A Theory of Pretense in Public Goods Provision

Huy Nguyen*

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Abstract

A player decides to help, bystand, or pretend to help in providing a public good in two games: the volunteer's dilemma and the public goods game. Pretending does not contribute, but it costs less than helping and can confer prestige. If actual contribution is less than claimed contribution, some claimants may be doubted as fakes and shamed. When pretense is possible, both the individual's chance to help and the expected level of good provision are strictly less than when pretense is not possible. Whether pretense occurs does not depend on group size. Pretenders dilute the prestige from helping and discourage actual helpers. If pretense causes negative externalities, an organization would actually benefit from anonymizing contributors. Introducing authenticated help at a premium can eliminate pretense. Extensions on asymmetry and incomplete information reveal that equilibria can exist where help, bystand, and pretend are all played.

Keywords: pretense, volunteer's dilemma, public goods game, information, signaling, social image, altruism

JEL Classifications: D71, D74, D83, H41

*University of California, Santa Barbara; huy@umail.ucsb.edu. I express gratitude to my advisor Theodore Bergstrom and committee members Gary Charness and Peter Kuhn for their exemplary mentoring. I thank UCSB for the Graduate Opportunity Fellowship 2015-2016, which funded this research. I appreciate Rodney Garratt, Zachary Grossman, Lones Smith, Charles Stuart, Lissette Wilensky, and my colleagues at UCSB for helpful comments.

1 Introduction

Recent developments in economic theory (Bénabou and Tirole 2006; Andreoni and Bernheim 2009) have emphasized the importance of image motivation.[1] People derive satisfaction from appearing virtuous to others, as a positive image confers real benefits such as influence and access to resources.[2] An overlooked aspect of image motivation is that it incentivizes pretense. If we value what others think, then appearing to be helpful can be just as good as actually helping. In fact, while pretense has been studied in some contexts like quality uncertainty or entry deterrence, it has not received much attention in public economics.[3] Motivated by these considerations, my paper explores the effects of social image concerns in a fundamental economic context—public good provision—when pretense is possible.

I introduce a player's decision to help, bystand, or pretend to help in providing a public good in two classic games: the volunteer's dilemma and the public goods game. Pretending does not contribute, but it costs significantly less than helping and can confer prestige. Conversely, a player suspected to be pretending or outright exposed suffers a shame cost. This pretense and prestige dynamic occurs in many real-life scenarios: U.N. states endorsing sustainability (e.g. 2015 Paris Agreement) but not reducing emissions; charities spending only a fraction on aid; reneging on blood/marrow/organ donation (i.e. fake donor registration); donating a minimum $1 to join a sponsors list; in social media (e.g. Facebook), 'liking' and 'sharing' causes without donation.

Such pretense may hinder social progress, so it is important to understand the role of pretense in public goods provision. In each of these examples, time, distance, or bureaucracy can make distinguishing a helper from a pretender difficult, if not impossible. Indeed, anyone in modern society has almost certainly witnessed an instance where a CEO, politician, coworker, or neighbor extols a virtue but does not follow their words. A formal model can help us identify the conditions conducive to pretense and the equilibrium amount of pretending. Consequently, we might design countermeasures to reduce pretense and improve good provision.

Thus I ask: How do potential contributors behave when pretending is an option? How do the equilibrium proportions of helpers, bystanders, and pretenders vary with group size, cost-benefit ratios, or asymmetric preferences? What are the consequences of pretense, and how might an organization mitigate them?

In Section 2, I explore pretense in the context of a volunteer's dilemma game. In addition to help or bystand, players can instead pretend to help in order to gain a *prestige* benefit.

[1] In the dictator game, one player of two decides how to split a sum of money. A 'rational' player takes all, yet many split 50-50 not only out of fairness but also because they wish to be *perceived* as fair by the observers.

[2] Indeed, Maslow's (1943) hierarchy of needs lists self-esteem and self-respect as major motivations after physiological needs.

[3] A person may disclose or mimic strength to deter conflict. In an I.Q. contest, players signaled over-confidence to deter entry and under-confidence provoke entry (Charness et al. 2013).

Pretending is costly but still much cheaper than helping.[4] Initially, the audience is naïve and, if the good is provided, confers prestige to everyone who appeared helpful. If the good is not provided, all claimants are exposed as fakes and suffer shame. Helpers also experience *warm glow*, a psychological benefit, which bystanders and pretenders do not experience.[5]

In Section 2.1 (existence and uniqueness), I note key assumptions and solve for pure- and mixed-strategy equilibria. In pure strategy equilibria, one player helps while all others pretend. In mixed-strategy equilibria, players mix either on help/pretend or help/bystand; which of the two they select depends on the cost-benefit ratio and, surprisingly, not on group size. Broadly two types of societies exist, one that sufficiently values image and one that does not. In other words, helping is always an option in both societies, but the alternative action is pretend in the image-oriented society and bystand in the other.

Our intuition would believe that prestige can only facilitate a public good's provision, but when prestige incentivizes pretending, the individual's chance to help and the group's chance of provision are both *less than* when people do not pretend.

In Section 2.2 (comparative statics), I show that an individual's chance to help increases with benefits and decreases with costs; as group size increases, it converges to zero. This is consistent with the original game. Three new dimensions of warm glow, prestige, and shame add novel results. Warm glow encourages only helping. Prestige incentivizes helping but also incentivizes pretending. Shame deters pretending. When pretense causes negative externalities, an organization may—surprisingly—prefer to reduce or even eliminate the prestige associated with helping. While an organization may not be able to directly choose prestige, they could instead control information structures like keeping donors anonymous.

In Section 2.3 (endogenous prestige), I consider a variant where players are sophisticated and aware that some who appear helpful may be pretenders. Consequently, prestige is now endogenous rather than exogenous. A key finding here is that the presence of pretenders discourages players from helping because their prestige is diluted by fakes, much like a lemons market. Discouraged helpers are thus willing to pay a premium to distinguish their help as authentic, provided the premium is not too costly. If an organization could introduce an authentication option, this would also minimize pretense.

In Section 3, I explore pretense in the context of a simplified public goods game where players can help by donating 1 unit toward the pot, bystand, or pretend to help. As before, pretending costs less than helping, appears indistinguishable, and can confer prestige. Prestige in the public goods game is always endogenous, and therefore discounted, because everyone can see the difference between the pot size and the number of help claims.

[4] Mimicry costs some effort to convince others. Further, most people are averse to lying because it exacts some cognitive or emotional toll. This is true even when lying is undetectable, absent strategic motives, or leads to improved monetary outcomes for everyone (Gneezy et al. 2013; Abeler et al. 2014; Erat and Gneezy 2011).

[5] Egoistic utility from giving, as opposed to *pure* altruism for the recipient's welfare. The colloquial term was first coined by Andreoni in 1989.

In Section 3.1 (existence and uniqueness), I prove that there exist two symmetric, pure-strategy Nash equilibria—either all help or all bystand—and zero asymmetric, pure-strategy equilibria. Essentially, players treat the game environment as a shop to 'buy' prestige and warm glow, either all buying or all not buying. There exists also one symmetric, mixed-strategy equilibrium. Whereas mixed-strategy equilibria in the volunteer's dilemma mix on two actions, the sole mixed strategy in the public goods game mixes on all three actions. This is because in a public goods game, a marginal helper always increases the size of the public good. A marginal helper increases the belief that a claimant is a helper while a marginal pretender decreases this belief.

In Section 3.2 (increasing/decreasing returns), I recompute the equilibria and prove that increasing returns to the pot has the same set of equilibria as constant returns. The forms vary slightly, of course, but no new equilibria are created. This is because the 'more is better' property forces boundary solutions: either all help or all bystand. I then prove that decreasing returns creates two asymmetric, pure-strategy equilibria. Decreasing returns makes possible interior solutions. There can exist a threshold number of helpers before which 'help' is preferred but after which its alternative, either 'bystand' or 'pretend,' is preferred.

In Section 3.3 (incomplete information), I treat players as having (i.i.d.) random but private valuations of both prestige and warm glow.[6] In equilibrium, players select one of three actions based on their preferences. Generally, players who care little about looking good bystand while those who care much about feeling good help; those who value looking good, but not feeling good, pretend. An equilibrium consisting of all three actions can thus be sustained this way. Two graphical examples illustrate this partition, one for the volunteer's dilemma and one for the public goods game.

In Section 4, I discuss my findings in the context of prominent literature and suggest subsequent avenues of exploration. To my best knowledge, my paper is the first to model pretense in public goods provision.

2 Volunteer's Dilemma

The volunteer's dilemma (Diekmann 1985) is an n-player game in which each player i benefits b_i from the provision of a public good if at least one player volunteers to pay the cost c_i, where $0 < c_i < b_i$. Each gains from the good's provision but prefers to let someone else pay for the good, a phenomenon known among social psychologists as *diffusion of responsibility* or the *bystander effect* (Darley and Latane 1968). Examples include helping an injured victim, taking out the trash, or enforcing a social norm such as confronting a smoker at a

[6] One might argue that players should vary in the cost of helping, but warm glow serves the same purpose.

restaurant.[7] Various authors have since covered extensions like asymmetric costs (Diekmann 1993), incomplete information (Weesie 1994), or cost sharing (Weesie and Franzen 1998). The studies closest to my paper incorporate behavioral elements like warm glow (Andreoni 1990; Bergstrom et al. 2015) and prestige from helping (Harbaugh 1998; Andreoni and Petrie 2004) but do not model the strategic *pretense* of helping.

A problem occurs, and n witnesses simultaneously decide whether to help (H), bystand (B), or a third option: pretend to help (P). Helping costs c and, if at least one person helps, confers three benefits: first, everyone consumes a material benefit $b > 0$; secondly, helpers and pretenders earn a prestige $g > 0$; thirdly, helpers enjoy a warm glow $w > 0$. Thus, I decompose the helper's benefit into material, prestige, and warm glow components.

Bystanding costs zero and contributes nothing; a bystander receives the material benefit b only if someone else helped. Pretending costs less than helping but more than zero, $0 < c_P < c$, and confers prestige g only if someone else helped.[8] If no one helped, the public good is not provided so all pretenders are exposed; exposed pretenders gain no prestige and suffer a shame cost s. A player's payoff from each action, conditional on whether or not someone else offers to help, is summarized below:

	u_i (if someone else helps)	u_i (if no one else helps)
help (H)	$b + g + w - c$	$b + g + w - c$
bystand (B)	b	0
pretend (P)	$b + g - c_P$	$-c_P - s$

where

b = material benefit	g = prestige	w = warm glow
c = cost	c_P = pretend cost	s = shame

Table 1. Actions and Payoffs

All parameters have positive value. H is the best response if no one else helps, so it cannot be strictly dominated. An assumption is needed to ensure that neither B nor P is weakly dominated.

Assumption 1. $0 < c_P < g < g + w < c < b$.

$c < b$ is the original volunteer's dilemma assumption. $g + w < c$ means that prestige and warm glow alone, without any material benefit, do not incentivize a bystander to help; else, H weakly dominates B. $c_P < g$ means that pretending is profitable if someone else helps; else, B weakly dominates P.

[7] Even animals like penguins and marmots are known to volunteer serving as a lookout for predators (Dawkins 1976).

[8] Prestige here is an abstract, non-rival benefit that makes more sense duplicated than divided equally. All else equal, three volunteers generate more prestige than one volunteer.

Initially, I model prestige g as fixed and exogenous. In later sections, I explore heterogeneity, where $g_i \sim U[0, \bar{g}]\ \forall\ i$, and endogenous prestige, where g is discounted by the Bayesian probability an observed helper is actually helping.

2.1 Existence and Uniqueness

Proposition 1. *Given Assumption 1, there exist n pure-strategy Nash equilibria where exactly one player helps and everyone else pretends.*

Proof. Given that all others pretend—and therefore do not help—the one helper receives positive utility $u_i = b + g + w - c > 0$. Deviating from action H would cause $u_i \leq 0$ because the public good would not be provided. Given that someone else helps, action P yields the greatest utility. □

Pretenders not only free ride the material benefit, but also share in the prestige.[9]

Proposition 2.
a. *Given Assumption 1, exactly one of two symmetric, mixed-strategy Nash equilibria exists: 1) players mix on help/bystand and no one pretends; or 2) players mix on help/pretend and no one bystands.*
b. *The condition that decides which equilibrium exists is:* $\dfrac{c-g-w}{b} \geq\leq \dfrac{c-c_P-w}{b+g+s}$. *If the left side is strictly lesser, then the help/bystand equilibrium exists. If the right side is strictly lesser, then the help/pretend equilibrium exists. In the coincidence that both sides are equal, either the help/pretend equilibrium exists or the help/bystand equilibrium exists.*
c. *A three-way, symmetric, mixed-strategy Nash equilibrium where players mix on all three actions–help, bystand, and pretend–with positive probability does not exist.*

Proof of Proposition 2: See Appendix.

The individual's probability of helping and the group's probability of provision in each equilibrium are summarized below:

	individual helps (p_H^*)	group provides ($\hat{P}^*$)
help/bystand (H/B)	$1-\left(\dfrac{c-g-w}{b}\right)^{1/(n-1)}$	$1-\left(\dfrac{c-g-w}{b}\right)^{n/(n-1)}$
help/pretend (H/P)	$1-\left(\dfrac{c-c_P-w}{b+g+s}\right)^{1/(n-1)}$	$1-\left(\dfrac{c-c_P-w}{b+g+s}\right)^{n/(n-1)}$

Table 2. Equilibrium Probabilities

[9] Whether this is socially wasteful is a matter of perspective, as the resources expended to pretend are compensated by the utility gained from shared prestige.

The condition that decides which equilibrium exists does not depend on n, group size. This condition also appears in Table 2; equivalently, only the equilibrium with the smaller probability to help exists. Proposition 2.b implies that when the help/pretend equilibrium exists, both the individual's probability of helping and the group's probability of provision are *strictly less than* if pretending were not an option. In other words, the possibility of pretending can only worsen public good provision!

2.2 Comparative Statics

Proposition 3

a. *Given Assumption 1, within the help/bystand and within the help/pretend equilibria,* $\partial p_H^*/\partial b$, $\partial p_H^*/\partial g$, $\partial p_H^*/\partial w > 0$ *and* $\partial p_H^*/\partial c < 0$. *Within only the help/pretend equilibrium,* $\partial p_H^*/\partial c_P$, $\partial p_H^*/\partial s > 0$.

b. *As* $n \longrightarrow \infty$, $p_H^* \longrightarrow 0$ *and* $\hat{P}^* \longrightarrow 1 - max\left\{\frac{c-g-w}{b}, \frac{c-c_P-w}{b+g+s}\right\}^{n/(n-1)}$.

Proof. Refer to Table 2. Marginal effects are deduced from each parameter's sign (+ or −) and position (numerator or denominator). $c_P < g < g + w < c$ implies the numerators are positive, $c - g - w > 0$ and $c - c_P - w > 0$. Then, $c < b$ implies the denominators are greater, so $p_H^* \in (0, 1)$. Asymptotics are also deduced by each exponent's convergence (either 0 or 1) as $n \longrightarrow \infty$. □

Asymptotic probabilities p_H^* and $\hat{P}^*$ are consistent with the original volunteer's dilemma. Notice that the expected utility is always $E(u_i) = E(u_i|H) = b + g + w - c$, which is unaffected by c_P or by whether one equilibrium exists or the other.

Proposition 4. *Given Assumption 1, when prestige is sufficiently large, only the help/pretend equilibrium exists.*

Proof. Proposition 2.b states that only the help/pretend equilibrium exists when $\frac{c-g-w}{b} > \frac{c-c_P-w}{b+g+s}$. Rearranging this expression in terms of prestige g yields:

$$0 < g^2 + (b + w - c + s) \cdot g + s(w - c) - b \cdot c_P \quad (1)$$

The right side of Expression 1 is a convex parabola $f(g)$ with a negative vertical intercept at $f(0) = s(w - c) - b \cdot c_P < 0$. This means somewhere in the domain $g > 0$, there exists a unique threshold g^T where $f(g^T) = 0$ crosses the horizontal axis. Specifically, this crossing point occurs at $g^T = (-B + \sqrt{B^2 - 4C})/2$ where $B = b + w - c + s$ and $C = s(w - c) - b \cdot c_P$. When prestige exceeds this threshold, the help/pretend equilibrium exists. □

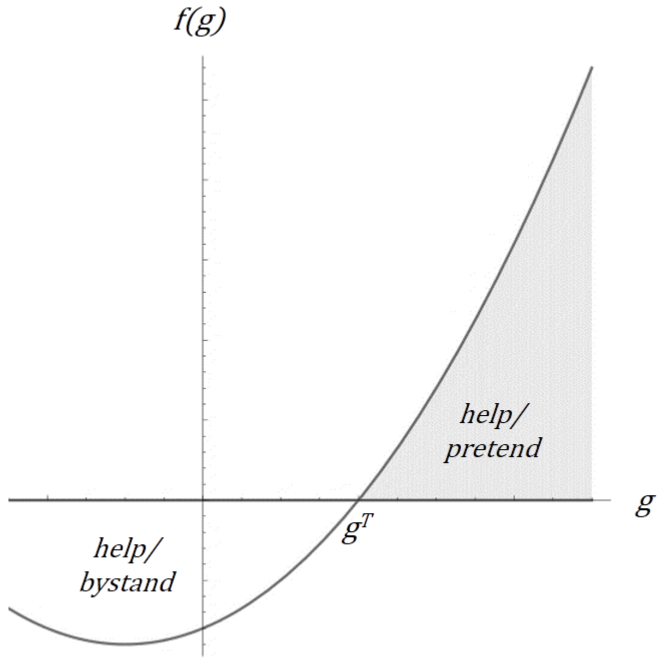

Fig. 1. Large Values of Prestige Induce a Help/Pretend Equilibrium

Prestige encourages contribution but also pretense. Societies that strongly value prestige develop a norm to appear helpful, even if that means pretending. Societies that care little for prestige develop a norm to bystand if one does not help. In some cases, pretense may waste social resources. For example, registering as an organ or marrow donor but reneging when asked can disrupt a clinic's plan. Our intuition would lead us to believe that prestige can only improve outcomes. However, when pretense imposes negative externalities, society may fare better if people value prestige less!

2.3 Endogenous Prestige

Assumption 2. g is discounted by $p_H^*/(p_H^* + p_P^*)$.
I now relax the assumption that all claimants earn prestige when the public good is provided. If audiences are sophisticated, they would discount the prestige g by the Bayesian probability an observed helper is actually helping. This discount factor is $p_H^*/(p_H^* + p_P^*)$, where p_H^* and p_P^* are the equilibrium probabilities that a player helps or pretends. This makes prestige endogenous, affected by the composition of helpers versus pretenders in equilibrium.

Proposition 5. *Given Assumptions 1-2, exactly one of three symmetric, mixed-strategy Nash equilibria exists: 1) players mix on help/bystand and no one pretends; 2) players mix* *on help/pretend and no one bystands; or 3) players mix on all three actions—help, bystand, and pretend–with prositive probability.*

Proof of Proposition 5: See Appendix.

Proposition 6. *Given Assumptions 1-2, both a player's probability of helping and the group's probability of provision in the help/pretend equilibrium are strictly less than under Assumption 1 only.*

Proof. When the equilibrium mixes only on actions H/B, any helpers are clearly helpers. When the equilibrium mixes only on H/P, prestige g is discounted. In this case, $p_P^* = 1 - p_H^*$ so the discount factor simplifies to p_H^*. Equating $E(u_i|H) = E(u_i|P)$ simplifies to:

$$\frac{c - c_P - w}{b + g \cdot p_H + s} = (1 - p_H)^{n-1} \in (0, 1) \tag{2}$$

A discounted $g \cdot p_H < g$ makes the denominator smaller, the fraction larger, and thus p_H smaller on the right side. □

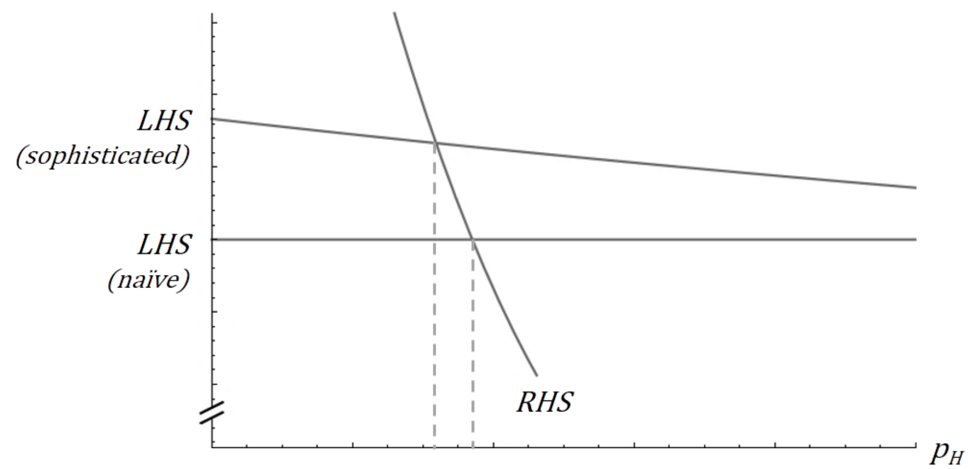

Fig. 2. Audience Sophistication Reduces Provision

Fig. 2 illustrates how Equation 2 determines equilibrium p_H^* depending on whether the left side uses endogenous prestige $g \cdot p_H$ (sophisticated audience) or exogenous prestige g (naïve audience). The presence of pretenders has a double-negative effect on provision. First, pretenders do not contribute. Second, pretenders dilute prestige, which discourages helpers who realize their help may be doubted as fake. This discouragement shares a structural similarity to the market for lemons (Akerlof et al. 1970).[10]

[10] Akerlof, Spence, and Stiglitz shared the 2001 Nobel Prize in Economics for their work in asymmetric information, of which the market for lemons was a central idea.

In an environment where audiences are sophisticated and some players pretend, helpers feel frustrated to be pooled with pretenders. Suppose now those helpers have the option to authenticate (A) their help. Authenticating costs more than helping ($c_A > c$), but it appears distinct from any other action and guarantees that the helper earns an undiluted prestige $g > g \cdot p_H$.[11]

Proposition 7.
a. *Given Assumptions 1-3, when authenticating is an option, exactly one of two symmetric, mixed-strategy Nash equilibria exists: 1) players mix on authenticate/bystand and no one pretends or unauthenticated-helps; or 2) players mix on unauthenticated-help/pretend and no one bystands or authenticates.*
b. *If the premium is less than the prestige loss, the authenticate/bystand equilibrium exists. If the premium is greater than the prestige loss, the help/pretend equilibrium exists.*

Proof. Helpers who are pooled with pretenders authenticate if $E(u_i|A) > E(u_i|H)$. This occurs when:

$$(c_A - c) < g(1 - p_H^*) \tag{3}$$

The left side of Expression 3 is the extra cost to authenticate while the right side is the extra benefit from restoring full prestige. If marginal benefit outweighs marginal cost, players pay this premium. If so, players mix on actions A/B (i.e. they play H and P with probability zero). If the premium is too costly, no one plays A and the equilibrium reverts to mixing on H/P. Mixing on A/P cannot be an equilibrium because players can deduce any claimant who does not authenticate help must be a pretender. The case where both sides of Equation 3 are exactly equal occurs on a set of measure zero in parameter space. □

When pretense is widespread and problematic, an organization may consider introducing an authentication option at a premium to minimize pretending behavior. This is an alternative solution to minimizing prestige itself via, for example, anonymizing donors.

3 Public Goods Game

The public goods game (Samuelson 1954) is a scenario where each of n members in a group can voluntarily contribute some or all of their private resources toward a common pool. This sum contribution is then multiplied by a given fraction $m \in (1/n, 1)$ and awarded identically to each member. The constraint $m > 1/n$ means that, should one contribute, the gain in group welfare outweighs the loss in individual welfare. That $m < 1$ means players face a

[11] Authentication is relevant only in a help/pretend equilibrium. In a help/bystand equilibrium, any help is already believed.

strategic tension between a lesser, private welfare versus a greater, public welfare. Else, donating 1 unit would immediately return $m \geq 1$ to the donor. The dominant strategy in this game is to donate zero, which shares some similarity to an n-player prisoner's dilemma. Regardless what others do, donating always reduces one's own material payoff.

Numerous experimental studies, however, show that people do donate for a variety of reasons: altruism, in that helpers feel a warm glow, but only when others are also helping; efficiency, in that helpers seek to increase the economic pie; or confusion, in that helpers choose arbitrarily (Andreoni 1995; Houser and Kurzban 2002). Extensions reveal that the threat of punishment (Fehr and Gächter 2000) or exclusion (Cinyabuguma et al. 2005; Charness and Yang 2014) can pressure players to donate at the social norm.

A group of n players convenes over a public good project. Each player simultaneously chooses one of three actions: 1) help (H), which costs one unit and contributes one unit ($c_i = x_i = 1$) toward the project; 2) bystand (B), which costs nothing and contributes nothing ($c_i = x_i = 0$), or pretend (P) to help, which costs less than helping ($0 < c_i = c_P < 1$) but contributes nothing ($x_i = 0$). The sum contribution $\hat{X} = \sum_{i=1} x_i$ in the project, sometimes colloquially called the pot, is then scaled by a known multiplier $m \in (1/n, 1)$ and enjoyed identically by each leader.[12] This composes the material benefit.

Players who claim to help also receive a prestige benefit $g > 0$ if the group believes the claimant helped and no prestige if the group believes the claimant pretended. This belief occurs with chance β, the conditional probability that a claimant is a helper from an audience's—not player's—perspective. A helper also enjoys a warm glow $w > 0$. A bystander receives neither prestige nor warm glow. While it is true players have different β_i based on private information of their own action, when n is large, we can use the approximation $\beta_H \approx \beta_P \approx \beta_N = X/k$, where k is the number of claimants.[13] I use this approximation to focus on salient features and keep the model tractable.

For example, suppose four players each possessing one coin are asked to donate. One player abstains while three others walk up to a box and gesture a donation. After all players have (simultaneously) acted, the donation box is revealed to contain only two coins! From an audience's perspective, each claimant has a $\beta = 2/3$ chance to be a helper and a $1 - \beta = 1/3$ chance to be a pretender. These probabilities are not independent. If one claimant is believed to be a pretender, the other two must therefore be helpers. These inferences are clear enough that we can treat all players as sophisticated.[14] A player i's payoffs from each action, with benefits decomposed into material versus image, is summarized below:

[12] I normalize cost and contribution to 1 because m already acts as a scalar.

[13] We might also ask: whose good opinion do players value? Each β_i could be weighted by a respect ω_i accorded to i based on i's action. Likely, people care most about the judgment of helpers. To a lesser extent, people also care for the judgment of bystanders and, least of all, pretenders.

[14] It would be inconceivably naïve to see two coins and believe all three were helpers.

	material	prestige (if believed)	warm glow
help (H)	$m(1+\sum_{j\neq i} x_j)-1$	g	w
bystand (B)	$m(0+\sum_{j\neq i} x_j)-0$	0	0
pretend (P)	$m(0+\sum_{j\neq i} x_j)-c_P$	g	0

where

m = multiplier	g = prestige	w = warm glow
x_j = player j's contribution	c_P = pretend cost	

Table 3. Actions and Payoffs

3.1 Existence and Uniqueness

Helping costs 1 but returns some material benefit via m. Let $c_H = 1 - m$ then denote the cost of helping. The prestige g payoffs to H and P are equal because both groups look indistinguishable and are subject to the same judgment. g varies based on the fraction of helpers among the population of k claimants. Players take this into account when choosing between H/P (but not H/B), since their choice alters the belief $\beta = X/k$.

Proposition 8.
a. *If a symmetric, pure-strategy Nash equilibrium exists, then either all players help or all players bystand. An equilibrium where all players pretend cannot exist.*
b. *An asymmetric, pure-strategy Nash equilibrium cannot exist.*
c. *If a symmetric, mixed-strategy Nash equilibrium exists, then it mixes on all three actions–help, bystand, pretend. An equilibrium that mixes on only two actions cannot exist.*

Proof of Proposition 8.a. All help is an equilibrium if each player prefers H to either B or P given that all others play H. When everyone helps, $\beta = n/n = 1$ and deviating from H to B does not change this belief $\beta = (n-1)/(n-1) = 1$. Deviating from H to P, however, reduces the belief to $\beta = (n-1)/n < 1$. $H \succeq B$ and $H \succeq P$, respectively, when:

$$g + w \geq c_H \tag{4}$$

$$\frac{g}{n} + w \geq c_H - c_P \tag{5}$$

If $w \geq c_H - c_P$, Expression (5) is true independent of n; else, n must be sufficiently small such that $H \succeq P$. This threshold is:

$$n \leq \frac{g}{c_H - c_P - w} \tag{6}$$

All bystand is an equilibrium if each player prefers B to either P or H given that all others play B. $B \succeq P$ because when all others bystand, everyone could see that the alleged helper donated nothing to the pot; a pretender would earn $-c_P < 0$.[15] $B \succeq H$ when $g + w \leq c_H$. All pretend cannot be an equilibrium because an empty pot exposes all claimants as fakes; each pretender would earn $-c_P < 0$. □

Essentially, the option of guaranteed prestige and warm glow collapses this game into a shop where players can 'buy' looking and feeling good via their contribution. This forms the other equilibrium where everyone plays H.

Proof of Proposition 8.b: See Appendix.
Proof of Proposition 8.c: See Appendix.

3.2 Increasing/Decreasing Returns

I now consider the case where the material benefit b has increasing or decreasing returns in the number of donors. Let $b = m(X)^r$ where $r > 1$ and $X = \sum_{i=1} x_i$ is the pot. Helping costs 1 but returns some material benefit via $m(X)^r$. Let $c_H(X) = 1 - m(X^r - (X-1)^r)$ denote the (discounted) cost of helping for the X^{th} helper. For emphasis, $c_H(\cdot)$ is the cost function to help, not a product. Increasing returns means that each marginal helper contributes more to the pot than the previous. So, $\partial b/\partial X > 0$ and $\partial c_H(X)/\partial X < 0$. Decreasing returns implies the opposite.

Proposition 9.
a. *Under increasing returns to scale, exactly one of three Nash equilibria exist: 1) a symmetric pure strategy where all help; 2) a symmetric pure strategy where all bystand; or 3) a symmetric mixed strategy on all three actions—help, bystand, and pretend.*
b. *Under decreasing returns to scale, exactly one of five Nash equilibria exist: 1) a symmetric pure strategy where all help; 2) a symmetric pure strategy where all bystand; 3) an asymmetric pure strategy where some help while the rest pretend; 4) an asymmetric pure strategy where some help while the rest bystand; or 5) a symmetric mixed strategy on all three actions—help, bystand, and pretend.*

Proof. An all-help equilibrium exists when $H \succeq B$ and $H \succeq P$, which occurs when:

$$g + w \geq c_H(n) \tag{7}$$
$$\frac{g}{n} + w \geq c_H(n) - c_P \tag{8}$$

[15] Because any deviation from B is to H, the off-equilibrium-path belief of anyone not bystanding must be $\beta = 1$. I assert not only that $\beta = 1$ sustains the equilibrium, but also the stronger claim that the unique $\beta = 1$ must be deduced if players are rational and know others are rational.

As before, this may depend on n being sufficiently small. $B \succeq P$ is always true so the addition of $B \succeq H$ results in an all-bystand equilibrium. All pretend cannot be an equilibrium. An asymmetric, pure-strategy equilibrium where X players help while the rest $(n-X)$ bystand would require that $E(u_H|H) \geq E(u_H|B)$ and $E(u_B|B) \geq E(u_B|H)$, which implies:

$$c_H(X) \leq g + w \leq c_H(X+1) \tag{9}$$

Under increasing returns, this is impossible because $c_H(X) > c_H(X+1)$. Under decreasing returns, an interior solution is possible because $c_H(X) < c_H(X+1)$. There can exist a threshold $X^* = \sum_i x_i < n$ number of helpers before which H is preferred but after which B is preferred. This leads to an asymmetric, pure-strategy Nash equilibrium where some players help while the rest bystand. An asymmetric equilibrium where X players help while the rest $(n-X)$ pretend would require that $E(u_H|H) \geq E(u_H|P)$ and $E(u_P|P) \geq E(u_P|H)$, which implies:

$$c_H(X) \leq \frac{g}{n} + w + c_P \leq c_H(X+1) \tag{10}$$

Under increasing returns, this is again impossible because $c_H(X) > c_H(X+1)$. Under decreasing returns, an asymmetric equilibrium is possible because $c_H(X) < c_H(X+1)$, provided that $X < n$. An asymmetric equilibrium in B/P cannot exist because P is never played without H. Lastly, an asymmetric equilibrium where X players help, $(k-X)$ pretend, and $(n-k)$ bystand requires that:

$$\frac{g(X+1)}{k+1} \leq 1 - m[X^r - (X-1)^r] - w \leq \frac{g \cdot X}{k} \tag{11}$$

$$\frac{g}{k} \leq 1 - m[X^r - (X-1)^r] - c_P - w \leq \frac{g}{k} \tag{12}$$

$$\frac{g \cdot X}{k+1} \leq c_P \leq \frac{g \cdot X}{k} \tag{13}$$

Expression (11) implies $g(k-X) \leq 0$, which is true only if $k = X$. This would mean all claimants are helpers and there are no pretenders, which is sufficient to refute a three-way asymmetry. For completeness, Expression (12) occurs only by coincidence and Expression (13) implies $g \cdot X \geq 0$, which is always true. So, an asymmetric, pure-strategy equilibrium in $H/B/P$ cannot exist for any $r > 0$.

As in Proof of Proposition 8.c, no closed-form solution exists for a symmetric, mixed-strategy Nash equilibrium for general n. Still, we can derive the equilibrium for $n = 2$ to show that a solution exists for general returns to scale. Equating $E(u_i|H) = E(u_i|P) =$

$E(u_i|B)$ yields:

$$0 = p_H \cdot m \cdot (2^r - 2) - p_P \cdot g/2 + g + w - 1 + m \tag{14}$$

$$0 = p_H \cdot g/2 - c_P \tag{15}$$

$$\implies (p_H^*, p_P^*) = \Big[\frac{2 \cdot c_P}{g}, \frac{2(g + w - 1 + m)}{g} + \frac{4 \cdot m \cdot c_P \cdot (2^r - 2)}{g^2}\Big] \tag{16}$$

I demonstrate examples for increasing ($r = 1.1$) and decreasing returns ($r = 0.9$) when $n = 2$. A mixed-strategy equilibrium exists when $p_H, p_P, p_B \in (0, 1)$ and parameters satisfy the condition:

$$\frac{g}{n} + c_P < 1 - m(2^r - 1) - w < g \tag{17}$$

$$r = 0.9 \quad m = 0.7 \quad g = 0.4 \quad w = 0.05 \quad c_P = 0.1 \quad \implies (p_H^*, p_P^*, p_B^*) \approx (.50, .30, .20)$$

$$r = 1.1 \quad m = 0.6 \quad g = 0.4 \quad w = 0.05 \quad c_P = 0.1 \quad \implies (p_H^*, p_P^*, p_B^*) \approx (.50, .47, .03)$$

□

3.3 Incomplete Information

The base model assumes every player is identical. A more realistic assumption would be that players are similar in preferences, but each varies in some individual way. Because prestige and warm glow are behavioral terms introduced to the model, it is most natural to introduce heterogeneity via g and w. Let each player have fixed benefits of prestige $g_i \sim U[0, \bar{g}]$ and warm glow $w_i \sim U[0, \bar{w}]$ that are identically and independently distributed uniformly. Individuals know their own preferences but not those of others; they know only the distribution from which these preferences are drawn.

To derive an equilibrium, I use the Bayesian approach for games of incomplete information (Harsanyi 1967-1968). A strategy here maps a player i's type (g_i, w_i) to an optimal action (H, B, or P).[16] In a Bayesian equilibrium, each player best-responds to maximize expected utility given the stochastic preferences (g_j, w_j, where $j \neq i$) of other players.

Proposition 10. *Under heterogeneity and incomplete information, a unique equilibrium exists where players choose exactly one of three actions—help, pretend, bystand—based on their type.*

Proof of Proposition 10: See Appendix.

[16] As shown in Weesie 1994, randomization among alternatives is unnecessary for an equilibrium to exist and is thus discarded.

While the strategy functions are complex and parameter-dependent, we can visualize a mapping of types to optimal actions. A rectangle of dimensions $\bar{g} \times \bar{w}$ represents the set space for $g_i \sim U[0, \bar{g}] \times w_i \sim U[0, \bar{w}]$. This type space is divided by three concentric rays into distinct regions corresponding to the three actions—help, bystand, or pretend. Every player uses this same mapping to play a pure strategy corresponding to their given type.[17]

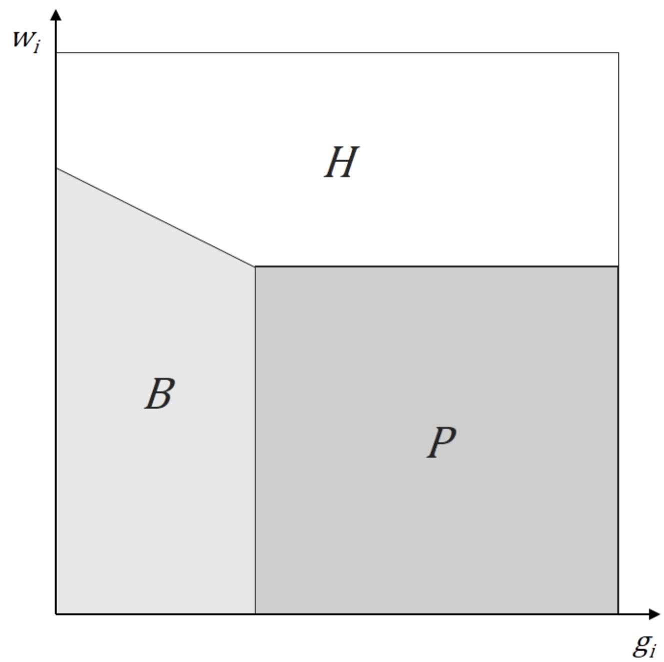

Fig. 3. Mapping from Preferences to Action (PGG)

At the indifference between B/P, any increase in prestige g_i would tip a marginal player to P. This is because warm glow does not exist in either B or P, but prestige matters in P. Likewise, at the indifference between H/B, any increase in g_i or w_i tips a marginal player to H. In fact, this threshold's slope is flatter than 45° because $\partial[E(u_i|H) - E(u_i|P)]/\partial g_i < \partial[E(u_i|H) - E(u_i|P)]/\partial w_i$. A helper receives warm glow with certainty and prestige only with some probability $\beta < 1$. The margin of H/P is separated by only w_i, with warm glow encouraging helping. Both helpers and pretenders are subject to the same judgment β so any difference must lie in w_i.

In this hypothetical illustration, the areas of H, B, and P are respectively 35%, 25%, and

[17] In fact, I conjecture that a valid partition always exists for any ratio of $H : N : P$. That is, given three distinct angles of rays originating from a central point P and given a desired ratio of $H : N : P$, I claim that there always exists at least one placement of P, either inside or outside the rectangle, that partitions the type space into the desired shares. However, the geometric proof for this is beyond the scope of this paper. I simply mention this for interested mathematicians.

40%.[18] A player's chance to help is 1 if their type falls in H and 0 otherwise. However, from an observer's perspective, it appears as if $p^* = 35\%$ per individual. For comparison, the equilibrium in the volunteer's dilemma, when audiences are sophisticated, resembles Fig. 3. When audiences are naïve, however, the equilibrium resembles Fig. 4. The H/B threshold's slope is exactly 45° because $\partial[E(u_i|H) - E(u_i|B)]/\partial g_i = \partial[E(u_i|H) - E(u_i|B)]/\partial w_i$. The H/P threshold's slope is flatter than 45° because $\partial[E(u_i|H) - E(u_i|P)]/\partial g_i < \partial[E(u_i|H) - E(u_i|P)]/\partial w_i$. Pretenders already receive g_i if the public good is provided, so it takes greater prestige than warm glow to tip a marginal pretender into helping.

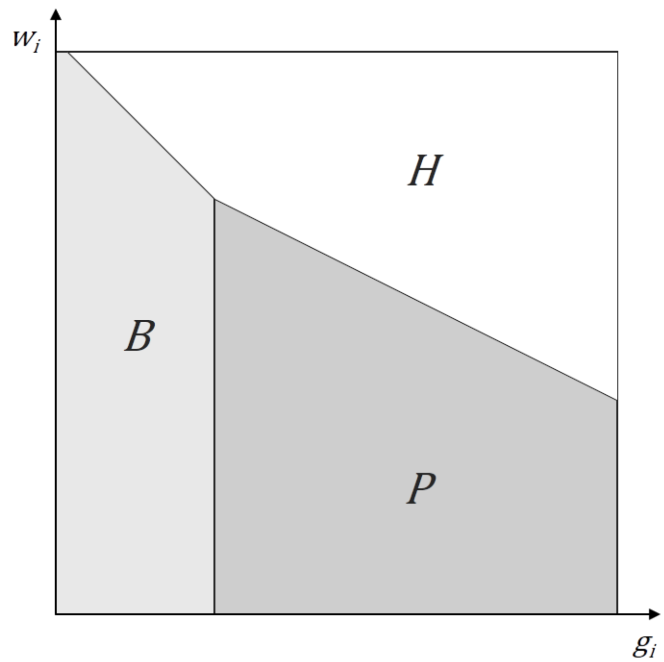

Fig. 4. Mapping from Preferences to Action (VOD, naïve)

4 Discussion

The key insight of this paper is that in many public goods contexts—economic, political, social—a 'help' signal may not correspond to a 'help' action. That is, people sometimes pretend to contribute, potentially free riding a positive image without paying the full help cost. How do decision makers behave when pretending is an option? How do cost-benefit ratios, group size, or asymmetry affect outcomes? What are consequences to pretense and how might we mitigate them? Until now, this has been a gap in the public goods literature.

I fill this gap by modeling two games, the volunteer's dilemma and the public goods game,

[18] These examples approximate the mean populations of altruistic, selfish, and 'reluctant' types across previous studies: Lazear et al. (2012), Dana et al. (2006), Dana et al. (2007).

with a third alternative: pretending to help. Pretending appears identical to helping, contributes zero toward the public good, and costs significantly less than helping ($0 < c_P < c_H$). Secondly, I decompose the benefits from helping (or pretending) into material (b), prestige (g), and warm glow (w). These three factors comprise the primary motivations for contributing toward a public good. Helping confers a material benefit via a stronger public good and also via prestige and warm glow. Pretending confers only prestige and only if the good is provided. Bystanding costs nothing and confers nothing.

In Diekmann's (1985) original, n-player volunteer's dilemma, there are n pure-strategy Nash equilibria where exactly one player helps and everyone else bystands. When I introduce the pretend option, these same equilibria instead become exactly one player helping while everyone else pretends.[19] The intuition is that if provision succeeds, pretenders can also share in prestige (Result 1). Whether this outcome is socially wasteful is a matter of perspective, as the resources expended to pretend are compensated by the utility gained from prestige. Like the original model, the pretense model has a unique, symmetric, mixed-strategy Nash equilibrium where everyone mixes on help/bystand. Unlike the original model, it additionally has one where everyone mixes on help/pretend. Exactly one of these two equilibria must exist (Result 2a).

Regarding the effect of group size, in both equilibria as n grows toward infinity, the probability an individual helps shrinks toward zero and the likelihood the good is provided approaches one minus the cost-benefit ratio. This is consistent with Diekmann's model (Result 3c). Our intuition might assume that whether the mixed-strategy equilibrium is help/bystand or help/pretend would depend on group size. Surprisingly, it does not and depends only on comparing cost-benefit ratios. Specifically, the equilibrium with the greater cost-benefit ratio dominates. A somber logic implies that when the help/pretend equilibrium is played, the public good is even *less* likely to be provided than when the help/bystand equilibrium is played (Result 2b)!

These cost-benefit ratios include the novel term, prestige. For sufficiently high prestige, the help/pretend mix dominates. From an anthropological view, this means that societies which value image heavily develop a norm to appear helpful, which motivates pretending (Result 4). The rise of social media, for instance, has created audience effects and norms of philanthropy. At the same time, the ease of forwarding solicitations can make pretense an attractive option. Telecommunications enables people to not only *not* help, but furthermore *pretend* to help. If pretense becomes widespread and problematic, an organization may actually prefer to reduce the prestige accorded to helpful behavior, perhaps by anonymizing donors.

The pretending volunteer's dilemma shares some structural features to Akerlof's (1970) famous market for lemons. Akerlof argued that when buyers cannot distinguish between a

[19]With endogenous prestige, the cost of pretending must be cheaper than the benefit of diluted prestige; else, the rest would bystand.

high-quality car ('peach') and a low-quality car ('lemon'), they will in expectation pay only the average price of a peach and lemon. Sellers, on the other hand, know the quality of their car. Given the lower, average price at which buyers would buy, 'lemons' sell while 'peaches' leave the market. Similarly, when the audience is sophisticated, they discount prestige by the probability an observed helper is actually helping. Thus, the presence of pretenders has a double-negative effect. First, pretenders do not help. Second, pretenders dilute prestige and discourage potential helpers from helping. Discouraged helpers may fear that their help will be doubted as fake (Result 5).

Interestingly, discouraged helpers facing a diluted prestige are willing to pay extra costs, up to the prestige loss, to authenticate their help. When an authentication option is introduced, either all mix on authenticate/bystand or all mix on help/pretend (Result 6). While it may seem strange, people sometimes do pay a premium to authenticate their help. A worker might work in an inconvenient but public space for visibility. A philanthropist might increase donation to a higher bracket to avoid being listed among lower-tier donors who may be donating the bare minimum (e.g. $100+ group instead of $1 to $99 group). This result is supported by data from Harbaugh's (1998a) work on philanthropy and prestige, which shows that when donations are both tiered and publicized, most donations bunch at the minimum required for inclusion in each tier.

I extend pretense to a simplified public goods game. In Samuelson's (1954) original, n-player public goods game, the dominant strategy is to donate zero. In my pretender's public goods game, this is also a Nash equilibrium, but there is a second outcome where everyone donates. This occurs if prestige and warm glow are sufficiently high to offset material costs.[20] In essence, the game simplifies into a shop where individuals may purchase looking and feeling good via their donation (Results 8a). There are no asymmetric, pure-strategy Nash equilibria (Result 8b). A second, striking difference is that the pretender's public goods game creates a mixed-strategy Nash equilibrium where none had originally existed. Distinctly, this mixed-strategy equilibrium is unique, symmetric, and three-way on help, pretend, and bystand. There are no two-way mixed-strategy equilibria (Results 8c).

Adjusting the returns to scale on the public good generates curious results as well. Relative to constant returns, increasing returns neither creates new equilibria nor destroys existing ones (Results 9a). Decreasing returns on the other hand creates two new, asymmetric equilibria—some help while others bystand or some help while others pretend! An asymmetric equilibrium cannot exist where some bystand while others pretend, nor where all three actions are played (Results 9b). The intuition is that decreasing returns creates *interior* solutions where, before a threshold number of helpers, helping is preferred but after which the alternative is preferred. Increasing returns creates *boundary* solutions and therefore no asymmetric equilibria.

[20] In some cases, it may also depend on n being sufficiently small.

In an incomplete information equilibrium, a two-dimensional space exists that maps types to actions (Result 10). Recent work on 'reluctant' helpers by Lazear et al. (2012), Dana et al. (2006, 2007) suggest that people can be grouped into altruistic, selfish, or 'reluctant' types. Reluctant helpers are motivated by prestige but would rather not help if unobserved. These types correspond closely to helpers, bystanders, and pretenders in my heterogeneity model. The work of these authors estimate that about 35% of the population are altruistic, 25% are selfish, and 40% are reluctant. In crafting Figs. 6 and 7, I took special effort to equate the areas of the three regions to these percentages.

My theory on pretense in public goods games enables several extensions. To construct a comprehensive theory, I made the simplifying assumption that players in the three public goods games (constant, increasing, and decreasing returns) contribute either 1 unit or 0 units. An extension might consider how the pretender's public goods model changes when players can contribute any continuous fraction of their endowment. Within this fraction, perhaps players can further decide how much is real and how much is fake. For example, a player endowed with 10 coins could submit 3 real coins and 4 (cheaper) fake coins.

Experimentally, when pretending is an option, should we expect an increase in *observed* donation levels, but a decrease in *actual* donation levels? If so, this would imply that some helpers were relucant and also that some bystanders would 'buy' prestige if only prestige were cheaper. Relative to donation levels found by Isaac and Walker (1988a, 1998b), does the pretender's public goods game decay more rapidly over time? In an environment where pretending is possible, to what extent would introducing an authenticate option improve donation levels?

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Appendix

A1. Proofs

Proof of Proposition 2. Let p_H be the probability an individual helps, p_B the probability of bystanding, and p_P that of pretending. With three actions, there potentially exist up to four symmetric, mixed-strategy equilibria: 1) mixing only on actions B/P (i.e. $p_B + p_P = 1, p_H = 0$); 2) mixing only on H/B; 3) mixing only on H/P; or 4) mixing on all three actions $H/B/P$.[21]

Mixing only on B/P is not an equilibrium because $p_H = 0$ means the public good is not provided and thus each player prefers action H to either B or P. Mixing only on H/B is an equilibrium if and only if there exists a p_B^* (and implicitly $p_H^* = 1 - p_B^*$) that makes a player indifferent between H and B but still weakly prefer either to P. Given that each other player helps with probability $1 - p_B^*$, a helper guarantees a payoff of $b + g + w - c$. A bystander receives b only if someone else helps, which occurs with probability $1 - (p_B)^{n-1}$. A pretender receives $b + g - c_P$ if someone else helps and $-c_P - s$ if no one else helps. In terms of expected utility, this translates to:

$$E(u_i|H) = E(u_i|B) \geq E(u_i|P) \tag{18}$$

$$b + g + w - c = b[1 - (p_B)^{n-1}] \geq (b + g)[1 - (p_B)^{n-1}] - s(p_B)^{n-1} - c_P \tag{19}$$

$$\implies p_B^* = \left(\frac{c - g - w}{b}\right)^{1/(n-1)} \geq \left(\frac{c - c_P - w}{b + g + s}\right)^{1/(n-1)} \tag{20}$$

If instead mixing only on H/P were an equilibrium, then:

$$E(u_i|H) = E(u_i|P) \geq E(u_i|B) \tag{21}$$

$$b + g + w - c = (b + g)[1 - (p_P)^{n-1}] - s(p_B)^{n-1} - c_P \geq b[1 - (p_P)^{n-1}] \tag{22}$$

$$\implies p_P^* = \left(\frac{c - c_P - w}{b + g + s}\right)^{1/(n-1)} \geq \left(\frac{c - g - w}{b}\right)^{1/(n-1)} \tag{23}$$

The right side of Inequality 20 is p_P^* in the H/P equilibrium; conversely, the right side of Inequality 23 is p_B^* in the H/B equilibrium. At least one of these inequalities must be true, but generally not both at once. The condition that decides which equilibrium exists is:

$$\frac{c - g - w}{b} \gtreqless \frac{c - c_P - w}{b + g + s} \tag{24}$$

If the left side is strictly greater, then the H/P equilibrium exists. If the right side is strictly greater, then the H/B equilibrium exists. In the coincidence that both sides are

[21]A game with k actions has up to $2^k - k - 1$ symmetric, (non-pure) mixed-strategy equilibria.

equal, the equilibrium mixes on either H/P (and $p_B^* = 0$) or H/B (and $p_P^* = 0$). However, this happens on a set of measure zero in parameter space.

Suppose a three-way, symmetric, mixed strategy exists. Given that each other player helps, pretends, or bystands with respective probabilities p_H, p_P, p_B, the expected utility from each action is:

$$E(u_i|H) = b + g + w - c \tag{25}$$

$$E(u_i|P) = (b+g)[1-(1-p_H)^{n-1}] - s(1-p_H)^{n-1} - c_P \tag{26}$$

$$E(u_i|B) = b[1-(1-p_H)^{n-1}] \tag{27}$$

Equating $E(u_i|B) = E(u_i|P)$ yields $p_H^* = 1 - \left(\frac{g - c_P}{g+s}\right)^{1/(n-1)}$. However, $E(u_i|P)$ evaluated at p_H^* equals $E(u_i|H)$ on a set of measure zero in parameter space. So, a three-way, symmetric, mixed strategy does not exist. □

Proof of Proposition 5. An equilibrium that mixes only on H/B exists when:

$$E(u_i|H) = E(u_i|B) \geq E(u_i|P) \tag{28}$$

$$b + g + w - c = b[1-(p_B)^{n-1}] \geq (b+g)[1-(p_B)^{n-1}] - s(p_B)^{n-1} - c_P \tag{29}$$

$$\implies p_B^* = \left(\frac{c - g - w}{b}\right)^{1/(n-1)} \geq \left(\frac{c - c_P - w}{b+g+s}\right)^{1/(n-1)} \tag{30}$$

An equilibrium that mixes only on H/P exists when:

$$E(u_i|H) = E(u_i|P) \geq E(u_i|B) \tag{31}$$

$$b + g \cdot p_H + w - c = (b + g \cdot p_H)[1-(p_P)^{n-1}] - s(p_P)^{n-1} - c_P \geq b[1-(p_P)^{n-1}] \tag{32}$$

$$\implies p_P^* = \left(\frac{c - c_P - w}{b + g \cdot p_H + s}\right)^{1/(n-1)} \geq \left(\frac{c - g \cdot p_H - w}{b}\right)^{1/(n-1)} \tag{33}$$

When both Inequalities 30 and 33 are false, then the three-way mixed strategy equilibrium exists. Given that each other player helps, pretends, or bystands with respective probabilities p_H, p_P, p_B, the expected utility from each action is:

$$E(u_i|H) = b + g\frac{p_H}{p_H + p_P} + w - c \tag{34}$$

$$E(u_i|P) = (b + g\frac{p_H}{p_H + p_P})[1-(1-p_H)^{n-1}] - s(1-p_H)^{n-1} - c_P \tag{35}$$

$$E(u_i|B) = b[1-(1-p_H)^{n-1}] \tag{36}$$

Subtracting $E(u_i|B)$ from each and equating $E(u_i|H) = E(u_i|P) = E(u_i|B)$ yields:

$$0 = b(1-p_H)^{n-1} + g\frac{p_H}{p_H+p_P} + w - c \tag{37}$$

$$0 = g\frac{p_H}{p_H+p_P}[1-(1-p_H)^{n-1}] - s(1-p_H)^{n-1} - c_P \tag{38}$$

Equilibrium (p_H^*, p_P^*) must satisfy this system of two equations. There is no general, closed-form solution due to the interaction between exponents and fractions, but I prove that such a solution exists via a numerical example:

$$n = 2 \quad b = 5 \quad g = 2 \quad w = 1 \quad c = 4 \quad c_P = 1 \quad s = 1$$

$$0 = \frac{2 \cdot p_H}{p_H+p_P} - 5 \cdot p_H + 2 = \frac{2 \cdot p_H^2}{p_H+p_P} - 2 \tag{39}$$

$$\implies (p_H^*, p_P^*, p_B^*) \approx (.74, .13, .13) \tag{40}$$

□

Proof of Proposition 8.b. An asymmetric, pure-strategy Nash equilibrium, if it exists, must take one of four forms: H/B, H/P, B/P, or $H/B/P$. Suppose one exists in only H/B such that X players help and the rest $(n-X)$ bystand. Given this composition, a helper must prefer $H \succ B$ and a bystander must prefer $H \prec B$. $E(u_H|H) \geq E(u_H|B)$ and $E(u_N|H) \leq E(u_N|B)$, respectively, imply:

$$g + w \geq c_H \tag{41}$$

$$g + w \leq c_H \tag{42}$$

However, this generally cannot be true simultaneously–contradiction! Either H is preferred or B is preferred. Likewise, $E(u_H|H) \geq E(u_H|P)$ and $E(u_P|H) \leq E(u_P|P)$, respectively, imply:

$$\frac{g}{n} + w \geq c_H - c_P \tag{43}$$

$$\frac{g}{n} + w \leq c_H - c_P \tag{44}$$

A player's decision between H/P does not depend on the composition of helpers and pretenders. An asymmetric equilibrium in only B/P cannot exist because P is never played without H. Lastly, consider an asymmetric equilibrium in $H/B/P$. Given the composition that X players help, $(k-X)$ pretend, and $(n-k)$ bystand, no type should switch to any other type.[22] This implies:

$$\frac{g(X+1)}{k+1} \leq c_H - w \leq \frac{g \cdot X}{k} \tag{45}$$
$$\frac{g}{k} \leq c_H - c_P - w \leq \frac{g}{k} \tag{46}$$
$$\frac{g \cdot X}{k+1} \leq c_P \leq \frac{g \cdot X}{k} \tag{47}$$

Expression 45 implies $g(k-X) \leq 0$, which is true only if $k = X$. This would mean all claimants are helpers and there are no pretenders, which is sufficient to refute a three-way asymmetry. For completeness, Expression 46 occurs only by coincidence and Expression 47 implies $g \cdot X \geq 0$, which is always true. So, an asymmetric, pure-strategy equilibrium in $H/B/P$ cannot exist and thus no asymmetric, pure-strategy equilibrium can exist at all. □

Proof of Proposition 8.c. A mixed-strategy equilibrium exists when $H \succeq B$ but $H \preceq P$, which occurs when:

$$\frac{g}{n} + c_P \leq c_H - w \leq g \tag{48}$$

Suppose everyone else plays H with probability p_H and P with $p_P = 1 - p_H$. A rational player calculates the (expected) payoffs from playing H versus P.

$$E(u_i|H) = \sum_{X=0}^{n-1} \binom{n-1}{X} (p_H)^X (p_P)^{n-1-X} \cdot \left(m \cdot X - c_H + \frac{g(X+1)}{n} + w\right) \tag{49}$$
$$E(u_i|P) = \sum_{X=0}^{n-1} \binom{n-1}{X} (p_H)^X (p_P)^{n-1-X} \cdot \left(m \cdot X - c_P + \frac{g \cdot X}{n}\right) \tag{50}$$

Players are indifferent when $E(u_i|H) = E(u_i|P)$, which occurs only by coincidence when $g \cdot n + w = c_H - c_P$. The terms p_H and $E(X)$, which depends on p_H, drop out, so a mixed strategy in only H/P cannot exist. This also applies for mixing on only H/B or on only B/P. Now suppose everyone plays H with probability p_H, P with probability p_P, and B with $p_B = 1 - p_H - p_P$. A symmetric, mixed-strategy equilibrium in $H/B/P$ requires that

[22] This notation matches our previous X being the pot and k being claimants.

$E(u_i|H) = E(u_i|B) = E(u_i|P)$ given equilibrium (p^*_H, p^*_B, p^*_P).

$$E(u_i|H) = \sum_{i=0}^{n-1} \left[\sum_{j=0}^{n-1-i} \frac{(n-1)!(p_H)^i (p_P)^j (p_B)^{n-1-i-j}}{i!j!(n-1-i-j)!} \Big(m \cdot i - c_H + \frac{g(i+1)}{i+j+1} + w \Big) \right] \tag{51}$$

$$E(u_i|P) = \sum_{i=0}^{n-1} \left[\sum_{j=0}^{n-1-i} \frac{(n-1)!(p_H)^i (p_P)^j (p_B)^{n-1-i-j}}{i!j!(n-1-i-j)!} \Big(m \cdot i - c_P + \frac{g \cdot i}{i+j+1} \Big) \right] \tag{52}$$

$$E(u_i|B) = m[E(X)] = m(n-1)p \tag{53}$$

While no closed-form solution exists for general n, we can derive the symmetric, mixed-strategy equilibrium for $n = 2$.[23] This system of equations becomes:

$$E(u_i|H) = p_H \cdot m - p_P \cdot g/2 + g + w - c_H \tag{54}$$

$$E(u_i|P) = p_H(m + g/2) - c_P \tag{55}$$

$$E(u_i|B) = p_H \cdot m \tag{56}$$

$$\implies (p^*_H, p^*_P) = \Big[\frac{2 \cdot c_P}{g}, \frac{2(g + w - c_H)}{g}\Big] \tag{57}$$

$p^*_H > 0$ always. $p^*_P > 0$ requires $g + w > c_H$. $p^*_H + p^*_P < 1$ requires $g/2 + w < c_H - c_P$. These are exactly the (strict) condition for a mixed-strategy equilibrium from Inequality 48. These parameters, for example, satisfy the condition ($0.30 < 0.35 < 0.4$). $m > 1/n$ maintains that the pot does not decay contributions. The equilibrium probabilities are:

$$g = 0.4 \quad w = 0.05 \quad c_H = 1 - m = 0.4 \quad c_P = 0.1 \quad \implies (p^*_H, p^*_P, p^*_B) = (.50, .25, .25)$$

□

Proof of Proposition 10. Let the binary function $S_i(g_i, w_i) \in \{0, 1\}$ determine the probability a player i helps (H) or not ($\neg H$) given their type. Further, if $S_i = 0$ for a player, then a second binary function $T_i(g_i, w_i) \in \{0, 1\}$ determines the probability i pretends (P), as opposed to bystand (B). Then, i's expected payoff is a function of their type and the

[23] One could argue that with $n = 2$, each player can infer exactly the other's action. However, I continue to use the Bayesian belief of an external audience to be consistent with the model for general n.

vectors $S = (S_1, S_2, ..., S_n)$, $T = (T_1, T_2, ..., T_n)$ that map types to actions.

$$E(u_i|H) = m(1 + \sum_{j \neq i} x_j) - 1 + w_i + g_i E(\beta) \tag{58}$$

$$E(u_i|B) = m(0 + \sum_{j \neq i} x_j) - 0 \tag{59}$$

$$E(u_i|P) = m(0 + \sum_{j \neq i} x_j) - x_P + g_i E(\beta) \tag{60}$$

where $E(\beta)$ is the *expected* belief that a claimant is a helper. It is the expected population of helpers out of the expected population of helpers and pretenders:[24]

$$E(H_{pop}) = \int_0^{\bar{w}} \int_0^{\bar{g}} S_i(g_i, w_i) dg_i dw_i \tag{61}$$

$$E(P_{pop}) = \int_0^{\bar{w}} \int_0^{\bar{g}} (1 - S_i(g_i, w_i)) T_i(g_i, w_i) dg_i dw_i \tag{62}$$

$$E(\beta) = \frac{E(H_{pop})}{E(H_{pop}) + E(P_{pop})} \tag{63}$$

A player i selects the action which yields the greatest expected utility. In equilibrium, the functions S_i, T_i must hold true given all other S_j, T_j where $j \neq i$. The equilibrium is unique because S, being a vector of functions, inputs a set of parameters and outputs exactly one threshold between helping (H) and not ($\neg H$) and thus one expected likelihood of provision $E(P^*)$. Vector T in turn inputs this $E(P^*)$ to output one threshold between pretending (P) and bystanding (B). Thus, the incomplete information equilibrium is unique. □

[24] β in the base model is determined in equilibrium. Here $E(\beta)$ is an expected value because preferences are private and random so no one can ex-ante deduce the realized value of β.