Foreclosure Delays and Default Incentives

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Abstract

The recent financial crisis brought with it an enormous increase in mortgage default and foreclosure filings. The influx created a massive congestion problem for banks and the courts. How does such congestion, and the consequent delays, affect a household's decision to enter delinquency? Interestingly, I find two competing effects of foreclosure delays. There is a moral hazard effect, where households realize they can skip payments with limited punishment. Without delays this effect is absent and as delays increase this effect increases mortgage default. There is also an insurance effect, where increased delays provide households with a better chance of keeping their homes during unemployment spells. A household with few liquid assets that expects to have a long unemployment spell would not expect to survive delinquency prior to foreclosure. However, with delays, this same household can now expect to withstand a longer unemployment spell. With this insurance effect, delays reduce the households incentive to default today. After describing the model results, I use the Survey of Consumer Finances to find some empirical evidence in support of the model.

Keywords: Mortgage Default, Delinquency, Foreclosure Delay

JEL Classification: D14, E21, E61, G21, R3

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1. Introduction

During the recent housing market crisis there was an unprecedented increase in mortgage delinquencies. Delinquency rates increased to over ten percent and remained above that level for several years. While there were also large increases in delinquencies among other forms of debt, such as commercial and credit card loans, no other form of debt had delinquencies rise by such a large amount and stay high for so long. To put into perspective how important this increase in delinquencies might be from the lender's point of view, an eight percentage point increase in the delinquency rate on single-family residential mortgages represents a loss of about $4.5 billion in monthly cash flow for lenders.[2]

The large volume of delinquencies led to significant delays in the foreclosure process. Foreclosure timelines pre-recession averaged 11 months in states where lenders could avoid the court system and 16 months in states where they could not, but these timelines increased to 19 and 28 months, respectively, following the crisis. Some states, like New York and Florida, have had extremely long foreclosure times (see Cordell et al. (2015) and Zhu and Pace (2015)). As of Q3 2016, New York and Florida have average foreclosure timelines of 35.2 and 34.1 months, respectively. Foreclosure delays matter for defaulters because they provide a period of free rent for households, increasing the value of foreclosure (see Ambrose et al. (1997)). During the recent crisis, the value of these foreclosure delays, which averaged eight extra months of delinquency, represents $41.2 billion in terms of avoided rent for borrowers who end up foreclosing.[3] In addition to providing free rent, these delays extend the foreclosure process for households providing additional time to possibly cure the delinquency spell. Overall, delays have remained persistently high following the housing market crisis and households now make default decisions taking delays into account.

This paper studies how foreclosure delays affect a homeowner's decision to enter delin-

[2]This back of the envelop calculation is based on an average monthly mortgage payment of $1,061 with 53.53 million residential mortgages.

[3]This back of the envelop calculation is based on 4.12 million seriously delinquent properties avoiding eight months of $1,250 rent payments (the U.S. average monthly rent). This rent is what the household would have paid had they been foreclosed on eight months earlier. The assumption is that the vast majority of serious delinquencies end in foreclosure.

quency. The previous literature examining this question dates back to Ambrose et al. (1997), who built the effects of foreclosure delays into an option pricing model of mortgage default. They find that delays increase the probability of default because "free rent" increases the likelihood that the default put option is "in the money." More recent empirical literature, examining the effects of foreclosure delays during the recent housing market crisis, is collectively ambiguous about the effects of delays. Some research finds that delays increase defaults (see Gerardi et al. (2013), Zhu and Pace (2015)), some finds that delays have no effect (see Ghent and Kudlyak (2011)), and some finds that delays lead to higher loan modification rates or cures (see Collins et al. (2011)). Calem et al. (2015) finds that while delays might not improve a household's likelihood of exiting delinquency, delays have a positive effect on household balance sheets and their ability to pay non-mortgage debt.

Some recent quantitative literature has also examined the effects of delays. Chatterjee and Eyigungor (2011) find that foreclosure delays had a large effect, increasing the number of foreclosures during the crisis, but mitigated some of the home price decline because it kept homes off of the for-sale market. One issue with these findings is that delinquency is not reversible in their model. Delays only act to reduce the cost of default by providing households a period of free rent, but households cannot cure their delinquency spells once they make their initial default decision. Herkenhoff and Ohanian (2015) allow for a reversible delinquency episode. They show that households use the delinquency process as an implicit loan, wherein as long as households eventually pay off their missed payments, the household can exit the foreclosure process and return to current status on their mortgage. Their paper focuses on the employment effects of foreclosure delays whereas my paper focuses on the mortgage default effects.

In my paper, I add to this literature by developing a structural model of the household's default decision with delays. In the model, households are endowed with mortgages, which are finite-length, collateralized debt contracts that provide housing services. Households receive Markov income shocks which may "trigger" delinquency spells. This is in line with the recent literature finding that default occurs because households with negative equity

receive a shock that effects their ability to pay the mortgage – possibly an unemployment spell, a divorce, a health shock, etc. This is referred to as the "double trigger" theory of default.[4] Households default on their mortgages by skipping payments but by entering delinquency they incur foreclosure risk. The level of foreclosure risk when delinquent depends on foreclosure delays. Without delays, a household in delinquency will be foreclosed on quickly, but with delays, households expect foreclosure to occur after a long delinquency spell. Based on the household's knowledge of foreclosure risk and future expectations about income risk, they optimally choose whether or not to default.

I find two competing effects of foreclosure delays. There is a moral hazard effect, where households realize they can skip payments with limited punishment. Without delays, this effect is absent and as delays increase this effect increases mortgage default. There is also an insurance effect, where increased delays provide households with a better likelihood of keeping their homes during unemployment spells. A household with few liquid assets that expects to have a long unemployment spell would not expect to recover their mortgage prior to foreclosure. Thus, without delays, it is optimal to default early. However, with delays, the same household can now expect to withstand a longer unemployment spell and can use liquid assets to avoid the foreclosure process temporarily while they find new employment. Under this insurance effect, delays reduce the households incentive to default today. To my knowledge, this is the first paper in which these two effects coexist to explain mortgage default.[5] My paper explains why the empirical literature is ambiguous on the effects of delays. I also provide a model with which policy makers can design foreclosure relief programs to limit the moral hazard problem.

To provide some empirical support for the model findings, I use the Survey of Consumer Finances (SCF) to compare short-term and seriously delinquent households. I find that both households with no delinquency and households with only short-term delinquency

[4] For references, see Foote et al. (2008),Garriga and Schlagenhauf (2009),Campbell and Cocco (2015),Corbae and Quintin (2015), Hatchondo et al. (2015), Gerardi et al. (2015)).

[5] Chetty (2008) finds similar effects with unemployment duration when increasing unemployment insurance benefits. In that paper, both effects operate in the same direction by increasing unemployment duration.

share the same unemployment duration, while households with serious delinquency have an average unemployment duration twice as long as the other two groups. This matches what the model suggests, that some households enter delinquency to offset the income loss from an unemployment spell and remain in delinquency until their unemployment spell has ended. I also find that the use of short-term delinquency increased among low asset deciles during the crisis. This suggests that some households are using delinquency because of the moral hazard effect; when delays are long enough it is optimal to enter delinquency for a short duration. Lastly, I examine what factors predict delinquency using a linear probability model. I find that having less than one month worth of mortgage payments in liquid assets increases the likelihood of entering delinquency by 19.3% to 25.8%. The effect of unemployment spells on delinquency varies much more over the sample I study, varying between 5.6% and 18.8%. Interestingly, I find that negative equity is a statistically significant predictor of delinquency in only one year of the sample, supporting the claims of Foote et al. (2008) and Bhutta et al. (2010) that negative equity alone is not enough to generate default.

This paper is organized as follows. In Section 2, I provide some details about the foreclosure process, foreclosure timelines, and the penalties of default for homeowners. In Section 3, I develop a model of the household's default decision with foreclosure delay. In Section 4, I solve the model numerically and discuss the model mechanics. I show how default decisions change when foreclosure delay increases. In subsection 4.5, I simulate how delinquency and foreclosure dynamics might change when labor market risk increases and home prices decrease. In section 5, I use the SCF to provide empirical support for some of my model findings. Lastly, in Section 6, I conclude.

2. The Foreclosure Process and Foreclosure Timelines

2.1. Steps of the Foreclosure Process

The details of the foreclosure process differ greatly across states but the key steps that a mortgagor faces are similar. After missing a mortgage payment, the mortgagor enters delinquency. Most lenders have a late-payment grace period after which a late fee will be

assessed and a letter will be sent to the borrower notifying them of their delinquency status. In some states, like California, law requires that the mortgage servicer personally contact the delinquent mortgagor by phone or in person 30 days before filing an initial notice of default. The mortgagee will sometimes provide the mortgagor an option to make partial payment to avoid having the delinquency referred to collections.

After about 90 days of delinquency, the lender sends the delinquent borrower an initial notice of default which formally starts the foreclosure process. In 2014, the Consumer Financial Protection Bureau servicing rules changed and now require a mortgage servicer to wait until 120 days delinquent before sending the borrower an initial notice of default. There are two main foreclosure processes that have emerged in the U.S. – judicial and statutory foreclosure. In states with judicial foreclosure, the proceeding must go through the court system and a judge or court official will oversee the proceedings. In states with statutory foreclosure, the lender can proceed without a court official based on state law and a "power of sale" clause. As of 2008, non-judicial or statutory foreclosure is available in 35 states and is the usual procedure in 28 states (Ghent (2012)). For a more detailed description of how judicial and statutory states differ, see the discussions in Gerardi et al. (2013) and Ghent (2012).

During the foreclosure process, the lender attempts to sell the property. A notice of sale is sent to the mortgagor prior to the auction which lists the time and place of the foreclosure auction. In California, the notice of sale must be recorded at least 20 days before the sale date and published once a week for three consecutive weeks. The borrower is allowed to cure the delinquency up to five days prior to the foreclosure sale, after which date the home is either sold at auction or ownership is given to the foreclosing lender and the home is said to be real-estate owned ("REO"). After the sale of the property, the foreclosed borrower is supposed to vacate the property. The new owner will typically either offer a cash-for-keys deal, offering money in exchange for vacating the property, or the owner will take steps to evict the borrower. In some states, borrowers are offered one last chance to cure their delinquency spell. In these states borrowers have a redemption period, during which they

can pay off their debt owned on the mortgage and any accrued fees and reclaim their home. Redemption periods are offered in nine states and across these states the redemption period averages 103 days (Cutts and Merrill (2008)).

If the lender cannot recover the full remaining balance of the debt owed through foreclosure auction, then in some states the lender can request a deficiency judgment. Lenders can request deficiency judgments in 38 states, called recourse states, and cannot request them in 10 states, called non-recourse states (Ghent and Kudlyak (2011)). However, while most states are recourse, because it is costly for banks to pursue deficiency judgments in terms of legal costs and time and because many foreclosed borrowers have very little non-housing wealth, lenders rarely seek deficiency judgments (Leland (2008) and Brueggeman and Fischer (2011)). Ghent (2012) argues that while deficiency judgments may not always be pursued, the threat of recourse has significant effects in deterring foreclosure.

2.2. Foreclosure Timelines

The steps of the foreclosure process outlined above can take a significant amount of time. Cordell et al. (2015) estimates foreclosure times during the crisis and finds that the overall time in foreclosure can last anywhere from 11 to 16 months.[6] They find that foreclosure delay increased this timeline to 19 to 28 months. Cordell et al. (2015) also breaks down the timeline into time taken at each step of the foreclosure process. They find that the majority of time occurs between initial notice of default and foreclosure auction, the step that most depends on whether a state has a judicial or statutory foreclosure process. Prior to delays, this phase would take about 5 months in non-judicial states and about 10 months in judicial states, but by 2012 these timelines had increased to over 10 and 20 months, respectively.

There are many impediments that can further impede the foreclosure process. The homeowner can postpone the foreclosure process through bankruptcy, through requested negotiation (in some states), or by challenging the banks' legal rights to foreclose on the

[6] Zhu and Pace (2015) also estimates foreclosure times during this period using similar data to Cordell et al. (2015).

property. Moreover, following the robo-signing scandal in 2010, when it became known that many banks were foreclosing on properties without legal merit, many foreclosures were frozen and paperwork requirements for foreclosure were increased to ensure future foreclosures were not invalid. This led to a further increase in foreclosure timelines, particularly in judicial states. Following all the controversy surrounding improper foreclosures and the large number of distressed homeowners in delinquency, many states started to enact laws to protect homeowners. Some of these laws, like the 2013 California Homeowner Bill of Rights, have led to permanent increases in the duration of the foreclosure process. With longer foreclosure processing times becoming the new norm, it is important to consider how a lengthened foreclosure process might impact the homeowner's default decision.

2.3. Delinquency Cures and Penalties

While some delinquencies end in foreclosure, Herkenhoff and Ohanian (2015) show that many delinquent mortgagors exit the foreclosure process and cure their delinquency spell. Using data from Lender Processing Services, Inc. (LPS), they find that delinquency is often temporary, with about 40% of 60 day delinquent households transitioning to current status between 2001-2003. They also find that the cure rate declines as homeowners remain in delinquency, with this transition rate to current status falling to 10.5% for 90 day late households. Among mortgagors in foreclosure, they find that a completed foreclosure is about equally as likely as a cure, supporting findings from Pennington-Cross (2010). Thus, unlike the majority of the literature, this paper studies how default decisions change when the delinquency process is reversible.

There are many ways in practice that mortgagors cure their delinquency spell. Cutts and Merrill (2008) discuss this in detail. The most common way delinquent borrowers exit delinquency is by accepting an alternative repayment plan. These plans typically spread the accrued missed payments and late fees over a period of time during which the borrower pays an additional amount in addition to regular monthly payments. A second option is a loan modification. In a loan modification, the total balance owed or the time horizon for the mortgage may be altered to improve the borrower's ability to pay the mortgage.

These are most common during longer duration delinquency spells. Most mortgage servicers are incentivized to minimize foreclosure and negotiate workout plans to keep the borrower in their homes. For example, Freddie Mac servicers are offered $250 for each successful repayment plan, $400 for approved loan modifications, $275 for deed-in-lieu of foreclosures negotiated, and $1,100 for short-sales executed.

Entering and exiting delinquency is not without consequence for the borrower. The main consequence is a reduction in the borrower's FICO score. Borrowers with high FICO scores can expect to see a large FICO score penalty relative to borrowers with lower FICO scores. The FICO score penalty also lasts longer for borrowers with higher scores. For borrowers with a FICO score around 720, the reduction in their score can take about three years to fully recover, whereas borrowers with a score around 680 can expect their scores to fully recover in about nine months. These FICO score penalties typically do not limit a borrowers ability to acquire credit, but they will increase the cost of credit.

Foreclosure has much larger and longer lasting consequences than delinquency. To begin with, a foreclosed borrower becomes temporarily ineligible from many types of mortgages. Fannie Mae and Freddie Mac will not approve a new mortgage within four years of foreclosure, and the American Bankers Association (ABA) says it can take three to seven years to obtain a new mortgage. If a borrower is found to have strategically defaulted, Fannie Mae will not approve a new mortgage within seven years of foreclosure. These borrowers will also be ineligible from Federal Housing Authority (FHA) loans for up to seven years, with the exact time depending on the borrowers credit history after foreclosure among other criteria. In addition to being ineligible for many mortgages, foreclosed borrowers will have a flag on their credit history that lasts for seven years. This flag can have significant consequences when applying for other loans, leases, or even employment. Upon foreclosure, a borrower's FICO score will go down by about 100 points. The borrower's FICO score will recover over time but it takes about seven years to fully recover.

Loan modifications and alternative exits from foreclosure have more modest penalties than foreclosure. Short sales and deeds in lieu of foreclosure lead to similar effects on a

borrower's FICO score as a foreclosure. The benefit to these alternatives to foreclosure is that the borrower avoids the foreclosure flag on their credit history and may have an easier time applying for new mortgages. Some lenders still require a period of two to seven years after short sale and deeds in lieu of foreclosure before they are willing to offer a new loan. Loan modifications have much more minor effects on credit. There is only a minor impact to a borrower's FICO score, in addition to the penalty from having been delinquent.

3. The Model

To study how delays impact a households default decision, I develop a model of the households decision. The canonical models used to study mortgage default without delays fall into two categories. One strand of the literature uses an option pricing model in which households "ruthlessly" default when the expected value of default exceeds the value of continuing to pay the mortgage (Kau et al. (1993, 1994); Kau and Keenan (1995); Vandell (1995)). The other strand of literature uses models in which households with negative equity default on their mortgages following a secondary shock that impacts their ability to pay the mortgage(Foote et al. (2008); Garriga and Schlagenhauf (2009); Campbell and Cocco (2015); Corbae and Quintin (2015); Hatchondo et al. (2015); Gerardi et al. (2015)). These theories are often called "double trigger" theories of default. The model I develop is most similar to these double trigger models. The essential ingredients I include are an income shock, that pushes some households to default, and incomplete insurance markets which make default optimal in certain scenarios. I then use the model to see why households default and how that decision depends on the level of delays.

3.1. Environment

Time t is discrete with an infinite horizon. There is a unit mass of households who value consumption $c \in \mathbb{R}^+$, housing space $h \in \{h_0, h_1\}$, and discount at rate $\beta \in [0, 1)$. Households have preferences over their lifetime expected utility, represented by

$$E \sum_{t=0}^{\infty} \beta^t u(c, h).$$

Households can accumulate assets $a \geq -\underline{a}$ that generate a risk-free rate of return r. Households face income risk. Income, w, evolves over time according to a common discrete Markov process $F(w', w)$.

Households are endowed with mortgages at date $t = 0$. Mortgages are collateralized debt contracts which give the household housing space of size $h_1 > h_0$ each period for payment $m(z, j)$, where $z \in \mathcal{Z} = \{1, \ldots, Z\}$ represents the mortgage type (ARM, FRM, etc.) and j represents the number of mortgage payments previously made. Mortgages require a household to make a finite series of payments, with total number of payments $J(z)$. Households have until period $t = N$ to make all $J(z)$. If they fail to do so, they immediately complete their foreclosure.[7] The mortgage structure z characterizes the size of the down-payment $\mu(z)$, the amortization schedule $a(z, j)$, and the nominal mortgage interest rate $r^m(z, j)$. Based on these, the mortgage payment is $m(z, j) = l(z, j)r^m(z, j) + a(z, j)$, where $l(z, j)$ is the loan amount remaining after j previous payments. The loan amount begins at $l(z, 0) = (1 - \mu(z))x(0)h_1$, where $x(t)$ is the exogenous price per unit of housing space, and the loan amount evolves according to $l(z, j + 1) = l(z, j) - a(z, j)$. Mortgagors are not allowed to refinance their mortgages or receive home equity lines of credit.

Households are either current, if they are up-to-date on mortgage payments, or delinquent, if they missed a payment in the previous period. Each period the mortgagor chooses whether or not to make their mortgage payment. If the payment is made, the mortgagor continues to the next period as a current homeowner with one less remaining payment. If the payment is not made, the mortgagor enters delinquency and faces a probability $p_f(1)$ of foreclosure. The probability of foreclosure changes over the duration of the delinquency spell $p_f(d) \in [0, 1]$, where $d \in \{0, 1, \ldots, \bar{d}\}$ is the number of consecutive periods delinquent and $\bar{d}$ is the maximum number of consecutive periods delinquent such that the homeowner's foreclo-

[7] This assumption allows me to solve the model using backward induction starting in the last period in which the household can pay their mortgage. The model can also be solved using backward induction starting at the last mortgage payment. Both algorithms provide similar numerical results but the model with a fixed number of periods provides a simple environment in which to embed a time-varying exogenous home price. I choose a number of periods sufficiently large such that this parameter does not affect the household's decisions.

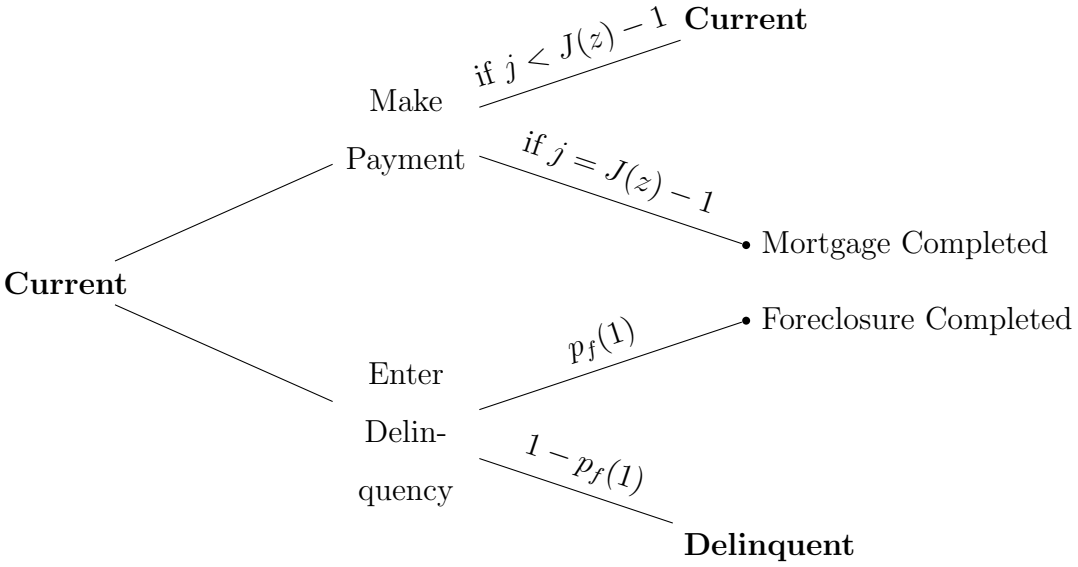
sure is completed with certainty if delinquent $\bar{d}$ periods, $p_f(\bar{d}) = 1$. Herkenhoff and Ohanian (2015) use a similar foreclosure probability to model foreclosure timelines. Delinquency is reversible and the mortgagor can exit delinquency by paying the lender the missed payment with interest and a late fee. The total payment to exit delinquent is $(1+r^m(z,j)+\Delta)^d m(z,j)$, where Δ represents the late fee. Note that to exit delinquency, I assume households only have to make one payment, not the full series of missed payments. This resembles a loan modification that extends the term of the mortgage by d periods. This is a tractable way to model the loan modification and these types of loan modifications are quite common in practice. When a mortgagor finishes making their $J(z)$th payment, their mortgage is paid off and they can keep housing space h_1 indefinitely with no monthly payment.

If a delinquent household is foreclosed, the mortgagor must forfeit housing services h_1 and instead receive housing services h_0 forever at no cost.[8] If foreclosed at date t, a household is given its home equity if positive, $eq(z,j;t) = \max\{x(t)h_1 - l(z,j), 0\}$.[9] This model is essentially a life-cycle model while the households have mortgages, with number of mortgage payments made replacing age, and a Bewley Model when mortgages are terminated, either through foreclosure or final payment.

The tree below describes a single model period for a household who starts the period current. Each period households observe their income shock and then choose whether or not to make their payment. A current homeowner can pay his mortgage and remain a current homeowner in the following period if there are remaining payments required or complete the mortgage if $j = J(z) - 1$. Alternatively, a current homeowner can skip his mortgage payment and enter delinquency. In this case he transitions to being a delinquent homeowner with probability $1 - p_f(1)$ or is foreclosed with probability $p_f(1)$.

[8] I assume foreclosed households receive h_0 units of housing space at no cost for computational simplicity. The difference between h_0 and h_1 is what is most important. Suppose $h_0 = h_1$, then households will have no incentive to pay their mortgages and all households will foreclose. As the spread between h_1 and h_0 grows, households will be more willing to pay their mortgages even though foreclosed households receive free housing. The spread between these two values captures the non-pecuniary cost of default.

[9] This assumption implies mortgages are non-recourse.



3.2. Household Decisions

Homeowners can either be current with value $v^c(a, w, z, j; t)$, delinquent with value $v^d(a, w, z, j, d; t)$, paid off with value $v^b(a, w, h_1)$, or foreclosed with value $v^b(a, w, h_0)$. A current homeowner's maximization problem can be described by $v^c(a, w, z, j; t) = \max\{C^C, C^D\}$ where, for $j < J(z) - 1$,

$$
\begin{aligned}
C^C(a, w,z, j; t) &= \max_{c,a'} u(c, h_1) + E_{w'|w}\beta\ v^c(a', w', z, j + 1; t + 1) \\
&\quad \textbf{s.t. } c + a' + m(z, j) = (1 + r)a + w \\
C^D(a, w,z, j; t) &= \max_{c,a'} u(c, h_1) + E_{w'|w}\beta\ [p^f(1)v^b(a' + eq(z, j; t), w', h_0) + \\
&\qquad (1 - p^f(1))v^d(a', w', z, j, 1; t + 1)] \\
&\quad \textbf{s.t. } c + a' = (1 + r)a + w.
\end{aligned}
$$

Current homeowners can decide whether to make their mortgage payment and remain current, if $C^C \geq C^D$, or not pay their mortgage payment and enter delinquency, if $C^C < C^D$. The decision rule $g_d^C(a, w, z, j; t) = 1$, where the subscript represents the choice variable and the superscript represents the current state C or D, characterizes the a current household's decision to enter delinquency and $g_d^C(a, w, z, j; t) = 0$ reflects the decision to not enter delinquency. For $j = J(z) - 1$ the problem is identical except that if the household makes the

payment, they transition into the paid off problem, $v^b(a', w', h_1)$, instead of remaining in the mortgage problem, $v^c(a', w', z, j+1; t+1)$.

A delinquent homeowner's maximization problem can be described by $v^d(a, h, z, j, d; t) = \max\{D^C, D^D\}$ where, for $j < J(z) - 1$ and $d < \bar{d}$,

$$\begin{aligned} D^C(a, w, z, j, d; t) &= \max_{c,a'} u(c, h_1) + E_{w'|w}\beta\, v^c(a', w', z, j+1; t+1) \\ &\textbf{s.t. } c + a' + (1 + r^m(z, j) + \Delta)^d m(z, j) = (1 + r)a + w \\ D^D(a, w, z, j, d) &= \max_{c,a'} u(c, h_1) + E_{w'|w}\beta\, [p^f(d+1)v^b(a' + eq(z, j; t), w', h_0) + \\ &\qquad (1 - p^f(d+1))v^d(a', w', z, j, d+1; t+1)] \\ &\textbf{s.t. } c + a' = (1 + r)a + w. \end{aligned}$$

Similar to current homeowners, delinquent homeowners will choose to exit delinquency if $D^C \geq D^D$ or remain in delinquency if $D^C < D^D$. The decision rule $g_d^D(a, w, h, z, j, d; t) = 1$ characterizes the a delinquent household's decision to remain delinquency and $g_d^d(a, w, h, z, j, d; t) = 0$ reflects the decision to exit delinquency. If $d = \bar{d}$, the homeowner will transition into the foreclosed problem $v^b(a, w, h_0)$ with certainty instead of having a probability of remaining delinquent.

Households without mortgages are simple. They are infinitely-lived Bewley agents with fixed housing spaces. They manage their stock of assets to self-insure against future income shocks. Their maximization problem can be described by $v^b(a, w, h)$, where the only difference between paid and foreclosed households is the level of housing services h:

$$\begin{aligned} v^b(a, w, h) = &\max_{c,a'}\ u(c, h) + E_{w'|w}\beta\, v^b(a', w', h) \\ &\textbf{s.t. } c + a' = (1 + r)a + w. \end{aligned}$$

4. Model Results

In each period, households solve a constrained optimization problem in which they must allocate disposable income between current consumption, next period assets, and paying their mortgage. If the household's disposable income $w + (1 + r)a$ is less than their

mortgage payment $m(z,j)$, they will not be able to make a mortgage payment and involuntarily enter delinquency. If the household's disposable income is greater than their mortgage payment, they may still choose to enter delinquency. Using the value functions $C^C(a,w,z,j;t)$ and $C^D(a,w,z,j;t)$ evaluated at the optimal asset choice, $a'^*_{cc}(a,w,z,j;t)$ when choosing to stay current and $a'^*_{cd}(a,w,z,j;t)$ when choosing to enter delinquency, I can derive an equation describing when households enter delinquency[10]:

$$\begin{aligned} u(w+(1+r)a-a'^*_{cd}(\cdot),h_1)-u(w+(1+r)a-a'^*_{cc}(\cdot)-m(z,j),h_1) \\ > \ \beta E_{w'|w}\beta[v^c(a'^*_{cc}(a,w,z,j;t),w',z,j+1;t+1) \\ -(p^f(1)\ v^b(a'^*_{cd}(\cdot)+eq(z,j;t),w',h_0) \\ +(1-p^f(1))\ v^d(a'^*_{cd}(\cdot),w',z,j,1;t+1))] \end{aligned} \tag{1}$$

This equation shows that households are trading off the cost of a potential reduction in housing services in the future, on the right hand side (RHS), with the benefit of increased resources today, on the left hand side (LHS). Delays change this default equation by changing the continuation value on the right hand side of the inequality. With no delays, $p^f(1)=1$, households transition directly into foreclosure, $v^b(a'^*_d(\cdot)+eq(z,j;t),w',h_0)$, whereas with delays they can remain delinquent for some time.

Equation 1 illustrates several reasons why a household might strategically default. If the household's disposable income $w+(1+r)a$ is sufficiently small that their budget will only provide a small level of consumption when choosing to pay, then the LHS will be large and this equation will hold. Households value the housing services they are guaranteed by paying their mortgage, as illustrated on the RHS, but with concave utility, the marginal value of increased resources will be greater than the expected reduction in housing services. In addition, it may also be optimal to default if foreclosure risk is low. The RHS of equation

[10]This equation holds when the household has more than one payment remaining, $j < J(z)-1$. For $j=J(z)-1$, the household will transition into the paid off state when making their payment instead of remaining current.

1 can be rewritten as:

$$\beta E_{w'|w}\left[v^c(\cdot) - v^d(\cdot) + p^f(1)\left(v^d(\cdot) - v^b(\cdot,\cdot,h_0)\right)\right]$$

With low foreclosure risk $p^f(1)$, the RHS is smaller so long as $v^d(\cdot) > v_b(\cdot,\cdot,h_0)$, which is true in general.

Lastly, if labor market risk is high, the homeowner may not expect to be able to make all of their future mortgage payments before defaulting. They may receive an income shock and become involuntarily delinquent for several periods leading to a foreclosure. In this case, the value of being current in the future $v^c(\cdot)$ is smaller which which reduces the RHS. It may be optimal to default today even if the homeowner can afford payments.

Assets play two roles for homeowners. Assets help to smooth consumption against income shocks, as is typical in the self-insurance incomplete markets literature (Bewley (1977) and Aiyagari (1994)), and assets insure against future ability to make mortgage payments. Once a homeowner completes their final payment or has a high expectation of being able to complete all mortgage payments, the value of assets declines and they chose to accumulate fewer assets.

4.1. Numerical Parametrization

I solve the model numerically using the parametrization described in this section. Details about the numerical algorithm used to solve the model are included in appendix A. Households have utility $u(c,h) = \log(c) + \gamma\log(h)$. Households can either be employed and have high income, w_{high}, or be unemployed and have low income, w_{low}. This low income can be interpreted as unemployment benefits. The transition matrix for this income process is chosen to be:

$$F(w'|w) = \begin{bmatrix} 0.96 & 0.04 \\ 0.75 & 0.25 \end{bmatrix}.$$

The probabilities in this transition matrix are chosen so that the invariant distribution of this Markov process has an employment probability of 0.949 and an unemployment probability

of 0.051. Households have one type of mortgage, a fixed-rate mortgage, that requires the mortgagor to make two payments with a 4.5% nominal interest rate and 20% down payment. The owned housing size is set to $h_1 = 1$, the foreclosed housing size is $h_0 = 0.5$, and the initial house price is $x(0) = 1$. The fixed monthly payment for a fixed rate mortgage with these characteristics that is obtained in period $t = 0$ is $m = 0.427$. I set the level of the hig income w_{high} to be equal to this mortgage payment and the low income equal to half a mortgage payment. The other parameters values are listed in the table below.

This numerical parametrization is intended to illustrate the model mechanisms. To make these mechanisms more transparent, I use a small number of periods (N=6) in which the household needs to make these two mortgage payments ($N = 6$). The periods can be thought of as years using the discount rate assigned $\beta = 0.96$.

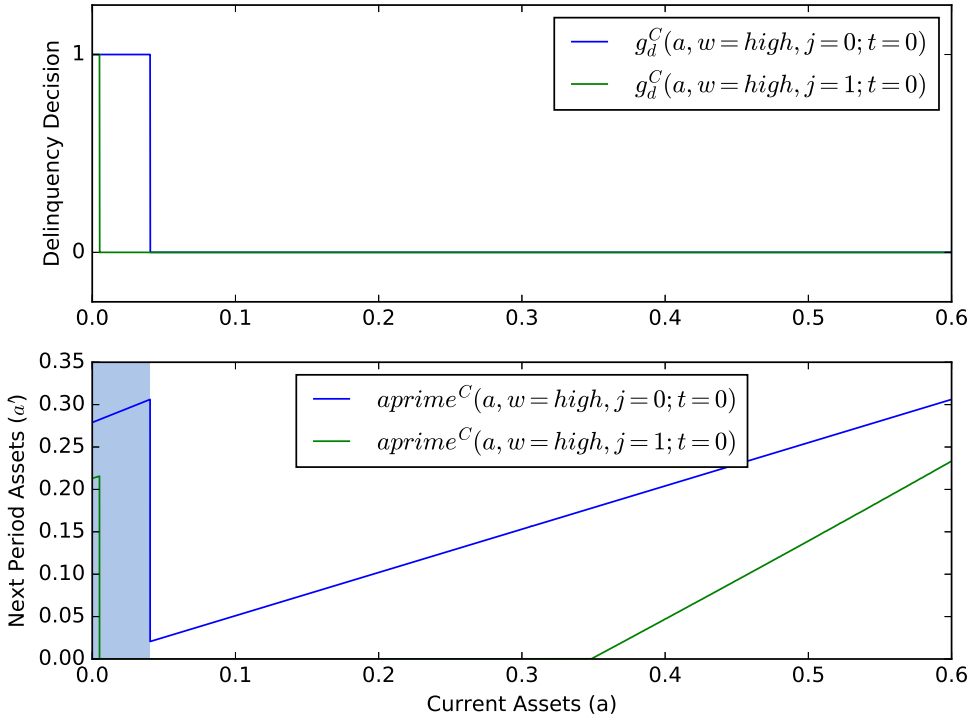
Parameter	Value	Description
β	0.96	discount rate
$\underline{a}$	0	borrowing constraint
γ	2	relative value of housing services
w_{high}	0.427	high income value
w_{low}	0.214	low income value
$p_f(1)$	0.1	foreclosure probability with $d = 1$
$p_f(2)$	0.3	foreclosure probability with $d = 2$
$p_f(3)$	1	foreclosure probability with $d = 3$

4.2. Numerical Results

With this parametrization, high income households rarely choose to enter delinquency. Decision rules for current and high income mortgagors in period $t = 0$, with both one payment remaining ($j = 1$) and two payments remaining ($j = 0$), are displayed in figure 1. The top panel of this figure shows the discrete delinquency decision rule, $g_d^C(a, w_{high}, j; t = 0)$, equal to 1 when delinquent and 0 when not. The bottom panel of this figure shows the next period asset decision rule, $a'(a, w_{high}, j; t = 0)$, with shading to indicate the region in which households with two remaining payments choose to be delinquent. The green lines show decisions when the mortgagor has one payment remaining. The top panel shows

that just about every high income household with one payment remaining chooses to make their final mortgage payment. The few mortgagors that don't make their final payment choose a high level of assets to carry into the next period. These households are entering delinquency because the marginal utility of additional resources outweighs the temporary risk of delinquency. Instead of paying their mortgage, they accumulate enough assets such that in the next period they can make their final payment and still have a moderate level of consumption. For the households that choose to make their final mortgage payment, assets are no longer as valuable since their mortgages are paid off, and the borrowing constraint binds for some of these households.

Figure 1: Decision Rules - Current and High Income



The blue lines in figure 1 show decision rules for a high income households with two remaining payments. The delinquency region is larger than for households with only one payment remaining. In addition, the households who are delinquent with a high income accumulate a large amount of assets. These households enter delinquency because of future

income risk. If the household makes their mortgage payment today, they will still need to make one more payment in the future and they do not know what their income level will be. With a high income, the household would want to make the final payment tomorrow, but with a low income the household might not pay. If the household makes a payment and ends up foreclosed in the future, they would not recover the amount paid today towards their mortgage because of negative equity. Thus, instead of making a payment today, it is optimal to skip the current payment and increase assets so that the household is more resilient to future income shocks. Then, in the next period, start making payments if the household retains a high income.

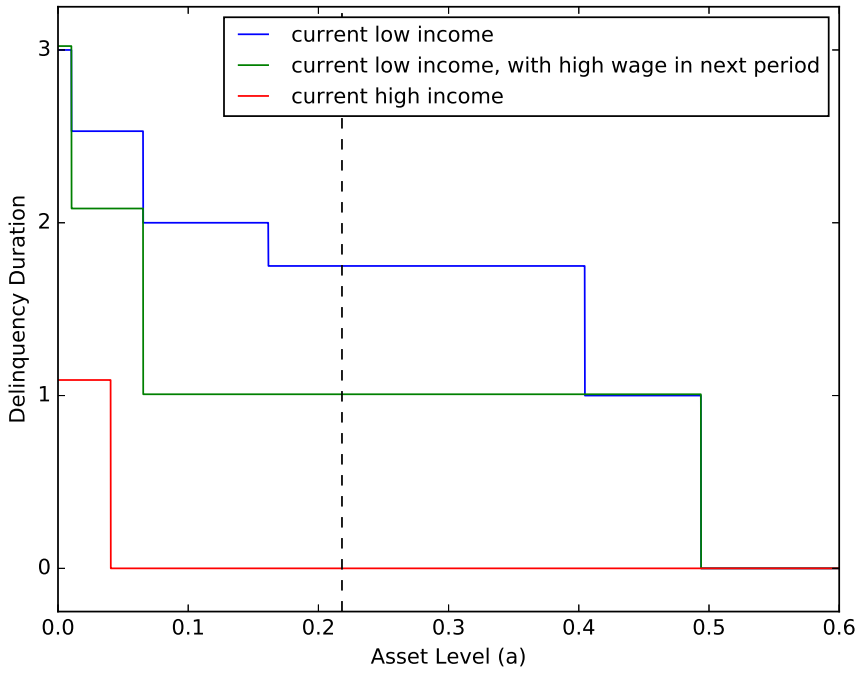
The household's expected delinquency duration reveals their strategy. Figure 2 shows expected delinquency durations for households with two remaining payments. An expected duration of one period implies that the household will exit delinquency in the next period regardless of what income shocks it receives, an expected duration of three periods, the maximum duration of delinquency, means that the household will remain in delinquency until foreclosure regardless of its income shocks, and an expected duration between these two values implies that the duration of delinquency depends on the series of income shocks. The red line in figure 2 shows expected delinquency duration for high income households. These households enter delinquency expecting to remain delinquent for 1.09 periods– one period if they receive a high income in the next period, with probability 0.96, and 2.5 periods if they receive a low income, with probability 0.06.[11] With reversible delinquency, delays reduce the risk of foreclosure while delinquent which allows the household to strategically default to insure themselves against income risk. If their income remains high, they exit delinquency quickly, but if they receive a poor labor market outcome, they remain delinquent and possibly foreclose. In essence, they are smoothing consumption across the housing services good and

[11] If the household receives a low income in the next period, they will remain delinquent until they return to a high income. With a low income, their probability of remaining with a low income is 0.5. When this happens, they have a delinquency duration of 3 periods and then foreclose with certainty. Their probability of returning to high income is also 0.5, in which case their delinquency duration is 2 periods. Thus, when they receive a low income in the next period, their expected delinquency duration is 0.5*2 + 0.5*3 = 2.5.

the consumption good, reducing expected housing services and increasing consumption.

The ability of to enter and exit delinquency, with a low risk of foreclosure, allows the household to strategically default to insure themselves against income risk. As long as their income remains high, they will exit delinquency quickly.

Figure 2: Expected Delinquency Duration with Two Remaining Payments

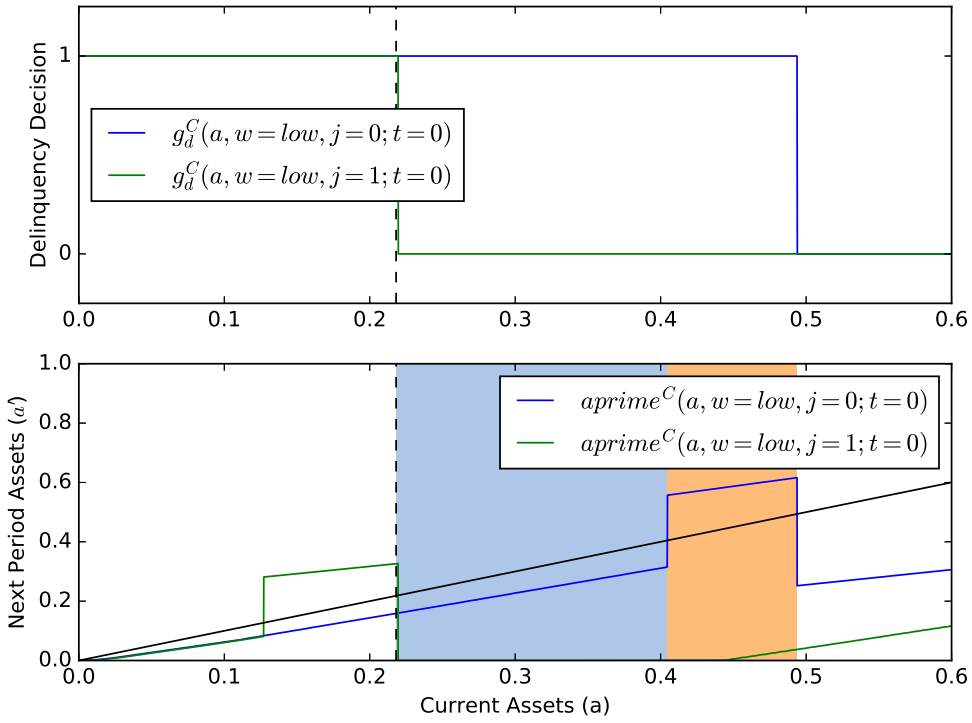


For low income households, delinquency occurs much more often. Decision rules for current and low income mortgagors are displayed in figure 3. As in the previous figure, the top panel shows the delinquency decision rule, $g_d^C(a, w_{low}, j; t = 0)$, equal to 1 when delinquent and 0 when not. With a low income, there is a possibility of involuntary delinquency. The dashed black lines show the asset level below which low income households are involuntarily delinquent. With only one payment remaining, all households to the right of this dashed black line choose to make their final mortgage payment.[12] With two remaining payments, many households to the right of the black dashed line choose to skip their mortgage payment.

[12]With the exception of the first few grid points, because of log utility.

There are two reasons for these delinquencies that can be better understood by observing the household's asset behavior.

Figure 3: Decision Rules - Current and Low Income



The bottom panel of figure 3 shows this asset behavior. The shading indicates regions where the household chooses to be delinquent with two remaining payments. The bottom panel also includes a 45 degree line, indicated by the solid black line. When households choose an asset position above the 45 degree line, they are choosing to increase their asset position relative its current level. With only one remaining payment, shown with the green line, households who just barely cannot afford their last payment, to the left of the dashed black line, choose a high enough level of assets so that they will make that final payment in the next period regardless of their future income shock. To the immediate right of the dashed black line, households make their last payment and choose the minimum asset level. They do this because with their mortgage paid off, assets are now only useful for self-insurance against income shocks which pushes their optimization problem to a corner solution. With

two remaining payments, shown with the blue line, the household's decision rule is continuous around the dashed black line. The function is smooth until the shaded yellow region, where households choose a much higher level of next period assets.

In the shaded blue region, households are entering delinquency to *insure* themselves against income risk. If these households continue to receive low income in the future, it is not optimal to pay the mortgage, and thus the optimal strategy is to wait until receiving a high income before paying the mortgage. Figure 2 shows that this how the households behave in this model. The blue line shows the unconditional expected delinquency duration and the green line shows the expected delinquency duration conditional on receiving a high income in the next period. The green line is equal to one for all households in the blue shaded region, meaning that they will exit delinquency with certainty if they receive a high income in the next period. The blue line in this region is greater than one, indicating that they will remain in delinquency if they receive a low income in the next period. The key feature of the model that allows this behavior is reversible delinquency. Without foreclosure delays and reversible delinquency, households could not use delinquency to insure against income risk. I call this the *insurance* effect of delays.

In the shaded yellow region of figure 3, households are entering delinquency because of *moral hazard*. With foreclosure delays, households expect to be able to survive short delinquency spells with high likelihood. With this expectation, they can skip their current mortgage payments, delaying those payments to the future, and increase current disposable income. Overall, by entering delinquency and accepting this limited foreclosure risk for a short period of time, the household increases its expected lifetime utility. The households in the shaded yellow region choose to increase assets relative to their current level to ensure they can exit delinquency in the next period regardless of their income level. Figure 2 confirms that the household will exit delinquency regardless of its income in the following period. The blue line shows that the unconditional expected delinquency duration is one period. As long as this household avoids foreclosure during its one period delinquency spell, this strategy benefits the household. I call this the *moral hazard* effect of delays because delays limit

foreclosure risk during delinquency– incentivizing this risky behavior.

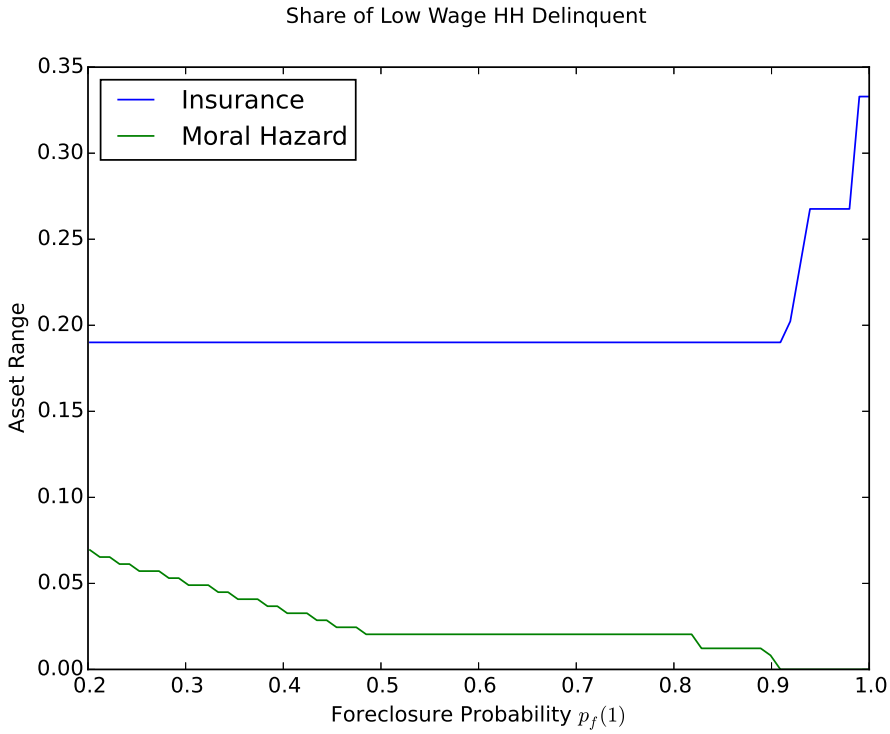
4.3. Effect of Delays

To better understand how the level of foreclosure delays affects household default, I vary the foreclosure probability in the first period of delinquency between 0.2 and 1. Figure 4 displays the numerical results for current and low income households with two remaining payments. The blue line displays the size of the blue shaded region in figures 1 and 3, which represent the insurance effect.[13] When households are guaranteed to foreclose in their first period of delinquency, $p^f(1) = 1$, the number of households defaulting because of future labor market risk is at its highest level. This occurs because households who are vulnerable to foreclosure if they receive a low income are now going to be foreclosed on in the future with a very high likelihood. If households are ever involuntarily delinquent for even one period, they will have no ability to cure their delinquency spell. Households lose their ability to use delinquency as insurance against income risk and will now be more likely to default early on because with negative equity, it is best to foreclose with as few payments made as possible. As the foreclosure probability decreases, the size of this region decreases until around a foreclosure probability of 0.9 after which the size of this region is constant. Overall, with the insurance effect, delays have a non-increasing effect on delinquency for low income households.

The green line in figure 4 shows the size of the yellow shaded region in figure 3 for various foreclosure probabilities. With the highest level of delays, which generate a foreclosure probability of 0.2 in the first period of delinquency, the size of the shaded region is the largest. As delays decrease, the size of this shaded region decreases and eventually, around a foreclosure probability of 0.9, the moral hazard effect is eliminated. Under the moral hazard effect, delays have a non-decreasing effect on delinquency for low income households. The moral hazard effect is trading off consumption smoothing over time with a current risk of foreclosure. As the risk of foreclosure increases, this trade off is less likely to be beneficial,

[13]The size of a region is defined as the difference between the highest asset value in the shaded region and the lowest asset value. This assumes a uniform distribution of households across assets.

Figure 4: Effects of Delay - Current and Low income



and eventually the risk is so high that no household will accept the risk of foreclosure for consumption smoothing purposes.

4.4. Lifetime Foreclosure Probabilities

One possible benefit of delays discussed in the literature is that delays can provide households an improved ability to avoid foreclosure. In my model, this is true, mechanically, since delays reduce the probability of foreclosure, but my model can provide some additional insight about how delays and assets interact to affect a households ability to avoid foreclosure. Without delays, assets reduce foreclosure likelihood because they protect against involuntary foreclosure following an income shock. With delays, assets also allow more strategic behavior, described earlier as the insurance effect and the moral hazard effect.

To understand the interaction between assets and foreclosure probabilities, I calculate lifetime foreclosure probabilities for each household based on their initial income level and their endowed level of assets. I calculate these probabilities by simulating the households

decisions under each possible series of income shocks. Define $w_i^N \in w^N$ as a particular income series i of length N from the set of all possible income series w^N. Then, define $g_d(a;t|w_i^N) \in \{0,1\}$ and $d(a;t|w_i^N) \in \{0,1,\ldots,\bar{d}\}$ as the delinquency decision and number of periods delinquent, respectively, for a household with initial asset level a, in period t, that observed income series w_i^N. The foreclosure probabilities can then be written as

$$\widehat{p_f}(a;t|w_i^N) = \begin{cases} p_f(d(a;t|w_i^N))g_d(a;t|w_i^N) & \text{if } t = 0 \\ \left[\prod_{i=0}^{t-1}(1-\widehat{p_f}(a;i|w_i^N))\right] p_f(d(a;t|w_i^N))g_d(a;t|w_i^N) & \text{if } 1 \leq t < N \text{ and } d(a;t|w_i^N) < \bar{d} \\ \left(1-\left[\sum_{i=0}^{t-1}(\widehat{p_f}(a;i|w_i^N))\right]\right) & \text{if } 1 \leq t < N \text{ and } d(a;t|w_i^N) = \bar{d} \\ \left(1-\left[\sum_{i=0}^{t-1}(\widehat{p_f}(a;i|w_i^N))\right]\right) g_d(a;t|w_i^N) & \text{if } t = N \end{cases}$$

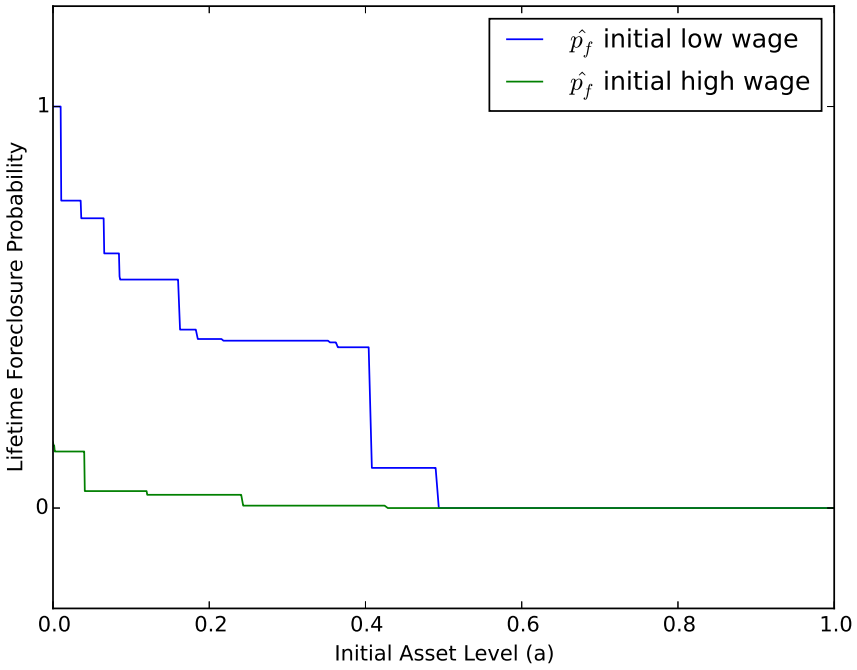
The lifetime foreclosure probability is then calculated as

$$\widehat{p_f}(a,w_0) = \sum_{w_i^N \in w^N} \mathbb{P}(w_i^N) \left\{ \sum_{t=0}^{N} \widehat{p_f}(a;t|w_i^N) \right\}.$$

where w_0 is the initial income level in period 0 and $\mathbb{P}(w_i^N)$ is the probability of income shock series w_i^N. This lifetime probability is defined such that $\widehat{p_f}(a,w_0) \in [0,1]$ and $\widehat{p_f}(a;t|w_i^N)$ can be interpreted as the probability of foreclosure in period t with income series w_i^N. See appendix B for details on the numerical algorithm used to solve these lifetime foreclosure probabilities.

Foreclosure probabilities depend largely on initial starting income in period 0. Figure 5 shows the lifetime probability of foreclosure in the model for a household endowed with a new loan in period 0 and initial assets displayed on the horizontal axis. The blue line shows the probabilities for a low initial income household and the green line shows the probabilities for a high initial income household. Low initial income households with the lowest initial asset level will foreclose with certainty. As initial assets increase, the foreclosure likelihood decreases monotonically until it eventually reaches zero. This line shows the importance of

Figure 5: Lifetime Foreclosure Probability



initial assets and initial wage in determining whether a household ends up foreclosed or paid off. High initial income households very rarely foreclose, only if they have very low initial assets and receive an unlikely sequence of poor income shocks.

4.5. Simulations

To better understand how delays affected mortgage default during the Great Recession, I use my model to simulate the increased labor market risk and decreased home prices seen during that period. I simulate increased labor market risk by holding the income levels constant but changing the transition matrix from

$$\begin{bmatrix} 0.96 & 0.04 \\ 0.75 & 0.25 \end{bmatrix} \longrightarrow \begin{bmatrix} 0.94 & 0.06 \\ 0.5 & 0.5 \end{bmatrix}.$$

These changes to the transition matrix represent the increase in job separation rates and the significant increase in the duration of unemployment that occurred during the Great

Recessionn. These changes to the transition matrix imply that instead of 5.1% of mortgage holders having a low income in steady state, 10.7% of mortgage holders have a low income. The second shock I simulate is a large decline in home prices. Figure 6 shows the calibrated series that enters the model.

In the previous numerical example, there were only a limited number of periods to try and illustrate clearly how default behavior changes with delays. In this simulation, it is desirable to study a longer time horizon and thus I change several of the parameter values to accommodate this. I increase the total number of periods to $N = 10$ and the total number of mortgage payments to $J(z) = 4$, representing 10 semi-annual periods with 2 years worth of payments. Because I hold the initial home price fixed at $x(0) = 1$, home size at $h = 1$, and the mortgage interest rate at 4.5%, each mortgage payment is now $m = 0.223$. I adjust the incomes to be $w_{low} = 0.5m$ and $w_{high} = m$. Because periods are now semi-annual, I adjust β to 0.98. Lastly, I adjust the delinquency process to allow 5 possible periods of delinquency, with $p_f(1) = p_f(2) = .1$, $p_f(3) = .3$, $p_f(4) = .5$, and $p_f(5) = 1$. This means homeowners can expect survive a full year of delinquency with 81% probability and will be foreclosed on with certainty in the third year of delinquency. All other parameters are identical to the prior numerical example.

I start by simulating the benchmark economy in which home prices are constant at $x(t) = 1 \ \forall \ t$ and the income process is identical to the numerical example prior. Figure 6 shows the time paths for aggregate delinquency and aggregate foreclosure in each period by initial income. Table 1 shows the effects aggregated over the lifetime for both initial low and initial high income homeowners. The dashed lines show how these time paths change when I introduce both the home price shock and the labor market shock. These shocks increase lifetime foreclosure probabilities by 5.3% for initial low income homeowners and by 0.07% for initial high income homeowners. I then decompose the effects for initial low income homeowners and find that the majority of the effects are caused by the increased labor market risk. For initial low income homeowners, 67.9% of the increase in foreclosures is due to increased labor market risk. For initial high income, 100.0% of the increases is due

Figure 6: Model Simulation

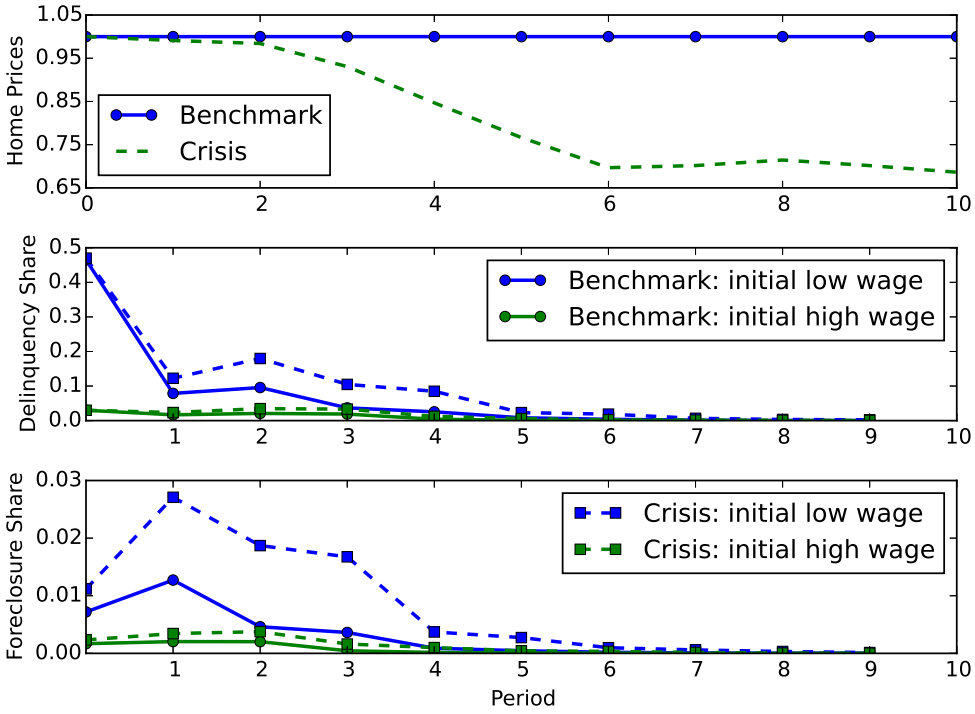
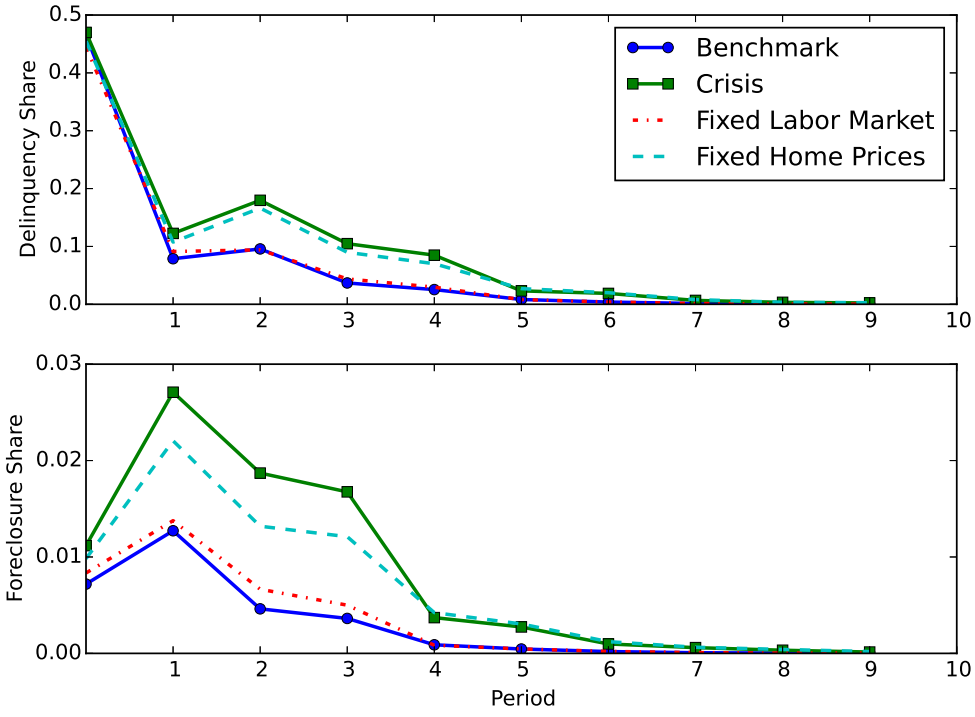


Figure 7: Model Simulation - Decomposition for Initial Low Income



to increased labor market risk. The time series decomposition is shown in figure 7.

Table 1: Model Simulation - Aggregate Results by Initial Income Level

	Lifetime Foreclosure Prob		Lifetime Delinquency Prob	
	Initial High Income	Initial Low Income	Initial High Income	Initial Low Income
Benchmark	0.009	0.076	0.078	0.478
Crisis	0.016	0.129	0.101	0.491
Fixed Labor	0.009	0.080	0.077	0.462
Fixed Home Prices	0.016	0.112	0.101	0.480

5. Empirical Findings

The model makes several predictions that I corroborate in the data. One of these predictions is that moral hazard households will only remain delinquent for a short duration whereas the delinquency duration for insurance households will depend their unemployment duration. Another prediction is that there will be more moral hazard delinquent households when foreclosure delays are high. I use the survey of consumer finances (SCF) to substantiate these model predictions and better understand how short-term delinquent households differ from seriously-delinquent households in the data.

The SCF is a triennial cross-sectional household survey of U.S. families that is sponsored by the Federal Reserve Board in cooperation with the U.S. Treasury Department. The survey collects detailed data on household assets, debts, incomes, and demographic characteristics including detailed questions about residential mortgages, other lines of credit, and asset portfolios. The survey has been conducted by the National Opinion Research Center (NORC) at the University of Chicago since 1992. Participation is voluntary and about 4,500 to 6,500 households are surveyed in each year. The majority of interviews are face-to-face and the median interview length in 2004 was eighty minutes. For further details about the SCF, see appendix section C.

Since the SCF is a repeated cross-section, this limits the ability to observe delinquency cures, asset dynamics, and repeated delinquency spells. What can be observed is whether a household has been delinquency in the past 12 months, if that delinquency spell is more

or less than 2 months, the asset level at the time of survey, and how many weeks in total a household has been unemployed in the past 12 months.

5.1. Sample Selection

I use the individual surveys between 2001 and 2013. I restrict the sample to households with a head of age 24 to 65 years who have a loan to value (LTV) ratio less than 2.5. With these restrictions, there are 20,130 observations remaining. Table 2 shows the number of observations, the percentage of households with a loan, and the short-term and serious delinquency rates by year.[14] Short-term delinquency is defined as less than two months delinquent and serious delinquency is defined as two months delinquent or more. The loan rate is above 80% in each year, slightly higher in the first three years of the data. The short-term delinquency rate is 12.19% in 2001, peaks at 18.58% in 2007, and declines back down to below its 2001 level in 2013. The time series for the serious delinquency rate is a bit different. The serious delinquency rate starts at 7.72%, peaks at 11.95% in 2010, and remains about 2.5% higher than its 2001 level in 2013. The overall time series for delinquency matches that of other data on single-family residential mortgage delinquencies which shows a persistently high level of delinquencies following the initial crisis in the late 2000s.

Table 2: Survey of Consumer Finances, Sample Size and Delinquency Rates

Year	Num. of Obs.	Loan Rate	Short-Term Del. Rate	Serious Del. Rate
2001	3,435	84.14%	12.19%	7.72%
2004	3,560	84.01%	13.23%	9.75%
2007	3,418	85.12%	18.58%	7.44%
2010	5,111	81.77%	13.02%	11.95%
2013	4,605	80.93%	11.95%	10.25%

Notes: Sample includes households with head aged 24 to 65 years who have a LTV ratio less than 250 percent. Delinquency is defined as being late on any type of debt over the past 12 months. Short-term is less than two months, seriously delinquent is two months or more. Loan rate is the share of all observations that have any type of loan. Statistics are calculated using all implicates in each year, weighted using revised weights. See appendix C for more details.

Table 3 shows summary statistics for the pooled SCF sample. As a result of multiple imputation in the SCF, there are five observations for each household in the SCF. These

[14] One of the main problems with the SCF data is that the delinquency question is not exclusive to single-family residential mortgages. Households are delinquent if they have missed payments on any of the loans they have.

statistics are calculated using all implicates, weighting each observation using the revised weights divided by five to account for the five implicates.[15] Dollar values are inflation adjusted and in 2010 dollars. The demographic variables include age, years of education, marital status and race. Years of education is top-coded at 17 for more than college and race is equal to one if not white. Table 3 also includes summary statistics on some income, balance sheet, and housing variables. The sample mean for income is about $85,000, higher than the U.S. average overall. For 20.1% of the sample either the household head or spouse was unemployed or looking for work within twelve months of the survey date. Looking at both the head and the spouse, the average amount of time spent unemployed and looking for work was 4.9 combined weeks. The average net worth of households is $437,494.9 and the average level of liquid assets is $25,418.53.[16] The average level of home equity is $95,370 and the average LTV ratio is 0.485. There are 13,739 households with a mortgage and 13.7% percent of mortgages are adjustable rate mortgages (ARM). The vast majority of mortgages have a long time until maturity, with an average of 25.0 years of payments remaining on primary mortgages. The average monthly payment is $1,232.41.

Table 4 displays sample means for many of the same variables conditional on a households delinquency status. Seriously delinquent households tend to be younger, less educated, less married, and less white than both short-term delinquent households and households with no delinquency. Among income and balance sheet variables, seriously delinquent households have fewer liquid assets, less net worth, and lower incomes on average than short-term delinquent households. Moving from short-term delinquent to non-delinquent households further improves these variables. Non-delinquent households tend to have positive deviations from their normal annual income, where as short-term and seriously delinquent households have incomes 4.03% and 12.00% below their normal annual income, respectively.[17] Interestingly,

[15] These weights are revised Kennickell-Woodburn consistent weights. They account for systemic deviations from CPS estimates of home-ownership by racial/ethnic groups.

[16] Liquid assets includes balances at checking accounts, savings accounts, money market accounts, and call accounts at brokerages.

[17] Normal annual income is what a household would have expected to earn if it had been a normal year.

non-delinquent and short-term delinquent households have just about the same average unemployment durations at 4.35 and 5.11 weeks, respectively, whereas seriously delinquent households have much longer average unemployment durations at 10.33 weeks. This supports the model implication that delinquency duration will depend on unemployment duration for households who enter delinquency because of the insurance effect. For households who enter delinquency because of moral hazard, delinquency duration will always be short-term.

5.2. Delinquency Rates by Liquid Asset Decile

The model predicts that in years with foreclosure delays, some households will be more likely to enter delinquency for a short-duration because of the moral hazard effect. To find evidence of this in the SCF, I look at short-term and overall delinquency rates across liquid asset deciles in the various years of the SCF. In years with delays, I expect to find higher short-term delinquency rates among low asset deciles relative to other deciles. Households in these low asset deciles benefit most from deferring mortgage payments because of their relative wealth. For high asset deciles, the benefit of skipping payments is outweighed by the expected cost of losing access to a high level of housing services.

Figure 8 shows overall delinquency rates, which includes both short-term and seriously delinquent households, by liquid asset decile. The lines in this figure show the difference between each decile's delinquency rate and the lowest decile's delinquency rate in that specific year. I do this to control for level differences in delinquency rates across years. For example, in 2004 the lowest decile's delinquency rate was 25.8% whereas in 2007 that same decile's delinquency rate was 32.2%. This figure shows that the relative relationship between liquid asset decile and delinquency is about the same in each year of the sample. Each year, the highest delinquency rate occurs in the second liquid asset decile, which is between 1 and 5 percentage points higher than the lowest asset decile. After the second liquid asset decile, delinquency rates decline as liquid asset decile increases. The top decile has a delinquency rate more than 20 percentage points less than the lowest liquid asset decile in each year. This result suggests that the aggregate shocks that occurred in 2007 and 2010 did not affect relative delinquency rates across liquid asset decile. These delinquency rates by decile are

Table 3: Survey of Consumer Finances, Summary Statistics

Variable Name	Mean	St Dev.	Min	Max
Demographic Variables				
Age of Head	44.61	11.33	24	65
Education	13.56	2.68	0	17
Marital Status	0.62	0.48	0	1
Race	0.32	0.47	0	1
Income and Balance Sheet Variables				
Liquid Assets	$25,418.53	$246,619.40	$0	$1.00e+08
Liq. Assets $<$ 1 mo.	0.08	0.28	0	1
Net Worth	$437,494.9	$2,699,899	$-2.27e+08	$1.16e+09
Income	$84,167.03	$309,513.9	$0	$3.56e+08
Unemployed	0.21	0.41	0	1
Unemployment Duration	4.9	13.29	0	104
Housing and Mortgage Variables				
Home Equity	$95,370.04	$24,9940	$-2.51e+07	$5.81e+07
LTV	0.49	0.36	0	2.4
Years Remaining	24.99	7.61	1	50
Regular Payment	$1,232.42	$1,974.17	$20	$2,030,000
ARM	0.14	0.34	0	1

Table 4: Survey of Consumer Finances, Sample Means by Delinquency Status

Variable Name	No Delinquency	Short-Term Del.	Seriously Del.
Demographic Variables			
Age of Head	45.15	42.59	42.06
Education	13.67	13.22	12.99
Marital Status	0.63	0.59	0.55
Race	0.30	0.39	0.41
Income and Balance Sheet Variables			
Liquid Assets	$ 29,850.61	$10,244.56	$2,112.46
Liq. Assets $<$ 1 mo.	6.56%	14.01%	24.87%
Net Worth	$508,877.70	$199,724.9	$52,398.15
Income	$91,846.43	$59,593.27	$41,260.1
Income % Dev	4.72%	-4.03%	-12.00%
Unemployment Duration	4.35	5.11	10.33
Housing and Mortgage Variables			
Home Equity	$108,720.4	$52,505.71	$20,989.08
LTV	0.46	0.56	0.69
Years Remaining	24.77	25.68	26.68
Regular Payment	$1,269.12	$1,067.84	$1,045.80
ARM	6.68%	8.29%	9.07%

Notes: Sample includes households with head aged 24 to 65 years who have a LTV ratio less than 250 percent. Delinquency is defined as being late on any type of debt over the past 12 months. Short-term is less than two months, seriously delinquent is two months or more. Sample Means are calculated using pooled SCF data from 2001-2013. All implicates are used, weighted using revised weights. Dollar values are inflation adjusted an in 2010 dollars. See text for variable definitions and see appendix C for more details on SCF imputation and weights.

driven by idiosyncratic shocks that did not change across these years.

Figure 9 shows the same relative delinquency rates across decile as in the previous figure except that instead of looking at overall delinquency rates, this figure shows short-term delinquency rates. For short-term delinquency, there is much less variation across liquid asset deciles than with overall delinquency. The top decile has a delinquency rate between 5 and 10 percentage points less than the lowest asset decile compared to the previous figure which had delinquency rates more than 20 percentage points less. The main result in this figure is that short-term delinquency rates increased relatively more for deciles two through four in 2007 and 2010 than they did in 2004 and 2013. These are the years of the sample which were most affected by aggregate shocks that increased delays and increased labor market turmoil. Short-term delinquency occurs more for low asset deciles relative to other deciles in these years.

5.3. Predicting Delinquency

Based on the double trigger model of default, negative equity is a necessary but not sufficient condition for default. In addition to negative equity, a household will default if they recive an unemployment shock or some other shock that affects their ability to pay the mortgage. With delays, this changes somewhat, because there are now alternative strategic motives that do not occur without delays. I can test which factors predict delinquency and how this changes across years of the SCF, some of which have delays and some of which do not. To determine whether or not the default decision changes structurally overtime, I estimate a linear probability model of delinquency in each year of the SCF sample. I predict whether a household is delinquent as a function of some demographic variables, unemployment status, loan to value ratio, and whether the household has less than one months worth of mortgage payments in liquid assets.[18] This regression is similar to what Gerardi et al. (2015) estimate for the 2007-2009 SCF panel. I differ by estimating this model for each individual year of the SCF and including some alternative regression specifications.

[18]Demographic variables include age, marital status, education, and race

Figure 8:

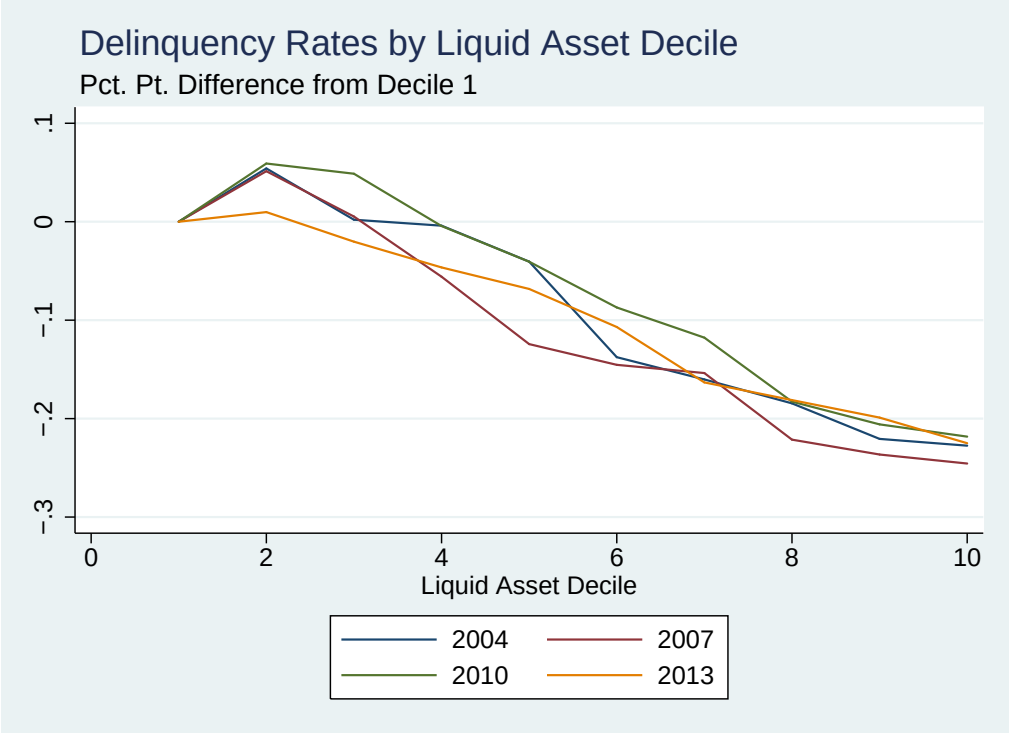


Figure 9:

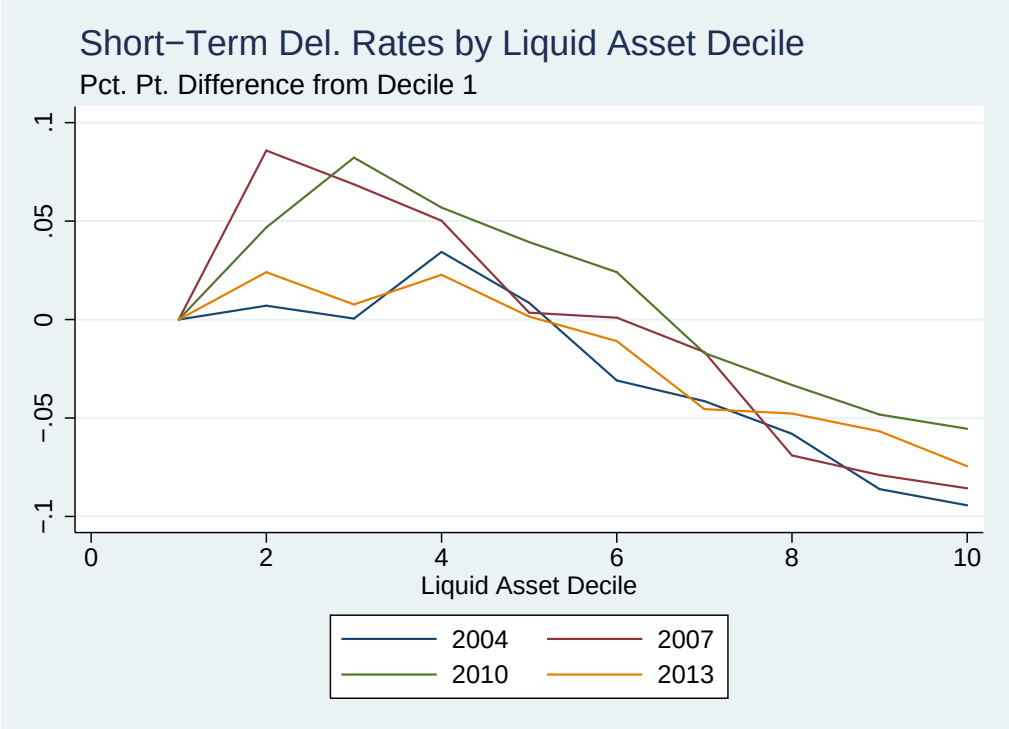


Table 5 displays the baseline results with bootstrapped standard errors. Unemployment status, equal to one when the head of household or the spouse has been unemployed in the past 12 months, is a statistically significant predictor of delinquency in each year. The magnitude of the unemployment effect varies greatly across each year, increasing a household's default probability by between 5.6% and 18.8%. Having insufficient liquid assets to cover one month of mortgage payments is also a positive and statistically significant predictor of default, increasing a household's default probability by between 19.3% to 25.8%. Table 6 shows the results when including a negative equity indicator in the regression. Interestingly, the coefficient on the negative equity indicator is only statistically significant in one year. In every other year, the coefficient is small and not statistically significant. This suggests that negative equity alone is a poor predictor of delinquency in the SCF.

I also test the effectiveness of several other potential default predictors. I estimate the model including a "double trigger" interaction term, equal to one when a household has both negative equity and an unemployment spell. Unfortunately this "double trigger" only occurs 128 times across all years of this SCF sample. I also estimate the model including an interaction term between low liquid assets and unemployment. There are 308 households that have both low liquid assets and unemployment in the pooled sample. The coefficient on this term, when included in the baseline regression, varies greatly across years but is positive and statistically significant in 2001, 2007, and 2013. Lastly, I test whether replacing the unemployment indicator with an unemployment duration indicator improved the regression's predictive power. I test two indicators, one equal to one if the household's unemployment duration was greater than the median and one equal to one if the household's unemployment duration was greater than eight weeks. Neither of these indicators improved upon the predictive power of the original unemployment indicator.

Trying to predict short-term or seriously delinquent homeowners using the previous linear probability model gives mixed results. With short-term delinquency as the dependent variable, the R-squared value falls to between 0.016 and 0.051 in each year, compared with between 0.084 and 0.130 in the baseline regression for overall delinquency. Additionally, the

coefficient on unemployed is only statistically significant at the 95% level in two years. This suggests that the prior regression which predicted overall delinquency well does not predict short-term delinquency as well. Other factors besides unemployment, liquidity constraints, and loan to value ratios are important in determining why households have a short-term delinquency spell. With serious delinquency as the dependent variable, the R-squared values are similar to the baseline regression and the key coefficients remain statistically significant and positive in just about every year. These factors includes in this regression – unemployment, liquidity constraints, and loan to value ratios – are important when determining why households become seriously delinquent.

6. Conclusion

In the aftermath of the foreclosure crisis, foreclosure processing times have remained persistently long. I investigate how these foreclosure delays affect the household's default decision. I find two competing effects. There is a moral hazard effect, where households enter delinquency for a short duration because of limited foreclosure risk. Delays reduce the risk of foreclosure when delinquent and thus increase the number of households that enter delinquency for a short duration. If foreclosure risk is high enough, the moral hazard effect will be eliminated. There is also an insurance effect, where households enter delinquency because they are uncertain about future income. They enter delinquency following a negative income shock and they stay in delinquency until they receive a positive income shock. Delays increase the duration of the delinquency spell which increases the household's likelihood of receiving a positive income shock and recovering their mortgage prior to foreclosure. When households expect to be foreclosed on quickly, they are more susceptible to income shocks, increasing the number of households that enter delinquency today.

These competing effects pose difficulties for a policy maker trying to assist distressed homeowners without providing the wrong incentives. Increasing delays and instituting "right-to-cure" laws, when given to all homeowners, can introduce a moral hazard prob-

Table 5: Survey of Consumer Finances, LPM Baseline Results.

	2001	2004	2007	2010	2013
Unemployed	0.088***	0.108***	0.056**	0.111***	0.188***
	(0.023)	(0.028)	(0.022)	(0.020)	(0.028)
Liq. Assets $<$ 1 mo.	0.213***	0.258***	0.193***	0.200***	0.214***
	(0.033)	(0.028)	(0.026)	(0.022)	(0.021)
LTV	0.191***	0.088**	0.094***	0.150***	0.126***
	(0.036)	(0.037)	(0.031)	(0.026)	(0.029)
Constant	0.080	0.165*	0.381***	0.240***	0.051
Demographic Controls	Y	Y	Y	Y	Y
N	1,719	1,809	1,845	2,394	2,032
R-squared	0.105	0.130	0.084	0.121	0.129

Dependent Variable: Delinquent (1 = Delinquent on any loan in the past 12 months; 0 otherwise).
Bootstrapped standard errors in parenthesis. Significance levels: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. Two-tailed test.

Table 6: Survey of Consumer Finances, LPM Results with Negative Equity.

	2001	2004	2007	2010	2013
Unemployed	0.090***	0.108***	0.056**	0.111***	0.188***
	(0.022)	(0.028)	(0.022)	(0.020)	(0.027)
Liq. Assets $<$ 1 mo.	0.200***	0.258***	0.193***	0.199***	0.214***
	(0.033)	(0.028)	(0.026)	(0.022)	(0.021)
LTV	0.147***	0.089**	0.092***	0.127***	0.118***
	(0.038)	(0.035)	(0.033)	(0.027)	(0.029)
Negative Equity	0.215***	-0.012	0.009	0.034	0.012
	(0.071)	(0.087)	(0.073)	(0.037)	(0.026)
Constant	0.086	0.164*	0.382***	0.255***	0.057
Demographic Controls	Y	Y	Y	Y	Y
N	1,719	1,809	1,845	2,394	2,032
R-squared	0.111	0.131	0.084	0.121	0.129

Dependent Variable: Delinquent (1 = Delinquent on any loan in the past 12 months; 0 otherwise).
Bootstrapped standard errors in parenthesis. Significance levels: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. Two-tailed test.
Notes: Sample includes households with head aged 24 to 65 years who have a LTV ratio less than 250 percent and have a mortgage. Standard errors are bootstrapped and adjusted for imputation uncertainty. See appendix C for more details on bootstrapping procedure.

Table 7: Survey of Consumer Finances, LPM for Seriously Delinquent.

	2001	2004	2007	2010	2013
Unemployed	0.044***	0.063***	0.078***	0.097***	0.136***
	(0.017)	(0.022)	(0.016)	(0.014)	(0.020)
Liq. Assets $<$ 1 mo.	0.170***	0.144***	0.145***	0.153***	0.144***
	(0.026)	(0.023)	(0.023)	(0.017)	(0.018)
LTV	0.108***	0.012	0.047**	0.112***	0.102***
	(0.020)	(0.022)	(0.020)	(0.021)	(0.022)
Constant	-0.005	0.090	0.115**	-0.025	0.031
Demographic Controls	Y	Y	Y	Y	Y
N	1,719	1,809	1,845	2,394	2,032
R-squared	0.036	0.084	0.100	0.122	0.141

Dependent Variable: Seriously Delinquent (1 = Delinquent for two months or more on any loan in the past 12 months; 0 otherwise).
Bootstrapped standard errors in parenthesis. Significance levels: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. Two-tailed test.

Table 8: Survey of Consumer Finances, LPM for Short-Term Delinquent.

	2001	2004	2007	2010	2013
Unemployed	0.045**	0.044*	-0.022	0.014	0.052***
	(0.022)	(0.025)	(0.017)	(0.014)	(0.019)
Liq. Assets $<$ 1 mo.	0.044*	0.114***	0.049**	0.046***	0.070***
	(0.024)	(0.023)	(0.020)	(0.017)	(0.018)
LTV	0.079***	0.076**	0.046*	0.037**	0.024
	(0.030)	(0.033)	(0.027)	(0.017)	(0.020)
Constant	0.091	0.074	0.267***	0.265***	0.02
Demographic Controls	Y	Y	Y	Y	Y
N	1,719	1,809	1,845	2,394	2,032
R-squared	0.036	0.051	0.022	0.019	0.016

Dependent Variable: Short-Term Delinquent (1 = Delinquent for less than two months on any loan in the past 12 months; 0 otherwise).
Bootstrapped standard errors in parenthesis. Significance levels: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. Two-tailed test.
Notes: Sample includes households with head aged 24 to 65 years who have a LTV ratio less than 250 percent and have a mortgage. Standard errors are bootstrapped and adjusted for imputation uncertainty. See appendix C for more details on bootstrapping procedure.

lem incentivizing mortgagors to enter delinquency.[19] This finding is consistent with my model and the findings of Gerardi et al. (2013) and Zhu and Pace (2015). However, a policy that targets distressed homeowners who have long unemployment spells can reduce default through the insurance effect. When homeowners do not expect to be able to survive a delinquency spell they may default preemptively, as shown in my model, and policy assisting these households could actually reduce default and alleviate some foreclosures.[20] Overall, delays can have mixed effects on default decisions depending on how policy is designed.

[19] "Right-to-cure" laws, enacted recently in several states, allow delinquent homeowners to request a delay in their foreclosure proceedings.

[20] A policy maker would need to worry about exactly how to target distressed homeowners. If unemployment duration becomes a signal for distressed status, then in equilibrium this could change search incentives for households. However, it is unlikely that households would choose to reject a job offer only because they wish to signal distressed status and temporarily avoid mortgage payments.

References

Aiyagari, SR, "Uninsured idiosyncratic risk and aggregate saving," *The Quarterly Journal of Economics*, 1994.

Ambrose, BW, RJ Buttimer Jr, and CA Capone, "Pricing mortgage default and foreclosure delay," *Journal of Money, Credit, and*, 1997.

Barillas, Francisco and Jesús Fernández-Villaverde, "A generalization of the endogenous grid method," *Journal of Economic Dynamics and Control*, 2007, *31*, 2698–2712.

Bewley, T, "The permanent income hypothesis: A theoretical formulation," *Journal of Economic Theory*, 1977.

Bhutta, N, H Shan, and J Dokko, "The depth of negative equity and mortgage default decisions," 2010.

Brueggeman, W.B. and J.D. Fischer, *Real Estate Finance and Investments*, 14th ed., New York: McGraw-Hill Irwin, 2011.

Calem, PS, J Jagtiani, and WW Lang, "Foreclosure delay and consumer credit performance," 2015.

Campbell, JY and JF Cocco, "A model of mortgage default," *The Journal of Finance*, 2015.

Chatterjee, Satyajit and Burcu Eyigungor, "A quantitative analysis of the US housing and mortgage markets and the foreclosure crisis," *Federal Reserve Bank of Philadelphia Working Paper*, 2011, (11).

Chetty, Raj, "Moral Hazard versus Liquidity and Optimal Unemployment Insurance," *Journal of Political Economy*, 2008, *116* (2), 173–234.

Collins, J. Michael, "SCFCOMBO: Stata module to estimate errors using the Survey of Consumer Finances," 2015.

_ , Ken Lam, and Christopher E. Herbert, "State mortgage foreclosure policies and lender interventions: Impacts on borrower behavior in default," *Journal of Policy Analysis and Management*, mar 2011, *30* (2), 216–232.

Corbae, Dean and Erwan Quintin, "Leverage and the Foreclosure Crisis," *Journal of Political Economy*, feb 2015, *123* (1), 1–65.

Cordell, Larry, Liang Geng, Laurie S. Goodman, and Lidan Yang, "The Cost of Foreclosure Delay," *Real Estate Economics*, nov 2015, *43* (4), 916–956.

Cutts, AC and W Merrill, "Interventions in mortgage default: Policies and practices to prevent home loss and lower costs," *Borrowing to live: Consumer and mortgage*, 2008.

Foote, CL, K Gerardi, and PS Willen, "Negative equity and foreclosure: Theory and evidence," *Journal of Urban Economics*, 2008.

Garriga, C and D Schlagenhauf, "Home equity, foreclosures, and bailouts," *Manuscript, Federal Reserve Bank of St.*, 2009.

Gerardi, K, KF Herkenhoff, LE Ohanian, and PS Willen, "Can't Pay or Won't Pay? Unemployment, Negative Equity, and Strategic Default," 2015.

_ , L Lambie-Hanson, and PS Willen, "Do borrower rights improve borrower outcomes? Evidence from the foreclosure process," *Journal of Urban Economics*, 2013, *73* (1), 1–17.

Ghent, AC, "The Historical Origins of America's Mortgage Laws," *Research Institute for Housing America Research*, 2012.

_ and M Kudlyak, "Recourse and residential mortgage default: evidence from US states," *Review of Financial Studies*, 2011.

Hatchondo, JC, L Martinez, and JM Sánchez, "Mortgage defaults," *Journal of Monetary Economics*, 2015.

Herkenhoff, KF and LE Ohanian, "The Impact of Foreclosure Delay on US Employment," *NBER Working Paper*, 2015.

Kau, JB and DC Keenan, "An overview of the option-theoretic pricing of mortgages," *Journal of Housing Research*, 1995.

_ , _ , and T Kim, "Transaction costs, suboptimal termination and default probabilities," *Real Estate Economics*, 1993.

_ , _ , and _ , "Default probabilities for mortgages," *Journal of urban Economics*, 1994.

Leland, J., "Facing Default, Some Walk Out on New Homes," feb 2008.

Pennington-Cross, A, "The duration of foreclosures in the subprime mortgage market: a competing risks model with mixing," *The Journal of Real Estate Finance and Economics*, 2010.

Vandell, KD, "How ruthless is mortgage default? A review and synthesis of the evidence," *Journal of Housing Research*, 1995.

Zhu, Shuang and R. Kelley Pace, "The Influence of Foreclosure Delays on Borrowers' Default Behavior," *Journal of Money, Credit and Banking*, sep 2015, *47* (6), 1205–1222.

Appendix

A. Solution Algorithm

The code written to solve the model has two parts. The first part uses endogenous grid method to solve the non-mortgage problem, which is a standard Bewley model with an additional utility term describing utility from housing, which is fixed. The second part of the code solves the mortgage problem by backward induction. There are N periods in which a household can make mortgage payments. Starting in period N, every household will end up in the non-mortgage problem with housing space of h_1 if they complete their final mortgage payment or h_0 if they do not. I solve the model in this period then the solve the model for the remaining $N-1$ periods using backward induction. The code is described in detail below.

Step 1: Solve the no mortgage value function, $v_b(a, w, h)$, using endogenous grid method. I use the exact algorithm from Barillas and Fernández-Villaverde (2007), with an additional constraint in their step 2 to ensure $c^* > 0$.

Step 2: Solve mortgage problem value functions, $v_c(\cdot)$ and $v_d(\cdot)$, for period $t = N$. This period is the final period to make mortgage payments and every mortgage holder will transition to $v_b(\cdot)$ with either a housing level h_1, if they make their final payment, or h_0, if they do not. I linearly interpolate v_b, solved in step 1, over the grid on a to formulate the homeowner's continuation value in their objective function. I then solve for the optimal next period assets a' for each grid point using Brent's method. The structure of the objective function changes depending on the households state and I describe that next.

Step 2a: Solve $CC(a, w, j; t = N)$

- It is possible that households cannot afford to make their current payment is if disposable income is too low. I call these households involuntarily delinquent. I use the following inequality to check if a household is involuntarily delinquent: $(1+r)\cdot a+w-m > \underline{a}$.
  - If false, the household is involuntarily delinquent. Set $CC = -99999$, so that CD will always be larger than CC. Move to step 2b.
  - If true, continue.
- Set up the objective function:

$$-u\left((1+r)\cdot a + w - m - a', h_1\right) - \beta \cdot \text{expectation}$$

  The expectation depends on the value of j.
  - If $j = J-1$ (one payment remaining), household will advance to $E_{w'|w}[v_b(a', w', \mathbf{h_1})]$.
  - If $j < J-1$ (more than one payment remaining), household cannot complete mortgage, will be foreclosed on instantly, and advance to $E_{w'|w}[v_b(a'+\mathbf{eq(j, d = 0; N)}, w', \mathbf{h_0})]$.
- Minimize the objective function, with choice variable a', using Brent's method. Searching for values of $a' \in [\underline{a}, ((1+r)\cdot a + w - m - 0.000001)$.

Step 2b-2d: Solve $CD(a, w, j; t = N)$, $DC(a, w, j, d; t = N)$, $DD(a, w, j, d; t = N)$

- Solve objective functions using Brent's method. The upper bound for the convex choice set for a' varies depending on whether the household is making a payment and if there are late fees.
- For CD, the objective function is:

$$-u\left((1+r)\cdot a + w - a', h_1\right) - \beta E_{w'|w}[v_b(a' + eq(j, d=0; N), w', \mathbf{h_0})]$$

- For DC, check if involuntarily delinquent. If not, objective function is similar to CC except instead of mortgage payment m, household must pay $(1+r_m)^d m$.
- For DD, objective function is:

$$-u\left((1+r)\cdot a + w - a', h_1\right) - \beta E_{w'|w}[v_b(a' + eq(j, d; N), w', \mathbf{h_0})]$$

Step 2e: Use solutions to CC, CD, DC and DD to define decision rules and value functions.

- Define $v_c(\cdot)$ and $v_d(\cdot)$ as follows:

$$v_c(a, w, j; t) = \max\{CC(a, w, j; t), CD(a, w, j; t)\}$$
$$v_d(a, w, j, d; t) = \max\{DC(a, w, j, d; t), DD(a, w, j, d; t)\}$$

- Define delinquency decision when current as $g_d^c(a, w, j; t) = 1$ if $CD(a, w, j; t) > CC(a, w, j; t)$, else $g_d^c(a, w, j; t) = 0$.
- Define delinquency decision when delinquent as $g_d^d(a, w, j, d; t) = 1$ if $DD(a, w, j, d; t) > DC(a, w, j, d; t)$, else $g_d^d(a, w, j, d; t) = 0$.
- Define next period assets decision rule when current as:

$$g_a^c(a, w, j; t) = (1 - g_d^c(a, w, j; t))CC(a, w, j; t) + g_d^c(a, w, j; t)CD(a, w, j; t)$$

- Define next period assets decision rule when delinquent as:

$$g_a^d(a, w, j, d; t) = (1 - g_d^d(a, w, j, d; t))DC(a, w, j, D; t) + g_d^d(a, w, j, d; t)DD(a, w, j, d; t)$$

Step 3: Solve mortgage problems, $v_c(\cdot)$ and $v_d(\cdot)$, for remaining periods $N-1, N-2, \ldots, 1$. Algorithm to solve CC, CD, DC, and DD is identical to before, except households do not necessarily end up in v_b, they can transition to v_c or v_d as well. The objective functions are listed below.

- For CC(a,w,j;t):

$$\text{If } j = J-1,\ -u\left((1+r)\cdot a + w - m - a', h_1\right) - \beta E_{w'|w}[v_b(a', w', h_1)]$$
$$\text{If } j < J-1,\ -u\left((1+r)\cdot a + w - m - a', h_1\right) - \beta E_{w'|w}[v_c(a', w', j+1; t+1)]$$

- For CD(a,w,j;t):

$$-u\left((1+r)\cdot a+w-a',h_1\right)-\beta E_{w'|w}[p_f(1)v_b(a',w',h_0)+(1-p_f(1)v_d(a',w',j,1;t+1)]$$

- For DC(a,w,j,d;t):

$$\text{If } j=J-1,\ -u\left((1+r)\cdot a+w-(1+r_m)^d m-a',h_1\right)-\beta E_{w'|w}[v_b(a',w',h_1)]$$
$$\text{If } j<J-1,\ -u\left((1+r)\cdot a+w-(1+r_m)^d m-a',h_1\right)-\beta E_{w'|w}[v_c(a',w',j+1;t+1)]$$

- For DD(a,w,j,d;t):

$$-u\left((1+r)\cdot a+w-a',h_1\right)-\beta E_{w'|w}[p_f(d{+}1)v_b(a',w',h_0)+(1-p_f(d{+}1)v_d(a',w',j,d{+}1;t{+}1)]$$

B. Solving Lifetime Foreclosure Probabilities

To understand how foreclosure risk varies across households with different income and asset levels, I calculate each household's weighted average lifetime foreclosure probability in period $t=0$, written as $\widehat{p_f}(a_0,w_0)$, where a_0 and w_0 are the households assets and incomes, respectively, in period $t=0$.

Step 1: To calculate this probability, I first define the set of possible income sequences that a household could encounter, W^N. This set has cardinality $|W|^N$.

Step 2: Next, I simulate the household's delinquency time paths conditional on initial asset level a_0 and specific income sequence $w_i^N=(w^t)_{t=0}^N \in W^N$.

- delinquency: $g_d(t|a_0,w_i^N)\in\{0,1\}$
- delinquency duration: $d(t|a_0,w_i^N)\in\{0,1,\ldots,\bar{d}\}$

Step 3: Then, I determine the household's foreclosure probability in each period t, defined as $\widehat{p_f}(t|a_0,w_i^N)$, conditional on initial asset level a_0 and specific income sequence w_i^N. These probabilities represent the likelihood a household is foreclosed on exactly in period t, conditional on having survived foreclosure until period t.

$$\widehat{p_f}(t|a_0,w_i^N)=\begin{cases} p_f(d(t|a_0,w_i^N))g_d(t|a_0,w_i^N) & \text{if } t=0 \\ \left[\prod_{i=0}^{t-1}(1-\widehat{p_f}(i|a_0,w_i^N)\right]p_f(d(t|a_0,w_i^N))g_d(t|a_0,w_i^N) & \text{if } 1\le t<N \text{ and } d(t|a_0,w_i^N)<\bar{d} \\ \left(1-\left[\sum_{i=0}^{t-1}(\widehat{p_f}(i|a_0,w_i^N))\right]\right) & \text{if } 1\le t<N \text{ and } d(t|a_0,w_i^N)=\bar{d} \\ \left(1-\left[\sum_{i=0}^{t-1}(\widehat{p_f}(i|a_0,w_i^N))\right]\right)g_d(t|a_0,w_i^N) & \text{if } t=N \end{cases}$$

where $p_f(d)$ is the foreclosure probability for a household that has been delinquent d consecutive periods.

Step 4: Lastly, I combine the foreclosure probabilities across t for a given income series w_i^N and then across all income series $w_i^N\in W^N$ for a given initial asset level a_0 and income level w_0.

$$\widehat{p_f}(a_0, w_0) = \sum_{\{w_i^N \in W^N : (w_0) = w_0\}} \left[\mathbb{P}(w_i^N) \left\{ \sum_{t=0}^{N} \widehat{p_f}(t|a_0, w_i^N) \right\} \right]$$

where $\mathbb{P}(w_i^N)$ is the probability of income series w_i^N such that $\sum_{\{w_i^N \in W^N : (w_0) = w_0\}} \mathbb{P}(w_i^N) = 1$.

C. Discussion on Survey of Consumer Finances

The Survey of Consumer Finances (SCF) combines two techniques to randomly sample the U.S. population. The first is a standard geographically based random sample to match broad characteristics of the overall population, such as home ownership. The second technique is a supplemental sample selected to disproportionately include wealthy families. Due to the complexity of the questions asked in the survey, there is a concern about missing values in the survey. Most missing observations are replaced using a multiple imputation procedure which creates five imputations for each missing value. The five separate values attempt to approximate the distribution of missing data by drawing repeatedly from an estimate of the conditional distribution of the data. The imputations are then weighted accordingly to their likelihood of being drawn from the estimated conditional distribution. The simple statistics described in this paper will be estimated using the SCF provided weights divided by 5. The regression results described will take into account imputation error and sample variability error by using the SCFCOMBO module in Stata. This module uses both imputation uncertainty and bootstrapped standard errors to correctly estimate the regression results. For more details on the SCFCOMBO module, see Collins (2015).