-child consumption relative to the number of children, and the population externality is a loss from lower per-child consumption, a lower ω tends to reduce the utility value of the externality. Thus real reductions in children's utility and the real population externality might be substantial, but with low ω, parents would simply not care much about this.

To judge the size of the effect, we assume $\omega = .25$ in modifications of the base case and the Jones-Schoonbroodt case. The modified Barro-Becker case has $\theta = .95$ and $b = .8$, which results in a ratio of natural to optimal steady-state populations of 1.4 and a steady-state child tax of 6.8 percent. The modified Jones-Schoonbroodt case has $\theta = 1.25$ and $b = 2$, which results in a ratio of natural to optimal steady-state populations of 1.3 and a steady-state child tax of 15.5 percent. Even in these cases the population externality is substantial, although there are (θ, b) pairs with low enough ω so the population externality would be small. As noted in section I, however, values $\omega \approx 1$ seem plausible and values substantially less than one may be difficult to square with small families in which parents devote substantial resources to ensuring children's consumptions.

Sensitivity Analysis–Technology

The true cost of reducing emissions is uncertain. Table 1 reports steady-state results when f is parameterized assuming it costs 2 percent of output to reduce emissions by 25 percent ($f(0.75) = 0.98$), instead of 3 percent as in the base case. This reduces the population externality but not greatly: the optimal child tax falls from 21.1 percent in the base case to 18.0 percent. The reason is that when it is less costly to reduce emissions, incomes and hence fertility are higher at any given population, and steady state is reached only when population is so high and the emissions ratio so low that incomes are close to incomes in the base case. That is, the income reductions needed to choke off population growth doesn't change much when the output cost of reducing emissions falls.[40]

Finally, we examine the abatement-cost specification. Under base-case utility assumptions, the critical backstop is $f^B = 0.507$ so we consider backstops of 0, 0.4, and 0.6. Comparing the first two rows of table 3 with the first two rows of table 1 shows

[40]In detail, steady-state natural income is slightly higher than in the base case, 0.418 instead of 0.408, but steady-state optimal income is lower, 0.665 instead of 0.721. (The smaller difference between $\tilde{y}^*_{ss}$ and $\tilde{y}_{ss}$ means the real externality terms $f'(e)e$ in (18) are lower.) Steady-state emission ratios are much reduced, however: $\tilde{e}_{ss}$ is only 0.059 instead of 0.101 in the base case, and $\tilde{e}^*_{ss}$ is 0.233 instead of 0.386. Consequently, $\tilde{N}_{ss}$ and $\tilde{N}^*_{ss}$ are roughly two-thirds greater than in the base case, but their ratio remains about four.

the effects of changing from a Cobb-Douglas to an abatement-cost specification with no backstop. Steady-state natural and optimal populations fall by a bit less than half and factor shares of emissions rise, but optimal child taxes change little, rising from 21.1 percent to 24.9 percent of income.

Table 3. Steady States with Abatement-Cost Production at Different Backstops

Case	Regime	$\tilde{N}_{ss}$	$\tilde{e}_{ss}$	$\tilde{y}_{ss}$	$f'\tilde{e}/f$	$\{\tau/y\}^*_{ss}$
$f(0) = 0$	natural	4.84	.190	.374	.842	
	optimal	1.58	.586	.805	.431	.249
$f(0) = .4$	natural	15.6	.059	.438	.168	
	optimal	2.84	.325	.702	.348	.201
$f(0) = .6$	natural	∞	0	.532	0	
	optimal	∞	0	.532	0	0

Comparing rows of table 3, a higher backstop means smaller income reductions as well as lower population externalities, but the effect is not great as long as $f(0) < 0.507$ (so adjusted population is constant in steady state). From table 3, the ratio of the natural to the optimal steady-state populations varies between about three and five when $f(0) < 0.507$.

If $f(0) > 0.507$, adjusted population grows forever and the population externality vanishes in the limit, quite a different long-run outcome than when $f(0) < 0.507$. The paths of the economy for the first few periods after transition, however, can be remarkably similar. Table 2 shows this. With $f(0) = 0.6$, abatement costs at any $\tilde{e}$ are reduced by 60 percent compared with abatement costs with $f(0) = 0$, and the optimal child tax in the transition period is similarly about half of the tax when $f(0) = 0$. With $f(0) = 0.6$, the tax peaks in the third period after transition at 10 percent, which is still about half the tax in the base case, then goes to zero in steady state.[41]

[41] In the effective-backstop case with $f(0) = 0.6$, both $\tilde{N}_{ss}$ and $\tilde{N}^*_{ss}$ go to infinity but their ratio

CONCLUSION

In Malthus, consumption tends to subsistence and total population tends to a constant. In Solow-type neoclassical growth models, additional population produces additional output under constant returns so total population can rise without bound. In the current paper, output is produced under constant returns from labor and total emissions, but a cap makes emissions a fixed common-property resource. This introduces a Malthusian element: as population and hence labor grow, the relative amount of the fixed factor falls, which drives down per-capita output and limits population growth.[42]

Without productivity growth, living standards and population under a cap converge to steady-state constants. With exogenous factor-augmenting productivity growth, living standards rise over time and population may rise or fall in steady state.

The focus of the current paper is on the population externality when a cap limits total emissions. The corrective Pigovian tax is about 10 percent of a parent's lifetime income per child in steady state given utility curvatures at the low end of those estimated empirically. The Pigovian tax rises sharply with utility curvatures, so that even relatively moderate curvatures imply Pigovian taxes substantially above 10 percent of a parent's lifetime income per child in steady state.

converges to about 3.4. Also, population growth remains positive as the emissions ratio converges to zero: $\tilde{n}_t$ converges to 1.16 and actual (not adjusted) population growth $n_t = \tilde{n}_t(\alpha/\lambda)$ converges to 1.03. In the limit, n_t^* converges to the same limit of 1.03. This is substantially lower than actual population growth of $n^+ = 1.52$ in the uncapped economy.

[42]The logic is general. If land is a fixed common-property resource, greater population would raise the ratio of labor to land and could reduce per-capita income.

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