

**Policy Competition, Factor Mobility and Multiple Policy Instruments:  
Existence and Non-Existence of Equilibrium**

by

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**Abstract**

Most existing models of fiscal competition between states within federations or regional unions share at least two common features. First, they focus on inter-jurisdictional competition in but one policy instrument, for example, taxes, public goods or environmental quality. The second is that the models capture policy competition as a game and analyze the nature of the Nash equilibrium without considering existence. We recognize that jurisdictions wish to choose efficient policy packages (Non-malevolence Theorem) and this allows us to examine the existence of equilibrium when there are multiple policy instruments. Sufficient conditions for existence are established followed by three examples. In the first, the sufficient conditions are satisfied, guaranteeing existence. For the second example, the sufficient conditions are not met but an equilibrium exists, while in the third example there is no equilibrium. The analysis shows that existence is by no means assured in fiscal competition models and much depends on the particular specification of the model employed.

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## 1. Introduction

Models of inter-regional fiscal competition are common in the urban residential location literature where ‘voting with feet’ in response to jurisdictional public policy choices is a salient feature. They are also employed to examine federal structures, or regional unions of semi-independent states, where factors of production (labor and capital) move freely between states, as well as between the federation or regional unions and the rest of the world, in search of the highest returns.

The models used in most papers in this literature share certain general features. First, they focus on inter-jurisdictional competition in but one policy instrument, e.g., taxes, public goods and environmental quality. Some models might include two instruments, for instance a public good and a tax, but a balanced budget condition effectively reduces the degrees of freedom to one so that competition occurs over one policy instrument. There are rare examples where multiple instruments have been contemplated. Wildasin (1988), for instance, characterizes a model in which states compete using taxes or public expenditures (but not both together). He finds differences in equilibria depending on which policy instrument is used.

The second feature of these models is that the policy competition that arises between states is captured as a game in which equilibria are within-state efficient, but not globally efficient. This is because each state’s policy choices affect the distribution of the mobile factor across states, as well as the total supply of the factor to the region. If states pursue self-interest and act competitively they ignore these inter-state effects, or fiscal externalities, when making their policy choices. Fiscal externalities are, therefore, a common feature of these models that leads to global inefficiency. The presence of externalities in turn leads to various efficiency arguments for corrective policies (eg. matching grants, taxes, inter-state transfers).

Third, it is often supposed that within the federation or regional union there are but two states, each endowed with a concave production technology, and that there is a common internal factor market with a fixed supply of a mobile factor of production (labor or capital) that allocates itself across states to satisfy an equal per capita return condition (see Boadway and Flatters (1982) and Myers (1990)). In other papers, various

degrees of 'attachment to home' are allowed within the fixed-sized common factor market (eg. Myers 1994).

Some models of fiscal competition also allow the common internal factor market to be linked with the world factor market through migration. In Oates and Schwab (1988) this is done by allowing migration between the federation and the world to satisfy a condition that the return within the common factor market must equal some given world return. Alternatively, it is reasonable to allow the regional mobile factor supply to expand as the common internal payoff increases relative to the factor's world payoff. The magnitude of this response is captured by a parameter reflecting the degree of integration between regional and world factor markets.

Finally, fiscal competition models commonly adopt one of two assumptions about government behavior. Some, for example the Boadway and Flatters paper, and Myers (1990), suppose that each government is non-malevolent and represents the interests of the mobile factor. Others, for instance Wildasin (1991), assume that there is a group of politically dominant permanent residents, a mobile factor of production (eg. guest workers) and that the government's interests are synonymous with the permanent residents (the mobile factor is disenfranchised).

Apart from earlier papers by Westhoff (1977) and Bewley (1981) there has been little concern about the existence of equilibrium for fiscal competition models. This is a curious oversight for there is little purpose in examining the efficiency properties of equilibria that do not exist. Exceptions here include the paper by Wildasin (1991a) where specific examples are constructed in which existence is assured. However, these papers do not attempt to provide general proofs of existence of equilibrium in fiscal competition models.

Here we focus on the existence issue for models of fiscal competition. This is done by constructing a model of inter-state fiscal competition incorporating many of the general features noted above. The analysis is also undertaken for two types of government. For the first, we suppose that within each state there is a group of permanent residents as well as a generic mobile factor (capital or labor) which migrates between states and also between the union and the rest of the world (similar to the

Wildasin (1991a) case). For the second type of government, we suppose that all citizens are potentially mobile and that governments represent their interests (as in Boadway and Flatters (1982)). The recognition that fully informed and non-malevolent states will always wish to choose policy packages that are within-state efficient allows us to model states that choose multiple policy instruments.

The model choice is guided by two considerations, the first is that it captures the important features of the models used in the literature. The second is that the model seems to us to be a close approximation to what we observe in practice, for example, in existing federations such as the US, Australia and Canada, and in emerging regional unions such as the European Union where factor markets are becoming increasingly integrating internally, as well as externally.

The discussion of existence proceeds as follows. First, we establish that a fully informed non-malevolent government will always wish to choose policies that are within-state efficient. This insight allows us to limit our search for existence to sets of feasible policies that are efficient. We then establish various concavity and continuity properties for the mobile factor supply functions. By appealing to well known results we then show that the existence of an equilibrium for the permanent resident government fiscal competition model is assured if the state payoff function is a concave function of the supply of the mobile factor. It is also shown that this requirement is met if the cost function associated with the policies adopted by states is quasi-convex.

Concavity is only a sufficient condition for existence, it is still possible that an equilibrium exists even without quasi-convexity of the cost function. To show this, we employ an example with two symmetric states and a fixed supply of the mobile factor of production. Three particular cases are considered. In the first, the cost function is convex and existence of a symmetric equilibrium is guaranteed (the symmetric equilibrium is often the one examined in the fiscal competition literature because of its convenient properties). The second example highlights that concavity is merely a sufficient condition for existence. Here, the cost function is not convex, and though the symmetric equilibrium does not exist, there is, nevertheless, an equilibrium. In the third example there is no equilibrium.

The analysis of the permanent resident government is followed by a discussion of existence for the case where governments represent the interests of the mobile factor. Here we argue that existence is relatively easy to establish.

Thus, the main contribution of the paper is to provide sufficient conditions for existence of equilibrium in fiscal competition models where governments represent the interests of permanent residents, and to show that existence is straightforward to establish when governments care about the welfare of the mobile factor. A further insight is that the issue of what is the strategic variable, taxes or public expenditures, is a misplaced concern. As noted, whatever the instruments available, it is in the interest of state policy makers to choose the vector of policy instruments to achieve their objective as inexpensively as possible<sup>1</sup>.

The paper is organized as follows. Section 2 presents our results on Non-malevolence; the desire of states to choose policies that are within-state efficient. Section 3 starts the development of the model of fiscal competition by discussing how states make policy choices and factors of production move between states and internationally. In Section 4, we present two Lemmas on concavity and continuity that are the basis of our existence results. Section 5 provides the main results on existence. Section 6 provides the brief discussion on existence for the case of the mobile factor government. Section 7 concludes.

## **2. Non-Malevolence**

We proceed with a model of fiscal competition that we think of as representing a federation or regional union of  $i = 1, \dots, I$  semi-independent states in which the fully informed government of each state makes policy choices that optimize some well-behaved measure of state welfare. Importantly, state policy choices can be rationalized as the optimization of a concave objective function. The citizens of the state to which this discussion applies are ‘rational’ and the government is both ‘democratic’ and ‘non-

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<sup>1</sup> See Donald Whitman (1989).

malevolent'. The implication is that the policies chosen are 'within-state efficient'.

These four features are defined as:

- (i) A citizenship of a state is 'rational' if the probability that each citizen votes to retain the existing government is a strictly increasing function of its current well-being.
- (ii) A government is 'democratic' if its objective is a function of citizen welfare only<sup>2</sup>, and is strictly increasing function of its probability of re-election.
- (iii) A government is 'non-malevolent' if its objective is a non-decreasing function of the welfare of each citizen of the state. A government is malevolent if it is not non-malevolent.
- (iv) A policy is 'within-state efficient' if there is no other feasible policy choice that will improve the welfare of one citizen without diminishing the welfare of any other citizen.

**Theorem 1 (Non-Malevolence):** *In states with rational citizens, non-malevolent, democratic governments always choose within-state efficient policies.*

**Proof:** A policy is within-state inefficient if there is another one that improves the welfare of at least one state citizen without diminishing that of any other state citizen. For this reason, for every inefficient policy there is at least one feasible policy choice for which the probability of a favorable re-election vote is higher, and, thus, the probability of government re-election is higher. For this reason, a government would never make a within-state inefficient policy choice unless it was malevolent//.

This does not imply that the policies adopted by Non-malevolent governments of states engaged in fiscal competition are globally efficient. As observed in the Introduction, they are likely to be inefficient from a federal or regional perspective because they ignore fiscal externalities. Within-state efficiency implies policy choices

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<sup>2</sup> The implication of this restriction is that government is not motivated to pursue policies that increase its own welfare at the expense of all its citizens, e.g., exploiting natural resource wealth and transferring it to numbered Swiss bank accounts.

that are efficient, as measured by within-state welfare, given the policy options available to the state. Each state government will wish to do the best it can by its citizens conditional on the constraints that it faces. However, non-malevolence precludes vindictive, or bad-spirited, actions of government designed to punish, or bedevil, a particular citizen or groups of citizens.

### 3. State Strategies and Mobility

We now focus on the strategies adopted by each state and the conditions that describe mobility between states and internationally. In this regard, each state is supposed to have a mobile factor of production,  $n_i$ , that can be thought of as capital or labor. State specific output  $y_i$ , sold at a constant unit price, is a differentiable, strictly quasi concave function of the state's mobile factor supply, and hence we define  $y_i = f_i(n_i)$ . The mobile factor receives a return  $w_i(n_i)$  which is equal to the value of its marginal product. With constant unit price output  $w_i(n_i)$  is equal to marginal product,  $f_i'(n_i)$ , which is declining in mobile factor supply.

The policies (strategies) of each state are contained in the set  $S_i \subset \mathbb{R}^K$ .  $S_i$  is a convex, compact set  $\forall i$ . A state has a vector of policies  $\sigma_i = (\sigma_{i1}, \dots, \sigma_{iK}) = (\sigma_{ik}) \in S_i$ . Let  $p_i(\sigma_i, w_i(n_i))$  be the payoff to the mobile factor of production in state  $i$  when the policy choice is  $\sigma_i$  and mobile factor supply is  $n_i$ . The payoff function, which from now on is written as  $p_i(\sigma_i, n_i)$ , is strictly quasi concave in  $\sigma_i$ , monotonically decreasing in  $n_i$  and continuous in both arguments. Suppose also that  $p_i(\sigma_i, 0) = \infty$  for all  $\sigma_i$ , that  $N$  is the entire (finite) world supply of the mobile factor and  $p_i(\sigma_i, n_i)$  is positive and finite for all  $n_i \in (0, N]$ .

Regional unions have common internal factor markets. To capture this, we assume that the mobile factor moves without restriction within the region to maximize its payoff. For a given regional mobile factor supply, if the pay-off is higher in state  $i$  than other states  $-i$ , the mobile factor will move from  $-i$  to  $i$ . This depresses the return in state  $i$  and raises the return in  $-i$  until payoffs are equal (no arbitrage opportunities remain). In

equilibrium, there is a common payoff  $p$  received by the mobile factor resident within the region regardless of its state of residence,

$$p_i(\sigma_i, n_i) = p_{-i}(\sigma_{-i}, n_{-i}) = p \quad \forall i. \quad (3.1)$$

International migration can also be incorporated. The magnitude of this response is captured by a parameter that reflects the degree of integration between the regional and world factor market. We capture this notion in a general way with the function

$$n_i + n_{-i} = n(p_i(\sigma_i, n_i)), \quad (3.2)$$

where  $n(\cdot)$ , the total regional mobile factor supply, is strictly concave and monotonically increasing in  $p_i$ , the payoff in state  $i$  (from (3.1) this is also the common internal payoff). In the discussion that follows, we use (3.2) to describe the supply of the mobile factor to the region.

#### 4. Concavity and Continuity

The mobile factor equilibrium conditions (3.1) and (3.2) can be written as the implicit function,

$$h(\sigma_i, \sigma_{-i}, n_i) = p_i(\sigma_i, n_i) - p_{-i}(\sigma_{-i}, n(p_i(\sigma_i, n_i)) - n_i) = 0.$$

It follows from the relationship between  $p_i$  and  $n_i$  that  $h(\cdot)$  is monotonically decreasing in the state specific mobile factor supply,  $n_i$ . Since  $p(\cdot)$  is continuous in all its arguments, the  $h(\cdot)$  function is continuous as well. These observations lead to the following lemmas.

**Lemma 1.** *For all  $\sigma_i$  and  $\sigma_{-i}$  there is a unique  $n_i$  that satisfies  $h(\sigma_i, \sigma_{-i}, n_i) = 0$ .*

**Proof:**

$$h(\sigma_i, \sigma_{-i}, 0) = \infty$$

$$h(\sigma_i, \sigma_{-i}, n(p_i(\sigma_i, n))) = -\infty$$

Since  $h(\cdot)$  is monotonically decreasing and continuous in  $n_i$ , there is a unique

$$n_i = n_i(\sigma_i, \sigma_{-i})$$

that satisfies  $h(\sigma_i, \sigma_{-i}, n_i) = 0$ . //

Now define  $L(\sigma_{-i}, n_i) = \{\sigma_i \mid h(\sigma_i, \sigma_{-i}, n_i) = 0\}$  as the (level) set of state  $i$  strategy vectors that satisfy the equilibrium condition for neighbor strategy  $\sigma_{-i}$  and mobile factor supply,  $n_i$ . Another way to define the level set is as follow: the set of values of  $\sigma_i$  that satisfies the equation  $n_i^0 = n_i(\sigma_i, \sigma_{-i}^0)$  where the 0 superscript indicates fixed values of  $n$  and  $\sigma_i$ . From this, then  $L(\sigma_{-i}^0, n_i^0) = \{\sigma_i \mid n_i(\sigma_i, \sigma_{-i}^0) = n_i^0\}$ . Notice that  $p_i(\sigma_i, n_i)$  is the same for all  $\sigma_i \in L(\sigma_{-i}, n_i)$  for a given  $n_i$  //

**Lemma 2.**  $n(\mathbf{s}_i, \mathbf{s}_{-i})$  is a concave function of  $\mathbf{s}_i$  and continuous in both arguments.

**Proof.** (a) **Concavity**

Define  $\sigma_i^\lambda = \lambda\sigma_i^0 + (1-\lambda)\sigma_i^1$  for  $\sigma_i^0, \sigma_i^1 \in L(\sigma_{-i}, n_i)$  and  $0 \leq \lambda \leq 1$ .

$$h(\sigma_i^\lambda, \sigma_{-i}, n_i) \geq 0$$

because  $p_i(\cdot)$  is concave in  $\sigma_i$ , thus  $p_i(\sigma_i^\lambda, n_i) > p_i(\sigma_i^0, n_i)$  and  $h(\cdot)$  is continuous and monotonically increasing in  $p_i$ .

There exists a unique  $n_i^\lambda$  such that

$$h(\sigma_i^\lambda, \sigma_{-i}, n_i^\lambda) = 0.$$

Because  $h(\cdot)$  is monotonically decreasing in  $n_i$

$$n_i^\lambda \geq n_i.$$

Therefore  $n_i(\sigma_i^\lambda, \sigma_{-i}) \geq n_i(\sigma_i, n_i)$  and  $n_i(\cdot)$  is strictly quasi concave in  $\sigma_i$ .

(b) **Continuity**

We show that the function  $n_i(\bullet)$  is continuous in  $\sigma_i$  for fixed  $\sigma_{-i} = \bar{\sigma}_{-i}$ .

The same argument applies to showing that  $n_i(\bullet)$  is continuous in  $\sigma_{-i}$ .

Suppose  $n_i(\bullet)$  is not a continuous function of  $\sigma_i$ . Then there are at least two distinct sequences

$$(\sigma_i^k)_k \in S_i \text{ and } (\sigma_i^m)_m \in S_i$$

that converge to the same value

$$\lim_{k \rightarrow \infty} (\sigma_i^k)_k = \lim_{m \rightarrow \infty} (\sigma_i^m)_m = \bar{\sigma}_i \in S_i$$

such that

$$\lim_{k \rightarrow \infty} n_i((\sigma_i^k)_k, \bar{\sigma}_{-i}) = n_i'$$

and

$$n_i' < n_i''$$

$$\lim_{m \rightarrow \infty} n_i((\sigma_i^m)_m, \bar{\sigma}_{-i}) = n_i''.$$

From the definition of  $h(\cdot)$

$$h(\sigma_i^k, \bar{\sigma}_{-i}, n_i(\sigma_i^k, \bar{\sigma}_{-i})) = h(\sigma_i^m, \bar{\sigma}_{-i}, n_i(\sigma_i^m, \bar{\sigma}_{-i})) = 0 \quad \forall k \text{ and } m.$$

By the continuity of  $h(\cdot)$

$$\lim_{k \rightarrow \infty} h(\sigma_i^k, \bar{\sigma}_{-i}, n_i(\sigma_i^k, \bar{\sigma}_{-i})) = h(\bar{\sigma}_i, \bar{\sigma}_{-i}, n_i') = 0$$

and

$$\lim_{m \rightarrow \infty} h(\sigma_i^m, \bar{\sigma}_{-i}, n_i(\sigma_i^m, \bar{\sigma}_{-i})) = h(\bar{\sigma}_i, \bar{\sigma}_{-i}, n_i'') = 0.$$

But

$$h(\sigma_i', \bar{\sigma}_{-i}, n_i') > h(\bar{\sigma}_i, \bar{\sigma}_{-i}, n_i').$$

The contradiction proves the assertion. //

## 5. Existence

As discussed earlier, there are different specifications for government objectives. One is a specification in which government interests coincide with those of the mobile factor. For this case, discussed in Section 6, existence is relatively straightforward to demonstrate. Another is a specification in which the government seeks to enhance the welfare of a group of permanent (immobile) residents.<sup>3</sup> For this type of government proving existence is much more complicated and, for the most part, not so optimistic.

Since this case is the focus of the paper it is the one that we consider first. We proceed with a discussion of the general theory of existence for this model. This is

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<sup>3</sup> Examples of this sort of model are those that focus on competition for factors of production. They often have as their concern, either commodity or capital taxation.

followed by an example designed to demonstrate that our results provide sufficient and not necessary conditions for existence. The example also reveals how the mobile factor's preferences for public goods can be pivotal in determining whether an equilibrium exists.

## 5.1 General Theory

Suppose that state welfare is equivalent to the welfare of homogeneous permanent (immobile) residents who benefit from the presence of the mobile factor and pursue policies to attract it. The benefit to a state can be expressed as a strictly quasi concave, continuous function of the quantity,  $n_i$ , of the mobile factor supply in the state,  $B_i(n_i)$ . In most models, the benefit is the residual of the contribution of the mobile factor to state output over the return paid to the factor<sup>4</sup>,

$$B_i(n_i) = f_i(n_i) - w_i(n_i)n_i. \quad (5.1)$$

The cost incurred by state  $i$  in attracting a given quantity of the mobile factor depends on the values of its policy choices as well as those of its neighbors (if they are continuous variables) and the quantity of the mobile factor attracted. This can be seen from an example in which states each have just two instruments: a per unit subsidy  $s_i$  paid directly to the mobile factor (a negative value implies a tax) and a local public good  $q_i$  provided for the benefit of the mobile factor. Thus,  $\sigma_i = (s_i, q_i)$  and the cost incurred by state  $i$  to attract a particular quantity of the mobile factor is  $c_i = s_i n_i + q_i$ . The cost is dependent on the value of the subsidy (tax), the quantity of the mobile factor and the level of public good provision. From Lemma 1,  $n_i$  is also a function of the policies adopted by state  $i$  and its neighbors. Therefore, for our more general case of multiple policies the cost is

$$c_i(\sigma_i, \sigma_{-i}, n_i). \quad (5.2)$$

From Theorem 1 a non-malevolent government will wish to act efficiently and hence seek the policy vector that minimizes its costs for a given value of  $n_i$ . Thus, for a given  $n_i$  the state's policy choices will solve the minimization problem

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<sup>4</sup> The permanent residents are residual claimants.

$$\begin{aligned} & \min_{(\sigma_i)} c_i(\sigma_i, \sigma_{-i}, n_i) \\ \text{s.t.} \quad & n_i(\sigma_i, \sigma_{-i}) \geq n_i. \end{aligned} \quad (5.3)$$

Assuming that  $c_i(\cdot)$  is quasi-convex in the strategies<sup>5</sup> then the concavity of  $n_i(\cdot)$  established in Lemmas 1 and 2 assures that there is a unique minimum cost for achieving all feasible values of  $n_i$ , for a given value of  $\sigma_{-i}$ . This guarantees the existence of a cost function,

$$C_i(n_i | \sigma_{-i}) = \min_{\sigma_i} \{c_i(\sigma_i, \sigma_{-i}, n_i)\}. \quad (5.4)$$

$C_i(\cdot)$  is positive or negative for different values of  $n_i$  (given  $\sigma_{-i}$ ) depending on the particular mix of policies within the policy vector,  $\sigma_i$ . For example, in the case noted above where states have two policies, a tax/subsidy and a public good,  $C_i(\cdot)$  is negative if  $s_i$  is negative (the mobile factor is taxed) and  $s_i n_i > q_i$ . In such a case, the government of state  $i$  redistributes from the mobile factor to its permanent residents, reducing the total payment to the mobile factor below its competitive return,  $w_i(n_i)$ . The first of our numerical examples presented below is just this case. Alternatively, if  $C_i(\cdot)$  is positive the mobile factor earns its return and receives a subsidy from the residual income of the permanent residents.

State welfare,  $V_i(n_i)$ , is the difference between the benefit (5.1) and the minimum cost to produce that benefit, given by (5.4):

$$V_i(n_i) = B_i(n_i) - C_i(n_i | \sigma_{-i}). \quad (5.5)$$

Because  $n_i$  is a continuous function of the strategies state welfare is also a function of those strategies,

$$W_i = V_i(n_i(\sigma_i, \sigma_{-i})) = B_i(n_i(\sigma_i, \sigma_{-i})) - C_i(n_i(\sigma_i, \sigma_{-i}) | \sigma_{-i}) = W_i(\sigma_i, \sigma_{-i}). \quad (5.6)$$

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<sup>5</sup> In the simple two policy example we know that  $c_i = s_i n_i + q_i$  which is linear in the policies (and hence quasi-convex). However, if costs were something like  $c_i = n_i \sqrt{s_i} + q_i$  then they would not be quasi-convex in the strategies.

**Theorem 2 (Existence).** *If  $V_i(\cdot)$  is a continuous and strictly quasi concave function of  $n_i$  for all states  $i$ , then a Nash equilibrium exists.*

**Proof:** If  $V_i(\cdot)$  is a continuous and strictly quasi concave function of  $n_i$  for all states  $i$ , then, because  $n_i(\cdot)$  is continuous and strictly quasi concave in  $\sigma_i$  for all states,  $W_i(\cdot)$  is continuous and strictly quasi concave in  $\sigma_i$  for all states. The policy game with interstate competition for a mobile factor is then defined on a non empty, convex and compact Euclidian strategy space,  $S$ , and each state's objective is a continuous and strictly quasi concave function of its own strategies. It follows<sup>6</sup> that a Nash equilibrium exists for the fiscal competition game.//

Since  $B_i(\cdot)$  and  $C_i(\cdot)$  are continuous in  $n_i$  and  $B_i(\cdot)$  is a quasi concave function of  $n_i$ , one case in which  $V_i(\cdot)$  will be a continuous and strictly quasi concave function of  $n_i$  is where the cost function is a strictly quasi convex function of  $n_i$ . Thus, convexity of the cost function is a sufficient condition for existence of a Nash equilibrium to the fiscal competition game.

Of course, the convexity of  $C_i(\cdot)$  is a sufficient and not a necessary condition for existence. An equilibrium may still exist even without a convex cost function. Moreover, even if the sufficient condition for existence holds there may still be more than one Nash equilibrium.

## 5.2 Example

A two state ( $i = 1,2$ ) and two policy (tax/subsidy and public good) example is now presented in which there is a fixed mobile factor population of size  $n$  in the region (complete isolation from the rest of the world). One aim is to show how a strong preference by the mobile factor for the public good can lead to non convexity in the cost function, and hence non-existence. The example also highlights that our theorem on existence provides sufficient, and not necessary, conditions for existence.

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<sup>6</sup> See Mas-Collell, et.al, p 253.

In the example, both states are the same; this is a symmetric example in which the technologies (production functions) are the same in both states (a case often considered in the fiscal competition literature because of its convenient properties). The fixed mobile factor assumption implies that  $n_2 = n - n_1$ . States provide a subsidy (tax)  $s_i$  and a public good  $q_i$  for the mobile factor ( $\sigma_i = (s_i, q_i)$ ).

The mobile factor payoff is assumed to be a quasi-linear function of its net income,  $x_i = w_i(n_i) + s_i$ , and consumption of the public good,

$$p_i = x_i + q_i^\beta. \quad (0 < \beta < 1) \quad (5.7)$$

The output of a state is a simple exponential function of its mobile factor supply,  $f_i = Fn_i^\alpha$ , where  $\alpha < 1$  and  $F$  is a positive constant, the same for both states. With this technology the return to the mobile factor is

$$w_i(n_i) = \alpha Fn_i^{\alpha-1}. \quad (5.8)$$

If  $s_i$  is the subsidy (tax) chosen by state  $i$ , the mobile factor equilibrium condition is

$$\alpha Fn_1^{\alpha-1} + s_1 + q_1^\beta = \alpha Fn_2^{\alpha-1} + s_2 + q_2^\beta. \quad (5.9)$$

Henceforth, the example is carried forward from the point of view of state 1. Since the states are identical, the analysis and conclusions apply to state 2 as well as to state 1. Notice that the benefit function in this example is  $B_i(n_i) = (1 - \alpha)Fn_i^\alpha$  which is a concave, continuous function of  $n_i$ .

The payoff  $P_1$  to the permanent residents of state 1 is equal to this benefit, less the cost associated with its policies. Therefore,

$$P_1 = (1 - \alpha)Fn_1^\alpha - c_1, \quad (5.10)$$

where  $c_1 = s_1 n_1 + q_1$ .

We know that a non-malevolent government wishes to adopt within-state efficient policies. This means that state 1 will choose to achieve its objective, given  $s_2$  and  $q_2$ , at the minimum possible cost. Therefore, the optimal policies of the state are found at the tangency of the boundary of the level set for a given value of  $n_1$  and the isocost line for a

given value of  $n_1$ . The value of  $s_1$  and  $q_1$  that produce a given value of  $n_1$  are those combinations that leave the value of  $P_1$  constant (if  $n_1$  is fixed so is  $w_1(n_1)$ ).

Thus, along the boundary of the level set  $s_1 + q_1^\beta$  must be constant and the slope of the boundary of the level set is

$$\frac{dq_1}{ds_1} = -\frac{1}{\beta} q_1^{1-\beta}. \quad (5.11)$$

The slope of the isocost line, considering that the price per unit of mobile factor of the public good is  $1/n_1$ , is  $-n_1$ . Equating the slope of the boundary of the level set with the slope of the isocost line gives the efficient level of public good provision (the one consistent with cost minimization),

$$q_1 = (\beta n_1)^{\frac{1}{1-\beta}}. \quad (5.12)$$

Because the states are non-malevolence both provide the efficient level of public good. Notice that the larger is  $\beta$  the larger is the level of public good for every size of the mobile factor. Employing the mobile factor equilibrium condition (5.9) the efficient level of subsidy (tax) is

$$s_1 = \alpha F[(n - n_1)^{\alpha-1} - n_1^{\alpha-1}] - (\beta n_1)^{\frac{\beta}{1-\beta}} + s_2 + q_2^\beta. \quad (5.13)$$

Substituting the equations for the cost minimizing policy choices, (5.12) and (5.13), into the state 1 payoff function, (5.10), yields the payoff to state 1 as a function of its mobile factor supply, conditional on the policies of its neighbors (and state 1 acting efficiently):

$$P_1 = F n_1^\alpha - \alpha F n_1 (n - n_1)^{\alpha-1} + \beta^{\frac{\beta}{1-\beta}} (1 - \beta) n_1^{\frac{1}{1-\beta}} - n_1 (s_2 + q_2^\beta). \quad (5.14)$$

To examine concavity of the payoff function, and hence existence, we need the first and second derivatives of the state 1 payoff function with respect to  $n_1$ :

$$\frac{\partial P_1}{\partial n_1} = \alpha F n_1^{\alpha-1} - \alpha F (n - n_1)^{\alpha-1} - \alpha(\alpha-1) F n_1 (n - n_1)^{\alpha-2} + (\beta n_1)^{\frac{\beta}{1-\beta}} - (s_2 + q_2^\beta) \quad (5.15)$$

and

$$\frac{\partial^2 P_1}{\partial n_1^2} = \alpha(\alpha - 1)F[n_1^{\alpha-2} + (n - n_1)^{\alpha-3}(2n - \alpha n_1)] + \frac{\beta^{\frac{1}{1-\beta}}}{1-\beta} n_1^{\frac{2\beta-1}{1-\beta}}. \quad (5.16)$$

The result here is convenient because the second derivative is independent of the state 2 strategy choice. To be more specific, consider the case in which the parameters are  $\alpha = 0.5$ , and both  $F$  and  $n$  are 10. With these values fixed, we can examine what happens to the sign of the second derivative, and hence existence, when the mobile factor's preference for the public good changes. A particularly handy specification is for  $\beta = 0.5$ . In this case, the second term of (5.16) is a constant and the second derivative is

$$\frac{\partial^2 P_1}{\partial n_1^2} = -2.5[n_1^{-1.5} + (10 - n_1)^{-2.5}(20 - 0.5n_1)] + 0.5 \quad (5.17)$$

Clearly, (5.17) is negative for all values of  $n_1 \in [0, 10]$ . Thus, the state 1 payoff function is strictly concave for all feasible values of  $n_1$ . Because the states are identical, the state 2 payoff function is similarly concave. The existence of a Nash Equilibrium is assured for this case and it is unsurprising that, in equilibrium, each state has an equal share of the fixed mobile factor supply ( $n_1 = n_2 = 5$ ). In this case, the equilibrium is

symmetric and the value of the public good is  $q_i = \left(\frac{5}{2}\right)^2 = 6.25$ , the subsidy is  $s_i = -\frac{2.5}{\sqrt{5}}$

(a tax) and the mobile factor return is  $w_i = \frac{5}{\sqrt{5}}$ . With these values the income (payoff)

to the permanent residents of each state is  $R_i = 1.46$  and the mobile factor monetary

reward is  $x_i = \frac{2.5}{\sqrt{5}}$ .

However, a  $\beta$  value of 0.6 demonstrates that global concavity of the state objective functions is a sufficient, but not necessary, condition for existence of an equilibrium. With  $\beta = 0.6$ , the second derivative of the state objective is positive for state mobile factor population in the range  $2.2 \leq n_i \leq 6.3$  and negative outside this range. The implication is that the symmetric equilibrium does not exist. Interestingly, this is a case often studied in the fiscal competition literature. Nonetheless, an outcome in which one

state has a mobile factor supply of approximately  $n_i = 1.9$  and the other  $n_{-i} = 8.1$  is an equilibrium, with policies

$\sigma_i = (s_i, q_i) = (-0.20, 1.35)$  and  $\sigma_{-i} = (s_{-i}, q_{-i}) = (-7.86, 52.37)$ . The payoffs to each state are 20.30 and 25.72 respectively.<sup>7</sup>

Finally, for  $\beta = 0.9$ , the second derivative of the payoff function is positive throughout the admissible range of  $n$ 's, except at the extreme values  $n_1 \rightarrow 0$  and  $n_2 \rightarrow 0$ . In this case, it can be shown that neither extreme value is an equilibrium. Thus, we conclude that an equilibrium does not exist in this case.

## 6. The Mobile Factor Government

As discussed, another model considered in the fiscal competition literature supposes that the government represents the interests of the mobile factor (no permanent residents). Examples here include the papers noted in the Introduction; Boadway and Flatters (1982), Wildasin (1988) and Myers (1990). As long as the mobile factor objective is strictly concave in the feasible strategies, the free mobility condition ensures the existence of a Nash equilibrium for this type of model. Furthermore, while there may be multiple equilibria, the most efficient outcome possible with the feasible policy instruments available can be supported as an equilibrium. We name this a 'conditionally efficient' equilibrium, defined as follows:

**Definition:** An equilibrium is 'conditionally efficient' if there are no feasible policies, i.e., no  $\sigma_i \in S_i$ , such that at least one citizen can be made better off without reducing the well-being of any other citizen.

**Theorem 3:** *If government policy is chosen to maximize the welfare of the mobile factor, conditionally efficient policies  $(\mathbf{s}_i^*, \mathbf{s}_{-i}^*)$  are a Nash Equilibrium of the interstate policy game.*

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<sup>7</sup> The example is a caution to those who employ the symmetric equilibrium in their analysis. States may both be the same, nonetheless, their equilibrium strategies may be quite different.

**Proof.** For every state  $i$ , let all other states choose efficient  $\sigma_{-i}^*$ . Then,  $\sigma_i^*$  is the value of state  $i$ 's policies that maximize  $p_i(\sigma_i, n_i(\sigma_i, \sigma_{-i}^*))$ . This establishes the theorem.//

Theorem 3, which is a generalization of Myers (1990), shows that any policy that is efficient, given the feasible policy instruments, can be supported as a Nash Equilibrium.

## 7. Conclusion

A number of studies examine public policy in the presence of inter-jurisdictional spillovers. These works, no matter their policy orientation, e.g., taxation, public expenditures or environmental quality, share certain characteristics and limitations. They generally deal with one policy instrument and, while they model inter-jurisdictional competition as a game and proceed to analyze the nature of the equilibria of that game, they do not take up the question of existence of equilibrium. We expand the scope of the analysis by addressing both limitations.

Rather than examine one policy instrument, we recognize that public policy is directed at achieving some objective (in this case a desired mobile factor supply) and that there are multiple instruments at the disposal of the governments (in the examples of this paper, direct subsidies (or taxes) and public expenditures). Any government that is not malevolent towards any of its citizens or group of citizens will choose only feasible policy bundles that are efficient. This recognition allows us to limit our search for equilibrium to only those sets of feasible policies that are efficient. We show that for the existence of an equilibrium it is sufficient that the government objective functions are continuous and strictly quasi concave in the desired mobile factor supplies.

We examine models in which inter-jurisdictional externalities are created by the migration of factors of production in response to public policies. The factors move between jurisdictions (vote with their feet) as long as there are differences in the payoffs to them. Given the factor payoff equality condition, we examine two polar specifications for policy objectives. One is policy concerned only with the welfare of permanent residents and another is policy concerned with the welfare of the mobile factor.

For the immobile resident welfare objective it is sufficient for the existence of equilibrium that the cost of efficient policies is a continuous and convex function of mobile factor supplies. We show, through use of examples that, while it is sufficient, cost convexity is not necessary. With mobile factor welfare objectives only, equilibrium always exists as long as there is free mobility. While there may be multiple equilibria, at least one is feasible efficient, i.e., efficient conditional on the feasible set of policy instruments.

## References

- Bewley, Truman (1981) A Critique of Tiebout Theory of Local Public Expenditures *Econometrica* 49:3 713-40
- Braid, R.M (1996). Symmetric Tax Competition with Multiple Jurisdictions in Each Metropolitan Area. *American Economic Review*. 86: 1279-90.
- Bucovetsky, S (1991). Asymmetric Tax Competition. *Journal of Urban Economics*. 30:167-81.
- Burbidge, J. B., and G.M. Myers (1994). Redistribution Within and Across the Regions of a Federation. *Canadian Journal of Economics*. 27:620-36.
- Mansoorian, A., and G.M Myers (1993). Attachment to home and efficient purchases of population in a fiscal externality economy. *Journal of Public Economics*. 52:117-32.
- Mas-Colell, A., M.D. Whinston and J.R. Green, (1995) *Microeconomic Theory*, Oxford University Press.
- Myers, G.M (1990). Optimality, Free Mobility and the Regional Authority in a Federation. *Journal of Public Economics*. 43:107-21.
- Oates, W.E., and R.M. Schwab (1988). Economic Competition Among Jurisdictions: Efficiency Enhancing or Distortion Inducing? *Journal of Public Economics*. 35:333-54.
- Rosen, J.B. (1965). Existence and uniqueness of equilibrium points for concave n-person games. *Econometrica*. 33:3.
- Wellisch, D., and D.E. Wildasin (1996). Decentralized income redistribution and Immigration. *European Economic Review*. 40: 187-217.
- Wellisch, D., (2000). *Theory of Public Finance in a Federal State*. Cambridge University Press.
- Westhoff, Frank (1977). Existence of Equilibria in Economies with a Local Public Good *Journal of Economic Theory* 14 84-112.
- Whitman, Donald (1989). Why Democracies Produce Efficient Results *The Journal of Political Economy* 97:6 1395-1424.
- Wildasin, D.E. (1988). Nash Equilibrium in Models of Fiscal Competition. *Journal of Public Economic*. 35: 229-40.
- Wildasin, D.E. (1991a). Income Redistribution in a Common Labor Market. *American Economic Review*. 81: 757-74

Wildasin, D.E. (1991b). Some Rudimentary 'Duopolity' Theory. *Regional Science and Urban Economics*. 21:393-421.

Wildasin, D.E., and J.D. Wilson (1991). Theoretical Issues in Local Public Economics. *Regional Science and Urban Economics*. 21: 317-31.