Liberal-Libertarian Optimal Tax Policy

By
Eduardo Zambrano

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Abstract

This paper derives the tax system implied a combination of liberal and libertarian ‘unfairness aversion’ principles, together with economic efficiency. The optimal policy equalizes the gains individuals obtain from the governmental activity, relative to how they would fare in its absence. Because industrious individuals would fare well either way there will be limits to how much redistribution is recommended by the optimal policy. To illustrate this conclusion I investigate the tax system this approach would recommend for the United States. The resulting “fair-and-efficient” policies would redistribute income via the adoption of a substantial tax credit per household while keeping marginal tax rates constant.

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2 Department of Economics, Orfalea College of Business, California Polytechnic State University, San Luis Obispo, CA 93407. Email: ezambran@calpoly.edu.
1 Introduction

Are ‘liberal’ and ‘libertarian’ principles of fairness compatible with each other and with standard notions of economic efficiency? If so, what kind of income tax policy would these principles jointly recommend? In this paper I attempt to answer these questions in the context of the labor-leisure choices of an equally-skilled population belonging to an economy in which there is a public ‘capital’ good that enhances the productivity of labor of all individuals in society. Because people vary in their preferences towards leisure and consumption, inequality in the distribution of income arises as a consequence. The basic public policy question to answer in this economy is: (i) how much public capital should the government provide?, and (ii) what should be the distribution of taxation in the economy?

These are normative questions, and for their evaluation one must adopt a methodology for ranking the economic states that follow from different public policies. In this paper I develop such methodology by combining liberal and libertarian principles of fairness, together with Pareto efficiency. According to the liberal fairness principle I call ‘Liberal Reward’, if two individuals have the same preferences and work the same amount of time but receive different after tax pay, this is deemed unfair and a transfer can be justified from the one that receives more pay to the one that receives less. According to the libertarian fairness principle I call ‘Laissez Faire Reward,’ if two individuals are such that the first one is worse off, after taxes, than in his autarchy position (how he would fare in the absence of public provision of the capital good) and the second one is worse off, after taxes, than in her autarchy position, this is deemed unfair and a transfer can be justified from the one that is better off to the one that is worse off.

The simultaneous consideration of these liberal and libertarian notions of fairness and efficiency will lead to an ordering over social policies that puts priority on enacting Pareto efficient reforms aimed at helping the individuals treated most unfairly in society, that is, where the optimal policy seeks to equalize the benefits individuals obtain from the capital good relative to how these same individuals would fare absent its public provision. Because industrious individuals would fare well regardless of whether or not the capital good was publicly provided there will be natural limits to how much redistribution is recommended by the optimal policy.

To illustrate this conclusion I investigate the tax system this approach would recommend for a society reminiscent of the US economy in 2011, with about 121 million households, and preferences for consumption and labor calibrated to reproduce the pre-tax distribution of labor income for that year, as released by the US Congressional Budget Office (CBO) in November 2014.

The characteristics of the implied tax system will depend in the end on one’s estimates of how much more productive government activity makes the private sector. If, for example, one estimates that the government’s activity would increase
‘potential output’³ by fifty percent, the optimal tax policy can be represented by a tax credit per household of about $41 thousand dollars per year and a constant marginal tax rate of about 48 percent. On the other hand, if one estimates, for example, that potential output only increases by about a third thanks to governmental activity, the optimal tax policy can be represented as a tax credit per household of about $21 thousand dollars per year and a constant marginal tax rate of about 24 percent.⁴ What remains constant is that, no matter what productivity gains the government activity generates, the optimal policy can be represented fairly well by a negative income tax.

That the negative income tax could be justified by appealing to such liberal and libertarian principles of fairness was not obvious. The need for these kinds of evaluations that combine the tools of modern economic analysis “with the notions of fairness of the common folk” has been loudly stated. For instance, Weinzierl (2012) writes that “conventional theory neglects the diverse normative criteria with which, as extensive evidence has shown, most people evaluate policy.”⁵ Thanks to recent advances in the literature of fair social choice, the tools are ripe for an examination of this sort.

1.1 Related Literature
This paper belongs to the optimal taxation literature, as in Ramsey (1927), Diamond and Mirrlees (1971), and Diamond (1998), among many others. The main difference between that strand of the literature and the present paper is that they work with respect to a given social objective as represented by a social welfare function whereas here I identify what that social objective should be from first principles.

Exactly how much tax progressivity one ought to have in a modern society is a topic that has garnered renewed interest, in part due to the documented increases in market income inequalities in the last few decades. Saez (2001) and Saez and Diamond (2011), for example, derive optimal tax rates for top earners under the assumption that the social planner has intrinsic preferences for redistribution that imply that the marginal social value of the consumption of top earners is negligible. Saez and Stantcheva (2015) develop a local theory of policy reform based on the development of ‘generalized social marginal welfare weights’ that can directly be applied to the income changes of different individuals due to the proposed tax reforms. Mankiw (2010) suggests that progressive taxation, while necessarily limited by the ‘just deserts’ principle, could be justified from a libertarian angle, if it can be shown that those with higher incomes benefit more from the governmental activity.

³ Potential output is defined to be the maximum income a society of identical individuals with perfectly inelastic labor supply could attain.
⁴ In either case, the Pareto efficient amount of the public capital will be provided, according to Samuelson’s rule – an amount that will vary slightly across both cases under consideration due to wealth effects.
⁵ Weinzierl 2012 p. 1, quoted in Maniquet and Fleurbaey 2015. Shefrin 2013 makes a similar point.
A recent and important literature aims to identify the preferences for redistribution the public has, with the hope of using this information in the formulation of social policy. That literature includes the work of Kuziemko et al. (2015), Gaertner and Schokkaert (2012) and D’Ambrosio and Clark (2015).

This paper is closest to the work by Fleurbaey and Maniquet (2006), Maniquet (2007), Maniquet and Sprumont (2010) and Fleurbaey and Maniquet (2011, Chs. 8 and 11). There are important differences between this work and the present paper, and I shall enumerate them here in some detail. First, I work in a context in which agents have indirect preferences over public policies that are induced from the preferences the agents have over leisure and the consumption of a private good. Maniquet (2007), Maniquet and Sprumont (2010) and Fleurbaey and Maniquet (2011, Ch. 8), following a large literature on public good cost sharing, treat these preferences over the public policies (the public good level and the tax policies) as primitives in their analysis. The advantage of my formulation is that it can be extended to the study of a variety of social policies, given the consumption-leisure preferences of the agents, whereas in those papers one has to postulate preferences over the public policies for each application. Second, Fleurbaey and Maniquet (2006 and 2011, Ch. 11) explicitly consider the effect of differences in skill on the fair and efficient taxes whereas here I deliberately wish to exclude these differences in skill, precisely to see how much redistribution can be justified on the basis of differences in preferences alone. Third, I adopt a variation of the standard robustness principle of Replication (which I call Blow Up) that allows the resulting social ordering functions to be definable, as in Bergson and Samuelson, through a specific aggregator function, but with a representation of the preferences of the agents of the Wold type. The practical advantage of this adoption is that it allows a unified treatment of the ranking of allocations in exchange and production economies. A secondary advantage is that this allows the comparison of allocations via the calculation of areas below certain demand curves, as in traditional welfare analysis. Fourth, I examine the specific implications of the resulting social order on what the fair and efficient tax rates ought to look like for the contemporary United States. Despite these numerous differences, it is obvious that the intellectual debt of the present paper to the work by Fleurbaey and Maniquet is immense: this paper would have been impossible to write without their work.

2 The Setting

Consider an economy with a set $N$ of agents that have the same skill but they differ in how they see the tradeoff between labor and leisure. All agents are endowed with one unit of time. The preference relation, $R_j$, of agent $j$ over leisure, $1 - l_j$, and a consumption good, $c_j$, is represented by an utility function $u_j(1 - l_j, c_j)$ that is strictly increasing in both arguments, twice differentiable, and strictly quasi-concave. Let $z_j = (1 - l_j, c_j)$, $z_N = (z_j)_{j \in N}$, and $R_N = (R_j)_{j \in N}$. The corresponding strict preference and indifference relations for agent $j$ are denoted $P_j$ and $I_j$, respectively. Let $\mathcal{R}$ denote the set of preferences satisfying these conditions.
There is a public capital good, the level of which is denoted as $g$, that determines the ability of all agents to transform labor into the consumption good, independent of the labor time. This public capital good is subject to non-rivalry and non-excludability unless specified otherwise. The public capital good’s cost function, $C(g)$, is assumed to be twice differentiable, strictly increasing, strictly convex and such that $C(0)=0$. Let $\mathcal{C}$ be the set of cost functions satisfying these conditions. An economy is then a list $E = (R_N, C)$ where $R_N \in \mathcal{R}^N$ and $C \in \mathcal{C}$. Let $\mathcal{E}$ be the set of all economies as defined above.\(^6\)

\[\text{Figure 2.1. Individual } i \text{ pays } t_i \text{ in taxes and takes advantage of the level of public capital good provided, } g.\]

We begin by studying a mixed economy, that is, a setting in which agent $j$ pays a tax $t_j$ (which can be negative) to finance the public good.

The decision problem for individual $j$ can be, therefore, expressed as follows:

$$\max_{c_j, l_j} u_j \left(1 - l_j, c_j, \right)$$

subject to

$$0 \leq c_j \leq g \cdot l_j - t_j,$$

$$0 \leq l_j \leq 1.$$  

\(^6\)This setting is based on the models developed by Fleurbaey and Maniquet 2011 in Chapters 8, 10 and 11.
with solution represented by the pair \( z'_j(g, t_j) = (1 - l'_j(g, t_j), c'_j(g, t_j)) \) or, more simply, by \( z'_j = (1 - l'_j, c'_j) \). We denote, for future reference, the cost of \( z_j \) in units of the consumption good by \( m(z_j) \), that is, \( m(z_j) = g \cdot (1 - l_j) + c_j \). Figure 2.1 represents such choice for individual \( i \).

### 3 Social Orderings

Given economy \( E \in \mathcal{E} \) and allocations \( z_N, z'_N \in Z(E) \) we write \( z'_N \mathcal{R}(E) z_N \) to denote that allocation \( z'_N \) is socially as least as good as \( z_N \) in economy \( E \). The corresponding strict social preference and social indifference relations are denoted \( \mathcal{P}(E) \) and \( \mathcal{I}(E) \), respectively.

In the determination of which social orderings are deemed acceptable we consider principles we would want any such social orderings to satisfy. We consider three kinds of principles: those related to efficiency in the allocation of resources, those related to fairness in the allocation of those resources, and those related to the robustness of those orderings to small changes in the environment in which the orderings are applied.

#### 3.1 Efficiency Principles

Efficiency is defined in the usual Paretian sense.

**Strong Pareto.** For all \( E \in \mathcal{E}, (z_N, z'_N) \in Z(E) \), if \( z'_i \mathcal{R}_i z_i \) for all \( i \in N \) then \( z'_N \mathcal{R}(E) z_N \). If, in addition, \( z'_i \mathcal{P}_i z_i \) for some \( i \in N \), then \( z'_N \mathcal{P}(E) z_N \).

### 4 Fairness

In this economy differences in consumption levels between the agents can be traced to: (1) differences in how they’re being taxed, and (2) differences in how hard they work.

#### 4.1 Strong Principles of Fairness

Our first principle of fairness is a first attempt to formalize a liberal notion of fairness in the context of the economy I study and stems from the following observation: Consider two agents that have equal levels of work but where one of the agents enjoys a more favorable level of consumption than another. Since we cannot trace their consumption differences to differences in skill, or to how hard they work, we can call this situation unfair, *from a liberal perspective*, and thus consider a social improvement (or at least not a worsening) the transfer of an amount of the consumption good from the favored to the less favored agent.\(^7\)

Formally, the principle can be expressed as follows:

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\(^7\)This is a version of the Pigou-Dalton transfer principle.
Equal Pay for Equal Work. For all $E \in \mathcal{E}, z_N, z'_N \in Z(E)$ if there exist $j, k \in N$ and $\Delta \in \mathbb{R}_{++}$ such that $l_j = l_k = l'_j = l'_k$,

$$c_j - \Delta = c'_j > c'_k = c_k + \Delta,$$

and for all $i \neq j, k, z_i = z'_i$, then $z'_N R(E) z_N$.

Figure 4.1 below illustrates the principle.

![Figure 4.1](image-url)

*Figure 4.1. These individuals work the same amount yet they do not consume the same. According to equal pay for equal work a move from $z_N$ to $z'_N$ is a weak social improvement.*

Our second principle of fairness regarding how the distribution of taxation ought to take place has libertarian roots: in an economy where a level $g$ of the public capital good is provided everyone has the opportunity, *in principle*, to profit equally from the existence of the public capital good. Consequently, if an individual works hard, and this is why he or she ends up in a more favorable position in the distribution of the consumption good in a society, this is not a legitimate reason to distribute resources away from the hard working person towards a person that works less, because the person that worked less could have worked as hard but chose not to. Thus, to the extent that any such redistribution is present in the status quo, a desirable tax reform would aim to reverse it.

Formally, the principle can be expressed as follows:

**Capitation.** For all $E \in \mathcal{E}, z_N, z'_N \in Z(E)$, if there exists $(g, t_N) \in \mathbb{R}_{++} \times \mathbb{R}^N$ with $\sum_i t_i \geq C(g), j, k \in N$ and $\Delta \in \mathbb{R}_{++}$ such that $z_j = z'_j(g, t_j), z_k = z'_k(g, t_k), l_j = l'_j, l_k = l'_k,$
\[ m(z_j) - \Delta = m(z'_j) > m(c'_k) = m(c_k) + \Delta, \]

and for all \( i \neq j, k, z_i = z'_i, \) then \( z'_k R(E)z_N. \)

Figure 4.2 below illustrates this principle. Capitation treats all individuals with equal access to the public good as equals and disregards the extent to which they differ by how hard they work in its approach to fair taxation.

\begin{figure}
\centering
\includegraphics[width=\textwidth]{fig42}
\caption{Individual k works more than j and is taxed more also but j could have worked as hard as k and chose not to. According to capitation a reduction in k’s taxes is a weak social improvement.}
\end{figure}

4.2 Fairness-efficiency tradeoffs
It turns out that these versions of the liberal and libertarian principles of fairness are too strong in the sense that they are each incompatible with Pareto efficiency.

**Theorem 1.** No social ordering function satisfies Strong Pareto and Equal Pay for Equal Work.

**Theorem 2.** No social ordering function satisfies Strong Pareto and Capitation.

All proofs are in the Online Appendix.

4.3 Moderate Principles of Fairness
A natural question follows: are there more moderate versions of the liberal and libertarian principles of justice that are compatible with economic efficiency?
A possible solution, in the liberal case, is a restriction to the application of the ideas behind Equal Pay for Equal Work to the case where two agents have the same preferences. Formally, the principle can be expressed as follows:

**Liberal Reward.** For all $E \in \mathcal{E}$, $z_N, z'_N \in Z(E)$, if there exist $j, k \in N$ such that $R_j = R_k$ and $\Delta \in \mathbb{R}_{++}$ such that $l_j = l_k = l'_j = l'_k$,

$$c_j - \Delta = c'_j > c'_k = c_k + \Delta,$$

and for all $i \neq j, k$, $z_i = z'_i$, then $z'_N R(E) z_N$.

Figure 4.3 illustrates this principle.

![Figure 4.3. Liberal reward.](image)

Our second version of libertarian fairness states that people have fundamental rights to the proceeds of their own labor that are tightly coupled with responsibilities in the procurement of whatever means they may need for their sustenance (including all capital goods). To develop this principle let’s first study the decision problem individuals face in a situation of *laissez-faire*.

**4.3.1 Laissez-faire choices**

The decision problem for individual $j$, when in a situation of laissez-faire, can be expressed as follows:

$$\max_{c_j, l_j, g_j} u_j(1 - l_j, c_j)$$

subject to

$$0 \leq c_j \leq g_j \cdot l_j - C(g_j),$$
$$0 \leq l_j \leq 1, 0 \leq g_j$$
with solution represented by the triple \((1 - l_j^L, c_j^L, g_j^L)\) or, more simply, by \((z_j^L, g_j^L)\).
We call this solution the laissez-faire position of agent \(j\).

![Figure 4.4. Choices in a laissez-faire world.](image)

Let \(\pi(l) = \max g \cdot l - C(g)\). It turns out that the decision problem under conditions of laissez-faire described above is equivalent to the following:

\[
\max_{c_j,l_j} u_j(1 - l_j, c_j)
\]
subject to
\[
0 \leq c_j \leq \pi(l_j), \\
0 \leq l_j \leq 1
\]
with solution represented by the triple \((1 - l_j^{LE}, c_j^{LE}, g_j^{LE})\), and where the value of \(g_j^{LE}\) is obtained by \(g_j^{LE} = \pi'(l_j^{LE})\), a fact that follows from an application of the Envelope theorem.\(^8\)

Figure 4.4 uses this representation of the decision problem faced by these individuals to find the laissez-faire position of individuals \(j\) and \(k\). The blue line depicts the function \(l \mapsto (1 - l, \pi(l))\), as \(l\) varies from 1 to 0. The green and red lines are indifference curves from the indifference maps of individuals \(j\) and \(k\) with preference relations \(R_j\) and \(R_k\) respectively. Individuals with different preferences will in general choose, in a situation of laissez-faire, different leisure-consumption bundles, as well as different levels of the public good, \(g\).

4.3.2 Fairness relative to a laissez-faire benchmark

\(^8\)When there is no ambiguity about which economy we are referring to, we simply refer to \((1 - l_j^{LE}, c_j^{LE}, g_j^{LE})\) as \((1 - l_j^L, c_j^L, g_j^L)\).
From a classical liberal perspective, if an individual in a situation of laissez-faire works hard, and this is why he or she ends up in a more favorable position in the distribution of the consumption good in a society, this is not a legitimate reason to distribute resources away from the hard working person towards a person that works less. On the other hand if, in the *mixed economy* (the one with taxation and public provision of the public good), an agent ends up with an allocation that is less favorable (in his own opinion) than his laissez-faire position whereas another agent ends up with an allocation that is more favorable (in her own opinion) than her own laissez-faire position we would call this situation unfair and thus consider a social improvement (or at least not a worsening) the transfer of an amount of the consumption good from the favored person to the less favored person.\footnote{This fairness principle is related to stand alone transfer, a principle introduced by Moulin 1987 to the literature on fair allocation.}

Formally, the principle can be expressed as follows:

**Laissez-Faire Reward.** For all $E \in \mathcal{E}, z_N, z'_N \in Z(E)$, if there exist $j, k \in N$ and $\Delta \in \mathbb{R}_{++}$ such that $l_j = l'_j, l_k = l'_k, c'_j = c_j - \Delta, c'_k = c_k + \Delta,$

$$z^L_k P_k z'_k P_k z^L_k \text{ and } \frac{z^L_j P_j z'_j P_j z^L_j}{z^L_j P_j z'_j P_j z^L_j}$$

and for all $i \neq j, k, z_i = z'_i$, then $z'_N R(E) z_N$.

Figure 4.5 below illustrates this principle.

It is worth noting at the outset that both Liberal Reward and Laissez-Faire Reward are compatible with each other, and with Strong Pareto. More on this in Section 6 below.
Figure 4.5. Individual k has been made worse off (relative to his laissez-faire position) by entering the mixed economy and landing at $z_N$ whereas the opposite is true for individual j. Laissez-faire reward thus dictates that a move from $z_N$ to $z_N'$ is a weak social improvement.

5 Robustness Principles
The robustness principles introduced below follow the general requirement that solutions to similar problems also need to be similar.

Our first robustness principle is related to (but weaker than) Arrow’s Independence of Irrelevant Alternatives condition from the social choice literature. Let the indifference curve that passes through $z_i$ of agent i with preference relation $R_i$ be defined as $I(z_i, R_i)$.

Hansson Independence. For all $E, E' \in \mathcal{E}, z_N, z'_N \in Z(E)$, if for all $i \in N$, 

$$I(z_i, R'_i) = I(z_i, R_i)$$

then $z_N R(E) z'_N \Leftrightarrow z_N R(E') z'_N$.

This principle says that the social ranking of two allocations remains unaffected by a change in individual preferences that do not affect the agent’s indifference curves at the bundles they receive in these two allocations.\(^{10}\)

Our second robustness principle states that when an agent has the same bundle in two allocations the ranking of these two allocations should remain the same if this agent were simply absent from the economy.

Separation. For all $E \in \mathcal{E}, z_N, z'_N \in Z(E)$, with $|N| \geq 2$ if there is $i \in N$ such that $z_i = z'_i$, then $z_N R(E) z'_N$ if and only if $z_{N \setminus \{i\}} R(R_{N \setminus \{i\}}, C) z'_{N \setminus \{i\}}$.

Our third robustness principle states that, in an economy obtained by blowing up an initial economy in size and productivity, blown up allocations should be ranked exactly as in the initial economy.\(^{11}\) Economy $E' = (R_{N'}, C') \in \mathcal{E}$ is said to be a blow up of $E = (R_N, C) \in \mathcal{E}$ if there is a positive integer $r$ and a mapping $\gamma: N' \rightarrow N$ such that for all $j \in N'$, $z^L_j E' = r z^L_j E$, for all $i \in N$, $|\gamma^{-1}(i)| = r$ and for all $j \in \gamma^{-1}(i)$, $R_j = R'_j$. Profile $R_{N'}$ is an $r$ -replica of $R_N$ for some positive integer $r$ if there exists a mapping $\gamma: N' \rightarrow N$ such that for all $i \in N$, $|\gamma^{-1}(i)| = r$ and for all $j \in \gamma^{-1}(i)$, $R_j = R'_j$. Similarly, allocation $z_{N'}$ is an $r$ -replica of $z_N$ for some positive integer $r$ if there

\(^{10}\) This principle was first used by Hansson 1973 in an abstract voting model and by Pazner 1979 in the context of fair division.

\(^{11}\) Blow up is a variation on the principle of Replication, which is used extensively in Maniquet and Fleurbaey 2011.
exists a mapping \( \gamma : N' \to N \) such that for all \( i \in N, \) \( |\gamma^{-1}(i)| = r \) and for all \( j \in \gamma^{-1}(i), \) \( z'_j = z_i. \)

**Blow Up.** For all \( E \in \mathcal{E}, z_N, z'_N \in Z(E), \) if \( E' \) is a blow up of \( E, z_N' \) is a replica of \( z_N \) and \( z'_N \) is a replica of \( z'_N \) then \( z_N R(E)z_N' \) if and only if \( z_N R(E')z'_N. \)

### 6 Fair Social Orderings

Consider the following utility representation \( u_i^E \) of preference relation \( R_i \) in economy \( E \in \mathcal{E}. \)

\[
    u_i^{LE}(z_i) = \lambda \iff z_i \lambda z_i^L
\]

This is the *laissez-faire equivalent income* function, as it measures the proportional share of the laissez-faire bundle for agent \( i \) in economy \( E, z_i^L, \) that would keep the agent indifferent between such share and \( z_i. \)

For future reference we denote this bundle by \( h_i^L(z_i). \) Figure 6.1 below illustrates.

*Figure 6.1* At \( z_i, \) individual \( j \) is as if he was consuming \( \lambda z_j^L. \) In other words: \( \lambda \) is the maximum amount \( j \) is willing to pay (in units of \( z_j^L \)) for the right to consume bundle \( z_i. \)

This representation, together with the maximin criterion, allows us to define the following social ordering:

**Laissez faire-Equivalent maximin \((R^L)\)**

For all \( E \in \mathcal{E}, z_N, z'_N \in Z(E), \)

\[
    z_N R^L(E)z_N' \iff \min\{u_i^{LE}(z_i)\} \geq \min\{u_i^{LE}(z'_i)\}
\]

This social ordering has a number of appealing properties, which are summarized in the theorem below.

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\(^{12}\)When there is no ambiguity about which economy we are referring to, we simply refer to \( u_i^{LE} \) as \( u_i^L. \)
Theorem 3. The laissez-faire equivalent maximin ordering $R^L$ satisfies strong Pareto, liberal reward, laissez faire reward, Hansson independence, separation and blow up. Conversely, if a social ordering $R$ satisfies strong Pareto, liberal reward, laissez faire reward, Hansson independence, separation and blow up, then for all $E \in \mathcal{E}, z_N, z'_N \in Z(E),$

$$\min_{i \in N} \{u_i^{L,E}(z_i)\} > \min_{i \in N} \{u_i^{L,E}(z'_i)\} \Rightarrow z_N P(E) z'_N$$

In other words, $R$ exhibits a maximin property with respect to laissez-faire equivalent incomes: If the smallest equivalent income is greater in one allocation, this allocation is preferred. The laissez faire equivalent maximin ordering $R^L$ satisfies this property but it is not the only one that does.\(^{13}\)

The interpretation is that, in its consideration of the fairness of an allocation, $R^L$ gives priority to the person treated most unfairly, where the degree of unfairness is measured by comparing what the person achieves in an allocation relative to his or her laissez-faire position to what others achieve in that allocation relative to their own laissez faire position.

6.1 Remarks

- To be sure, the equivalent income representation of preferences and the aggregation rule embedded in $R^L$ are not chosen arbitrarily: they follow from the consideration of the efficiency, fairness and robustness principles identified above.
- No interpersonal comparisons of well-being take place here, as these equivalent incomes are utility indices that merely represent ordinal and noncomparable preferences, as usual.
- Something, of course, is being compared: degrees of differential access to the resources available to the individuals in the economy. It is an approach that is based on what rights people have given the situations they face. To the extent that a person’s rights are being violated, to that same extent we say this person is deemed as disadvantaged in a society and restitution of those rights becomes the focus of policy evaluations.
- As far as what people have a right to in the present setting, the answer is: they have a right to “what they built” in their laissez-faire position. And since each person has a different laissez-faire position depending on their disposition towards work and leisure, the implication is that what they have original rights to, in terms of leisure-consumption bundles, differs among individuals.
- A way of thinking about how the fairness axioms interact in $R^L$ is that the liberal reward, together with Pareto efficiency and the robustness principles, drives the ordering to focus on the individual treated most unfairly in a given

\(^{13}\)This is just as in Fleurbaey and Maniquet 2011, p. 76.
allocation; and laissez faire reward determines the yardsticks to be used in the determination of who is being treated fairly or unfairly.

\[ U_j^i(z_j) = \lambda \]

\[ z_j = \lambda z_j^L \]

\[ h_j(p, u_j(z_j^L)) \]

\[ U_j^i(z_j) = A + B \]

\[ (p^* z_j^L) = 1 \]

**Figure 6.2. Two ways of calculating** \( u_i^L(z_i) \).

- The value \( u_i^L(z_i) \) admits an interpretation familiar from the Welfare Economics literature: it is the joint area under the inverse hicksian demand curves for leisure and for the consumption good (with prices measured in units of \( z_j^L \)) evaluated at \( h_i^L(z_i) \).\(^{14}\) Figure 5.2 below illustrates this calculation.
- This interpretation helps us understand better the social choices that arise from \( R^L \): under this criterion, *one does not add* areas under demand curves

\(^{14}\) Such area, as usual, measures the commodity value of \( h_i^L(z_i) \), that is, the maximum number of units of the numeraire basket (\( z_j^L \) in this case) that the individual is willing to sacrifice in exchange for being able to consume bundle \( h_i^L(z_i) \). Since the individual is indifferent between \( h_i^L(z_i) \) and \( z_i \), then this area represents the commodity value of \( z_i \) as well.
across individuals to determine the worth of a particular allocation (as it is often done in applied welfare economics). Rather, one compares areas under demand curves across individuals and identifies the worth of a particular allocation with the smallest calculated area.

7 Fair and efficient public policy
In Section 2 we studied the choices regarding consumption and leisure that individuals make in the mixed economy, that is, given a level of government expenditure $g$ and a distribution of the tax burden $t_N := \{t_j\}_{j \in N}$. In this Section we investigate what those levels $[g, t_N]$ ought to be according to social ordering $R^k$.

7.1 The first-best
We first use the social ordering $R^k$ to determine what public policy $[g, t_N]$ to adopt in an environment in which the government knows the preferences of all agents in the economy.

Given the policy $[g, t_N]$ let $V^k_i(g, t_i) := u^k_i(z^*_i(g, t_i))$ denote the advantage over laissez-faire of $[g, t_N]$ for agent $i$. Such value is a way of measuring the benefit each individual derives from the public policy relative to each individual’s laissez faire position. The problem for the government is therefore to find the policy $[g^f, t^f_N]$ such that

1. The sum of the marginal rates of substitution between the public good and the consumption good across individuals must equal the marginal cost of providing the public good (Samuelson’s rule).
2. The taxes collected from all individuals must cover the cost of the public good (Budget balance).
3. The advantages over laissez-faire from the public policy are equalized across individuals (Tax fairness).

Figure 7.1 illustrates how the first-best level of the public policy $[g^f, t^f_N]$ is determined in an economy with two individuals.

7.2 Second-best policies when preferences are not observable but income is observable
Even when the government is not able to observe the agent’s private information regarding their preferences, agents are often not able to hide their consumption levels, and therefore their wage income, from the government. In this situation the first-best optimal policy may be able to be implemented by the government through the establishment of a suitably defined income tax scheme.
Figure 7.1. The first best policy is the policy \([g^f, t_j^f, t_k^f]\). Such policy satisfies:

\[
\text{MRS}^k_{g,-t} + \text{MRS}^j_{g,-t} = \text{MC}(g^f), C(g^f) = t_j^f + t_k^f, \text{and } \lambda^j = \lambda^k.
\]

Figure 7.2 illustrates how this could be done for the case of two agents. In this Figure the public policy is the same as in Figure 7.1, that is, it is a first-best optimal policy. It is, however, implemented differently. Upon selecting \(g^f\) as the level of public good to be provided all agents face a budget constraint, before taxes, equal to the light blue line. The government then adopts the following income tax policy:

\[
\tau(y) = \begin{cases} 
\min\{y, t_j^f\} & \text{if } y \leq g^f \cdot l_j^f \\
t_j^f & \text{if } g^f \cdot l_j^f < y \leq g^f \cdot l_k^f \\
t_k^f & \text{if } y > g^f \cdot l_k^f 
\end{cases}
\]
This leads all agents to face the same after-tax budget constraint given by the light pink line. From this common menu agent $k$ chooses bundle $z^f_k$ and agent $j$ chooses bundle $z^f_j$, in accordance to their preferences.$^{15}$

![Diagram](https://via.placeholder.com/150)

**Figure 7.2.** Second-best optimal policies can sometimes implement the first best outcome.

### 8 An Extended Example: Fair and Efficient Taxes for the US Economy

In the example the unit of analysis is the household. Consider an economy composed of $N$ types of households with preferences given by the function

$$u_j(1 - l_j, c_j) = (1 - l_j)^{1 - \alpha_j} \cdot c_j^{\alpha_j}$$

---

$^{15}$Implementation of the first best via tax functions is not always possible in this way. In the example from Figure 7.2 neither agent envies the other agent’s allocation and that is why it is suitable to be implemented through an income tax function. See, e.g., Fleurbaey and Maniquet 2011, pp. 205-206.
with \( j = 1, \ldots, N \) and \( 0 < \alpha_j < 1 \). Let \( \omega_j \) be the proportion of households that are of type \( j \) in a population of \( H \) households. I calibrate these preferences to reproduce the pre-tax distribution of labor income for 2011, as released by the CBO in November 2014 (CBO 2014), and depicted in Figure 8.1 below. The CBO reports labor income, non-labor income, federal taxes and transfers for eight quantiles (therefore \( N=8 \)) from a population of 121.2 million households and from that information, together with an estimate of potential output per household for the economy, one can identify the \( \alpha_j \) consistent with that income and tax information.

![Figure 8.1 2011 US Labor Income](image)

The cost function for the public capital is given by \( C(g) = a \cdot g^{1+b} \) for \( a > 0 \) and \( b > 0 \). I calibrate the cost function so that \( g = 1 \) corresponds to a potential output per household of 600 thousand dollars a year. Coming up with an estimate of how much the government’s activity increases potential output’ is more difficult, as it involves the counterfactual “what would the US economy look like in 2011 if the

\[16\) Notice that these household’s preferences satisfy the Spence-Mirrlees single-crossing condition property in that an indifference curve of an agent crosses an indifferent curve of another agent once, at most.

\[17\) I provide details of how these \( \alpha_j \)'s were estimated in the Online Appendix.

\[18\) As reference, notice that labor income earned by the top 1% in the US reached its highest level in 2007, at 580 thousand dollars for the year.
government did not provide any public capital?” For illustrative purposes I make all my calculations under two baselines: a ‘liberal’ baseline in which government’s activity is believed to increase potential output by fifty percent, and a ‘conservative’ baseline in which government’s activity is believed to increase potential output by one third.

Table 8.1 below summarizes the information about preferences and technology parameters in the example.

<table>
<thead>
<tr>
<th>Preference Parameters across Households</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quantile</td>
</tr>
<tr>
<td>Bottom Quintile</td>
</tr>
<tr>
<td>Second Quintile</td>
</tr>
<tr>
<td>Middle Quintile</td>
</tr>
<tr>
<td>Fourth Quintile</td>
</tr>
<tr>
<td>81st to 90th Percentile</td>
</tr>
<tr>
<td>91st to 95th Percentile</td>
</tr>
<tr>
<td>96th to 99th Percentile</td>
</tr>
<tr>
<td>Top 1%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Technology Parameters, in Per Household terms</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{a}{H}$</td>
</tr>
<tr>
<td>Liberal baseline</td>
</tr>
<tr>
<td>Conservative baseline</td>
</tr>
</tbody>
</table>

Table 8.1. *The distribution of characteristics in the stylized 2011 US Economy*

Household type $j$ therefore makes the following choices in the mixed economy:

$$c_j^* = \alpha_j \cdot (g - t_j), \text{ and}$$
$$l_j^* = \alpha_j + (1 - \alpha_j) \frac{c_j^*}{g}$$

and the following choices in a situation of laissez-faire:

$$g_j^L = \left( \frac{\alpha_j}{a(\alpha_j + b)} \right)^{\frac{1}{b}}$$
$$c_j^L = \frac{b}{a^b} \left( \frac{\alpha_j}{(\alpha_j + b)} \right)^{\frac{1+b}{b}} , \text{ and}$$
$$l_j^L = \alpha_j \left( \frac{1+b}{\alpha_j + b} \right)$$

With this in hand we can compute the advantage over laissez-faire of a particular public policy for each household:
Recall that, by construction,

\[ V_j^L(g, t_j) = \left( \frac{a_j}{\alpha_j} \right)^{\frac{a_j}{b}} \left( \frac{a_j + b}{b} \right) \frac{g - t_j}{g^{1/\alpha_j}}. \]

8.1 Laissez Faire, USA

To illustrate the determination of the first best optimal policy for our stylized economy let’s first consider the situation of laissez faire in which those households would hypothetically find themselves according to our two baselines. Figure 8.2 illustrates. The figure reveals that there is considerable income variation in each of the baselines. This is due to two effects: (i) households with lower \( \alpha_j \) work less, and therefore produce less income than households with higher \( \alpha_j \), and (ii) households with lower \( \alpha_j \) build less of the capital good in a situation of laissez faire than those households with higher \( \alpha_j \). Building capital on one’s own term is so onerous that all households, even those with very high \( \alpha_j \), realize income levels that are far from the economy’s potential output of 600 thousand dollars a year in both baselines. The economy is very inefficient.
8.2 The first best fair-and-efficient policy

The first best fair-and-efficient policy is a collection of lump-sum taxes \((t_1, ..., t_8)\) and a public good level \(g\) such that:

\[
H \sum_{i=1}^{8} \omega_i \left( \alpha_i + (1 - \alpha_i) \frac{t_i}{g} \right) = a(1 + b)g^b,
\]

\[
H \sum_{i=1}^{8} \omega_i t_i = a \cdot g^{1+b}, \text{ and}
\]

\[
\left( \frac{a}{\alpha_i} \right)^{\frac{a_i+b}{b}} \frac{a_i+b}{b} \frac{g-t_i}{g^{1-a_i}} = \left( \frac{a}{\alpha_j} \right)^{\frac{a_j+b}{b}} \frac{a_j+b}{b} \frac{g-t_j}{g^{1-a_j}} \text{ for all } i, j \in \{1,2, ..., 8\}
\]

The first equation is Samuelson’s condition. The second condition ensures that the government’s budget is balanced. The third condition ensures that the advantages over laissez-faire from the public policy are equalized across households.

Table 8.2 below shows the first best policies for our two baselines, and where \(g\) is presented in terms of the maximum achievable income it allows a household in the mixed economy (equal, by construction, to \(600 \cdot g\)).

<table>
<thead>
<tr>
<th></th>
<th>Liberal Baseline</th>
<th>Conservative Baseline</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(In Thousands of Dollars)</td>
<td></td>
</tr>
<tr>
<td>Income</td>
<td>Taxes</td>
<td>Income</td>
</tr>
<tr>
<td>Bottom Quintile</td>
<td>0</td>
<td>-39</td>
</tr>
<tr>
<td>Second Quintile</td>
<td>29</td>
<td>-24</td>
</tr>
<tr>
<td>Middle Quintile</td>
<td>63</td>
<td>-9</td>
</tr>
<tr>
<td>Fourth Quintile</td>
<td>114</td>
<td>14</td>
</tr>
<tr>
<td>81st to 90th Percentile</td>
<td>175</td>
<td>40</td>
</tr>
<tr>
<td>91st to 95th Percentile</td>
<td>235</td>
<td>67</td>
</tr>
<tr>
<td>96th to 99th Percentile</td>
<td>335</td>
<td>115</td>
</tr>
<tr>
<td>Top 1%</td>
<td>547</td>
<td>229</td>
</tr>
<tr>
<td>Maximum Achievable Income</td>
<td>558</td>
<td>585</td>
</tr>
</tbody>
</table>

*Table 8.2 Fair and efficient optimal labor income and taxes*

The liberal baseline is based on the premise that governmental activity increases the productivity of the private sector much more than in the conservative baseline and, since in this economy the top earners benefit from the public capital more than the rest of the population they are taxed more heavily in the liberal baseline. Everybody benefits from the public capital provision, and the differential tax rates ensure that those benefits are shared equally, relative to their initial –laissez faire– conditions, and this requires in this case that a large fraction of households receive subsidies in both baselines. Because there are fewer gains from government activity in the conservative baseline, taxation of the top earners is much smaller in that case, and so are the magnitudes of the subsidies to the bottom earners. As a consequence of
these lower subsidies, the bottom earners work harder in the conservative baseline, and earn a higher income, than in the liberal baseline.

It is important to stress that the work disincentives that arise due to the differential taxation of the different kinds of households described above should not be called, “distortionary” as these policies are first-best Pareto efficient in the economy under consideration. There are many Pareto efficient allocations in this economy, and the one singled out here is the one consistent with the principles of efficiency, fairness and robustness principles studied in Sections 3-5.

8.3 Properties of the first-best tax system across baselines

As a way to understand the characteristics of the optimal taxes recommended above Figure 8.3 plots these taxes vs. the corresponding incomes against which they're levied.

Figure 8.3 The first-best fair and efficient tax policy can be represented as a negative income tax

In both cases the first-best policy can be represented as a negative income tax, namely, a tax credit per household plus a constant marginal tax rate on labor income. In the liberal baseline the approximation is a tax credit per household of about $41 thousand dollars a year and a marginal tax rate of 48% of labor income, whereas in the conservative baseline the approximation is a tax credit per household of about $21 thousand dollars per year with a constant marginal tax rate of about 24% of labor income. This is interesting, of course, because a constant marginal tax rate is being implemented in practice despite the strong progressive content of the fairness principles from which these optimal taxes were derived.
Figure 8.4 Fair and Efficient ‘first best’ redistribution
Underlying the optimal policies in each of the baselines are principles that limit to the size of the tax credit. Were they to be any higher and the taxes on the high income households needed to finance them would be high enough to be unfair to those earners. After all, high income earners deserve to keep “a big hunk” of their proceeds, by laissez-faire reward.

Whether these optimal policies call for very little or a lot of redistribution is, of course, in the eye of the beholder. It is exactly “the right amount” according to the principles discussed above for those baselines, but reasonable people can disagree. For reference one may wish to compare the differences in the distributions of pre and post tax income that arise under each of these baselines (Figure 8.4) against the differences in the distribution of pre and post tax (market) income for the 2011 US Economy (Figure 8.5).\(^9\) On the one hand, we learn that, as fractions of total income in the economy, the optimal policies across baselines are not too dissimilar.\(^{20}\) On the other hand, we learn that the optimal tax policies in our liberal and conservative baselines roughly ‘sandwich’ the 2011 US Tax System: the policies in the liberal baseline are clearly more progressive than those in the conservative baseline, and the 2011 US tax policies fall somewhere in between.

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\(^{19}\)The scope of what can be learned from these comparisons is limited by the fact that market income includes forms of non-labor income that are especially large for high-income earners. A detailed treatment of non-labor income requires a richer model, and possibly different normative principles, than what is discussed here.

\(^{20}\)In the sense that the average absolute distance between the liberal and conservative baselines of the fraction of economy wide income paid in taxes across the eight quantiles depicted in Figure 8.5 is less than 2.5 percentage points. In other ways they clearly differ, as Table 8.2 and Figure 8.3 show.
8.4 Second-best piecewise step income tax functions
Because in this economy all income differences can be traced to differences in household preferences, observing income is sometimes all one needs to know in order to design a tax policy that implements the first best. That this is sometimes possible was illustrated in Figure 7.2 for the case of two agents. This is, unfortunately, not the case for the first best policies depicted in Table 8.2, as these policies are not incentive compatible.21 Considerations of incentive compatibility require the resulting allocations implied by the income tax policy to be envy-free. Table 8.3 below presents piecewise step (second-best) fair-and-efficient policies for our two baselines.

<table>
<thead>
<tr>
<th>Tax Table – Liberal Baseline</th>
<th>Tax Table – Conservative Baseline</th>
</tr>
</thead>
<tbody>
<tr>
<td>(In Thousands of Dollars)</td>
<td></td>
</tr>
<tr>
<td>If your household income is</td>
<td></td>
</tr>
<tr>
<td>At least</td>
<td>But no greater than</td>
</tr>
<tr>
<td>0</td>
<td>16</td>
</tr>
<tr>
<td>16</td>
<td>41</td>
</tr>
<tr>
<td>41</td>
<td>65</td>
</tr>
<tr>
<td>65</td>
<td>102</td>
</tr>
<tr>
<td>102</td>
<td>149</td>
</tr>
<tr>
<td>149</td>
<td>198</td>
</tr>
<tr>
<td>198</td>
<td>293</td>
</tr>
<tr>
<td>293</td>
<td>531</td>
</tr>
</tbody>
</table>

Table 8.3 Piecewise step fair-and-efficient policies in the liberal and conservative baselines. The maximum achievable income are: 541 thousand dollars a year (liberal baseline) and 575 thousand dollars a year (conservative baseline).

To better understand these policies Figure 8.4 plots the piecewise step taxes vs. the maximal incomes against which they’re levied. We learn that, while the negative income tax is still a very good approximation of the relationship between taxes and income in the conservative baseline, this is less so in the liberal baseline: a representation with increasing marginal tax rates would provide a much better fit in this case.

Most interestingly, the source of the tax progressivity in this case is not the set of fairness principles adopted, when considered in isolation, but how they interact with the incentive constraints. This is because the limits imposed by the incentive

---

21 The problem, in the liberal baseline, is that, given the income tax policy implied by Figure 8.3, households types 2 through 7 would rather have the allocations of types 1 through 6, respectively. The same problem arises with households types 2 through 6 in the conservative baseline with regard to the allocation of types 1 through 5.
constraints to what can be redistributed away from certain households apply solely to the ‘middle class’ in this economy. An informal rendition of what is happening is the following: The ‘middle class’ households (everybody but the bottom quintile and the top 1%) are not content with the bundles they receive according to the first best policies in the sense that they envy bundles which involve lower income (and taxes) but more leisure, which they value greatly. To make them like the bundles they receive according to a particular policy we have to make those allocations sufficiently attractive, and that is achieved in this second-best treatment through the lowering of their taxes. This does not happen to the bottom 20% (because they’re already receiving the lowest taxes), or to the 1% (because they like the consumption good so much they don’t envy allocations that have a lot less of it). In sum, the tax progressivity arises not due to a desire to tax the rich disproportionately more ‘because that is what is fair’ but due to the need to tax the middle class households disproportionately less ‘because that is what is incentive compatible.’ As a result, the advantages over laissez faire are only equalized across the bottom 20% and the top 1% in the second-best allocation with the rest of the households obtaining a larger advantage. This is true in both baselines, but the effect is much more pronounced in the liberal baseline, and especially for the top 81%-99% of households.

![Income Tax Function (Liberal Baseline)](image1)

![Income Tax Function (Conservative Baseline)](image2)

*Figure 8.4 The piecewise step fair-and-efficient tax policies in the liberal and conservative baselines*

This picture is corroborated in Figure 8.5, which depicts the first-best and piecewise step average tax rates as functions of income in both baselines. We learn that, relative to the first-best, subsidies to the bottom 20 percent of households are much smaller as fractions of income in this second-best world, as are the average tax rates of the middle class. Not so for the average taxes of the top 1%, which are nearly the same.
Figure 8.5 Average tax rates in the first-best and the ‘piecewise step’ second-best world.
8.5 Second-best affine income tax functions

As discussed in Section 8.3, the first-best taxes can be described as a negative income tax, but this is not the same thing as saying that implementing said negative income tax would produce the first best allocation as a result. This, as discussed in Section 8.4, is due to the presence of incentive constraints. A natural question arises: what are the consequences, in the second-best world, to restricting the policy choices to negative income tax policies? Table 8.4 below provides the result.

<table>
<thead>
<tr>
<th>Second–Best Negative Income Tax</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>(In Thousands of Dollars)</strong></td>
</tr>
<tr>
<td><strong>Liberal Baseline</strong></td>
</tr>
<tr>
<td>$Taxes = -24 + 47.4% \cdot Income$</td>
</tr>
<tr>
<td><strong>Conservative Baseline</strong></td>
</tr>
<tr>
<td>$Taxes = -17 + 23.6% \cdot Income$</td>
</tr>
</tbody>
</table>

Table 8.4 Negative income tax fair-and-efficient policies in the liberal and conservative baselines. The maximum achievable income are: 551 thousand dollars a year (liberal baseline) and 581 thousand dollars a year (conservative baseline).

The tax credits per household and marginal tax rates are lower than those implied by the first-best policies although the changes are much more pronounced for the liberal baseline. In the liberal baseline the subsidies to the bottom 20 percent are larger and the marginal tax rates on the top 1 percent smaller than those implied by the piecewise step second best policies. The implication is that passing from the piecewise step policies to the negative income tax fair and efficient policies in the liberal baseline would benefit the bottom 20 percent and the top 1 percent at the expense of the middle class. In fact, the benefit to the bottom 20 percent is large enough to rank the negative income tax policies above the piecewise step policies according to social ordering $R^L$. To be sure, the negative income tax economy has smaller income and more leisure across all households than the piecewise step policies. It is preferred by the top 1% because of its substantial tax savings and by the bottom 20% because of the premia those households put on leisure.

Table 8.5 below summarizes how all households fare across the two second best environments, in terms of equivalent income, and relative to their laissez faire equivalent income under the first best policies.

The negative income tax fair-and-efficient policy has the feature that it distributes the social losses that the incentive constraints create (relative to the first best) in a near equal fashion. Many factors contribute to this conclusion, but this is in large part due to the fact that the first best policy admits a close affine representation, as shown in Section 8.3 above.
<table>
<thead>
<tr>
<th>Equivalent Incomes(*)</th>
<th>Liberal Baseline</th>
<th>Conservative Baseline</th>
</tr>
</thead>
<tbody>
<tr>
<td>PS</td>
<td>NIT</td>
<td>PS</td>
</tr>
<tr>
<td>Bottom Quintile</td>
<td>.959</td>
<td>.974</td>
</tr>
<tr>
<td>Second Quintile</td>
<td>.973</td>
<td>.979</td>
</tr>
<tr>
<td>Middle Quintile</td>
<td>.990</td>
<td>.981</td>
</tr>
<tr>
<td>Fourth Quintile</td>
<td>1.014</td>
<td>.983</td>
</tr>
<tr>
<td>81st to 90th Percentile</td>
<td>1.047</td>
<td>.984</td>
</tr>
<tr>
<td>91st to 95th Percentile</td>
<td>1.084</td>
<td>.985</td>
</tr>
<tr>
<td>96th to 99th Percentile</td>
<td>1.135</td>
<td>.985</td>
</tr>
<tr>
<td>Top 1%</td>
<td>.959</td>
<td>.974</td>
</tr>
</tbody>
</table>

(*) Relative to their laissez faire equivalent income under the first best policies

Table 8.5 Equivalent incomes are more equally distributed under the negative income tax (NIT) fair-and-efficient policy than under the piecewise step (PS) fair and efficient policy.

9 Conclusions
This paper provides a language in which the fairness and the efficiency of different public policies can be discussed and employs it to identify the optimal policies that would ensue in an environment in which different agents benefit differentially from the provision of public capital. The optimal policy strikes a balance between ‘taxing the rich’ (because they benefit the most from the provision of the public capital) and ‘not taxing the rich too much’ (because they have the right to keep ‘a hunk’ of the proceeds of their hard work). This balance arises from the simultaneous consideration of liberal and libertarian principles of fairness. It turns out that a version of these principles is compatible with the Pareto criterion, and that opens up the possibilities for a rich discussion of both efficient and fair social policies, a discussion that is sorely needed in modern society.

To illustrate the potential of the methodology I develop an example loosely based on the characteristics of the US economy in 2011 aimed at determining the optimal degree of tax progressivity implied by the methodology and find that the first best fair and efficient policy can be described as a negative income tax, a policy with a clearly conservative pedigree. The implied tax credits are, however, quite large, thus producing substantive average tax progressivity (albeit not at the margin). It is my hope that the example shows that the methodology is rich enough to be used in realistic applications aimed at the determination of second-best fair and efficient policies, the characteristics of which will depend on the nature of the policy instruments that are available to the decision maker. We thus have a language in which to investigate the extent to which different tax policies in general, and the negative income tax in particular, execute a first or second-best efficient compromise between what is efficient and what is fair.
References


Appendix

A.1 Proof of Theorem 1
Assume the existence of a social ordering function that satisfies Strong Pareto and Equal Pay for Equal Work. Consider an economy $E \in \mathcal{E}$ with allocations $z^a_N, z^b_N \in Z(E)$ where $z^a_i = z^b_i$ for all $i \neq j, k$ and $c^a_j > c^b_j > c^b_k > c^a_k$ with $c^b_j = c^a_j - \Delta$, $c^b_k = c^a_k + \Delta$ for $\Delta > 0$ and $l^a_k = l^b_k = l^a_j = l^b_j$.

Moreover, $z^a_N$ and $z^e_N$ are such that $z^a_k P_k z^b_k$, $z^a_j P_j z^b_j$, $z^a_k P_k z^d_k$ and $z^a_j P_j z^d_j$.

Figure 10.1 illustrates this construction.

We obtain the following:
- $z^b_N R(E) z^a_N$, by Equal Pay for Equal Work
- $z^a_N P(E) z^d_N$, by Strong Pareto
- $z^d_N R(E) z^e_N$, by Equal Pay for Equal Work
Assume the existence of a social ordering function that satisfies Strong Pareto and Capitation. Consider an economy $E \in \mathcal{E}$ with allocations $z^a_N, z^b_N \in Z(E)$ such that $z^a_i = z^b_i (g, t_i)$ for some $i = j, k$, $z^a_i = z^b_i$ for all $i \neq j, k$, and $m(z^a_i) > m(z^b_j) > m(z^b_k) > m(z^a_k)$ with $m(z^b_i) = m(z^a_i) - \Delta$, $m(z^b_k) = m(z^a_k) + \Delta$ for $\Delta > 0$ and $l^a_k = l^b_k, l^a_j = l^b_j$.

Consider $(g', t'_N)$ with $\sum_i t'_i = C(g')$ and allocations $z^a_N$ and $z^e_N$, different from $z^a_N$ and $z^b_N$, such that $z^e_i = z^a_i (g', t'_i)$ for all $i$, $z^a_i = z^e_i$ for all $i \neq j, k$ and $m(z^a_k) > m(z^d_k) > m(z^d_j)$ with $m(z^d_k) = m(z^e_k) - \Delta$, $m(z^d_j) = m(z^e_j) + \Delta$ for $\Delta > 0$ and $l^a_k = l^e_k, l^d_j = l^e_j$. Moreover, $z^a_N$ and $z^e_N$ are such that $z^a_N P_k z^b_k, z^e_N P_j z^b_j, z^a_N P_k z^d_k$ and $z^e_j P_j z^d_j$.

Figure A.2 illustrates this construction.

**Figure A.2. The Proof of Theorem 2.**

We obtain the following:

$z^b_N R(E) z^a_N$, by Capitation
The construction is illustrated in Figure A.3. The thick curves represent indifference curves for \( R_j = R_k \) as well as for \( R'_j = R'_k \), whereas the thin curves are indifference curves for \( R'_j = R'_k \).

Let \( E' = \left( (R_N \setminus \{ j, k \}), R'_j, R'_k, C \right) \in \mathcal{E} \) and notice that \( Z(E) = Z(E') \). We obtain the following:

\[
\begin{align*}
&z_N^a P(E) z_N^b, \text{ by Strong Pareto} \\
&z_N^a R(E) z_N^b, \text{ by Capitation} \\
&z_N^a P(E) z_N^b, \text{ by Strong Pareto} \\
&z_N^a P(E) z_N^b, \text{ by transitivity.}
\end{align*}
\]

The contradiction establishes the result. □

### A.3 Preparing for the Proof of Theorem 3

To prove Theorem 3 we need an extra definition and a couple of lemmas. The proofs of these lemmas are similar to the proofs of Theorems 3.1 and Lemma 4.1 in Fleurbaey and Maniquet (2011). Those theorems do not apply directly here, as they use assumptions that are different than the ones used in the present paper.

#### Liberal Priority.

For all \( E \in \mathcal{E}, z_N, z'_N \in Z(E) \), if there exist \( j, k \in N \) such that \( R_j = R_k \),

\[
l_j = l_k = l'_j = l'_k,
\]

\[
c_j > c'_j > c'_k > c_k,
\]

and for all \( i \neq j, k, z_i = z'_i \), then \( z'_N R(E) z_N \).

#### Lemma A.1.

If a social ordering function satisfies Strong Pareto, Liberal Reward and Hansson Independence then it satisfies Liberal Priority.

**Proof.** Fix \( E = (R_N, C) \in \mathcal{E} \) and consider \( z_N, z'_N \in Z(E) \) such that there exist \( j, k \in N \) such that \( R_j = R_k \),

\[
l_j = l_k = l'_j = l'_k,
\]

\[
c_j > c'_j > c'_k > c_k,
\]

and for all \( i \neq j, k, z_i = z'_i \).

Let \( R'_j = R'_k \) and \( z_j^1, z_j^2, z_j^3, z_j^4, z_k^1, z_k^2, z_k^3, z_k^4 \in Z(E) \) be constructed in such a way that

\[
I(z_j, R_j) = I(z_j, R'_j), \quad I(z_j, R_j) = I(z'_j, R_j), \quad I(z_k, R_k) = I(z_k, R'_k), \quad I(z'_k, R_k) = I(z'_k, R'_k),
\]

\[
z_j^1 l_j^1 z_j^2, z_j^3 l'_j^1 z_j^2, z_j^1 l'_j^1 z'_j^2, z_k^1 l_k^1 z_k^2, z_k^3 l'_k^1 z_k^2, z_k^1 l'_k^1 z'_k^2
\]

and

\[
c_j^2 = c_j^1 - \Delta > c_k^2 = c_k^1 + \Delta
\]

\[
c_j^4 = c_j^3 - \Delta > c_k^4 = c_k^3 + \Delta
\]

for \( \Delta > 0 \) and \( l_j^1 = l_k^1 = l_j^2 = l_k^2, l_j^3 = l_k^3 = l_j^4 = l_k^4 \).

The construction is illustrated in Figure A.3. The thick curves represent indifference curves for \( R_j = R_k \) as well as for \( R'_j = R'_k \), whereas the thin curves are indifference curves for \( R'_j = R'_k \).

Let \( E' = \left( (R_N \setminus \{ j, k \}), R'_j, R'_k, C \right) \in \mathcal{E} \) and notice that \( Z(E) = Z(E') \). We obtain the following:
Figure A.3. The Proof of Lemma A.1

\[ z_N' I(E') (z_{N\setminus(j,k)}, z_j^4, z_k^4), \text{ by Strong Pareto} \]

\[ (z_{N\setminus(j,k)}, z_j^4, z_k^4) R (E') (z_{N\setminus(j,k)}, z_j^3, z_k^3), \text{ by Liberal Reward} \]

\[ (z_{N\setminus(j,k)}, z_j^3, z_k^3) I (E') (z_{N\setminus(j,k)}, z_j^2, z_k^2), \text{ by Strong Pareto} \]

\[ (z_{N\setminus(j,k)}, z_j^2, z_k^2) R (E') (z_{N\setminus(j,k)}, z_j^1, z_k^1), \text{ by Liberal Reward} \]

\[ (z_{N\setminus(j,k)}, z_j^1, z_k^1) I (E') z_N, \text{ by Strong Pareto} \]

\[ z_N' I(E') z_N, \text{ by transitivity} \]

\[ z_N' I(E) z_N, \text{ by Hansson Independence. Then Liberal Priority holds} \]
Lemma A.2. If a social ordering function $R$ satisfies Separation and Blow Up then for all $E = (R_N, C) \in \mathcal{E}, E' = (R_N, C')$ such that $z_j^{E'} = rz_j^E$ for some rational numbers $q > 0,$

$$R(E) = R(E')$$

Proof. First, show that if $R$ satisfies Separation and Blow Up then it satisfies this Property: For all $E = (R_N, C) \in \mathcal{E}$ and $z_N, z'_N \in Z(E), r \in \mathbb{Z}_{++},$ if $E' = (R_{N'}, C) \in \mathcal{E}$ is such that $R_{N'}$ is an $r$ -replica of $R_N, z_{N'}$ is an $r$ -replica of $z_N$ and $z'_{N'}$ is an $r$ -replica of $z'_N$ then

$z_N R(E) z'_N \Leftrightarrow z_{N'} R(E') z'_{N'}.$

To see this, write $R_{N'}$ as $(R_{N^1}, R_{N^2}, \ldots, R_{N^r})$ such that $R_{N^s}$ is a $1$ -replica of $R_N$ for each $s \in \{1, \ldots, r\}.$ By Blow Up,

$$z_N R(E) z'_N \Leftrightarrow z_{N^s} R(R_{N^s}, C) z'_{N^s}.$$

Define $z_N^0, z_{N'}^1, \ldots, z_{N'}^r \in Z(E')$ as follows:

$$z_N^0 = (z_N, \ldots, z_N),$$
$$z_{N'}^1 = (z_{N^1}, z_N, \ldots, z_N),$$
$$z_{N'}^2 = (z_{N^2}, z_{N^1}, z_N, \ldots, z_N),$$

$$\vdots$$
$$z_{N'}^r = (z_{N'}, \ldots, z_N).$$

Notice that $z_N^0$ and $z_{N'}^1$ differ only in their first set of $r$ entries, $z_{N'}^1$ and $z_{N'}^2$ differ only in their second set of $r$ entries, and so on. Therefore, by Separation, for each $s \in \{1, \ldots, r\},$

$$z_N^{s-1} R(E') z_N^s \Leftrightarrow z_{N^s} R(R_{N^s}, C) z'_{N^s}.$$

Notice that $z_N^0, R(E') z_N^1, \Leftrightarrow z_N R(E) z_N^1$ and $z_{N'}^1, R(E') z_{N'}^2, \Leftrightarrow z_N R(E) z_{N'},$ and so $z_N^0, R(E') z_{N'}^2, \Leftrightarrow z_N R(E) z_{N'}^2$ and since also

$$z_N^2, R(E') z_N^3, \Leftrightarrow z_N R(E) z_N^3$$
$$z_N^3, R(E') z_N^4, \Leftrightarrow z_N R(E) z_N^4$$

$$\vdots$$
$$z_N^{s-1} R(E') z_N^s \Leftrightarrow z_N R(E) z_N$$

it then follows that

$$z_N^0, R(E') z_N^2, \Leftrightarrow z_N R(E) z_N^2$$
$$z_N^0, R(E') z_N^3, \Leftrightarrow z_N R(E) z_N^3$$

$$\vdots$$
$$z_N^0, R(E') z_N^r, \Leftrightarrow z_N R(E) z_N^r$$

Because $z_N^0 = z_N,$ and $z_N^r = z_N,$ we obtain

$$z_N^r R(E') z_N^r \Leftrightarrow z_N R(E) z_N^r,$$

which is what we wanted to show.

Now to complete the proof let $E = (R_N, C) \in \mathcal{E}, z_N, z'_N \in Z(E)$ and $p, q \in \mathbb{Z}_{++}.$ Let $R_{N^p}$ be a $p$ -replica of $R_N, z_{N^p}$ a $p$ -replica of $z_N, z'_{N^p}$ a $p$ -replica of $z'_N$ and $E^p = (R_{N^p}, C_p)$ a $p$ -replica of $E.$

By Blow Up,

$$z_N R(E) z'_N \Leftrightarrow z_{N^p} R(R_{N^p}, C_p) z'_{N^p}.$$
By the Property shown above,
\[ z_N R(R_N, C_p) z_N' \leftrightarrow z_{N,q} R(R_{N,q}, C_p) z_{N,q}'. \]

Let \( R_{N,q} \) be a \( q \)-replica of \( R_N \), so by the Property above we get
\[ z_N R(R_N, C_p) z_N' \leftrightarrow z_{N,q} R(R_{N,q}, C_p) z_{N,q}'. \]

By Blow Up,
\[ z_N R \left( R_N, C_p \right) z_N' \leftrightarrow z_{N,q} R \left( R_{N,q}, C_p \right) z_{N,q}'. \]

It follows that
\[ z_N R \left( R_N, C_p \right) z_N' \leftrightarrow z_{N,q} R \left( R_{N,q}, C_p \right) z_{N,q}' \leftrightarrow z_N R \left( R_N, C_p \right) z_N' \leftrightarrow z_{N,q} R \left( R_{N,q}, C_p \right) z_{N,q}' \]

that is,
\[ z_N R \left( R_N, C_p \right) z_N' \leftrightarrow z_N R \left( R_{N,q}, C_p \right) z_{N,q}', \]

which is what we wanted to show. \( \blacksquare \)

### A.4 The Proof of Theorem 3

It is relatively straightforward to verify that the laissez-faire equivalent maximin ordering \( R^E \) satisfies Strong Pareto, Liberal Reward, Laissez Faire Reward, Hansson Independence, Separation and Blow Up. I omit the proof here.

Conversely, assume a social ordering \( R \) satisfies Strong Pareto, Liberal Reward, Laissez Faire Reward, Hansson Independence, Separation and Blow Up, and pick \( E = (R_N, C) \) \( E \in \mathcal{E}, z_N, z_N' \in Z(E) \) such that \( \min_{i \in N} \{ u_i^{LE}(z_i) \} > \min_{i \in N} \{ u_i^{LE}(z_i') \} \). By Strong Pareto we only need to consider allocations \( z_N, z_N' \in Z(E) \) such that \( l_i = l_i' = \bar{l} \) for some \( \bar{l} \in (0,1) \) and all \( i \in N \). We want to show that \( z_N P(E) \vartriangleleft z_N' \).

It suffices to focus on the case with
\[ u_j^{LE}(z_j') > u_j^{LE}(z_j) > u_k^{LE}(z_k) > u_k^{LE}(z_k') \]

for some \( j, k \in N \) while for \( i \neq j, k, z_i' = z_i \). We study those allocations in economy \( E' = (R_N, C_q) \), where \( q \) is a positive rational number chosen so that
\[ u_j^{LE'}(z_j') > u_j^{LE'}(z_j) > 1 > u_k^{LE'}(z_k) > u_k^{LE'}(z_k'), \]

Create economy \( E'' \) by introducing two agents \( a \) and \( b \) into economy \( E' \) with preferences, \( R_a, R_b \), and bundles, \( z_a, z_b \), such that \( R_a = R_j, R_b = R_k, l_a = l_b = \bar{l} \) and
\[ u_j^{LE''}(z_j') > u_a^{LE''}(z_a) > 1 > u_b^{LE''}(z_b) > u_k^{LE''}(z_k'), \]

Let \( \Delta > 0 \) and \( c_a' \) be such that \( c_j' > c_j > c_a' > c_a \) with \( c_a' = c_a + \Delta \).

Let \( c_b' \) and \( c_k' \) be such that \( c_b > c_b' > c_k' > c_k \) with \( c_b' = c_b - \Delta \). Let \( z_a' = (c_a', 1 - \bar{l}) \), \( z_b' = (c_b', 1 - \bar{l}) \) and \( z_k' = (c_k', 1 - \bar{l}) \).

Figure A.4 illustrates this construction.
Figure A.4 The proof of Theorem 3

By Strong Pareto, Liberal Reward and Hansson Independence, Lemma A.1 applies. Therefore,

\[ (z'_N \setminus (j,k), z_j, z'_k, z'_a, z'_b) R(E'')(z''_N, z_a, z_b). \]

Notice that, \( c_a = c^*_a - \Delta, c_b = c^*_b + \Delta \) for \( \Delta > 0 \), \( l_a = l'_a, l_b = l'_b \) and

\[ u_{a}^{L,E''}(z''_a) > u_{a}^{L,E''}(z'_a) > 1 > u_{b}^{L,E''}(z'_b) > u_{b}^{L,E''}(z''_b), \]

Therefore, Laissez Faire Reward applies and

\[ (z'_N \setminus (j,k), z_j, z'_k, z_a, z_b) R(E'')(z''_N \setminus (j,k), z_j, z'_k, z'_a, z'_b). \]

By transitivity,

\[ (z''_N \setminus (j,k), z_j, z'_k, z_a, z_b) R(E'')(z'_N, z_a, z_b) \]

By Separation,

\[ (z'_N \setminus (j,k), z_j, z'_k) R(E') z'_N \]

By Strong Pareto,

\[ (z'_N \setminus (j,k), z_j, z'_k) P(E') z'_N, \]

that is,

\[ z_N P(R_N, C_q) z'_N. \]
By Separation and Blow Up, Lemma A.2 applies. Therefore \( \mathbf{R}(E) = \mathbf{R}(R_N, \mathbf{C}_q) \) and we get 

\( z_N \mathbf{P}(E) z'_N, \)

which is what we wanted to show. \( \blacksquare \)

### A.5 The preference parameters from the example

On November 12, 2014 the CBO made available the 2011 Distribution of Household Income and Federal Taxes for eight quantiles of market income from a population of 121.2 million households (CBO 2014). For each quantile we then know its labor income, business income, capital income, retirement income, government transfers, and federal taxes. Labor income includes cash wages and salaries, including amounts allocated by employees to 401(k) plans, employer paid health insurance premia, the employer’s share of payroll taxes for Social Security, Medicare, and Federal Unemployment Insurance, and the share of corporate income taxes borne by workers. The eight quantiles for which we have data are: the bottom quintile, the second quintile, the middle quintile, the fourth quintile, the 81st to 90th percentile, the 19st to 95th percentile, the 96th to 99th percentile, and the top 1%.

On the basis of this data I calculate the \( c^*_j \) component of household’s optimal choice 

\[ z^*_j = (1 - l^*_j, c^*_j) \]

by means of the formula

\[ c^*_j = \text{Market Income}_j + \text{Transfers}_j - (\text{Federal Taxes}_j - \text{Payroll Taxes}_j). \]

This estimate does not discount Payroll Taxes when calculating \( c^*_j \) under the presumption that it is ‘consumption’ in the same way that ‘employer paid health insurance premia’ is treated as consumption by the CBO, and therefore a component of Market Income (more on the treatment of Payroll Taxes in Section A.5.1 below).

Given that the maximum achievable labor income in the model economy is \( g \) thousand dollars a year I calculate the \( l^*_j \) component of household’s optimal choice 

\[ z^*_j = (1 - l^*_j, c^*_j) \]

by means of the formula

\[ l^*_j = \frac{\text{Labor Income}_j}{g}. \]

Each of these income groups faces different marginal income tax rates and these depend in complex ways on their demographic characteristics and on the phaseout of a variety of tax credits, deductions, cash and in-kind benefits that depend on earnings. In 2012 the CBO published estimates of the effective marginal tax rates for low and moderate income workers (CBO 2012).

Based on those calculations, and on the 2011 federal tax code, I estimate the effective marginal tax rates for each quintile as depicted below:
Effective Marginal Tax Rate across Households

<table>
<thead>
<tr>
<th>Quantile</th>
<th>$MTR_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bottom Quintile</td>
<td>7%</td>
</tr>
<tr>
<td>Second Quintile</td>
<td>18%</td>
</tr>
<tr>
<td>Middle Quintile</td>
<td>22%</td>
</tr>
<tr>
<td>Fourth Quintile</td>
<td>25%</td>
</tr>
<tr>
<td>81st to 90th Percentile</td>
<td>28%</td>
</tr>
<tr>
<td>91st to 95th Percentile</td>
<td>31%</td>
</tr>
<tr>
<td>96th to 99th Percentile</td>
<td>36%</td>
</tr>
<tr>
<td>Top 1%</td>
<td>38%</td>
</tr>
</tbody>
</table>

Table A.1

These tax rate estimates explicitly exclude Payroll Taxes and factor in all other Federal Taxes, State Taxes, and the reductions in benefits that come from the Supplemental Nutrition Assistance Program, the Temporary Assistance for Needy Families, the Housing Choice Voucher Program, Medicaid and the Children’s Health Insurance Program as earnings grow.

At the margin, a household $j$'s choice $z_j^* = (1 - l_j^*, c_j^*)$ satisfies

$$MRS_{1-l,c}(1 - l_j^*, c_j^*) = g \cdot (1 - MTR_j)$$

which in the example translates into

$$\frac{1 - \alpha_j}{\alpha_j} \frac{c_j^*}{1 - l_j^*} = g \cdot (1 - MTR_j)$$

from which a value of $\alpha_j$ can be recovered as follows:

$$\alpha_j = \frac{c_j^*}{c_j^* + g(1-MTR_j) \cdot g(1-l_j^*)}.$$  

In the example the maximum achievable income is identified to be equal to $g=600$ thousand dollars a year. This is then the formula used to calculate the preference parameters presented in Table 8.1.

A.5.1 The Treatment of Payroll Taxes

Because Social Security (Payroll) taxes are linked to benefits, including them in a calculation of marginal tax rates is problematic, and that is the reason why I don't include them in my calculations. The analysis completed above can nevertheless be redone by fully taking them into account in the calculation of the marginal tax rate (and suitably removing them from the calculation of $c_j^*$, as they would be 'a tax'). These considerations lead to the following table of marginal tax rates:
Effective Marginal Tax Rate across Households Including Payroll Taxes

<table>
<thead>
<tr>
<th>Quantile</th>
<th>( MTR_j )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bottom Quintile</td>
<td>19%</td>
</tr>
<tr>
<td>Second Quintile</td>
<td>30%</td>
</tr>
<tr>
<td>Middle Quintile</td>
<td>34%</td>
</tr>
<tr>
<td>Fourth Quintile</td>
<td>37%</td>
</tr>
<tr>
<td>81st to 90th Percentile</td>
<td>40%</td>
</tr>
<tr>
<td>91st to 95th Percentile</td>
<td>43%</td>
</tr>
<tr>
<td>96th to 99th Percentile</td>
<td>48%</td>
</tr>
<tr>
<td>Top 1%</td>
<td>50%</td>
</tr>
</tbody>
</table>

*Table A.2*

and formula for the calculation of \( c^*_j \):

\[
c^*_j = Market\ Income_j + Transfers_j - Federal\ Taxes_j.
\]

The corresponding preference parameters are depicted in the column called “Case B” from Table A.3. I reproduce the (default) parameters from Table 8.1 in Table A.3 in the column called Case A to facilitate comparisons.

<table>
<thead>
<tr>
<th>Quantile</th>
<th>Case A (Default)</th>
<th>Case B</th>
<th>Case C</th>
<th>Case D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bottom Quintile</td>
<td>0.06</td>
<td>0.06</td>
<td>0.06</td>
<td>0.05</td>
</tr>
<tr>
<td>Second Quintile</td>
<td>0.09</td>
<td>0.10</td>
<td>0.08</td>
<td>0.07</td>
</tr>
<tr>
<td>Middle Quintile</td>
<td>0.13</td>
<td>0.14</td>
<td>0.12</td>
<td>0.09</td>
</tr>
<tr>
<td>Fourth Quintile</td>
<td>0.19</td>
<td>0.20</td>
<td>0.17</td>
<td>0.13</td>
</tr>
<tr>
<td>81st to 90th Percentile</td>
<td>0.26</td>
<td>0.28</td>
<td>0.23</td>
<td>0.19</td>
</tr>
<tr>
<td>91st to 95th Percentile</td>
<td>0.34</td>
<td>0.36</td>
<td>0.31</td>
<td>0.24</td>
</tr>
<tr>
<td>96th to 99th Percentile</td>
<td>0.50</td>
<td>0.53</td>
<td>0.47</td>
<td>0.37</td>
</tr>
<tr>
<td>Top 1%</td>
<td>0.97</td>
<td>0.97</td>
<td>0.97</td>
<td>0.95</td>
</tr>
</tbody>
</table>

*Table A.3*

We learn that very little changes in the way of estimating these preference parameters as we vary the treatment we give to Payroll Taxes in these calculations.

**A.5.2 Further robustness checks**

Despite the matter being simple in principle, the determination of which marginal tax rates affect behavior is not a trivial matter in practice. This is so because taxpayers may not fully know what their effective marginal tax rate is. In the words of the CBO:

*If taxpayers misperceive their marginal tax rate (MTR), changes in their actual MTR may not have much effect on their decisions about how much to work. The income tax system does not make MTR’s readily apparent, and complex rules and interactions between the tax and transfer systems tend to further obscure those rates. Moreover, the average taxpayer may not fully understand how benefits*
are linked to income because of various exemptions and deductions and because of nonfinancial criteria for qualifying for a program. (...) In light of the difficulty people face when determining their marginal tax rate, the disincentives to work caused by high marginal tax rates may be partially mitigated.\textsuperscript{22}

This being the case it is important to inquire into how sensitive the (default) preference parameters presented in Table 8.1 are to different assumptions about how households perceive those marginal tax rates. I have already looked at two distinct possibilities but let’s look at two more. A possibility is that households simply look at Federal Income Tax tables and overlook the more subtle dimensions of the tax code and its interactions with the government programs to which they qualify on the basis of their earnings. Were that to be the case the relevant marginal tax rates would be as depicted in Table A.4 below:

<table>
<thead>
<tr>
<th>Quantile</th>
<th>$MTR_j$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bottom Quintile</td>
<td>10%</td>
</tr>
<tr>
<td>Second Quintile</td>
<td>15%</td>
</tr>
<tr>
<td>Middle Quintile</td>
<td>25%</td>
</tr>
<tr>
<td>Fourth Quintile</td>
<td>25%</td>
</tr>
<tr>
<td>81st to 90th Percentile</td>
<td>25%</td>
</tr>
<tr>
<td>91st to 95th Percentile</td>
<td>28%</td>
</tr>
<tr>
<td>96th to 99th Percentile</td>
<td>33%</td>
</tr>
<tr>
<td>Top 1%</td>
<td>35%</td>
</tr>
</tbody>
</table>

\textit{Table A.4}

The corresponding preference parameters consistent with these tax rates are depicted in the column called “Case C” from Table A.3.

A fourth possibility is that households treat their annual income tax bill, net of transfers, as a lump sum tax. The corresponding preference parameters consistent with this perception of the tax code are depicted in the column called “Case D” from Table A.3.

I do not wish to defend cases C and D as they are arguably very unrealistic but in any case the preference parameters are rather similar across all four cases presented in Table A.3 and the optimal policies discussed in Section 8 of the paper are nearly identical across these four depictions of the preferences of the households in this version of the US economy circa 2011.

\textsuperscript{22} CBO 2012, p.2.