

Borrowing Constraints, Search, and Life-Cycle Inequality¹

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Abstract

This paper quantifies the impact of borrowing constraints on consumption and earnings inequality in a life-cycle model with labor market search and endogenous human capital accumulation. I first show that following an unemployment spell, likely-constrained workers in the Survey of Income and Program Participation match to jobs that pay more per quarter when they receive an increase in their unemployment insurance. I then construct a life-cycle model with risk averse workers who face borrowing constraints, accumulate human capital endogenously, and search both on and off the job. I use indirect inference to estimate the model parameters, and show that wealth inequality causes both placement into lower-paying jobs as well as slower human capital accumulation when workers face borrowing constraints. A standard deviation decrease in initial wealth causes a larger decline in life-cycle consumption than a standard deviation decrease in initial human capital.

JEL Classification: E21, E24, J63, J64, D31, I32, J31

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1 Introduction

Households accumulate substantial amounts of debt by the time they enter the labor market. A worker at their first full-time job spends 18% of their income on debt payments, and of those workers more than 40% report denial of requested additional credit (Survey of Consumer Finances, 2013). This paper argues that borrowing constraints affect job placement, earnings, and on-the-job human capital growth. I construct a quantitative life-cycle model that considers risk-averse workers who face incomplete asset markets, must search for jobs, and can choose their rate of human capital accumulation. After estimating the model, I find that constrained individuals apply for lower-paying, more easily obtainable jobs than their wealthier unconstrained peers, and then choose to substitute future human capital growth for precautionary savings. Quantitatively, I show that a standard deviation decrease in wealth at age 23 decreases consumption and earnings growth by more than a standard deviation decrease in human capital.

I build a model to decompose the contribution of initial conditions to inequality when workers face borrowing constraints. I start with a life-cycle model of on-the-job directed search with wage posting, building off the work done by [Menzio et al. \(2016\)](#) and [Herkenhoff \(2014\)](#). I consider risk averse households, a natural borrowing constraint, and [Ben-Porath \(1967\)](#) human capital accumulation. Incomplete asset markets limit the ability of workers to directly substitute future income in order to smooth consumption. Directed search allows workers to choose the degree of income (and consumption) risk that they face by directing their search to jobs with an inverse relationship between wages and probability of employment in equilibrium. Human capital accumulation lets employed workers choose to allocate productive time between working and accumulating human capital.³ The model considers initial heterogeneity in wealth, human capital, and learning, and quantifies the impact of each on life-cycle inequality.

To motivate the theory, I use the Survey of Income and Program Participation (SIPP) to show evidence that borrowing constraints alter earnings following an unemployment spell. I exploit differences in replacement rates across states to estimate the differential effect that unemployment insurance replacement rates have on constrained and unconstrained households. I find that workers from the first quintile of the liquid wealth (liquid assets

³To my knowledge, this is the first paper to incorporate [Ben-Porath \(1967\)](#) into an environment with risk aversion and incomplete markets, but not the first among search models. [Bowlus and Liu \(2013\)](#) do the same for a model with linear preferences and explore the contributions of search and human capital to wage growth.

net of unsecured debt) distribution match to 6.3% higher paying jobs when given a 10% increase in unemployment insurance, despite nearly identical pre-spell earnings, tenure, and education.⁴ Over longer horizons, I find that the effects on wages appear to persist.

I estimate the model using indirect inference. Indirect inference creates a straightforward connection to the data by matching parameters from reduced-form specifications that approximate equilibrium outcomes of the model. I use findings from the SIPP as well as life-cycle earnings and job transition statistics from the Panel Study of Income Dynamics (PSID) and the National Longitudinal Study of Youth 1979 (NLSY) to discipline key features that determine the behavior of workers in my model. The moments from the SIPP yield inference on borrowing constraints, while moments from the PSID and NLSY provide inference on the correlations between wealth, human capital growth, and earnings over the life-cycle. I test the fit of the estimated model and find that the model fits the data well.

I find that constrained workers decrease their consumption risk by applying to jobs that offer shorter expected unemployment durations, but lower wages. Wealthy workers can smooth consumption and experience extended unemployment spells, while finding better employment. Though the estimation suggests that poor and wealthy individuals have similar productivities, wealth inequality causes large differences in first job placement. While the effect on initial job placement is transitory, differences in earnings persist. Unable to borrow against future income, constrained workers substitute intertemporally by allocating time to production, rather than human capital accumulation. The placement effect causes earnings inequality for 5 years, while the human capital effect persists for the life-cycle.

My findings suggest that borrowing constraints amplify inequality when workers face frictional labor markets. Differences in wealth among similarly productive individuals can lead to long-term differences in earnings and substantial lifetime consumption inequality. For the average individual, a standard deviation decrease in initial wealth depresses lifetime consumption by more than a standard deviation decrease in initial human capital. The difference in wealth causes consumption to decrease -10.5% , while the change in human capital causes consumption to change by -7.6% . Wealth operates primarily by changing the application strategy of workers (-2.2%), but also decreases earnings by changing average human capital (-1.1%). I also find that the average individual at the 10th percentile of the wealth distribution experiences a lifetime earnings increase of 3.5% when given the median wealth in the sample.

⁴[Herkenhoff et al. \(2016\)](#) find evidence that access to additional credit improves labor market outcomes following an unemployment spell.

The paper is organized as follows: in [Section 2](#) I review the literature and describe how previous work differs from mine. In [Section 3](#) I document liquidity effects on re-employment wages for constrained groups. In [Section 4](#), I construct a model that incorporates these findings, and show the equilibrium. In [Section 5](#), I explain the functional form and parameter assumptions, and my construction of targets for indirect inference. In [Section 6](#), I decompose the implications for life-cycle inequality, and compare my findings to the existing literature. Lastly, in [Section 7](#) I summarize my contributions and discuss routes for future work.

2 Related Literature

This paper addresses a question that relates to the literatures on labor market search, human capital, and inequality. Here, I summarize many of the most closely related papers.

[Graber and Lise \(2015\)](#) investigates facts about the age-profile of consumption and earnings variance within a model that features borrowing constraints, search, and human capital accumulation. They argue that such a model is required to match life-cycle facts about the variance in earnings and consumption (they increase roughly linearly) and the negative skewness of earnings changes. While both papers focus on inequality, I focus on inequality that results from initial conditions, while they focus on inequality in response to shocks over the life-cycle.⁵ Our models also differ in that I endogenize human capital accumulation as well as the match rate between workers and firms. They employ a “learning-by-doing” human capital accumulation technology⁶, which allows for an exogenous productivity drift while employed, and assume that workers receive draws from a wage distribution at an exogenous rate, both for tractability. As my findings indicate, endogenous human capital accumulation plays an important role in lifetime inequality in models with borrowing constraints. To my knowledge, [Bowlus and Liu \(2013\)](#) is the only other paper to include a model with both search and [Ben-Porath \(1967\)](#) human capital accumulation. They focus on the decomposition of earnings growth between search frictions and human capital accumulation. Agents in their model are risk-neutral, while my paper shows that risk aversion has a consequential effect on both search and human capital. Two more papers, [Bagger et al. \(2014\)](#) and [Yamaguchi \(2010\)](#) explore life-cycle wage dynamics in search models that feature human capital (through learning-by-doing). Both papers primarily focus on decomposing the contri-

⁵Their work is a follow-up to [Lise \(2013\)](#), which was similar, but without human capital accumulation.

⁶Recent evidence suggests that [Ben-Porath \(1967\)](#) human capital production accounts for life-cycle earnings growth 2-3 times better than learning-by-doing ([Blandin, 2016](#)). It’s unclear if these results would generalize to a frictional setting.

butions of wage bargaining and human capital growth to wage growth, and neither feature risk aversion. [Ji \(2017\)](#) primarily studies the impact that student debt has on aggregate outcomes, but also considers the effect on life-cycle profiles. He estimates a search model with borrowing constraints, risk-aversion, and a college entry decision, and analyzes the general equilibrium effects of two college debt repayment plans. He finds that individuals with student debt experience lower pay and shorter unemployment durations than non-borrowing peers for roughly 15 years. His model does not consider the dynamic effects through human capital accumulation.

There is also a growing literature primarily focused on identifying the short-term effects of student debt on labor market outcomes. [Gervais and Ziebarth \(2017\)](#) uses the Baccalaureate and Beyond 1993 Longitudinal Study and exploit a kink in subsidized stafford loan eligibility to show that an extra \$1,000 in student loan debt at graduation decreases earnings by 2.5%. [Luo and Mongey \(2017\)](#) and [Rothstein and Rouse \(2011\)](#) use variation in the ratio of grants to total loans across cohorts, but within institutions. Surprisingly, both papers find that debt increases earnings after graduation (1.21% in [Luo and Mongey](#) and \$978 in [Rothstein and Rouse](#)). My paper focuses on broader definitions of employment and debt because constrained individuals might be willing to take part-time employment to smooth consumption, and repayment of certain college loans only begin following employment.

One notable feature that distinguishes my model from much of the previous work is that my model allows both a distribution of human capital and a distribution of wealth that are determined endogenously. [Burdett and Coles \(2010\)](#) introduces risk averse agents and learning-by-doing human capital accumulation into the [Mortensen and Pissarides \(1994\)](#) model, but restricts agents to face credit markets characterized by autarky in order to recover the structure of optimal contracts. They show that firms optimally backload contracts in order to retain workers, which generates the prediction that initially low-wage workers will achieve faster rates of earnings growth as they age. In my model, this negative relationship between initial earnings and growth rates is decoupled as a result of low-wage workers substituting future earnings growth for precautionary savings. Others who have introduced risk-aversion into the [Mortensen and Pissarides \(1994\)](#) model include [Lentz and Tranaes \(2005\)](#), [Krusell et al. \(2009\)](#), and [Costain and Reiter \(2008\)](#). These papers feature distributions of wealth, but do not include human capital.

My model extends the block recursive search frameworks [Menzio and Shi \(2010\)](#) and [Menzio et al. \(2016\)](#). These are search frameworks that allow for endogenously determined distributions of agents. Follow-up work by [Herkenhoff \(2014\)](#), introduced directed search

with risk aversion into a life-cycle version of the block recursive search model, focused on the effects of credit access on the business cycle.⁷ Another paper, [Herkenhoff et al. \(2016\)](#), introduced human capital accumulation into this framework, but did so through learning-by-doing, and again restrict their exploration to aggregate fluctuations. [Chaumont and Shi \(2017\)](#) uses a closely related model with infinitely-lived agents to highlight the effects of unemployment risk on precautionary savings. They focus on cross-sectional dispersion, rather than life-cycle effects, and do not include human capital accumulation. They find that wealth effects alone play a small role in determining wage dispersion.

My paper also relates to the literature focused on identifying the causes of inequality. Broadly, the literature on life-cycle inequality focuses on assigning importance to initial conditions relative to shocks experienced in determining earnings or consumption variance. The most closely related, [Huggett et al. \(2011\)](#), studies both the relative importance of shocks and initial conditions, and decomposes the contribution of life-cycle inequality among initial conditions. Similar to my work, their model features heterogeneity in wealth, human capital, and learning, and allows earnings to grow through a [Ben-Porath](#) production function. They find that initial conditions (age 23) determine more than 60 percent of variation in lifetime utility, but that the bulk of this results from human capital inequality. Similarly, [Heathcote et al. \(2014\)](#) use a model with heterogeneity in preferences and productivity to decompose sources of inequality. They reach a similar conclusion as [Huggett et al.](#): productivity is the primary driver of earnings inequality. I find the opposite: that initial wealth plays a more important role in determining life-cycle inequality than heterogeneity in human capital. The difference is caused by my inclusion of frictional labor markets, which makes wealth have a first order effect on earnings.⁸

3 Empirical Regularities

Three key empirical regularities motivate the construction of my model. First, constrained individuals who receive more generous unemployment insurance replacement rates match to

⁷[Herkenhoff \(2012\)](#) was the first to consider risk averse workers facing borrowing constraints in this framework.

⁸Two more papers, [Keane and Wolpin \(1997\)](#), and [Heckman et al. \(1998\)](#) development dynamic models of schooling, work and occupational choice, as well as human capital accumulation. They both focus on decisions prior to the period analyzed by this paper and are complementary in that they find that initial heterogeneity play substantial roles in determining long-term outcomes.

higher-paying jobs following an unemployment spell.⁹ Second, among the full-time employed, initially wealthy individuals consistently receive more training throughout the life-cycle, suggesting a link between initial wealth and human capital accumulation. Third, I find that there are large, persistent differences in earnings among individuals with below median wealth and above median wealth. I use these findings as motivation as well as estimation targets for my model in [Section 5](#).

3.1 Re-Employment Elasticities

To explore the effects of borrowing constraints on labor market outcomes, I estimate the responsiveness of constrained (using liquid wealth as a proxy) individuals to changes in their unemployment insurance replacement rates. I find that the elasticity of the re-employment wage with respect to unemployment insurance amount is substantial for constrained individuals, but has no effect for unconstrained individuals. As a robustness check, I perform a similar exercise on employment-to-employment job transitions and find no effect. I use Survey of Income and Program Participation (SIPP) panels from 1990-2008, as well as data from state unemployment insurance laws provided by the Employment and Training Administration. I restrict my sample to 23 and older males who take up UI within one month of unemployment. More details on the construction of this data is available in [Section A.1](#).

3.1.1 Empirical Strategy

I do not have a direct measure of the degree to which each household is constrained, so I compare the labor market outcomes of individuals by quintiles of net liquid wealth (defined as liquid assets net of unsecured debt) in response to changes in unemployment insurance. This proxy has been used extensively in to quantify the effects of UI on labor market outcomes ([Browning and Crossley \(2001\)](#), [Bloemen and Stancaelli \(2005\)](#), [Sullivan \(2008\)](#), and [Chetty \(2008\)](#), among others). These papers find that unemployment insurance is used as a substitute for income during unemployment spells among illiquid households, which motivates the use of net liquidity as a proxy for borrowing constraints.

Individuals frequently misreport their level of unemployment benefits; therefore, I proxy for unemployment insurance by using the average weekly benefit over an unemployment spell

⁹While there is previous evidence for the effect of borrowing constraints (also known as liquidity effects in the literature) on unemployment outcomes, ([Herkenhoff et al. \(2016\)](#) on earnings, [Chetty \(2008\)](#) on durations, among others) the effects on re-employment earnings is sparse.

at the state-month level and frequency. This provides a credible source of exogenous variation that has been used extensively in the literature: unemployment insurance replacement rates vary within a state over time as a result of changes in legislation. I include potential UI duration, defined as the average number of weeks a cohort of unemployed individuals could receive UI, at a state-by-quarter level and frequency to capture any correlation between replacement rates and duration generosity for a state unemployment insurance system. [Table B.1](#) summarizes key employment and demographic characteristics by liquidity quintile and UI generosity. The table shows that individuals vary across the liquid wealth distribution, but do not vary by state UI generosity for characteristics that would be potential sources of concern. The first quintile shows no difference in previous wage, previous tenure, education or age, which would be areas of concern for the validity of the comparison.¹⁰

My approach to measure the effect of unemployment insurance on re-employment wages is to use a standard Mincer equation and bin the sample of unemployed individuals into quintiles of liquid wealth. I also use a linear spline of the previous annual wage to control for changes in behavior across the income distribution as well as for endogeneity with respect to ability, to the extent possible. I also include state fixed effects to control for endogeneity with respect to location choice. In other words, I exploit variation in unemployment insurance over time that is not the result of previous income, UI duration, or choice of location. Similar identification strategies are employed by [Engen and Gruber \(2001\)](#), [Chetty \(2008\)](#), among others. In each of the following equations, I include age, race, marital status, education, tenure, as well as state and year fixed effects. I also include interactions between net liquidity quantiles and each of industry, occupation (2-digit) and the log-wage spline. My main test uses the following specification:

$$\ln(Y_{i,j+1,s,t}) = \alpha_0 + \sum_{q=1}^5 \delta_0^q \times \ln(UI_{s,t}) + \sum_{q=1}^5 \delta_1^q \times UIDur_{s,t} \quad (3.1)$$

$$+ \delta_s + \delta_t + X_{i,j,t}\beta + \epsilon_{i,j+1,s,t} \quad (3.2)$$

where j is the previous job and $j + 1$, the next job, reported by individual i at time t in net liquidity quintile q . δ_0^q and δ_1^q are the effect of UI replacement rates and potential UI duration

¹⁰Selecting on unemployment insurance recipients may cause bias in my estimates; however, per [Table B.1](#), the rates of UI take-up do not vary across wealth quintiles, which suggest that endogenous take-up is not driving the following results that I find for individuals from the first quintile. Within the first quintile take-up in below median UI states is lower than in states above the median, counter to what we would expect if the recipients selected along liquidity needs.

for an individual in net liquid wealth quintile q at the start of a spell. A positive δ_0^q indicates that more generous unemployment insurance is associated with better employment outcomes for quintile q . A negative δ_1^q indicates that longer unemployment insurance durations result in worse re-employment outcomes.

3.1.2 Findings

My results show that constrained workers alter their search behavior when presented with additional unemployment insurance. Individuals from the first quintile of liquid wealth find jobs offering 6.34% higher pay the month after unemployment when they receive a 10% increase in UI (column 1 of [Table 3.1](#)). Column 2 shows that the effect is the same magnitude (6.28%) during the quarter following unemployment. The estimate is significant at the 5-percent level, using Taylor Linearized standard errors, (the suggested variance estimator for the SIPP's complex survey design) but only for the first quintile. I also find that longer potential UI is associated with a decline in wages, though only for the wealthiest population.

Given that employment is highly persistent, while the average unemployment spell in my sample is less than 25 weeks, an elasticity of 0.63 suggests that an additional source of income alters job search behavior. Prior to separation, these individuals had nearly identical labor market characteristics ([Table B.1](#)). Results clustered at the state level yield similar significance levels, and are reported in the online appendix. As a check on the credibility of my findings, I explore whether unemployment insurance generosity is predictive of job-to-job (J2J) wage changes. If there were some underlying state trend over time driving my results, it would be reasonable to expect to find a similar pattern among job-to-job wage changes. I use the same specification as [Equation 3.1](#), and include UI interacted with liquid wealth quintiles. This yields insignificant results for all coefficients of interest, and is reported in the online appendix.

Re-Employment Labor Income Regressions (by Net Liquidity)		
VARIABLES	(1)	(2)
	Log of Next Weekly Earnings	Log of Next Quarterly Earnings
Net Liq. Quintile 1 X Log of Avg. UI	0.634** (0.262)	0.628** (0.260)
Net Liq. Quintile 2 X Log of Avg. UI	0.223 (0.258)	0.303 (0.273)
Net Liq. Quintile 3 X Log of Avg. UI	0.00351 (0.351)	-0.0310 (0.279)
Net Liq. Quintile 4 X Log of Avg. UI	0.132 (0.205)	0.0194 (0.214)
Net Liq. Quintile 5 X Log of Avg. UI	0.174 (0.256)	0.109 (0.272)
Net Liq. Quintile 1 X Potential UI Weeks	-0.00102 (0.00985)	0.0160 (0.0112)
Net Liq. Quintile 2 X Potential UI Weeks	-0.00946 (0.00836)	-0.00847 (0.00701)
Net Liq. Quintile 3 X Potential UI Weeks	0.0155 (0.0174)	0.0113 (0.0172)
Net Liq. Quintile 4 X Potential UI Weeks	0.0117 (0.0112)	0.00507 (0.00983)
Net Liq. Quintile 5 X Potential UI Weeks	-0.0225** (0.00977)	-0.0217*** (0.00777)
Observations	2172	2311
R^2	0.359	0.381
State FE	X	X
Year FE	X	X
Qtile FE + Qtile X Wage Spline	X	X
Ind + Ind X Qtile FEs	X	X
Occ + Occ X Qtile FEs	X	X

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 3.1: Elasticities by net liquidity quintile. Column 1 reports employment outcomes during the first month following an unemployment spell. Column 2 reports employment outcomes for wages over the first quarter following an unemployment spell. Liquid wealth quintile refers to liquid assets net of unsecured credit.

3.2 Life-Cycle Profiles

To examine the correlation between initial wealth and lifetime earnings, I use the Panel Study of Income Dynamics (PSID) and National Longitudinal Survey of Youth 1979 (NLSY), and partition individuals into their wealth quantiles before entering the labor force. I detail the sample selection as well as the construction of these profiles in [Appendix A](#). I first use the NLSY to explore the correlation between initial wealth and training hours over the life-cycle.

Then I use the PSID to explore differences in earnings for individuals from different wealth quantiles. For both results, I use the following specification:

$$Y_{i,a,s,t} = \alpha_0 + \sum_{q=1}^5 (\delta_0^q \times \text{Age}) + \delta_s + \delta_t + X_{i,a,t}\beta + \epsilon_{i,a,s,t} \quad (3.3)$$

where $Y_{i,a,s,t}$ is the outcome of interest (either training hours or log-earnings) for individual i , at age a , in area s , in year t . Both regressions control for education level, race, marital status, state (or region in the NLSY), year, as well as the hours worked by individual. In each case, I weight the results by the provided sample weights.

3.2.1 Human Capital Accumulation

I use the NLSY79 to show a correlation between initial wealth (prior to entering the labor market), and training over the duration of this study (ages 25-54). The sample is restricted to individuals employed full-time, and wealth quintiles are permanent and defined before entering the labor market. Details of the sample selection are available in [Section A.3](#). The profiles show a clear correlation between initial wealth and time training ([Figure 3.1](#)). These

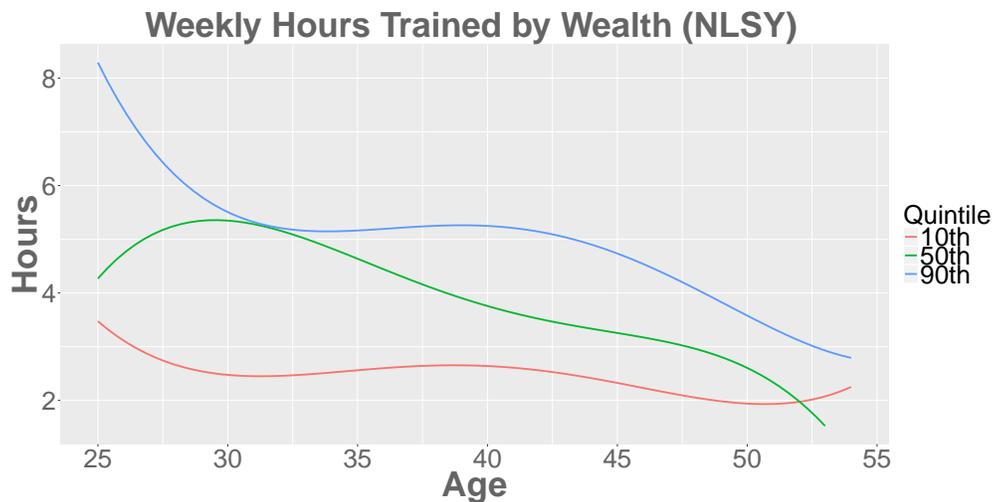


Figure 3.1: Training Hours Per Week by Wealth Quintile.

measures include training outside of work as well as training sponsored by employers. These profiles suggest that there is a correlation between wealth and human capital accumulation while working.

3.2.2 Earnings

There appear to be permanent earnings differences between individuals from different wealth strata. Figure 3.2 shows the average earnings profiles individuals by their liquid wealth prior to entering the labor market. The left panel shows high school educated individuals, and the right panel shows college educated individuals. Both show that individuals from the bottom of the wealth distribution experience persistently different earnings profiles from their wealthier peers. Details of the sample selection are available in Section A.2.

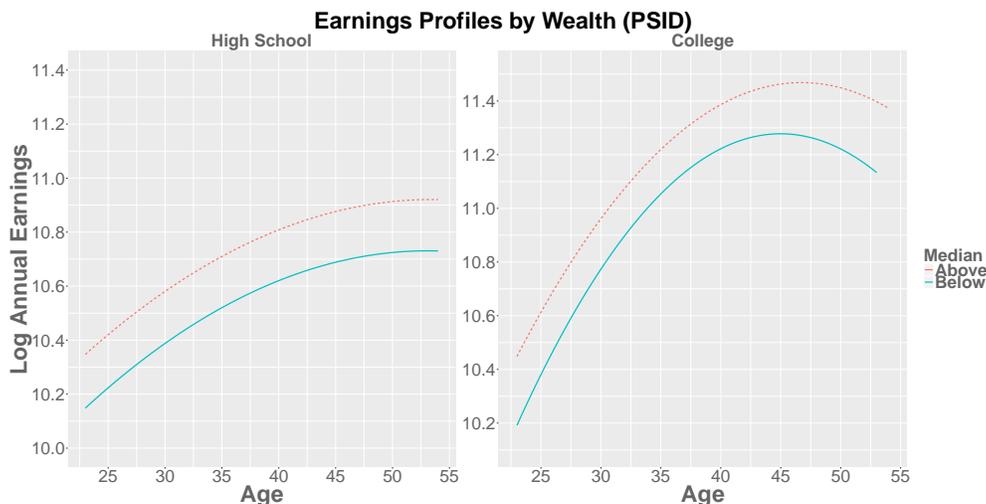


Figure 3.2: Earnings Profiles by Initial Wealth and Education.

4 The Model

4.1 Environment

Time is discrete and continues forever, while each agent lives deterministically for $T \geq 2$ periods. There is a continuum of both firms and workers, each of which discounts future value at the identical rate β . Each worker is born unemployed without benefits, and receives a draw from a correlated trivariate log-normal distribution $\Psi \sim LN(\psi, \Sigma)$ of wealth, human capital, and learning ability (a_0, h_0, ℓ) . Over the life-cycle, a worker may be in one of three employment states: employed, unemployed with unemployment insurance, and unemployed without unemployment insurance. Workers in each employment state are allowed to direct their search to contracts posted by firms.

Each worker is endowed with one indivisible unit of labor that they can enjoy as leisure

during unemployment or supply inelastically while employed. Leisure utility ν is assumed to be additively separable, $u(c) + (1 - e)\nu$, where e denotes employment status. Workers are risk-averse, with utility $u'(c) \geq 0$, $u'(0) = \infty$, and are allowed to smooth consumption over the life-cycle by borrowing and saving at rate r_F . They face a borrowing limit at each age, \underline{a}' , and are not allowed to default on any debt obligations, nor exit the terminal period T with negative asset holdings. While employed, workers are allowed to devote productive time τ to accumulating human capital through a [Ben-Porath](#) production function, $H(h, \ell, \tau, L)$, which is increasing in its first 3 arguments. L denotes the labor market status E or U . All workers face an *iid* human capital shock between periods, $\epsilon' \sim N(\mu_\epsilon, \sigma_\epsilon)$, that permanently alters human capital. This is modeled as $h' = e^{\epsilon'}(h + H(h, \ell, \tau, E))$.

Workers transition from employment to unemployment in one of two ways: with probability δ , they receive a separation shock and enter unemployment, and with probability $\lambda_E \leq 1$, they are allowed to search while employed for a new job. Employed workers receive $\mu(1 - \tau)f(h)$ as income each period, where μ is their piecerate wage, $(1 - \tau)$ the time left over after human capital decisions, and $f(h)$ is their productivity given their current human capital. If they receive an unemployment shock, workers receive unemployment benefits $b_{UI} = \min\{b\mu(1 - \tau)f(h), \bar{b}\}$, where b is the replacement rate, and \bar{b} is the maximum benefit allowed per quarter. I assume that unemployment benefits are drawn from a distribution $b \sim N(\mu_b, \sigma_b)$. Both the distribution of replacement rates and benefit cap are important for my identification strategy. Agents stochastically lose benefits with probability γ , and receive $b_L \leq b_{UI}$, which reflects opportunities to earn money outside the labor force.

Firms post vacancies at cost κ . These vacancies are one-firm-one worker contracts that specify the piecerate of output paid as earnings, μ . These contracts are assumed to be renegotiation-proof, and firms are not allowed to respond to outside offers, thus μ is fixed for the duration of the contract. Worker characteristics are assumed to be observable, and thus firms open vacancies into specific submarkets that are indexed by the observables of the worker. Thus, submarkets are identified by the following tuple: $(\mu, a, h, \ell, t) \in \mathbb{R}_+ \times \mathbb{R} \times \mathbb{R}_+ \times \mathbb{R}_+ \times \mathbb{R}_+$. In equilibrium, each submarket has a known probability of employment. Once matched, a firm receives $(1 - \mu)(1 - \tau)f(h)$ in profits each period. They continue until the match dissolves, either through exogenous separation or on-the-job search.

Following [Pissarides \(1985\)](#), I refer to submarket tightness as $\theta_t(\mu, a, h, \ell) = \frac{v(\mu, a, h, \ell)}{u(\mu, a, h, \ell)}$. The rate at which firms and workers match in each submarket is characterized by a constant returns to scale matching function, $M(u, v)$, where u is the number of unemployed searchers in the submarket and v is the number of firms posting vacancies in the submarket. I define

the probability at which firms meet workers as $\frac{M(u,v)}{v} = q(\theta_t(\mu, a, h, \ell))$, and the rate at which workers meet firms as $\frac{M(u,v)}{u} = p(\theta_t(\mu, a, h, \ell))$, both of which I assume to be invertible. I assume that within each submarket the free entry condition holds, meaning that firms compete away any expected profits within a submarket by opening additional vacancies.

The aggregate state of the economy is summarized by the following tuple: $\psi = (z, u, e, \rho)$. The first component is the current level of output in terms of the numeraire for a job in the economy, independent of human capital. The second component is a function that tracks the measure of workers with assets a , human capital h , learning ability ℓ , at age t , $u(a, h, \ell, t)$. The third determines the measure of employment for each of these same types. The last component is the stochastic process that determines newly born workers in each period. By restricting the equilibrium to be block recursive, decision rules do not depend on the distribution of workers or firms. I demonstrate this in [Section C.1](#). The aggregate state z is suppressed because the model is stationary.

4.2 Worker's Problem

4.2.1 Production, Savings, and Human Capital Accumulation

Each period is divided into two subperiods: job search, and production. During the production subperiod agents choose consumption and savings allocations (c and a'), and the employed workers choose the proportion of time to spend accumulating human capital, τ . All agents are subject to a borrowing constraint \underline{a}' , which changes with age. Following these decisions, age advances. Employed workers solve the problem given in [Equation 4.1](#).

$$W_t(\mu, a, h, \ell) = \max_{c, a' \geq \underline{a}', \tau \in [0,1]} u(c) + \beta E[(1 - \delta)R_{t+1}^E(\mu, a, h', l) + \delta R_{t+1}^U(b_{UI}, a', h', l)] \quad (4.1)$$

$$\text{s.t. } c + a' \leq (1 + r_F)a + \mu(1 - \tau)f(h) \quad (4.2)$$

$$h' = e^{\epsilon'}(h + H(h, \ell, \tau, E)) \quad (4.3)$$

$$\epsilon' \sim N(\mu_\epsilon, \sigma_\epsilon) \quad (4.4)$$

$$b_{UI} = \min\{b(1 - \tau)\mu f(h), \bar{b}\} \quad (4.5)$$

$$b \sim N(\mu_b, \sigma_b) \quad (4.6)$$

Human capital evolves according to $e^{\epsilon'}(h + H(h, \ell, \tau, E))$, where ϵ' is the human capital depreciation shock experienced at the start of the following period. The function H determines the accumulation of human capital and is a non-decreasing function of τ , $\frac{\partial H(h, \ell, \tau, E)}{\partial \tau} \geq 0$,

$\frac{\partial H^2(h, \ell, \tau, E)}{\partial \tau^2} \leq 0$. Human capital accumulation is realized before separation shocks, so any time spent accumulating human capital affects b_{UI} . Employed agents face a probability δ of separating from their current employer. Newly unemployed agents are assumed to have unemployment benefits for at least one period. Following period $T + 1$, employed utility is zero:

$$W_{T+1}(\mu, a, h, \ell) = 0 \quad (4.7)$$

Unemployed agents choose consumption and savings and receive benefit and human capital shocks ϵ' once age advances. Their problem is given in [Equation 4.8](#).

$$U_t(b_{UI}, a, h, \ell) = \max_{c, a' \geq \underline{a}'} u(c) + \nu + \beta E[(1 - \gamma)R_{t+1}^U(b_{UI}, a', h', \ell) + \gamma R_{t+1}^U(b_L, a', h', \ell)] \quad (4.8)$$

$$\text{s.t. } c + a' \leq (1 + r_F)a + b_{UI} \quad (4.9)$$

$$h' = e^{\epsilon'}(h + H(h, \ell, \tau, U)) \quad (4.10)$$

$$\epsilon' \sim N(\mu_\epsilon, \sigma_\epsilon) \quad (4.11)$$

where b_{UI} is their unemployment benefit. Unemployed agents face shocks to their benefits, which they lose with probability γ , and their human capital, which evolves according to $e^{\epsilon'}(h + H(h, \ell, \tau, U))$. Note that $H(h, \ell, \tau, U)$ is assumed to be zero for all unemployed agents. Following these decisions, age advances and unemployed workers receive benefits and human capital shocks. Unemployed agents without UI face a problem described by [Equation 4.12](#).

$$U_t(b_L, a, h, \ell) = \max_{c, a' \geq \underline{a}'} u(c) + \nu + \beta E[R_{t+1}^U(b_L, a', h', \ell)] \quad (4.12)$$

$$\text{s.t. } c + a' \leq (1 + r_F)a + b_L \quad (4.13)$$

$$h' = e^{\epsilon'}(h + H(h, \ell, \tau, U)) \quad (4.14)$$

$$\epsilon' \sim N(\mu_\epsilon, \sigma_\epsilon) \quad (4.15)$$

where $b_L \leq b_{UI}$. Without benefits, these workers have no probability of receiving benefits again without first becoming employed. Unemployed agents of both types die after T periods with certainty and thus their unemployment utility in period $T + 1$ is zero:

$$U_{T+1}(UI, a, h, \ell) = 0 \quad \forall UI \in \{b_{UI}, b_L\} \quad (4.16)$$

4.2.2 Job Search

Age advances and shocks are realized following the production period. Unemployed agents in the job search period solve the problem given by [Equation 4.17](#).

$$\begin{aligned} R_t^U(b_{UI}, a, h, \ell) &= \max_{\mu'} P(\theta_t(\mu', a, h, \ell)) W_t(\mu', a, h, \ell) \\ &\quad + (1 - P(\theta_t(\mu', a, h, \ell))) U_t(b_{UI}, a, h, \ell) \end{aligned} \quad (4.17)$$

where b_{UI} denotes their current level of UI and μ' denotes the application strategy $\mu'(w, a, h, \ell, t)$. For agents without unemployment insurance, $b_{UI} = b_L$. Employed workers are allowed to search on the job, and solve the problem given by [Equation 4.18](#).

$$\begin{aligned} R_t^E(\mu, a, h, \ell) &= \max_{\mu'} \lambda_E P(\theta_t(\mu', a, h, \ell)) W_t(\mu', a, h, \ell) \\ &\quad + (1 - \lambda_E P(\theta_t(\mu', a, h, \ell))) W_t(\mu, a, h, \ell) \end{aligned} \quad (4.18)$$

4.3 Firm's Problem

Firms produce using a single worker as an input. New firms post piece-rate wage contracts in submarkets characterized by (μ, a, h, ℓ, t) , each of which is assumed to be observable to the firm. Contracts dictate the share of revenue to be received by each side in the match. Wage contracts are assumed to be renegotiation-proof. A firm with a filled vacancy produces using technology $y = (1 - \tau)f(h)$, where τ is the time spent accumulating human capital by the worker that cannot be used in production. The firm retains a fraction $(1 - \mu)$ of this output as profits and pays the rest out in wages. Matches continue with probability $(1 - \delta)(1 - \lambda_E P(\theta_{t+1}(\mu', a', h', \ell)))$, the probability that the match does not separate exogenously and the worker does not find a new employer. Firms discount at the same rate as workers, β . The value function of a firm matched with a worker is given in [Equation 4.19](#).

$$J_t(\mu, a, h, \ell) = (1 - \mu)(1 - \tau)f(h) + \beta E[(1 - \delta)(1 - \lambda_E P(\theta_{t+1}(\mu', a', h', \ell))) J_{t+1}(\mu, a', h', \ell)] \quad (4.19)$$

$$h' = e^{\epsilon'}(h + H(h, \ell, \tau, E)) \quad (4.20)$$

where $a' = g_a(\mu, a, h, \ell)$ and $\tau = g_\tau(\mu, a, h, \ell)$ are the worker policy decisions over wealth and human capital accumulation. $\mu' = g_\mu(\mu, a', h', \ell)$ is the application strategy of the worker

conditional upon his asset and human capital policy rule. Profits from a filled vacancy at age $T + 1$ are zero:

$$J_{T+1}(\mu, a, h, \ell) = 0 \quad (4.21)$$

New firms have the option of posting a vacancy at cost κ in any submarket. Each submarket offers a probability of matching with a worker given by $q(\theta_t(\mu, a, h, \ell))$. In expectation, the value of opening a vacancy in submarket (μ, a, h, ℓ) is given by [Equation 4.22](#).

$$V_t(\mu, a, h, \ell) = -\kappa + q(\theta_t(\mu, a, h, \ell))J_t(\mu, a, h, \ell) \quad (4.22)$$

I assume that the free entry condition holds for every open submarket. Firms enter until the expected profits of a vacancy, $V_t(\mu, a, h, \ell) = 0$. This means that [Equation 4.22](#) can be rewritten as [Equation 4.23](#).

$$\kappa = q(\theta_t(\mu, a, h, \ell))J_t(\mu, a, h, \ell) \quad (4.23)$$

In equilibrium, this yields the following:

$$q(\theta_t(\mu, a, h, \ell)) = \frac{\kappa}{J_t(\mu, a, h, \ell)} \quad (4.24)$$

$$\theta_t(\mu, a, h, \ell) = q^{-1}\left(\frac{\kappa}{J_t(\mu, a, h, \ell)}\right) \quad (4.25)$$

Using the definition of the matching function, $\frac{M(u,v)}{u} = p(\theta_t(\mu, a, h, \ell))$ and $\frac{M(u,v)}{v} = q(\theta_t(\mu, a, h, \ell))$, the equilibrium job-finding rate for workers and firms in a submarket can be expressed as $p(\theta_t(\mu, a, h, \ell)) = \theta q(\theta_t(\mu, a, h, \ell))$.

4.4 Timing

The timing in the model is as follows:

1. Firms open vacancies in submarkets (μ, a, h, ℓ, t) .
2. Employed and unemployed workers search for vacancies in submarkets (μ, a, h, ℓ, t) .
3. Agents who receive job offers transition employment states. Agents who are not offered a job remain unemployed.
4. All agents make consumption and savings decisions. Employed agents allocate time between production and human capital accumulation.

5. Age advances. Agents receive human capital shocks, benefits shocks, and unemployment shocks in that order.

4.5 Equilibrium

A Block Recursive Equilibrium (BRE) in this model economy is a set of policy functions for workers, $\{c, \mu', a', \tau\}$, value functions for workers W_t, U_t , value functions for firms with filled jobs, J_t , and unfilled jobs, V_t , as well as a market tightness function $\theta_t(\mu, a, h, \ell)$.¹¹ These functions satisfy the following:

1. The policy functions $\{c, \mu', a', \tau\}$ solve the workers problems, W_t, U_t, R_t^E, R_t^U .
2. $\theta_t(\mu, a, h, \ell)$ satisfies the free entry condition for all submarkets (μ, a, h, ℓ, t) .
3. The aggregate law of motion is consistent with all policy functions.

5 Estimation

I use indirect inference to estimate the model. Indirect inference is a moment-matching approach based on targeting parameters from reduced-form models that make up an “auxiliary model” and capture important aspects of the underlying structural model. I select reduced-form equations in my auxiliary model to identify borrowing constraints and heterogeneity in earnings growth, the key mechanisms in my structural model. This approach is popular among papers estimating household response to risk (Güvenen and Smith, 2014), as well as those estimating search behavior over the life-cycle (Lise (2013), Bowlus and Liu (2013)). I discuss this methodology further in Section 5.2.4. In Section 5.5, I use decision rules from the estimated model to demonstrate the sources of identification.

To implement indirect inference, I preset functional forms and parameters that are ubiquitous throughout the related literature. These choices are detailed in Section 5.1. The remaining parameters are estimated by indirect inference by matching moments from the auxiliary model presented in Section 5.2.

¹¹A Block Recursive Equilibrium is one in which the first two “blocks” of the equilibrium, i.e. the individual decision rules, can be solved without conditioning upon the aggregate distribution of agents across states, i.e. the third block of the equilibrium. The aggregate state can then be recovered by simulation. For an extended discussion see Section C.2.

5.1 Empirical Preliminaries

5.1.1 Functional Form and Distributional Assumptions

I set the functional forms to those commonly used in the literatures on search and on inequality. I choose a power utility function of the following form:

$$u(c) = \begin{cases} \frac{c^{1-\sigma}-1}{1-\sigma} & \text{if } \sigma > 0, \sigma \neq 1 \\ \ln(c) & \text{if } \sigma = 1 \end{cases} \quad (5.1)$$

When agents are unemployed, I assume that they receive linear leisure utility, $u(c) + \nu$. I use the matching function from [Schaal \(2011\)](#), which is constant returns to scale and generates well-defined probabilities:

$$M(u, v) = \frac{uv}{(u^\eta + v^\eta)^{\frac{1}{\eta}}} \quad (5.2)$$

I assume the following functional form for production:

$$y = f(h) \quad (5.3)$$

$$f(h) = zh \quad (5.4)$$

where z is a scale factor. Linear production is a common restriction in the search literature when models do not consider physical capital. I assume that all workers face shocks to their human capital each period, $e^{\epsilon'}$ with $\epsilon' \sim N(\mu_\epsilon, \sigma_\epsilon)$. Employed workers accumulate human capital using [Ben-Porath \(1967\)](#) technology:

$$h' = e^{\epsilon'}(h + H(h, \ell, \tau, E)) \quad (5.5)$$

$$H(h, \ell, \tau, E) = \ell(h\tau)^{\alpha_H} \quad (5.6)$$

where ℓ is the learning ability of an individual endowed at the beginning of the life-cycle and can be thought of as a fixed effect (it is constant). τ is the fraction of productive time that an employed worker spends accumulating human capital. The fractional exponent reflects the fact that my model is quarterly, while previous work incorporating human capital is generally at an annual frequency.

Ben-Porath is widely employed among papers on human capital and inequality, which

allows for more straightforward comparisons between my findings and the findings of other papers on inequality. However, this assumption is a departure from much of the previous work in the search literature that incorporates human capital. With the exception of [Bowlus and Liu \(2013\)](#), models of search with human capital have assumed that human capital is accumulated through “learning-by-doing,” which means that human capital grows exogenously while employed. The learning-by-doing approach yields tractability, which papers like [Bagger et al. \(2014\)](#) and [Carillo-Tudela \(2012\)](#) exploit in order to decompose the variance of wage growth over the life-cycle. The empirical evidence is divided on which approach best fits the data. Recent evidence from [Blandin \(2016\)](#), who nests learning-by-doing and [Ben-Porath](#) within a single model and tests their predictions about life-cycle earnings finds that [Ben-Porath](#) fits the data roughly 4 times better than learning-by-doing. It is unclear if those results generalize to a model with labor market frictions. Within the context of my model, learning-by-doing causes trouble matching the downturn in earnings growth without other assumptions.

For unemployed workers, I assume that they are unable to invest in human capital and face only the i.i.d. human capital depreciation process.

$$h' = e^{\epsilon'}(h + H(h, \ell, \tau, U)) \quad (5.7)$$

$$= e^{\epsilon'} h \quad (5.8)$$

I assume that agents are subject to a natural borrowing constraint each period:

$$\underline{a}' = \sum_{j=t}^T \frac{b_L}{(1 + r_F)^j} \quad (5.9)$$

In each period t , \underline{a}' is the amount that any agent could repay if he were in the worst income state (b_L) in every period until the terminal date. Modeling borrowing constraints in this way is appealing because it never fully binds and is the least restrictive borrowing constraint in a model without the option to default. While natural borrowing constraints are common in the heterogeneous agent literature ([Huggett \(1993\)](#), and [Aiyagari \(1994\)](#) among many others), it is not ubiquitous. One commonly used alternative is [Kehoe and Levine \(1993\)](#), which allows default under penalty of future autarky. In most cases, adopting an alternate approach like these would yield tighter borrowing constraints in my model. Specifically, for agents approaching the borrowing constraint, this would yield larger borrowing premiums

and a tighter debt limit.

Lastly, I assume that initial conditions (a_0, h_0, ℓ) are drawn from a multivariate log-normal distribution, $\Psi \sim LN(\psi, \Sigma)$, with mean ψ and variance-covariance Σ , so that human capital and learning ability are both positive and each marginal distribution can be characterized by a shape and scale parameter. The initial distribution of wealth is shifted by $-\underline{a}'(t=0)$, the borrowing constraint in period 0. These are common assumptions when modeling inequality. I use a Gaussian copula with correlations $\rho_{AH}, \rho_{AL}, \rho_{HL}$ (the pairwise correlations between wealth, human capital, and learning, respectively) to generate correlated draws from this initial distribution. The preset functional forms and initial conditions are summarized in [Table 5.1](#).

Table 5.1: Preset Functional Forms and Distributions

Category	Symbol	Value or Function	Source
<i>Model Parameters</i>			
Utility Function	$U(c)$	$\frac{c^{1-\sigma}-1}{1-\sigma} + (1-e)\nu$	Power Utility
Production Function	$f(h)$	zh	
Human Capital Production	$H(h, \ell, \tau, E)$	$\ell(h\tau)^{\alpha_H}$	Ben-Porath (1967)
Human Capital Evolution	h'	$e^{\epsilon'}(h + H(h, \ell, \tau, E))$	
Matching Function	$M(u, v)$	$\frac{uv}{(u^\eta + v^\eta)^{\frac{1}{\eta}}}$	Schaal (2012)
Borrowing Constraint	\underline{a}'	$\sum_{j=t}^{j=T} \frac{b_L}{(1+r_F)^j}$	Natural Borrowing Limit at time t
<i>Distributional Parameters</i>			
Depreciation	ϵ'	$\epsilon' \sim N(\mu_\epsilon, \sigma_\epsilon)$	
Initial Conditions	Ψ	$\Psi \sim LN(\psi, \Sigma)$	
Mean	ψ	$[\mu_A \quad \mu_H \quad \mu_L]'$	
Variance	$\text{diag}(\Sigma)$	$(\sigma_A, \sigma_H, \sigma_L)$	
Correlation		$(\rho_{AH}, \rho_{AL}, \rho_{HL})$	

5.1.2 Preset Parameter Values

I select a subset of the parameters to be set to common values from the relevant literature. Agents in the model live for $T = 128$ quarters, covering the post-schooling and prime working ages, 25-54. I set the exogenous separation rate to match the average quarterly flows from employment to unemployment ([Shimer, 2012](#)), $\delta = 0.030$. An exogenous interest rate is required to for the equilibrium concept used to solve the model, so I set the risk-free rate to a quarterly $r_F = 0.012$, which generates an annual risk-free rate of about 5%. I set $\beta = \frac{1}{1+r_F}$, so that agents smooth consumption in expectation. The elasticity parameter of the matching function, η is set so that the elasticity of the job-finding probability of unemployed workers with respect to submarket tightness is on average 0.5, consistent with

the empirical exploration in Shi (2016). The cost of opening a vacancy, κ , is also set at 0.2 using the results from Shi (2016). I use a scale factor, z , equal to the average quarterly income in the PSID at age 25.

I assume that the unemployment insurance replacement rate distribution has mean $\mu_b = 0.42$, and $\sigma_b = 0.053$, which is a normal distributed approximation to the distribution of replacement rates allowed under most state UI systems in my data. I discretize this distribution and restrict draws to be two standard deviations or less to ensure that negative replacement rates are not possible, meaning that the range of possible replacement rates is [32%, 53%]. I also cap unemployment insurance at a weekly maximum of \$450, which is the average cap in my data. Both of these considerations are required for identification in my estimation procedure. I assume that unemployment insurance does not fluctuate with human capital depreciation, but can be lost with probability γ . I set $\gamma = 0.54$, which matches the expected max duration of UI in my data (≈ 24.1 weeks).

There are 15 parameters remaining to be estimated (shown in Table 5.3). The preset parameters are summarized in Table 5.2.

Table 5.2: Preset Parameter Values

Category	Symbol	Value or Function	Source
<i>Model Parameters</i>			
Discount Factor	β	0.9882	$\frac{1}{1+r_F}$
Risk Aversion	σ	2.0	Standard
Quarters	T	128	Standard
Elasticity of Matching Function	η	0.5	Shi (2016)
Vacancy Creation Cost	κ	0.2	Shi (2016)
Separation Rate	δ	0.030	Quarterly average 1968-2013
Scale Factor	z	18 165	Mean quarterly earnings (Age 25, PSID)
Quarterly Max UI (unscaled)	γ	1.29	Average UI cap
UI Loss Probability	γ	0.54	Sample max UI duration average
Risk Free Rate	r_F	0.0120	Annual rate of $\approx 5\%$
<i>Distributional Parameters</i>			
UI Replacement Rate Distribution	(μ_b, σ_b)	(0.42, 0.053)	Approx. sample distribution

5.2 Indirect Inference and Auxiliary Model

I estimate the remaining structural parameters of the model using indirect inference (Gourieroux et al., 1993). Indirect inference is a generalized method of moments (GMM) estimation technique in which the user selects a set of coefficients from a parsimonious “auxiliary model” composed of one or many reduced-form equations. Rather than matching unconditional

moments, indirect inference minimizes the distance between parameters from the auxiliary model and identical reduced-form estimations run on simulated data. It has also been widely applied in papers that analyze inequality through the lens of search models (Bowlus and Liu (2013), Lise (2013), Graber and Lise (2015), among others).

The primary advantage for my application is that I can select a set of reduced-form equations as an auxiliary model, whose relation to the data is clear, and then provide a structural interpretation using my model. If the auxiliary model yields inference on a mechanism in the data, then the model is replicating the data generating process by matching the conditional density. The resulting structural parameters are consistent with that mechanism, provided that the auxiliary models identify all the structural parameters. The technique allows me to easily deal with flaws in my estimation sample by inserting the same flaws into the model generated data. This allows me to deal with missing observations by sampling the simulated data at the same frequency.

My model has initial heterogeneity from three sources: differences in wealth, differences in initial human capital, and differences in learning ability, which are jointly distributed at the beginning of the life-cycle. I pick a set of reduced-form moments and estimate an auxiliary model in order to discipline this initial heterogeneity. In each specification, I denote the set of parameters to be matched through indirect inference with β_i , where i indexes the parameter or set of parameters. In my empirical specifications, I use an extensive set of controls that have no analog in my model. I denote these “nuisance” parameters δ .

With the exception of moments characterizing the borrowing constraint and initial wealth and earnings, I estimate my model on agents ages 25 to 54, two years after I start agents in the model (age 23). This is because I observe earnings at first jobs for very few agents, particularly for whom I also observe either wealth or proxies for human capital or learning ability. By matching my model to data on agents who are already employed, I allow for wage growth while still retaining inference on the structural parameters of interest.

5.2.1 Model Parameters

To identify and estimate the set of borrowing constraints in my model, I match the re-employment wage elasticities with respect to changes in unemployment insurance for individuals from each of the liquid wealth quintiles. Ex ante identification requires that conditional on observables, each individual’s borrowing constraint is identical. This is not an unreasonable assumption: a lender is likely to condition credit offered to a worker on their previous

employment history and demographic characteristics. This regression largely follows my approach in [Section 3](#), with two modifications. I drop potential UI duration, as this is identical for all agents in my model, and I use an interaction between liquid wealth quintile and log of last wage rather than a spline. The specification that I use in both the SIPP and the simulated data is given in [Equation 5.10](#).

$$\ln(W_{i,j+1}) = \beta_0 + \sum_{q=1}^5 \beta_1^q \times \mathbb{1}_q \ln(UI_i) + \sum_{q=1}^5 \beta_2^q \times \mathbb{1}_q \ln(W_j) + \beta_3 \times Age \quad (5.10)$$

$$+ X_{1,i} \delta_1 + \epsilon_{i,j+1} \quad (5.11)$$

where j indexes the job of a worker. I match the set of auxiliary parameters $\beta_0, \beta_1^q, \beta_2^q, \beta_3$, and treat the set of data controls δ as nuisance parameters. The set of controls, $X_{1,i}$ is identical to those in [Section 3](#). These moments directly connect the degree to which an individual is capable of smoothing consumption during an unemployment spell to their subsequent employment outcome, which yields inference on the degree to which a borrowing constraint is binding for individuals across the asset distribution.

For inference on the human capital evolution parameters, α_H and $(\mu_\epsilon, \sigma_\epsilon)$, I use six age bins of five years each (25-29, 30-34, 35-39, 40-44, 45-49, 50-54), and match within job earnings growth from the NLSY using [Equation 5.12](#). I use the estimate of the standard deviation from this regression to match the standard deviation of earnings growth, σ_L .

$$\Delta \ln(W_{i,j,a}) = \sum_{a=1}^6 \beta_4^a \mathbb{1}_{Age \in [a, a+1)} + \Delta X_{2,i} \delta_2 + \Delta \epsilon_{i,j,a} \quad (5.12)$$

In my model, human capital accumulation is the only source of earnings growth for individuals who stay with the same job, making this the appropriate analog to estimate α_H and $(\mu_\epsilon, \sigma_\epsilon)$.¹² I use a similar strategy to estimate the on-the-job search efficiency parameter, λ_E . I bin individuals using the same age categories and estimate a linear probability model whose dependent variable is whether an individual changed employers during the period,

¹²The use of this equation to identify α_H is identical to the approach taken by [Bowlus and Liu \(2013\)](#); this approach to human capital depreciation is similar to the approach used by [Huggett et al. \(2011\)](#), who restrict their sample to ages in which individuals are unlikely to accumulate human capital.

following [Equation 5.13](#).

$$\text{SameJob}_{i,j,a} = \sum_{a=1}^6 \beta_5^a \mathbb{1}_{\text{Age} \in [a, a+1)} + \delta_s + \delta_t + X_{3,i} \delta_3 + \epsilon_{i,j,a} \quad (5.13)$$

λ_E changes the rate at which individuals transition jobs, satisfying the requirement for ex-ante identification.

To identify leisure, I use a workers employment status at the time of interview to match the sample unemployment rate in the PSID. As in [Section 3](#), I restrict this sample to ages 25-54, to focus on prime age male workers. This takes the following form:

$$\text{URate} = \frac{1}{NT} \sum_{i=1}^N \sum_{t=25}^{54} \mathbb{1}_{U_{nemp}}. \quad (5.14)$$

where N is the number of individuals in the PSID and T is the number of years for which I record their observations (30). The panel is not balanced, but I drop individuals in my estimation at the same frequency as in the data.

5.2.2 Marginal Distribution Parameters

Having identified the borrowing limit at age $t = 0$ (23 in my model), the marginal distribution of wealth (μ_A, σ_A) can be estimated from the data by using the income profiles of individuals from different wealth quantiles. I use the distribution of the last observation of liquid wealth in the PSID for men prior to entering the labor market and divide the sample into deciles. I include the mean of each decile as auxiliary parameters in the model.

Likewise, the borrowing constraint and parameters characterizing the marginal distribution of wealth identified, the marginal distribution of human capital (μ_H, σ_H) can be identified by matching the distribution of initial earnings in the data. In the model, jobs are determined by a worker's application strategy, which is characterized upon first entering the labor market by a workers wealth and human capital (learning plays a small role in initial placement, but is separately identified below). Thus, I use the distribution of earnings at the first job observed in the PSID. I use the same sample restrictions as in construction the liquid wealth deciles, and match deciles of the initial earnings distribution.

The marginal distribution of learning ability, (μ_L, σ_L) are identified from the variance of earnings growth rates by age ([Equation 5.12](#)), and the average growth rate over the life-cycle, estimated using [Equation 5.15](#). σ_L influences the variability of earnings growth over the life-

cycle by changing the dispersion of human capital growth, while μ_L alters the average rate of human capital growth.

5.2.3 Correlations

The final three parameters to estimate are the correlations between initial wealth, human capital, and learning ability, $\rho_{AH}, \rho_{AL}, \rho_{HL}$. To identify these parameters, as well as average learning ability μ_L , I estimate two Mincer equations on panel data and stratify individuals by their initial wealth in the PSID and AFQT scores in the NLSY79 (a proxy for learning ability). In both cases, I use specification [Equation 5.15](#) on individuals ages 25 to 54.

$$\ln(Y_i) = \beta_7 + \sum_{q=2}^5 \mathbb{1}_q \beta_8^q + \beta_9 \text{Age}_i + \sum_{q=2}^5 \beta_{10}^q \times \mathbb{1}_q \text{Age}_i + X_{4,i} \delta_4 + \epsilon_i \quad (5.15)$$

where q refers to either the quintile of initial wealth or AFQT scores. I add $N(0, 0.15)$ measurement error in my model-generated analog to avoid singularity.

Relating wealth to the slope and intercept of earnings profiles allows me to discipline the correlations between wealth and human capital as well as learning ability (ρ_{AH} and ρ_{AL}). This, in conjunction with the liquidity effects estimated from the re-employment elasticities, disciplines the correlation between initial wealth and initial human capital. Intuitively, liquidity effects serve to depress initial earnings for low-wealth individuals, meaning that the variance in human capital is likely to be lower than previously estimated. The slope for individuals from different liquid wealth quintiles allow me to discipline the correlation between initial wealth and learning ability.

Using the same Mincer equation stratified by AFQT scores, I discipline the correlation between initial human capital and learning ability, ρ_{HL} , as well as the average learning ability in my sample, μ_L . The NLSY79 records Armed Force Qualifying Test (AFQT) scores, a standardized test that is often used as a proxy for ability. Here, it serves as a proxy for learning ability. I classify individuals into quintiles by their percentile scores using the national distribution and assess the average growth rate of their earnings. I discuss my sample restrictions and data construction in [Section A.3](#). Because learning ability acts as the dominant factor characterizing the slope for life-cycle profiles between ages 25 and 54, this yields inference on the correlation between human capital and learning ability, σ_{HL} as well as μ_L .

5.2.4 Implementation

Indirect inference can be implemented as either maximum likelihood, by minimizing a Gaussian objective function, or generalized method of moments. Because I use multiple datasets, the generalized method of moments approach is a more natural fit. This makes my estimation analogous to a seemingly unrelated regression (SUR) estimation. Indirect inference proceeds by first specifying an auxiliary model, (here, the specifications in: [Section 5.2.1](#), [Section 5.2.2](#), [Section 5.2.3](#)), and minimizing the distance between auxiliary parameters from the data and model simulations. Let T denote the number of observations, who need not be observed for every moment included in the auxiliary model. I largely follow the notation from [DeJong and Dave \(2011\)](#) in the following explanation of the procedure. I estimate the following:

$$\beta(Z) = \arg \max_{\delta} \Delta(Z, \delta) \quad (5.16)$$

$$\beta(Y, \theta) = \arg \max_{\delta} \Delta(Y, \delta) \quad (5.17)$$

where $Z = [z_1, \dots, z_M]$ and $Y = [y_1, \dots, y_M]$ are observed data and model generated data for observations $1, \dots, M$, respectively. Δ are specifications ([Equation 5.10](#) - [Equation 5.15](#)) characterizing the auxiliary model, θ the structural parameters of the model, and β the auxiliary parameters estimated from the auxiliary model.

$$\beta_S(Y, \theta) = \frac{1}{S} \sum_{j=1}^S \beta(Y^j, \theta) \quad (5.18)$$

where j is the j^{th} simulation of the model. The goal is to minimize the distance between the model generated auxiliary parameters and their empirical counterparts. I follow [DeJong and Dave \(2011\)](#) and minimize the following objective function:

$$\min_{\theta} \Gamma(\theta) = g(Z, \theta)' \times \Omega \times g(Z, \theta) \quad (5.19)$$

$$g(Z, \theta) = \beta(Z) - \beta_S(Y, \delta) \quad (5.20)$$

where Ω is a positive-definite weighting matrix and $g(Z, \theta)$ the moments constructed from the binding functions. For the weighting matrix, I choose the inverse of the variance of the sample moments $\text{var}(\beta(Z))^{-1}$. Like [Bowlus and Liu \(2013\)](#), I estimate the variance-

covariance matrix using the following:

$$\text{Var}(\hat{\theta}) = (1 + \frac{1}{S})[\frac{\partial g'}{\partial \theta} \Omega^{-1} \frac{\partial g}{\partial \theta}]^{-1} \quad (5.21)$$

where the jacobian matrix, $\frac{\partial g}{\partial \theta}$, is approximated using forward differences. For the model generated data, I average over $S = 100$ simulations for each iteration, and impose identical sample restrictions and attrition rates as in the observed data. I treat simulated data precisely the same as in my empirical analysis: I impose identical sample restrictions (where applicable) in my simulations, and force each sample to contain an identical number of observations as its empirical counterpart. To deal with missing data in the PSID and NLSY, I randomly drop observations at the same frequency as in the data by age. I do this by wealth and AFQT quantiles so that the data generating process from the structural model is as close as possible to that in the data. I simulate separate sets of data for each dataset used in the auxiliary model. I start agents at age 23 with no labor market experience (i.e., unemployed without unemployment insurance) and a random draw from the joint distribution of initial conditions.

5.3 Estimation Results

I use simulated annealing to estimate the model. This allows me to solve for a global minimum distance by sampling from the parameter space and comparing objective function values. With some positive probability, it accepts a new point at which the objective function is higher than previous, and then searches nearby points. This allows the algorithm to test areas of the parameter space that other approaches would have ruled out, giving credibility to the global solution. This solution method is commonly used in search papers that are estimated using indirect inference, like [Lise \(2013\)](#) and [Bowlus and Liu \(2013\)](#). The parameter estimates are reported in [Table 5.3](#). Notably, the standard errors fit tightly around the estimated values, with the exception of leisure utility. Because of the differences in scales, some of the parameters are not directly comparable with [Huggett et al. \(2011\)](#). However, the mean and standard deviation of human capital shocks are both within the 95% confidence intervals of [Huggett et al.](#)'s estimates. The human capital curvature is higher than estimated in previous search papers, but falls just below the bottom of the estimates from [Browning et al. \(1999\)](#), who put the range at $[0.5, 0.99]$, for models without search frictions.

Table 5.3: Estimation Results

Category	Symbol	Model Value		Comment
<i>Model Parameters</i>				
Subsistence Benefits	b_L	0.0108 [0.0075,0.0141]		
Borrowing Constraint	\underline{a}_t			Qtrly Period-0 Value (2011\$): \$2939
On-the-job Search Efficiency	λ_E	0.3775 [0.3765,0.3785]		
Human Capital Curvature	α_H	0.4977 [0.4251,0.5704]		BHH: [0.5, 0.99]
Leisure Utility	ν	0.0065 [-0.0742,0.0873]		
<i>Distributional Parameters</i>				
Dist. of ϵ'	$(\mu_\epsilon, \sigma_\epsilon)$	$\mu_\epsilon = -0.0175$ [-0.0239, -0.0111]	$\sigma_\epsilon = 0.1350$ [0.0975, 0.1724]	HVY: (0.029, 0.111)
Marg. Dist. of a_0	(μ_A, σ_A)	$\mu_A = 0.8458$ [0.6811, 1.0106]	$\sigma_A = 1.7556$ [1.7548, 1.7563]	Mean: \$35 011
Marg. Dist. of h_0	(μ_H, σ_H)	$\mu_H = -0.4278$ [0.1919, 0.1936]	$\sigma_H = 0.1928$ [0.0363, 0.5408]	Mean: \$3655
Marg. Dist. of ℓ	(μ_L, σ_L)	$\mu_L = -3.7251$ [-3.7253, -3.7249]	$\sigma_L = 0.2885$ [0.0363, 0.5408]	
Correlations	$\rho_{AH}, \rho_{AL}, \rho_{HL}$	$\rho_{AH} = 0.4691$ [0.4620, 0.4762]	$\rho_{AL} = 0.5811$ [0.5783, 0.5839]	$\rho_{HL} = 0.4697$ [0.4696, 0.4698] HVY: $\rho_{HL} = 0.655$

Notes: BHH refers to [Browning et al. \(1999\)](#), and HVY refers to [Huggett et al. \(2011\)](#).

5.4 Fit

The fit for each of the auxiliary parameters is presented in [Table B.3](#). There are 74 auxiliary parameters and 15 structural parameters, so the model should not be expected to perfectly fit each of the moments. It does fit a number of the auxiliary parameters well, matching the direction of the SIPP elasticities (though smaller in magnitude), and almost precisely matching the growth rates by initial conditions. Because the simulations are structured to precisely mirror the data generating processes of the auxiliary model specifications, I am able to test to see which simulated auxiliary parameters are statistically different from their empirical counterparts. These are also presented in [Table B.3](#). I show plots comparing the initial distributions of wealth and earnings in [Figure B.1](#). While the initial distributions visually fit well, the model predicts earnings more left skewed than the data and wealth more right skewed than the data. A comparison between the average earnings profile in the PSID and the average earnings profile generated by the model (a non-targeted moment) is shown in [Figure 6.6a](#).

5.5 Identification

I show that the auxiliary model draws inference from the intended mechanisms. The left panel in [Figure 5.1](#) shows that application strategies exhibit strong wealth effects as individuals approach the borrowing constraint. Additionally, individuals who enter unemployment with identical wages, but different UI replacement rates behave as predicted by [Equation 5.10](#). The dashed blue line and solid red line are two sets of identically productive workers, with the same previous wages. However, the dashed blue workers receive a replacement rate on their previous wage of (54%), while the red line workers receive a replacement rate of (32%). The dashed brown line represents workers who are nearly at the UI cap, and solid yellow workers who are at the UI cap. The right panel shows that low wealth workers are the only workers affected by changes in UI replacement rates.

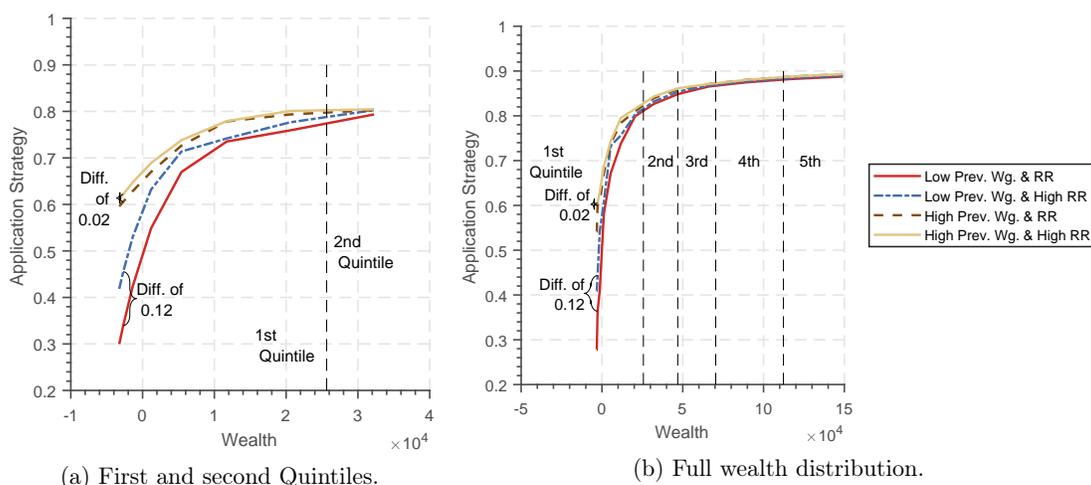


Figure 5.1: Application Strategy by UI and Wealth, Age 25.

[Figure 5.2](#) shows the human capital mechanism identified by the auxiliary model. The left panel shows that learning ability is associated with large differences in human capital growth, while the right panel shows that wealth is only weakly associated with human capital growth by age 25, consistent with the specifications in [Section 5.2.3](#). In the data, AFQT differences are associated with large growth rate differences, while wealth is weakly associated with growth rates after age 25, indicating that the auxiliary model identifies human capital growth correctly. The bottom panel shows that as workers age, they spend less time accumulating human capital, which suggests that using growth rates by age bins correctly identifies characteristics of human capital growth and depreciation ([Equation 5.12](#)).

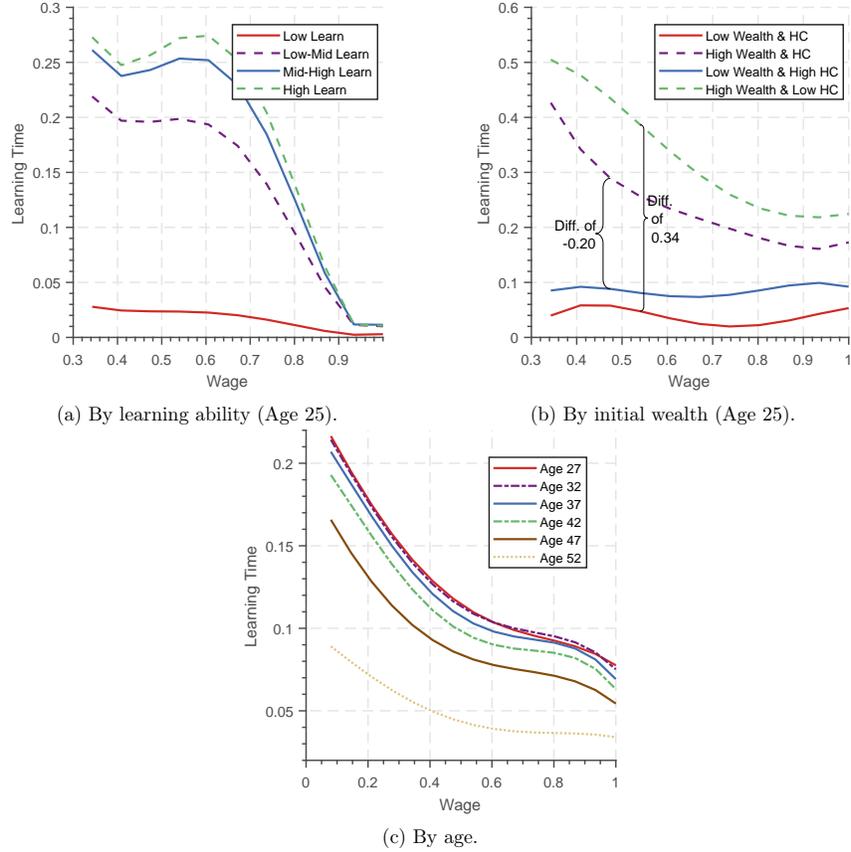


Figure 5.2: Human capital decision rules.

6 Findings

I now use the estimated model to address the central question posed in this paper: how do borrowing constraints interact with search frictions and human capital to alter life-cycle inequality? I start by exploring the key mechanisms in the model: worker application strategies and time allocations. I do this in [Section 6.1](#). Then I explore the contribution of each of these mechanisms contribute to earnings growth and dispersion over the life-cycle in [Section 6.2](#) through model simulations.

In [Section 6.3](#), I quantify how changes in initial wealth, human capital, and learning ability impact inequality. I do this two ways: first, in [Section 6.3.1](#) I compare the baseline simulation of the model from [Section 6.2](#) to simulations in which one of the initial conditions is either increased or decreased in isolation by a standard deviation. I extend this test by comparing these outcomes to simulations in which two of the initial conditions are altered by one standard deviation, in either the same or opposite directions. The interaction shows how the impact of changes in one initial condition can be magnified or mollified by changes

in the others. Then, in [Section 6.3.2](#), I explore outcomes for individuals from bottom quintile of each marginal distribution. With this as the baseline, I give each a “helicopter drop” of wealth, human capital, or learning ability, and compare outcomes.

6.1 Decision Rules

6.1.1 Savings

Age 25 agents in the model increase their savings approximately linearly as their wealth increases. [Figure 6.1](#) shows the contour sets from savings policy functions for agents who are employed or unemployed with UI. The top left figure shows savings rates for high learning ability employed individuals ([Figure 6.1a](#)). The top right panel, [Figure 6.1b](#), shows the savings decisions of unemployed individuals with different replacement rates, zoomed in on low-wealth agents. Like [Figure 5.1a](#), this shows that borrowing constraints can cause wealth effects for poor enough agents.

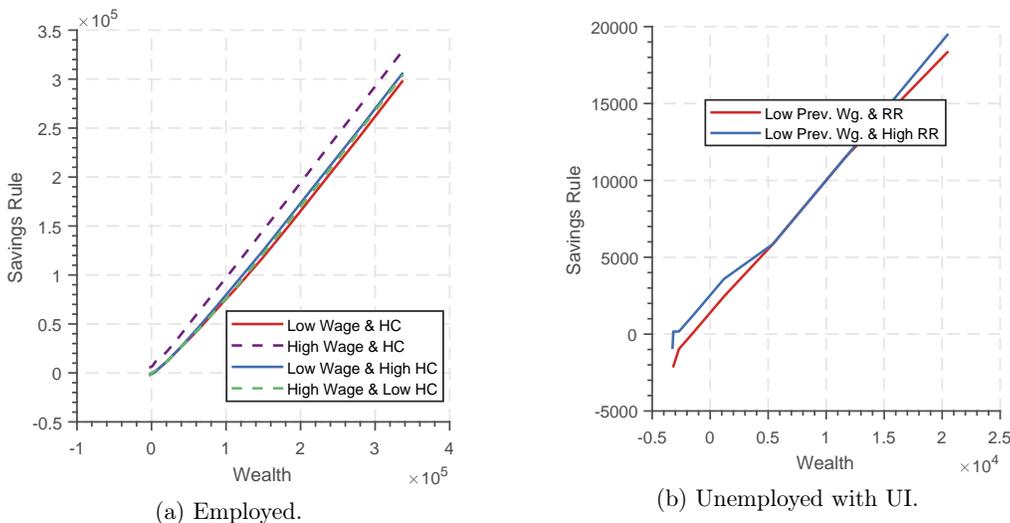


Figure 6.1: Savings rules by employment status.

6.1.2 Labor Market

In the labor market, low wage and low wealth agents respond in predictable ways. [Figure 6.2](#) shows contour plots for agents application rules. The left panel depicts the application strategy in different wealth and human capital states for workers employed at different piece-rate. There is a strong wealth effect at low wages: for the same level of human capital, low wealth agents apply for much more readily available jobs. This is seen by comparing the solid

red and dashed green lines. The right panel shows the application strategy of unemployed agents without UI, for both low and high learning agents.

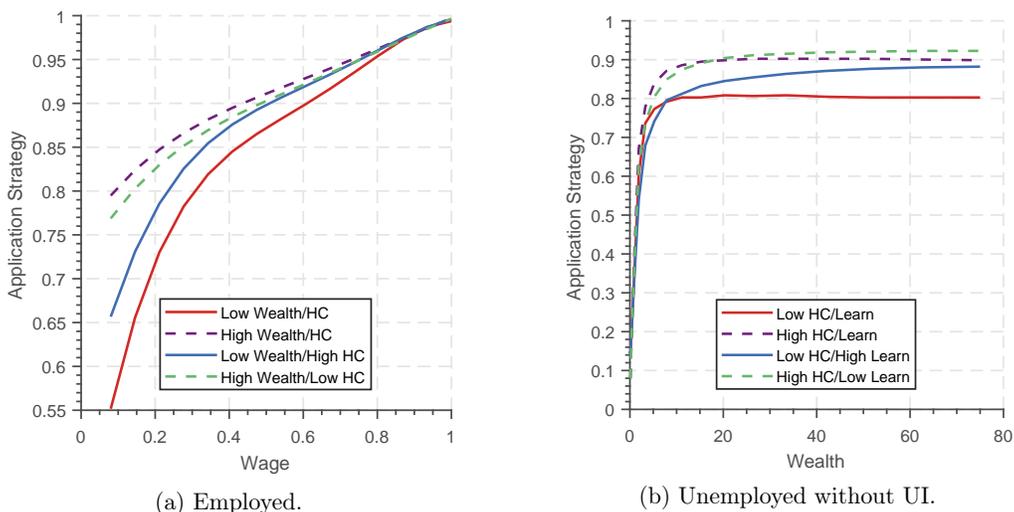


Figure 6.2: Application strategies by employment status for high ability agents.

In equilibrium, submarket tightness is decreasing across the wage distribution. Figure 6.3 shows that each agent type faces a decreasing probability of finding a job as they apply for higher paid positions for both low learning and high learning agents (left and right panel).

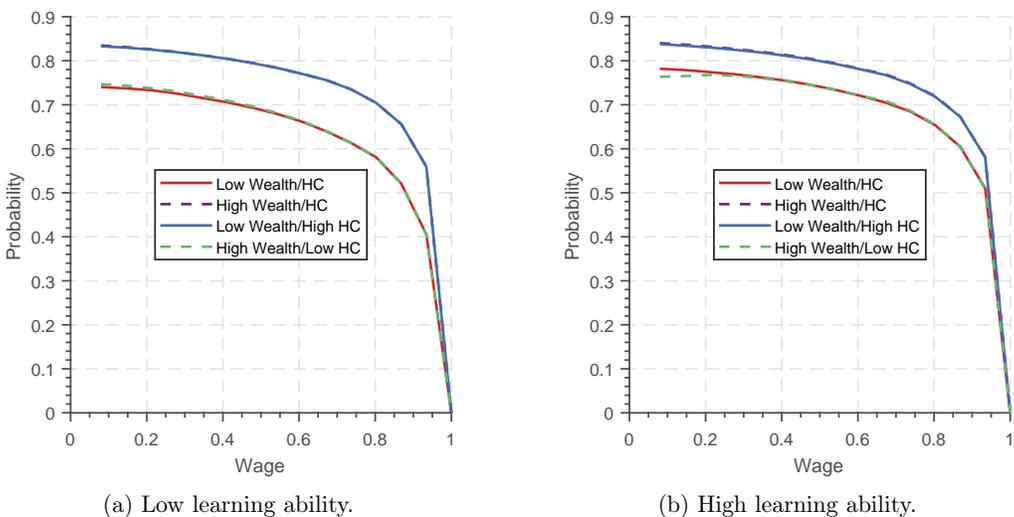


Figure 6.3: Employment probability by wage and learning ability.

6.1.3 Human Capital

The next figure (Figure 6.4) shows contour sets for time allocation decisions for different ages and state vectors. The top two panels show time allocation decisions for low and

high learning ability agents at age 25. These show clear evidence of the effect that wealth has on time allocation decisions: the dashed green line shows that a low human capital agent spends in excess of half his time accumulating human capital when he is wealthy, while an equally productive but poor agent spends less than 10% of his time accumulating human capital (the solid red line). Likewise, a low wealth, but highly productive agent (the blue line) spends virtually none of his time accumulating human capital, while a wealthy and productive agent (the dashed purple line) spends a larger fraction of his time, unless employed in very lucrative careers. Note that while there are some high wealth (green line), there are relatively few by the end of the life-cycle, contributing to the overall decline in accumulation time (Figure B.1b).

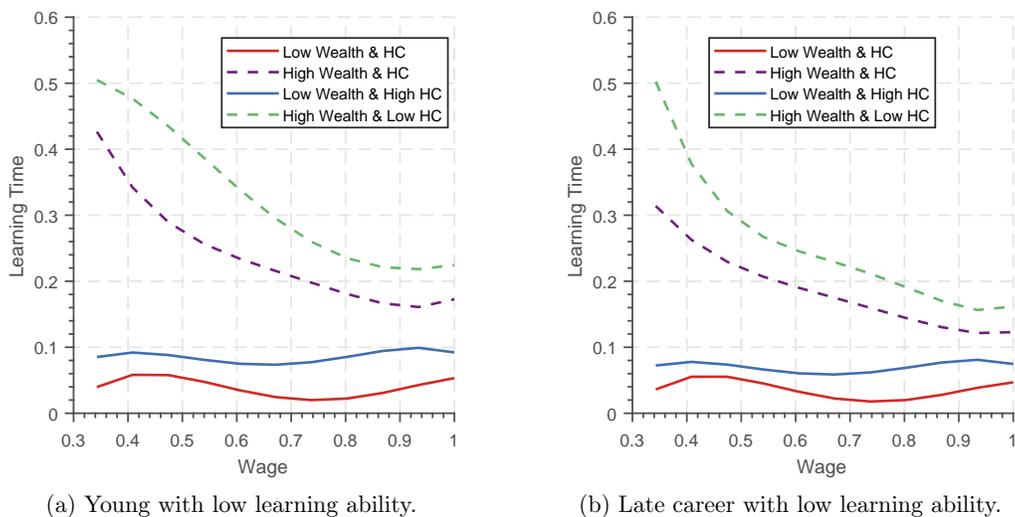


Figure 6.4: Time Accumulating Human Capital.

Some of the difference in human capital accumulation is due to a consumption smoothing motive, rather than a precautionary motive against unemployment shocks. A consumption smoothing motive would be present in a Huggett (1993), or Aiyagari (1994) heterogeneous agent model; a precautionary motive against unemployment shocks would not. In Figure 6.5, I vary unemployment risk of an age 30 worker by changing δ to $\delta = 0.01$ and $\delta = 0.05$. The figure shows that when unemployment risk decreases, the gap in human capital accumulation between a low wealth and medium wealth agent declines for the entire wage distribution. This shows that unemployment risk has an important role in human capital accumulation. The reason is that for unemployed workers, wealth has a first-order effect on application strategies (Figure 5.1) as a worker approaches the borrowing limit. To minimize the cost of a possible unemployment spell, low-wealth workers substitute intertemporally by decreasing

human capital accumulation.

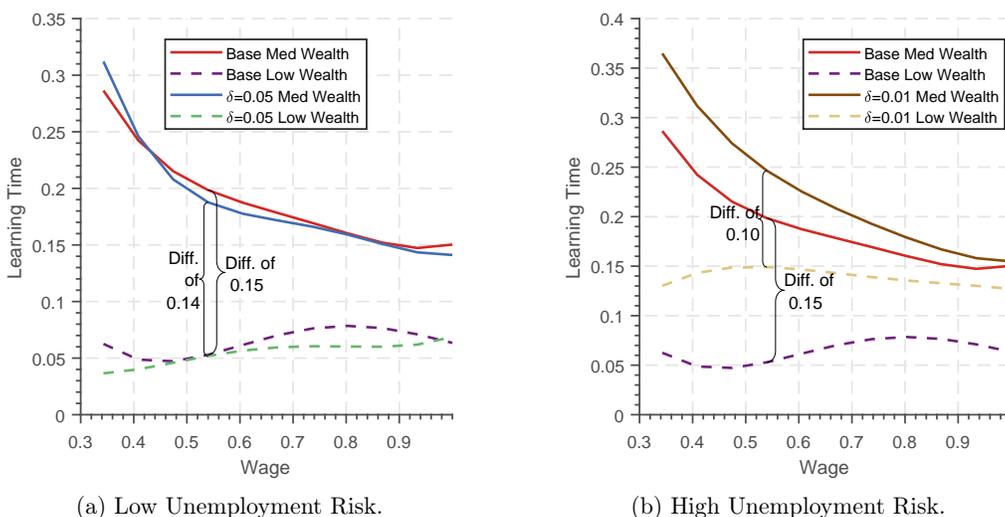


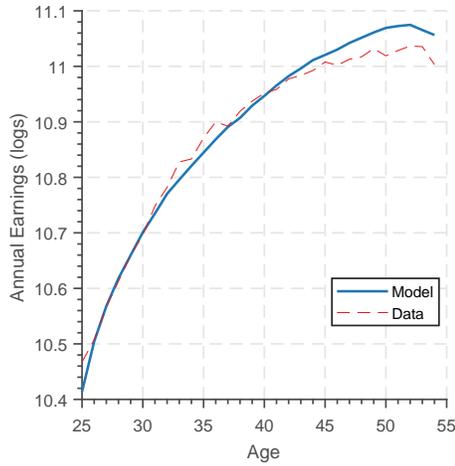
Figure 6.5: Human Capital Responses to Unemployment Risk.

6.2 Sources of Life-Cycle Earnings Growth

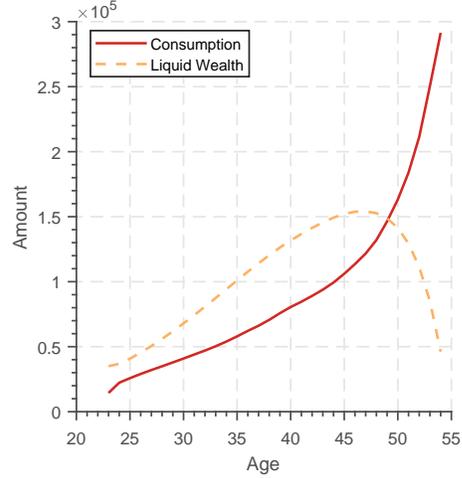
Agents in the economy experience substantial earnings growth during the first ten years of their working career, before remaining relatively flat until retirement (Figure 6.6a). Consistent with previous work on inequality, earnings profiles begin to decline as agents approach retirement. Because the model has no intensive margin, this could be due to either hours or wages, consistent with Rupert and Zanella (2015) who note that much of the decline in earnings in the PSID is due to hours. Consumption and wealth profiles roughly follow the same pattern, though agents decumulate and consume their savings rapidly at the end of the life-cycle (Figure 6.6b).

Over the life-cycle, the model predicts that growth comes from two sources in two distinct time periods. Agents initially move to jobs with higher piece-rates to increase their earnings, Figure 6.7a, and then devote substantial time during the middle of their careers to accumulating human capital, Figure B.1b. By the middle of their careers, they spend just enough time learning to maintain their human capital stocks.

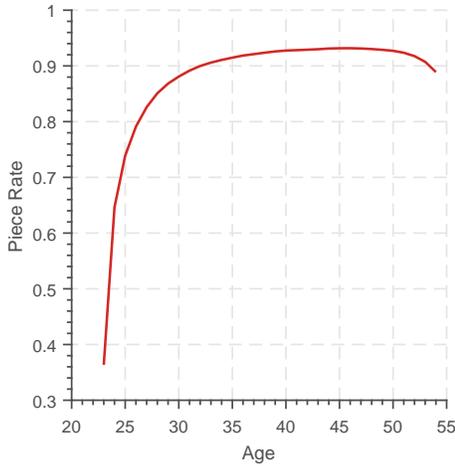
Agents respond in substantively different ways based on their employment status. Figure 6.7a shows that unemployed agents apply for low-paying jobs (relative to those employed), while their peers without unemployment insurance apply for even lower paying jobs. It would be easy to conclude that this is the result of differences in human capital, and indeed Figure 6.8a shows substantial differences in human capital among each of these groups. But,



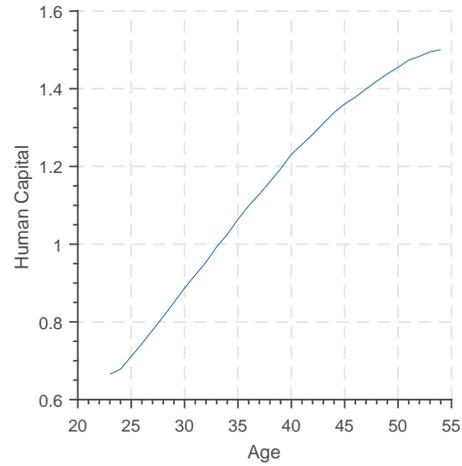
(a) Income.



(b) Consumption and Wealth.



(c) Piece-rate.



(d) Human Capital.

Figure 6.6: Profiles of key state and outcomes variables.

unemployed agents without UI simultaneously apply for jobs that offer the highest likelihood of employment, despite the lower human capital. (Figure 6.8b). This is a direct result of borrowing constraints. Unemployed agents with no UI also have disproportionately lower wealth than their peers. Rather than face additional consumption risk, they take low-paying jobs that offer high probabilities of employment.

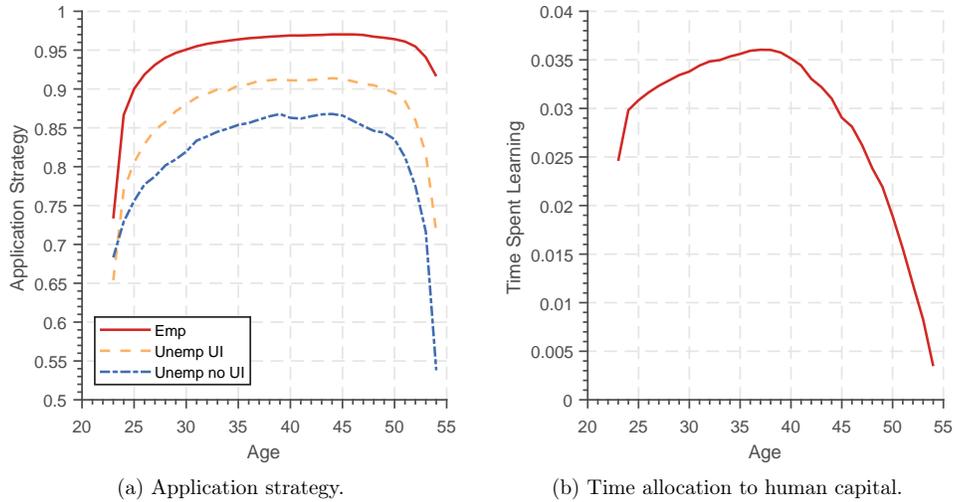


Figure 6.7: Decision rules by employment status.

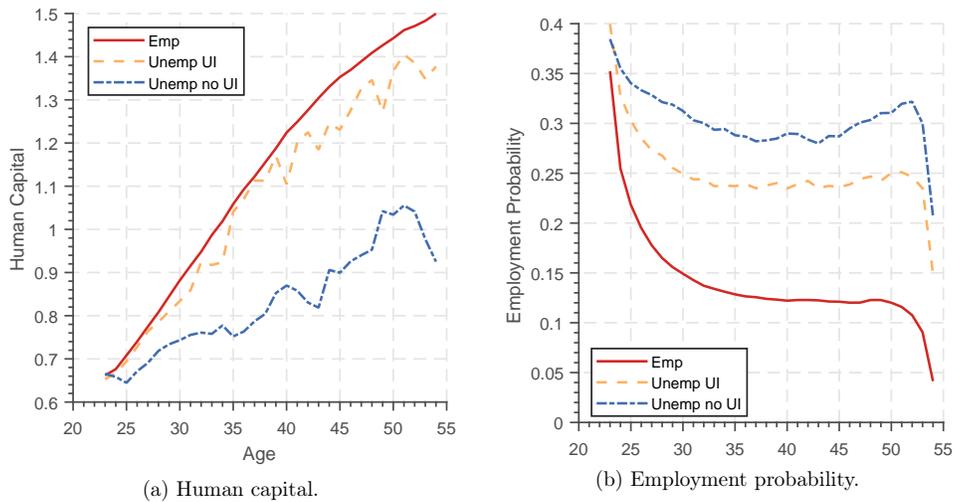


Figure 6.8: Human capital and job-finding rates by employment status.

6.3 Initial Conditions and Life-Cycle Inequality

I run two tests to assess the effects of initial conditions on life-cycle inequality. Test 1 compares outcomes of agents in the baseline simulation to agents who receive identical shocks, but whose initial conditions are changed by one standard deviation. After considering these results, I consider the outcomes of individuals for whom two of the initial conditions change simultaneously. Test 2 considers the impact of altering a 10th percentile agent's initial conditions. In test 2, each agent starts at the 10th percentile of the tested initial condition (with a correlated draw from the other two), and is compared with an individual at the median of the tested initial condition (the other two unchanged). The baseline as well as the counterfactuals for the tests are detailed in [Table 6.1](#).

Table 6.1: Initial Conditions

Variable	Test 1 Value	Test 2		
		10th Wealth	10th Human Capital	10th Learning
<i>Wealth</i>				
Baseline	35 010.8	-2127.9	8750.2	3989.6
Median		7546.5		
+1 St. Dev.	116 503.4			
-1 St. Dev.	5062.5			
<i>Human Capital</i>				
Baseline	3014.0	2671.9	2313.0	2657.1
Median			2962.2	
+1 St. Dev.	3654.8			
-1 St. Dev.	2485.9			
<i>Learning Ability</i>				
Baseline	0.025	0.020	0.021	0.017
Median				0.024
+1 St. Dev.	0.034			
-1 St. Dev.	0.019			

Notes: The table presents the initial conditions associated with the baseline as well as the comparison group for tests 1 and 2.

6.3.1 Test 1: Average Worker

For the average household a standard deviation change in any of the initial conditions is important. I find that a standard deviation decrease in initial wealth plays a larger role in determining both lifetime consumption and lifetime wealth than a standard deviation change in human capital, shown in [Table 6.2](#). The reason is that here wealth and human capital have similar effects: both human capital and wealth improve initial placement and lead to faster human capital growth. The relatively tight distribution of human capital needed to match the data makes a change in human capital relatively unimportant compared with changes in wealth. I also find that learning ability is the primary driver of inequality: a decrease in learning ability leads to little or no human capital growth throughout the life-cycle. This is because earnings growth is driven by both human capital accumulation and search frictions, making the average learning ability in the economy lower than in previous papers.¹³

Wealth plays a role through two channels: first, agents who start poor are rushed to

¹³In [Huggett et al. \(2011\)](#), they find that average learning ability is 0.321. They need such a large value to match earnings profiles, but here earnings growth is driven by search frictions. A model without search frictions would not be consistent with the empirical regularities from the SIPP.

Table 6.2: Test 1 Results

Change	Δ Consumption		Δ Earnings (%)	Δh (%)	$\Delta \tau$ (%)	$\Delta \mu'$ (%)
	(%)	HVY (%)				
<i>Wealth</i>						
+1 St. Dev.	+23.9	+7.1	+2.7	+1.8	+31.1	+1.0
-1 St. Dev.	-10.5	-1.6	-2.3	-1.1	-13.8	-1.1
<i>Human Capital</i>						
+1 St. Dev.	+9.7	+39.3	+10.0	+9.5	+0.8	+0.3
-1 St. Dev.	-7.6	-28.3	-7.8	-7.5	-0.8	-0.3
<i>Learning Ability</i>						
+1 St. Dev.	+24.7	+5.7	+29.7	+29.1	+32.3	+0.5
-1 St. Dev.	-15.0	-2.6	-17.9	-17.5	-27.8	-0.3

Notes: The table presents the change in lifetime utility (equivalent variation) for a one standard deviation change in each of the listed variables. When a variable is changed, the other variables are left unchanged.

find a job, consistent with the regularities that I found in the SIPP (Section 3). Then, they accumulate less human capital while employed. Figure 6.9b shows this effect for the average individual in the economy. An increase in wealth plays a small role early in the life-cycle, but has little dynamic effect as the average individual in the economy is not constrained. However, moving closer to the borrowing constraint has a tangible and dynamic effect on earnings. Until late in the life-cycle, these individuals have lower earnings than their wealthier counterparts. Human capital has a relatively small effect in both directions, and learning ability plays a big role in both directions (Figure 6.10b and Figure 6.11b respectively). While the initial wealth effect is important, wealth continues to have an effect by dynamically altering the accumulation of human capital. A single standard deviation change in wealth alters the accumulation of human capital drastically over the first 20 quarters of work, as shown in Figure 6.9b. A change in human capital does little to alter acquisition over the lifetime, as shown in Figure 6.10b. The interaction between wealth and human capital accumulation is substantial, causing a continued difference even late into the life-cycle. I consider the role of interactions in Table B.2. Among the notable results are that decreasing wealth and increasing learning ability, which could be thought of as roughly attending college, increase earnings substantially, but by less than a simple increase in learning ability (a 3% difference). Increasing human capital and learning simultaneously has a larger effect on earnings than the two changes added together.

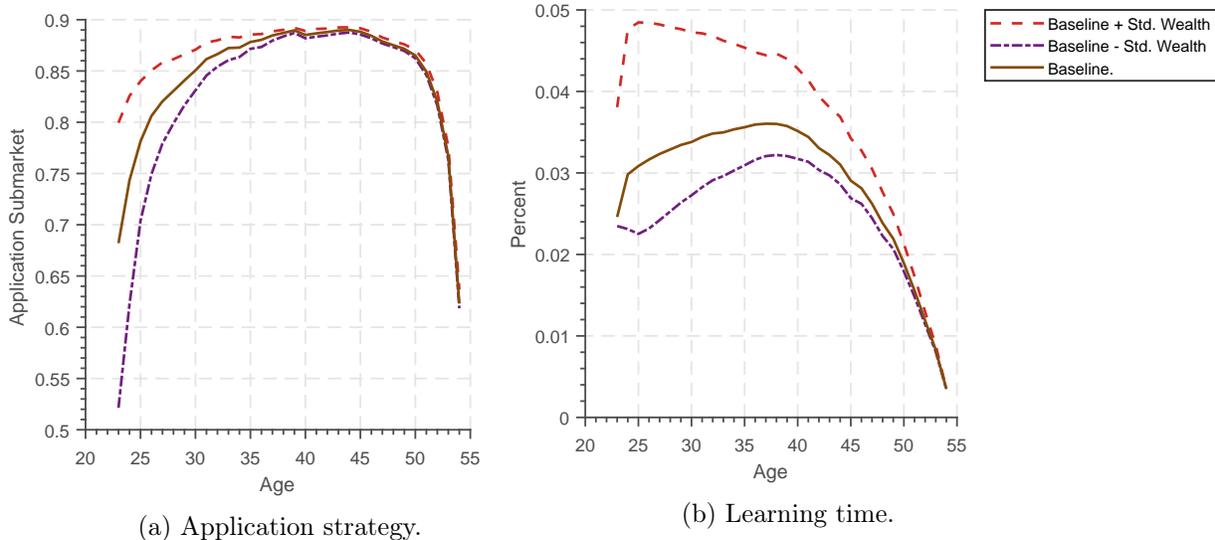


Figure 6.9: Decision rules after wealth change (Test 1).

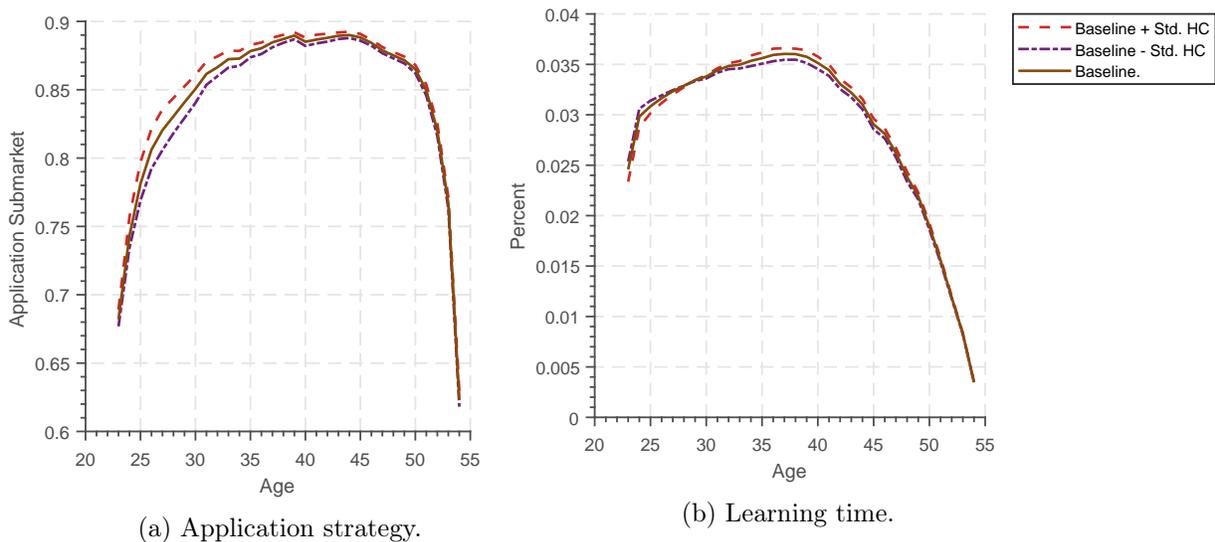


Figure 6.10: Decision rules after human capital change (Test 1).

6.3.2 Test 2: 10th Percentile Worker

I now assess the impact of changing initial conditions for individuals at the bottom of the distribution. I test how the outcomes of an individual from the 10th percentile of one initial condition and the corresponding values of the other two initial conditions changes when they shift from the 10th percentile to the median for that initial condition (leaving the other two unchanged). The initial conditions as well as their counterfactuals are summarized in [Table 6.1](#). The outcomes are summarized in [Table 6.3](#). [Table 6.3](#) indicates that wealth inequality is an important driver of earnings inequality among poor households. While the

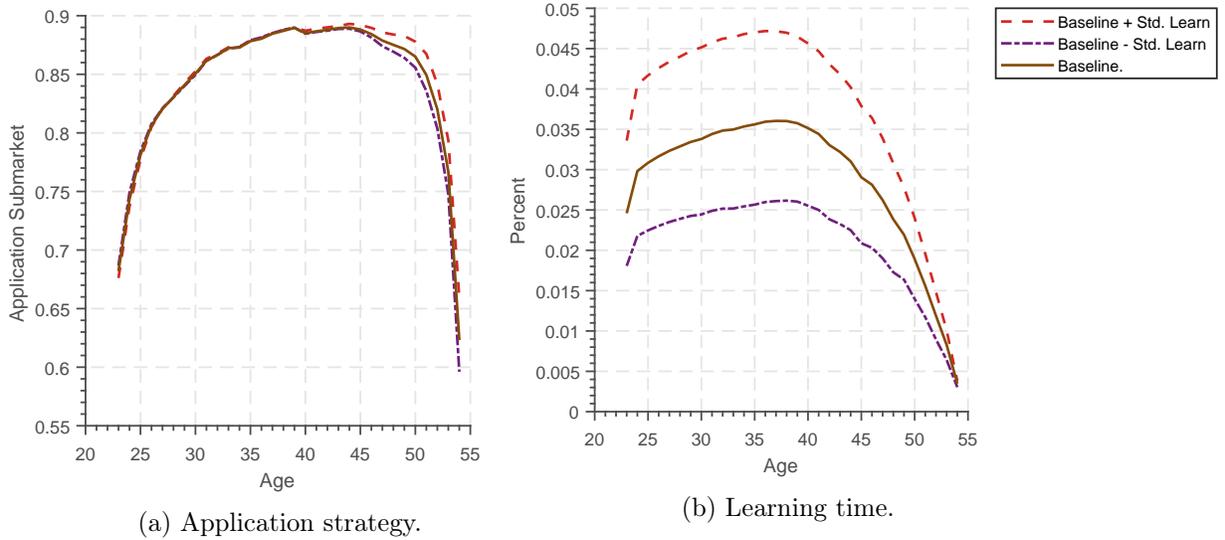


Figure 6.11: Decision rules after learning change (Test 1).

Table 6.3: Test 2 Results

Counterfactual	Percent Change				
	Δ Consumption	Δ Earnings	Δh	$\Delta \tau$	$\Delta \mu'$
<i>Wealth</i> : 10th \rightarrow 50th	+6.9	+3.5	+0.6	+4.7	+2.1
<i>Human Capital</i> : 10th \rightarrow 50th	+12.7	+11.8	+11.2	+4.1	+0.4
<i>Learning</i> : 10th \rightarrow 50th	+25.6	+27.4	+26.5	+59.7	+0.5

Notes: The rows represent comparisons between an individual from the 10th percentile of each initial condition (with correlated draws from the other two initial conditions), with an individual at the median of the tested initial condition.

impact of an increase in human capital is large, wealth increases lifetime earnings by 3.5%. For low wealth households, the human capital channel is less active, increasing by 0.6% over the life-cycle, compared with the estimates for the average household in [Section 6.3.1](#) of 1.8% for a standard deviation increase. Human capital is more important in determining earnings, though again much of this is from a direct change in their productivity. As before, an increase in learning ability plays the largest role in both consumption and earnings inequality (25.6% and 27.4%).

[Figure 6.12](#), [Figure 6.13](#) and [Figure 6.14](#) explore the mechanisms through which outcomes change. [Figure 6.12b](#) and [Figure 6.14b](#) shows that increases in wealth and learning ability cause large increases in time devoted to human capital accumulation, while changes in human capital ([Figure 6.13b](#)) plays little role on human capital accumulation over the life-cycle.

This shows that the effect of a change in human capital is largely direct: productivity

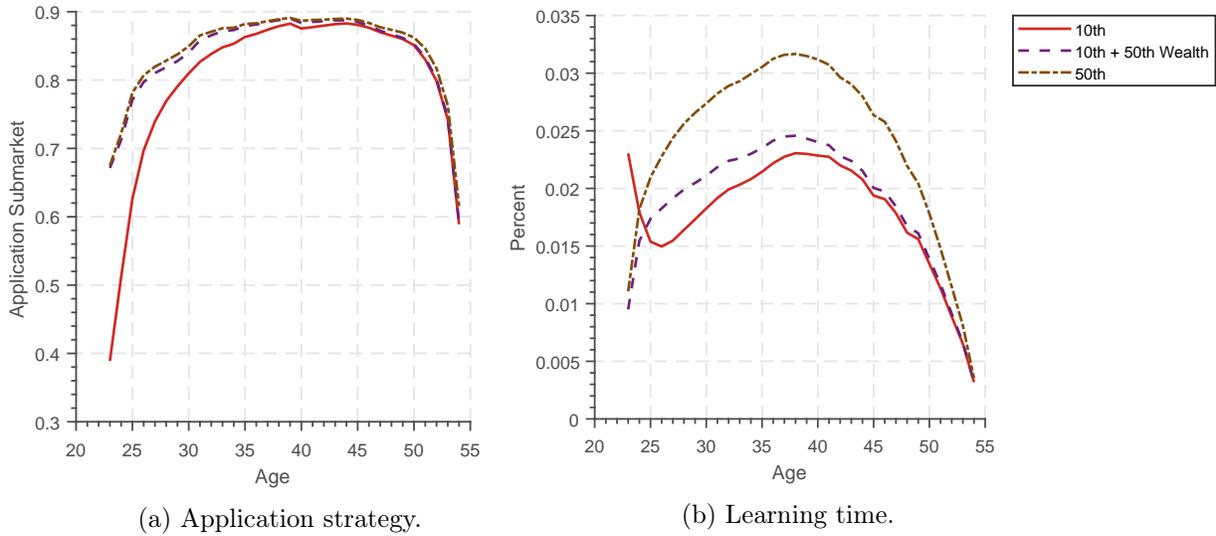


Figure 6.12: Changes in response to an increase in wealth.

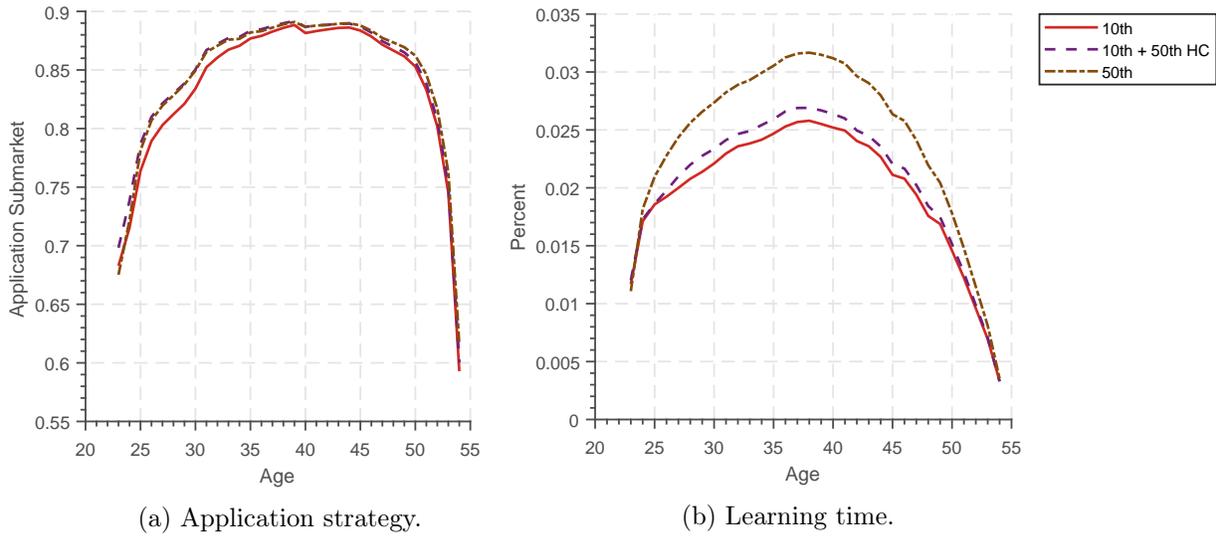


Figure 6.13: Changes in response to an increase in human capital.

increases directly translate into higher wages, but do not substantially alter decision rules. On the other hand, wealth and learning do alter decision rules. This is shown more clearly in [Figure 6.12a](#), [Figure 6.13a](#), and [Figure 6.14a](#), each of which shows the change in application strategy for unemployed individuals. For these agents, wealth plays a substantial role in determining the jobs to which a household applies. A 10th percentile household initially applies for a job that offers a piece-rate of around 40%. After a change in their initial wealth, they find jobs that offer nearly a 70% piece-rate. At the same time, they accumulate more human capital, leading to higher income and more productivity overall. These tests

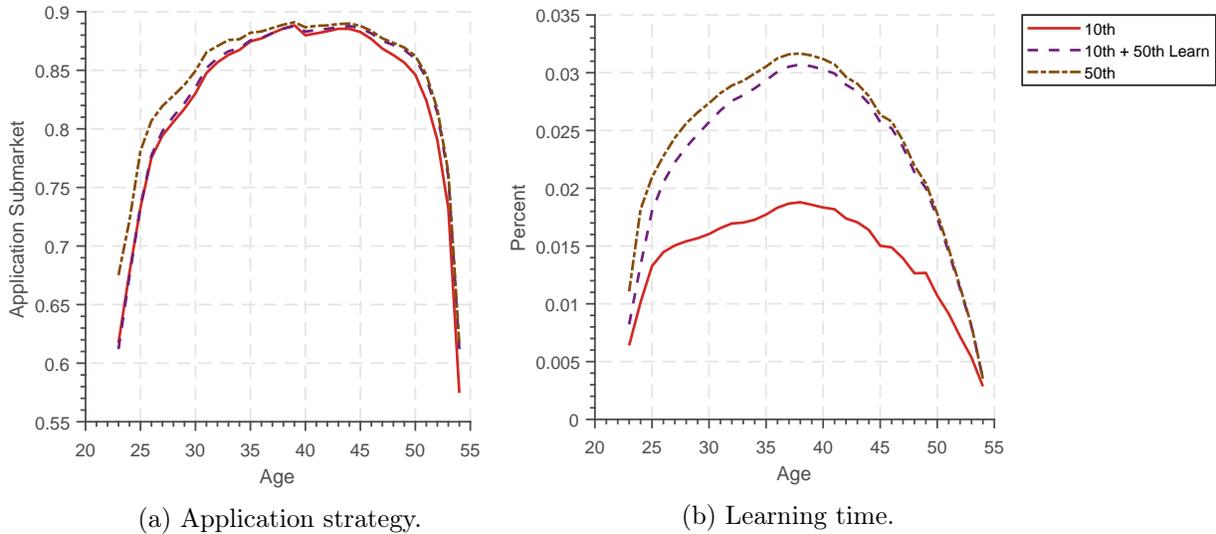


Figure 6.14: Changes in response to an increase in learning ability.

are strongly indicative of the importance of wealth in determining inequality, suggesting that decreasing the debt of a 10th percentile household may be enough to increase their productivity and decrease inequality in earnings or consumption.

7 Conclusion

In this paper, I develop a quantitative model of labor market search to study inequality. The model considers risk averse workers who borrowing constraints and frictional labor markets, and accumulate human capital using [Ben-Porath](#) production. I estimate this model and use it to quantify the impact that wealth inequality has on earnings and consumption. I find that borrowing constraints cause low wealth workers to accept lower-paying jobs, and accumulate human capital at a slower pace than their wealthier peers. Both effects are important. The model predicts that a standard deviation decrease in wealth decreases consumption and earnings growth by more than a standard deviation decrease in human capital. Among poor workers, I find that increasing wealth can lead to large increases in earnings.

Using the SIPP, I show that borrowing constraints affect labor market outcomes following an unemployment spell. Constrained workers in the SIPP match to higher paying jobs when given more generous unemployment insurance replacement rates. I also find evidence that this effect persists. These results help to discipline borrowing constraints when I estimate the model.

I use indirect inference to estimate the model. To do this, I pick reduced-form models that

identify key aspects of my structural model in the data. I target re-employment elasticities from the SIPP to gain inference on borrowing constraints, as well as life-cycle moments from the NLSY and PSID to identify the effects of wealth and human capital on growth, as well as their correlations. By matching these moments and treating the data in the same way, the model is asked to match the data generating process of the relevant mechanisms in the data. Despite substantially more moments than estimated parameters, the model fits the reduced-form moments well, indicating that the model can explain the key mechanisms in the data.

Quantitatively, I find that initial wealth has a larger effect on consumption inequality and earnings growth than initial human capital. A standard deviation decrease in initial wealth causes a -10.5% change in lifetime consumption, while a standard deviation decrease in human capital causes a change of -7.6% . Wealth has an important effect on earnings through both a worker's initial placement (-2.2%), as well as his human capital accumulation (-1.1%). For a worker at the 10th percentile of the initial wealth distribution, an increase to the median causes an increase in earnings of 3.5% .

My findings suggest that the importance of wealth inequality has previously been understated. I show that when workers face borrowing constraints and frictional labor markets, wealth occupies a similar role as human capital, and can alter productivity over the life-cycle. In terms of productivity growth, my counterfactual exercises suggest that increases in wealth lead to more productivity growth than increases in human capital. Policy and aggregate considerations are left out of this paper, but these findings suggest that policies aimed at helping constrained workers, both employed and unemployed, could decrease inequality. Furthermore, the effect of wealth on productivity indicates that they may increase aggregate productivity in the economy. Both questions of worthy of further research.

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A Data Construction

A.1 Survey of Income and Program Participation (SIPP)

I use the SIPP to assess the effect that liquidity has on labor market outcomes. The SIPP is a panel dataset with separate surveys conducted annually from 1984 to 1993, and then during 1996, 2001, 2004, and 2008. Each survey follows a household for 16 to 36 months, with interviews every four months for each “wave” of respondents. Each interview includes detailed information on the employment, income, and unemployment insurance reciprocity. Employment variables are coded down to a weekly frequency, which yields an extremely precise picture of a worker’s unemployment spells for the duration of the panel. In addition, each wave includes detailed information on special topics in “topical modules.” Although information on wealth is not available in the core questionnaire, it is included in some of the topical modules, averaging twice per panel.

My selection criteria is similar to the previous literature on the liquidity effects of unemployment insurance¹⁴. I first pool SIPP panels from 1990 to 2008. From these panels, I restrict my sample to unemployment spells for males age 23 and older with at least 3 months work experience, who took up UI within one month of job loss, and who are not on a temporary layoff¹⁵. For each individual, I observe race, marital status, age, years of education, as well as tenure, industry, occupation, and wage at their previous job. Demographic characteristics are shown in ???. This allows me to link 2,311 unemployment spells to a variety of measures of their wealth upon entering an unemployment spell. The selection of individuals who experience unemployment spells but do not report wealth is random, because questions on wealth are only asked during some waves of the panel.

The SIPP employs a stratified sample design whose primary sampling units changed in 1992, 1996, and 2004. I make use of this complex survey structure to obtain accurate estimates of subsample variance, while accounting for design change by specifying the primary sampling units during each design regime (1990-1991,1992-1993,etc.) with a unique identifier. That is, an individual from the first PSU in 1990 would not be assigned to the same variance strata as an individual from the first PSU in 2001. I weight all of my results using person weights for individuals at the start of their unemployment spells.

A.2 Panel Study of Income Dynamics (PSID)

The PSID is a panel that follows a group of households from the United States that ran yearly from 1968 to 1997, and in alternating years through the present. Because the PSID spans nearly 50 years, it has been frequently employed for researchers interested in exploring life-cycle effects within the United States ([Storesletten et al. \(2004\)](#) and [Rupert and Zanella \(2015\)](#), among others), as well as researchers interested in inequality ([Huggett et al. \(2011\)](#), [Güvenen \(2009\)](#), among others). In addition to this, the PSID began recording information on household wealth holdings in their “wealth supplements,” in 1984 repeated these questions

¹⁴See [Chetty \(2008\)](#) and [Meyer \(1990\)](#) for two examples using the same selection criteria.

¹⁵Selecting on an endogenous variable, like unemployment insurance, may lead to biased estimates ([Anderson and Meyer, 1997](#)). I discuss this in [Section 3.1.1](#)

in 1989, 1994, and 1999, and then in each subsequent interview. In the United States, this is the only publicly available dataset that contains multiple cohorts, long-term observations on earnings, and measures of household wealth at ages close to or before labor market entry¹⁶. In addition to these variables, the PSID includes rich observations on demographics, labor market experience, as well as family history and behavioral characteristics.

I employ sample restrictions similar to [Huggett et al. \(2011\)](#). First, I require that each individual be head of their household, male, and between the ages of 25 and 54. For constructing the distribution of wealth and earnings at first employment (moments 1 and 4), I require that the individual either be observed before entering employment, or that they report they entered employment during the previous year and the job is their first. I also require that these individuals be no younger than 23 and no older than 27. Over the life-cycle, I require that the individuals in my sample be strongly attached to the labor market: any individual in my sample must work at least 520 hours during the year and earn at least \$9,500 in 2011 dollars if they are 31 or older. If they are younger than 30, I lower this requirement to \$4,750, and 260 hours, to capture individuals who might choose part-time employment in order to have a steady income stream. I use the same sample restrictions when constructing profiles by initial liquid wealth quantile.

A.3 National Longitudinal Survey of Youth, 1979 (NLSY79)

The National Longitudinal Survey of Youth follows cohorts who were ages 14-22 in 1979 through the present. It was conducted annually from 1979-1994 and bi-annually from 1994 until now, and includes detailed information on labor market status, including current employer, weeks employed, unemployed, and out of the labor force, as well as any training received by the individual since the last interview. Earnings are recorded annually as well as hours worked. In addition, the NLSY recorded a standardized test score, the Armed Forces Qualification Test (AFQT) for every individual in the sample. This allows me to link individuals by their AFQT scores to their outcomes late in the life-cycle. In 1985, the NLSY began recording information on the wealth of individuals. Unfortunately, a large fraction of the sample had already become employed, making its usage challenging in my analysis. I use identical sample restrictions as [Section A.2](#).

A.3.1 Wealth Quantile Construction

I use net liquid wealth as a measure of liquidity in the PSID. I define this to be any liquid assets, including checking, savings, stocks, bonds, etc. net of any unsecured obligations, including credit cards and student debt. I define earnings to be exclusively labor earnings at an annual frequency, and always in 2011 dollars, identical to the definition that I use in my exploration of the SIPP. Unfortunately, prior to 2011, the PSID did not report the specific composition of the debt held by households other than a few aggregated categories.

To assign individuals to initial quintiles in the wealth distribution, I first exclude observations who do not meet the following characteristics: first, agents must be the head of their

¹⁶The NLSY79 contains information on wealth, but for few individuals before labor market entry.

household when I observe their assets; second, they must be age 30 or younger during a year in which I observe their assets; third, they must have no labor market experience, having earned no more than \$9,750 dollars (2011 dollars) or worked more than 520 hours (one standard deviation less than the sample average) during the previous year¹⁷. This subsample faces limitations, as few individuals have both observations on their assets at an age younger than 30 and simultaneously have observations on earnings at later ages. I also scale wealth before entering the labor market by the number of individuals in the household. I pool all individuals for whom I observe assets and adjust for growth over time.

Having run this regression, I assign individuals to quantiles within the distribution based on their observed liquid wealth. I assign individuals to the nearest quintile (in terms of their rank) within the distribution. Because the wealth data contains few observations on earnings for individuals, while simultaneously observing their wealth before age 30, I employ a strategy similar to a synthetic control method. I classify individuals into five quintiles as described above, and then using these generated quintiles, I run an ordered logit to classify individuals for whom I do not have observations on wealth, based on their observables. Qualitatively, this technique generates earnings profiles that exhibit the same correlations in earnings for the ages for which I have wealth observations, but allows me to match my model to earnings at ages greater than 50.

B Tables and Figures

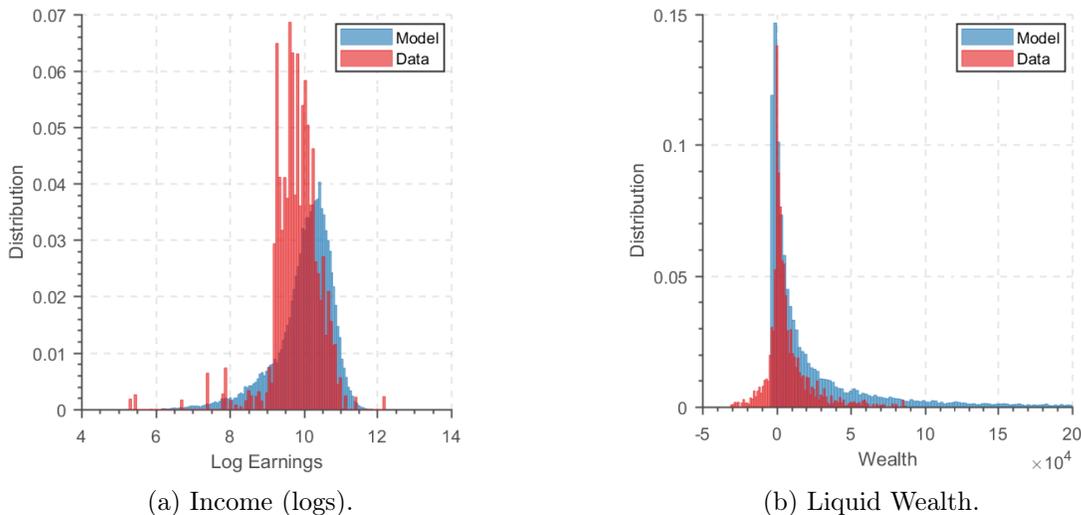


Figure B.1: Initial Distributions of Earnings and Wealth.

¹⁷Huggett et al. (2011) use a similar sample selection method.

Table B.1: Summary Statistics by Liquidity Quintile and UI Generosity

	Avg. State UI				Avg. State UI		
	< Med	> Med	P-Val		< Med	> Med	P-Val
White				Duration			
Q1	0.700	0.790	0.0593	Q1	17.27	19.59	0.0939
Q2	0.552	0.684	0.00718	Q2	18.66	20.16	0.215
Q3	0.589	0.683	0.166	Q3	17.52	19.90	0.146
Q4	0.810	0.835	0.553	Q4	18.48	19.78	0.385
Q5	0.896	0.891	0.873	Q5	17.66	19.31	0.285
HS Degree				UI Reported			
Q1	0.353	0.378	0.606	Q1	250.3	329.5	1.62e-90
Q2	0.332	0.452	0.0117	Q2	246.7	324.1	3.20e-95
Q3	0.405	0.415	0.875	Q3	249.6	327.5	4.33e-72
Q4	0.314	0.352	0.465	Q4	253.1	332.2	5.47e-83
Q5	0.332	0.263	0.193	Q5	251.7	336.0	7.81e-95
Coll. Degree				Prev. Ann. Wage			
Q1	0.112	0.0798	0.275	Q1	36391.3	36471.7	0.967
Q2	0.0317	0.0456	0.391	Q2	28051.0	31679.2	0.0357
Q3	0.0536	0.0650	0.664	Q3	31155.4	33229.5	0.323
Q4	0.170	0.127	0.253	Q4	44891.5	46128.4	0.701
Q5	0.154	0.210	0.171	Q5	62213.4	55497.1	0.197
Age				Prev. Tenure (wks)			
Q1	36.62	37.14	0.609	Q1	44.08	43.94	0.969
Q2	37.26	36.81	0.641	Q2	36.82	43.52	0.0338
Q3	37.37	36.13	0.234	Q3	41.80	48.97	0.0955
Q4	40.54	38.89	0.113	Q4	48.73	41.02	0.0329
Q5	43.92	43.93	0.996	Q5	47.67	50.08	0.512
Observations	1210	1144	2354	Observations	1210	1144	2354

Notes: Means are weighted and variance is corrected for the survey design. Number of observations is unweighted.

Table B.2: Test 1 Interaction Results

	Change		Consumption	Earnings
<i>Wealth</i>		<i>Human Capital</i>		
		<i>Learning Ability</i>		
↑		↑	32.9	12.7
↑		↓	16.9	-5.2
↑			49.0	33.3
↑		↑	8.8	-15.8
↓		↑	-0.7	7.6
↓		↓	-18.2	-10.2
↓			14.0	26.8
↓		↑	-25.4	-19.9
		↑	35.7	41.4
		↑	-6.5	-9.3
		↓	15.9	20.3
		↓	-21.5	-24.5

Notes: This table presents the change in earnings and consumption as two of the initial conditions are varied by one standard deviation.

Table B.3: Estimated Auxiliary Parameters

Slopes and Intercepts by Wealth (PSID)				Slopes and Intercepts by AFQT (NLSY)				Re-Employment Elasticities (SIPP)			
Var.	Data	Model	P-Val	Var.	Data	Model	P-Val	Var.	Data	Model	P-Val
Age	0.0219 (0.0030)	0.0262 (0.0007)	0.3320	Age	0.0206 (0.0047)	0.0244 (0.0016)	0.4145	Q1 x Ln(UI)	0.6974 (0.2492)	0.5581 (0.0529)	0.2912
Wealth Q2 x Age	0.0002 (0.0053)	0.0020 (0.0013)	0.3797	AFQT Q2 x Age	0.0003 (0.0012)	-0.0006 (0.0020)	0.2611	Q2 x Ln(UI)	0.3819 (0.2637)	0.4965 (0.0540)	0.3429
Wealth Q3 x Age	-0.0069 (0.0036)	0.0014 (0.0012)	0.0178	AFQT Q3 x Age	0.0013 (0.0016)	0.0013 (0.0019)	0.4118	Q3 x Ln(UI)	-0.0550 (0.2733)	0.4835 (0.0524)	0.0224
Wealth Q4 x Age	-0.0079 (0.0032)	0.0010 (0.0011)	0.0072	AFQT Q4 x Age	0.0054 (0.0010)	0.0041 (0.0019)	0.3039	Q4 x Ln(UI)	0.0724 (0.2046)	0.4498 (0.0508)	0.0308
Wealth Q5 x Age	-0.0082 (0.0031)	0.0040 (0.0008)	0.0002	AFQT Q5 x Age	0.0118 (0.0027)	0.0131 (0.0018)	0.4391	Q5 x Ln(UI)	0.1509 (0.2660)	0.2417 (0.0428)	0.3301
Wealth Q2	0.0072 (0.1616)	0.0588 (0.0452)	0.3852	AFQT Q2	0.0649 (0.0757)	0.1594 (0.0671)	0.1404	Q2	2.0829 (2.1167)	-0.2161 (0.2981)	0.1428
Wealth Q3	0.2564 (0.1139)	0.1767 (0.0430)	0.2616	AFQT Q3	0.1354 (0.0550)	0.2138 (0.0659)	0.1721	Q3	5.5140 (2.3008)	-0.7378 (0.2803)	0.0040
Wealth Q4	0.3759 (0.0972)	0.2930 (0.0381)	0.2563	AFQT Q4	-0.0016 (0.0367)	0.2420 (0.0641)	0.0011	Q4	5.6164 (2.5393)	-1.2227 (0.2348)	0.0010
Wealth Q5	0.4578 (0.0945)	0.3524 (0.0305)	0.1647	AFQT Q5	-0.1798 (0.0916)	0.1923 (0.0637)	0.0004	Q5	6.6118 (2.5393)	-2.1769 (0.2348)	0.0003
Cons.	9.6977 (0.1375)	9.7336 (0.0254)	0.2619	Cons.	9.5107 (0.1491)	9.7462 (0.0524)	0.0339	Cons.	-1.1684 (2.1378)	0.7406 (0.1956)	0.1802
Within Job Wage Growth (NLSY)				Job-Stay Rate (NLSY)				Age	-0.0009 (0.0018)	-0.0051 (0.0006)	0.0067
Var.	Data	Model	P-Val	Var.	Data	Model	P-Val	Wealth	0.0000 (0.0000)	4.0500×10^{-7} (0.0000)	0.0344
Cons.	0.0654 (0.0030)	0.0711 (0.0007)	0.4022	Age 25 - 29	0.7790 (0.0080)	0.7572 (0.0043)	0.0169	Q1 x Ln(LstWg)	0.3296 (0.2492)	0.3472 (0.0551)	0.4396
Age 30 - 34	-0.0328 (0.0029)	-0.0339 (0.0013)	0.4769	Age 30 - 34	0.8277 (0.0091)	0.8134 (0.0046)	0.0957	Q2 x Ln(LstWg)	0.4540 (0.0744)	0.4246 (0.0563)	0.3657
Age 35 - 39	-0.0318 (0.0035)	-0.0456 (0.0053)	0.0288	Age 35 - 39	0.8524 (0.0039)	0.8271 (0.0055)	0.4118	Q3 x Ln(LstWg)	0.3806 (0.0762)	0.4909 (0.0536)	0.1998
Age 40 - 44	-0.0430 (0.0038)	-0.0510 (0.0056)	0.2030	Age 40 - 44	0.8710 (0.0158)	0.8324 (0.0060)	0.0187	Q4 x Ln(LstWg)	0.3693 (0.0641)	0.5664 (0.0499)	0.0167
Age 45 - 49	-0.0571 (0.0042)	-0.0570 (0.0061)	0.2589	Age 45 - 49	0.8853 (0.0190)	0.8312 (0.0068)	0.0073	Q5 x Ln(LstWg)	0.3738 (0.0629)	0.8182 (0.0364)	0.0000
Age 50 - 54	-0.0589 (0.0043)	-0.0642 (0.0079)	0.4819	Age 50 - 54	0.8971 (0.0107)	0.8536 (0.0098)	0.0033				
Within Job Wage Growth Variance (NLSY)				Unemployment Rate (PSID)							
Var.	Data	Model	P-Val	Var.	Data	Model	P-Val				
Cons.	0.0168 (4.0856)	0.0672 (1.7467)	0.4955	Age 25 - 29	0.0311 (0.0013)	0.0562 (0.2331)	0.4545				

C Proofs

C.1 Existence of a Block Recursive Equilibrium

The existence proof of a block recursive equilibrium is shown by using backwards induction and at each stage of the life-cycle showing that agents decisions are not conditional on the distribution of workers across states. Throughout, I include aggregate productivity z in the aggregate state, though this is stationary in the model.

Because the value in $T+1$ for all agents is 0, the three worker value functions [Equation 4.8](#), [Equation 4.12](#), and [Equation 4.1](#) respectively, satisfy the following in period T .

$$U_T(b_{UI}, a, h, \ell; \psi) = u((1 + r_F)a + b_{UI}) \quad (\text{C.1})$$

$$U_T(b_L, a, h, \ell; \psi) = u((1 + r_F)a + b_L) \quad (\text{C.2})$$

$$W_T(\mu, a, h, \ell; \psi) = u(\mu f(h) + (1 + r_F)a) \quad (\text{C.3})$$

The optimal policy for the terminal period is known: agents will use all accumulated savings to purchase consumption, and spend no time accumulating human capital, because the gains would not be realized until the following period. Because the interest rate is assumed to be the world interest rate and taken as given, each of the value functions do not depend on the distribution of workers across states. Therefore, the distributions, ψ can be dropped from the state space and the value functions rewritten as $U_T(b_{UI}, a, h, \ell; \psi) = U_T(b_{UI}, a, h, \ell; z)$, $U_T(b, a, h, \ell; \psi) = U_T(b, a, h, \ell; z)$, and $W_T(\mu, a, h, \ell; \psi) = W_T(\mu, a, h, \ell; z)$. Since there is no new employment activity for workers of age T , the decision rules of these agents do not depend upon the distribution of agents in the economy. Now, consider the market tightness function for firms posting vacancies for workers who will be age T when they are first employed (i.e., are currently in the search subperiod of age T). Since the continuation value to the firm in period $T + 1$ is zero, the period T value of a vacancy is given by

$$J_T(\mu, a, h, \ell; \psi) = (1 - \mu)f(h) \quad (\text{C.4})$$

where again, I impose the optimal learning time of age T agents. The vacancy creation conditions can then be solved explicitly for every worker state:

$$V(\mu, a, h, \ell; \psi) = -\kappa + q(\theta_T(\mu, a, h, \ell; \psi))(1 - \mu)f(h) \quad (\text{C.5})$$

Free entry of firms yields the following:

$$\kappa = q(\theta_T(\mu, a, h, \ell; \psi))(1 - \mu)f(h) \quad (\text{C.6})$$

By assumption, q is invertible, and this is imposed in the calibration. Therefore, submarket tightness can be solved for any worker state:

$$\theta_T(\mu, a, h, \ell; \psi) = \begin{cases} q^{-1}\left(\frac{\kappa}{(1-\mu)f(h)}\right) & : \text{if } (1 - \mu)f(h) \geq \kappa \\ 0 & : \text{else} \end{cases}$$

This again does not depend upon the distribution of workers; thus, $\theta_T(\mu, a, h, \ell; \psi) = \theta_T(\mu, a, h, \ell; z)$. This means that the vacancy creation condition is known to workers without knowing the distribution of workers across the state space in the rest of the economy. Now, consider the search and matching decision of unemployed workers of age T :

$$\begin{aligned} R_T^U(b_{UI}, a, h, \ell; \psi) &= \max_{\mu'} P(\theta_T(\mu', a, h, \ell; \psi))W_T(\mu', a, h, \ell; \psi) \\ &\quad + (1 - P(\theta_T(\mu', a, h, \ell; \psi)))U_T(b_{UI}, a, h, \ell; \psi) \end{aligned} \quad (\text{C.7})$$

$$\begin{aligned} R_T^U(b_L, a, h, \ell; \psi) &= \max_{\mu'} P(\theta_T(\mu', a, h, \ell; \psi))W_T(\mu', a, h, \ell; \psi) \\ &\quad + (1 - P(\theta_T(\mu', a, h, \ell; \psi)))[\gamma U_T(b_L, a, h, \ell; \psi)] \end{aligned} \quad (\text{C.8})$$

Imposing the conditions for θ_T , as well as the value functions in the terminal production and consumption period yields the following

$$\begin{aligned} R_T^U(b_{UI}, a, h, \ell; \psi) &= \max_{\mu'} P(\theta_T(\mu', a, h, \ell; z))W_T(\mu', a, h, \ell; z) \\ &\quad + (1 - P(\theta_T(\mu', a, h, \ell; z)))U_T(b_{UI}, a, h, \ell; z) \end{aligned} \quad (\text{C.9})$$

$$\begin{aligned} R_T^U(b_L, a, h, \ell; \psi) &= \max_{\mu'} P(\theta_T(\mu', a, h, \ell; z))W_T(\mu', a, h, \ell; z) \\ &\quad + (1 - P(\theta_T(\mu', a, h, \ell; z)))[\gamma U_T(b_L, a, h, \ell; z)] \end{aligned} \quad (\text{C.10})$$

Note that neither the probabilities within each submarket, nor the continuation value depend on the distribution of workers across states. Therefore, the job search value functions are independent of the aggregate state and can be written $R_t^U(b_{UI}, a, h, \ell; \psi) = R_t^U(b_{UI}, a, h, \ell; z)$, and $R_t^U(b_L, a, h, \ell; \psi) = R_t^U(b_L, a, h, \ell; z)$, and the optimal application strategy is independent of the aggregate distribution of workers. Performing the same exercise for employed workers similarly yields

$$\begin{aligned} R_T^E(\mu, a, h, \ell; \psi) &= \max_{\mu'} P(\theta_T(\mu', a, h, \ell; \psi))W_T(\mu', a, h, \ell; \psi) \\ &\quad + (1 - P(\theta_T(\mu', a, h, \ell; \psi)))W_T(\mu, a, h, \ell; \psi) \end{aligned} \quad (\text{C.11})$$

$$\begin{aligned} R_T^E(\mu, a, h, \ell; \psi) &= \max_{\mu'} P(\theta_T(\mu', a, h, \ell; z))W_T(\mu', a, h, \ell; z) \\ &\quad + (1 - P(\theta_T(\mu', a, h, \ell; z)))W_T(\mu, a, h, \ell; z) \end{aligned} \quad (\text{C.12})$$

which again shows that the employed job searcher's value function does not depend on the aggregate distribution nor does the optimal application strategy, meaning $R_T^E(\mu, a, h, \ell; \psi) = R_T^E(\mu, a, h, \ell; z)$. Now consider the consumption, savings, and human capital decisions of age $T - 1$ unemployed workers:

Note that in this economy, the aggregate state is assumed to be $z_t = z \forall t$. To prove that

this exhibits a block recursive equilibrium, it must be the case that the value of an employed agent in the same time period is also independent of the distribution of agents across types. Consider the problem of an employed agent at time $T - 1$:

$$U_{T-1}(UI, a, h, \ell; \psi) = \max_{c, a'} u(c) + \nu + \beta E[(1 - \gamma)R_T^U(b_{UI}, a', h', \ell; \psi) + \gamma R_T^U(b_L, a', h', \ell; \psi)] \quad (\text{C.13})$$

$$\text{s.t. } c + a' \leq (1 + r_F)a + b_{UI} \quad (\text{C.14})$$

$$a' \geq \underline{a}' \quad (\text{C.15})$$

$$h' = e^{\epsilon'}(h + H(h, \ell, \tau, U)) \quad (\text{C.16})$$

$$U_{T-1}(b_L, a, h, \ell; \psi) = \max_{c, a'} u(c) + \nu + \beta E[R_T^U(b_L, a', h', \ell; \psi)] \quad (\text{C.17})$$

$$\text{s.t. } c + a' \leq (1 + r_F)a + b_L \quad (\text{C.18})$$

$$a' \geq \underline{a}' \quad (\text{C.19})$$

$$h' = e^{\epsilon'}(h + H(h, \ell, \tau, U)) \quad (\text{C.20})$$

Substituting in the age T value functions yields the following:

$$U_{T-1}(b_{UI}, a, h, \ell; \psi) = \max_{c, a'} u(c) + \nu + \beta E[(1 - \gamma)R_T^U(b_{UI}, a', h', \ell; z) + \gamma R_T^U(b_L, a', h', \ell; z)] \quad (\text{C.21})$$

$$\text{s.t. } c + a' \leq (1 + r_F)a + b_{UI} \quad (\text{C.22})$$

$$a' \geq \underline{a}' \quad (\text{C.23})$$

$$h' = e^{\epsilon'}(h + H(h, \ell, \tau, U)) \quad (\text{C.24})$$

$$U_{T-1}(b_L, a, h, \ell; \psi) = \max_{c, a'} u(c) + \nu + \beta E[R_T^U(b_L, a', h', \ell; z)] \quad (\text{C.25})$$

$$\text{s.t. } c + a' \leq (1 + r_F)a + b_L \quad (\text{C.26})$$

$$a' \geq \underline{a}' \quad (\text{C.27})$$

$$h' = e^{\epsilon'}(h + H(h, \ell, \tau, U)) \quad (\text{C.28})$$

Note that the neither the continuation values nor the prices depend on the aggregate distribution of workers, as debt is priced individually (in this case, with one price). This means that the consumption and savings rules of unemployed workers are independent of the distribution of workers, and the value functions can be written $U_{T-1}(\mu, a, h, \ell; \psi) = U_{T-1}(\mu, a, h, \ell; z)$ and $U_{T-1}(b_L, a, h, \ell; \psi) = U_{T-1}(b_L, a, h, \ell; z)$. By essentially the same argument, the value function during the consumption and savings period of an employed worker

can be written as

$$W_{T-1}(\mu, a, h, \ell; \psi) = \max_{c, a', \tau} u(c) + \beta E[(1 - \delta)R_T^E(\mu, a, h', \ell; \psi') + \delta R_T^U(b_{UI}, a', h', \ell; \psi')] \quad (\text{C.29})$$

$$\text{s.t. } c + a' \leq (1 + r_F)a + \mu(1 - \tau)f(h) \quad (\text{C.30})$$

$$a' \geq \underline{a} \quad (\text{C.31})$$

$$h' = e^{\epsilon'}(h + H(h, \ell, \tau, E; \psi)) \quad (\text{C.32})$$

$$b_{UI} = b(1 - \tau)\mu f(h) \quad (\text{C.33})$$

$$b \sim N(\mu_b, \sigma_b) \quad (\text{C.34})$$

$$\tau \in [0, 1] \quad (\text{C.35})$$

$$W_{T-1}(\mu, a, h, \ell; \psi) = \max_{c, a', \tau} u(c) + \beta E[(1 - \delta)R_T^E(\mu, a, h', \ell; z) + \delta R_T^U(b_{UI}, a', h', \ell; z)] \quad (\text{C.36})$$

$$\text{s.t. } c + a' \leq (1 + r_F)a + \mu(1 - \tau)f(h) \quad (\text{C.37})$$

$$a' \geq \underline{a} \quad (\text{C.38})$$

$$h' = e^{\epsilon'}(h + H(h, \ell, \tau, E; z)) \quad (\text{C.39})$$

$$b_{UI} = b(1 - \tau)\mu f(h) \quad (\text{C.40})$$

$$b \sim N(\mu_b, \sigma_b) \quad (\text{C.41})$$

$$\tau \in [0, 1] \quad (\text{C.42})$$

Again, neither the consumption, nor savings decisions depend on the distribution of workers across states. Furthermore, because human capital and learning are assumed to be observable, each worker state vector maps to a wage offer by the firm, independent of the distribution of human capital, learning, or wealth and wage. Thus, the human capital accumulation decision is independent of the distribution of workers, and the value function can be written $W_{T-1}(\mu, a, h, \ell; \psi) = W_{T-1}(\mu, a, h, \ell; z)$, and each of the decision rules are independent of the distribution of workers across states.

It's similarly easy to show that the value of a filled vacancy of a worker age $T - 1$ does not depend on the distribution of workers across states. The value function of the firm may be written

$$J_{T-1}(\mu, a, h, \ell; \psi) = (1 - \mu)(1 - \tau)f(h) + \beta E[(1 - \delta)(1 - P((\theta_T(\mu', a', h', \ell; \psi'))))J_T(\mu, a', h', \ell; \psi')] \quad (\text{C.43})$$

$$h' = e^{\epsilon'}(h + H(h, \ell, \tau, E; \psi)) \quad (\text{C.44})$$

$$\tau = g_\tau(\mu, a, h, \ell; \psi) \quad (\text{C.45})$$

$$a' = g_a(\mu, a, h, \ell; \psi) \quad (\text{C.46})$$

$$\mu' = g_\mu(\mu, a', h', \ell; \psi) \quad (\text{C.47})$$

Each of the employed worker decision rules do not depend on the distribution of workers across states. In addition, Θ_T , and J_T do not depend on the distribution as shown earlier.

Thus,

$$J_{T-1}(\mu, a, h, \ell; \psi) = (1 - \mu)(1 - \tau)f(h) + \beta E[(1 - \delta)(1 - P((\theta_T(\mu', a', h', \ell; z))))J_T(\mu, a', h', \ell; z)] \quad (\text{C.48})$$

$$h' = e^{\epsilon'}(h + H(h, \ell, \tau, E; z)) \quad (\text{C.49})$$

$$\tau = g_\tau(\mu, a, h, \ell; z) \quad (\text{C.50})$$

$$a' = g_a(\mu, a, h, \ell; z) \quad (\text{C.51})$$

$$\mu' = g_\mu(\mu, a', h', \ell; z) \quad (\text{C.52})$$

Therefore, the value function of a filled vacancy for a worker age $T - 1$ does not depend on the distribution of workers across states, $J_{T-1}(\mu, a, h, \ell; \psi) = J_{T-1}(\mu, a, h, \ell; z)$. From the free entry condition and the invertibility of $q(\theta)$, this yields

$$\theta_{T-1}(\mu, a, h, \ell; \psi) = \begin{cases} q^{-1}\left(\frac{\kappa}{J_{T-1}(\mu, a, h, \ell; \psi)}\right) & : \text{if } J_{T-1}(\mu, a, h, \ell; \psi) \geq \kappa \\ 0 & : \text{else} \end{cases}$$

and furthermore, $\theta_{T-1}(\mu, a, h, \ell; \psi) = \theta_{T-1}(\mu, a, h, \ell; z)$.

Finally, it remains to be shown that a worker who is searching during age $T - 1$ does not make decisions conditional on the distribution of workers. Similar to before, the value functions of unemployed searchers can be written

$$R_{T-1}^U(b_{UI}, a, h, \ell; \psi) = \max_{\mu'} P(\theta_{T-1}(\mu', a, h, \ell; \psi))W_{T-1}(\mu', a, h, \ell; \psi) + (1 - P(\theta_{T-1}(\mu', a, h, \ell; \psi)))U_{T-1}(\mu, a, h, \ell; \psi) \quad (\text{C.53})$$

$$R_{T-1}^U(b_L, a, h, \ell; \psi) = \max_{\mu'} P(\theta_{T-1}(\mu', a, h, \ell; \psi))W_{T-1}(\mu', a, h, \ell; \psi) + (1 - P(\theta_{T-1}(\mu', a, h, \ell; \psi)))U_{T-1}(b_L, a, h, \ell; \psi) \quad (\text{C.54})$$

Again, because the continuation values as well as the set of submarket tightnesses do not depend on the distribution, this can be written

$$R_{T-1}^U(b_{UI}, a, h, \ell; \psi) = \max_{\mu'} P(\theta_{T-1}(\mu', a, h, \ell; z))W_{T-1}(\mu', a, h, \ell; z) + (1 - P(\theta_{T-1}(\mu', a, h, \ell; z)))U_{T-1}(\mu, a, h, \ell; z) \quad (\text{C.55})$$

$$R_{T-1}^U(b_L, a, h, \ell; \psi) = \max_{\mu'} P(\theta_{T-1}(\mu', a, h, \ell; z))W_{T-1}(\mu', a, h, \ell; z) + (1 - P(\theta_{T-1}(\mu', a, h, \ell; z)))U_{T-1}(b_L, a, h, \ell; z) \quad (\text{C.56})$$

where once again, the application strategy is independent of the distribution of workers across states, and therefore $R_{T-1}^U(b_L, a, h, \ell; \psi) = R_{T-1}^U(b_L, a, h, \ell; z)$. Lastly, the same can

be shown of employed searchers of age $T - 1$:

$$\begin{aligned} R_{T-1}^E(\mu, a, h, \ell; \psi) &= \max_{\mu'} P(\theta_{T-1}(\mu', a, h, \ell; \psi)) W_{T-1}(\mu', a, h, \ell; \psi) \\ &\quad + (1 - P(\theta_{T-1}(\mu', a, h, \ell; \psi))) W_{T-1}(\mu, a, h, \ell; \psi) \end{aligned} \quad (\text{C.57})$$

$$\begin{aligned} R_{T-1}^E(\mu, a, h, \ell; \psi) &= \max_{\mu'} P(\theta_{T-1}(\mu', a, h, \ell; z)) W_{T-1}(\mu', a, h, \ell; z) \\ &\quad + (1 - P(\theta_{T-1}(\mu', a, h, \ell; z))) W_{T-1}(\mu, a, h, \ell; z) \end{aligned} \quad (\text{C.58})$$

where again, $R_{T-1}^E(\mu, a, h, \ell; \psi) = R_{T-1}^E(\mu, a, h, \ell; z)$; thus, all decision rules for actors in the model in period $T - 1$ do not depend on distributions. The proof can be repeated for ages $\{T - 2, \dots, 1\}$, and by the same logic as above, these value and policy functions will not depend upon the aggregate distribution of agents across states. Thus, the model exhibits a block recursive equilibrium.

C.2 BRE Discussion

A block recursive equilibrium in this economy is possible because of a few assumptions: first, the interest rate cannot depend on the distribution of assets. With this, firms and workers do not have to condition on the distribution of assets in their policy functions. Second, workers must be able to direct their search to submarkets, and in these submarket workers characteristics must either be observable, or be implied by sorting. This assumption allows firms to know the expected profits from opening a vacancy within a submarket, causing policy functions to no longer have to depend upon the distribution of workers across types. Third, the matching function must be constant returns to scale. This implies that the probability of a firm matching with a worker is a function only of the ratio of vacancies to unemployed searchers, which causes policy functions to no longer depend upon the distribution of workers within types. Finally, the probability that firms meet with workers must be invertible, which allows the recovery of the probability a worker meets with a firm in a submarket. With this, workers can select a submarket and know the wage offered and probability of employment.