Equilibrium Voluntary Disclosures, Asset Pricing, and Information Transfers *

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Abstract

We study a firm's manager's voluntary disclosure decisions and those disclosure decisions' asset pricing, cost of capital, and information transfer effects in a model where investors trade multiple securities. We: develop new asset pricing formulas when the manager makes no disclosure that impose testable cross-equation restrictions on firms' market values; develop a wide array of comparative statics; obtain surprising findings about nondisclosure's effects on investors' perceptions of uncertainty about firms' future cash flows; develop simple, interpretable expressions for firms' cost of capital; and show how no disclosure by one firm generates informational externalities on other firms.

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1 Introduction

In this paper we study a model of a firm's manager's equilibrium voluntary disclosure decisions when investors are risk-averse and can invest either in the disclosing firm's or in other firms' securities after the manager has made her disclosure decision. We produce an array of new findings. We develop simple, easily interpretable and testable, equilibrium asset pricing equations both when the manager discloses her information and when she makes no disclosure. We

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show exactly how the manager's equilibrium disclosure policy changes as: investors' aggregate risk-aversion changes, as the variance of the firm's future cash flows ("CF") changes, as the correlation between the manager's firm's and other firms' CF changes, and as the precision of the information the manager receives and discloses changes. We develop a wide array of intuitive comparative statics. We obtain surprising findings concerning the effects of a manager's nondisclosure on investors' perceptions of both the uncertainty about her firm's CF and the covariances between her firm's and other firms' CF. We show how nondisclosure by one firm generates informational externalities on other firms. We obtain new expressions for a firm's cost of capital that yield intuitive and (unlike some other disclosure models in the literature) sensible comparative statics.

We develop all of these results using conventional economic and statistical assumptions: investors are assumed to have constant absolute risk-averse ("CARA") preferences and to be price-takers on the securities' market[1]; equilibrium asset prices are determined by equating aggregate shareholders' demands for each asset to aggregate supply of each asset; firms' CF are multivariate normally distributed; the information the manager of the sometimes-disclosing firm gets is also normally distributed.

We start with the Dye [1985] and Jung and Kwon [1988] voluntary disclosure framework, which considers the natural situation where the manager of a firm sometimes privately receives information pertinent to valuing her firm which she may credibly disclose to or withhold from investors. The manager's objective in choosing to disclose the information she sometimes receives is based on whether disclosure yields a higher market price for her firm than does nondisclosure. Dye [1985] and Jung and Kwon [1988] studied a manager's disclosure decisions when investors are assumed to be risk-neutral in a single firm context. We extend their framework in two ways: we study a manager's disclosure decisions when investors are assumed to be risk-averse, and we study disclosures in a multi-firm context.

The new equilibrium asset pricing equations we derive when the manager makes no disclosure entail simple risk-based additive adjustments to the con-

[1] Extensions of the model to investors who are not price-takers and instead have market power would be straightforward.

ventional asset pricing equations that would apply were the manager of the sometimes-disclosing firm never to have received (and hence never to have voluntarily disclosed) any information. These risk-based additive adjustments result in asset pricing equations that impose cross-equation restrictions that are subject to empirical testing.

We show that when the manager adopts an equilibrium disclosure policy, i.e., her disclosure policy is the one investors expected her to adopt, and she does not make a disclosure, then this *always* causes investors' perceptions of both the variance of the manager's firm's CF and the absolute value of the covariance between her firm's CF and the CF of any other firm to increase from their prior values. This contrasts starkly with the conventional statistical result that were the manager to disclose her private information, then this *always* causes investors' perceptions of both the variance of the disclosing firm's CF and the covariance between the disclosing firm's and all other firms' CF to shrink toward zero relative to investors' prior beliefs.

These variance-increasing results of nondisclosure are unexpected. There is news in a firm's manager's decision not to disclose information during in a period, and that news causes investors to revise their beliefs about the distribution of the manager's firm's CF. Since new information of any sort (e.g., news of the manager's nondisclosure) about any random variable (e.g., a firm's CF) must reduce the expected value of the variance of that random variable, by the "law of total variance,"[2] one might have expected, contrary to our findings, that investors' learning of the manager's nondisclosure would reduce investors' uncertainty about the firm's CF.

An implication of the variance-increasing effect of nondisclosure is that it results in the firm's manager disclosing the information she receives more often (than she would disclose her information were investors risk-neutral) so as to diminish the price penalty risk-averse investors impose on the firm for the extra uncertainty its manager's nondisclosure generates.

We show how nondisclosure generates information transfers across firms: if the unconditional covariance between the sometimes-disclosing firm's and another firm's CF is positive, nondisclosure by "own" firm reduces investors' per-

[2] See, e.g., Weiss [2005].

ceptions of the other firm's expected CF, whereas if the unconditional covariance between the two firms' CF is negative, nondisclosure by "own" firm increases investors' perceptions of the other firm's future expected cash flows. At the same time these first moment information transfers occur, we show that second moment information transfers also occur: when the manager of "own" firm makes no disclosure during a period, investors' perceptions of the variance of any other firm's' CF also increase as long as the prior covariance between those two firms' CF is nonzero.

In addition, we produce a variety of new comparative statics results, showing how various endogenous components of the model (e.g., firms' cost of capital, the manager's equilibrium disclosure policy, firms' equilibrium "no disclosure" prices) change with changes in the model's exogenous parameters. We highlight just two comparative statics in this Introduction: 1. we show that when investors are risk-averse, a manager always discloses her private information (when received) less often as investors' prior beliefs about the variance of the manager's firm's CF declines. Since a primary reason investors' prior beliefs about the variance of a firm's CF declines is because of investors' receipt of information from other sources - e.g., analysts, newspapers, disclosures of other firms, etc., - this result can be recast as asserting that: information from these other sources and the manager's voluntary disclosures are always substitutes in that more information from these other sources leads to less voluntary disclosure by the manager. While this is a seemingly natural and intuitive result, this result *never* obtains when investors are risk-neutral.[3] 2. We show that information transfers between two firms, one of which is the sometimes disclosing firm, affects the nondisclosing firm's cost of capital. As an example of one such informational externality, we show that if the covariance between the two firms' CF is positive, then an increase in the probability that the sometimes disclosing firm's manager receives information reduces the cost of capital for the nondisclosing firm, whereas if the covariance between the two firms' CF is negative, the opposite result obtains.

It is also worth remarking that many of our comparative statics results are robust in that they hold even when we perturb the main assumptions underlying

[3]See point 4 following (7) for elaboration on this point.

our model.

We believe all of our results for risk-averse investors are new to the literature on voluntary disclosures.

This paper is part of the now substantial, and still growing, literature in accounting, economics, and finance that studies formal models of voluntary disclosure. Within this literature, there are several papers that study disclosure problems where investors are uncertain of a firm's manager's receipt of information, as originally developed by Dye [1985] and Jung and Kwon [1988]. These include single period models that consider the efficiency of investments in cash flow-generating assets and information acquisition (Pae [1999]; Pae [2002]; Pae and Hughes [2014]; Hughes and Pae [2004]), as well as multi-period models that show how the timing of disclosure affects investors' interpretations of firms' values (e.g., Einhorn and Ziv [2007], Beyer and Dye [2012], Guttman, Kremer and Skrzypacz [2014]). Some papers in this literature address cost of capital issues (e.g., Goto, Watanabe and Xu [2009], Gao [2010], Jorgensen and Kirschenheiter [2003], Cheynel [2013], Clinch and Verrecchia [2013]). Each of Gao [2010], Jorgensen and Kirschenheiter [2003], and Cheynel [2013] merit further comment. Gao [2010] identifies some conditions under which there may be a counterintutive relationship between additional disclosure and a firm's cost of capital. Jorgensen and Kirschenheiter [2003] focus on a disclosure problem distinct from the one we do: they are concerned about when managers will disclose the private information they have about their firms' riskiness. But, the methods they employ in studying their problem turn out to be useful in our analysis. Cheynel [2013] studies a large economy model of voluntary disclosures. While her paper differs in both setup and emphasis from the present paper, in those areas where our papers overlap - principally in the comparison of the systematic risk of firms who do or do not make voluntary disclosures - she concludes, as do we, that disclosing firms have lower systematic risk than nondisclosing firms.

This paper is also related to the empirical literature on voluntary disclosure. While a comprehensive description of all the points of contact between the present paper and this empirical literature is too extensive to document here, we mention several representative connections here.[4] Our results on the

[4] See, e.g., Healy and Palepu's [2001] survey for an enumeration of some additional connec-

relationship between a firms's voluntary disclosure policy and investors' prior beliefs about the distribution of the firm's future cash flows is related to Beyer, Cohen, Lys, and Walther [2010] (who emphasize that firms' accounting reports and disclosures should be considered in the context of the firms' entire information environments); our work on how voluntary disclosures influence firms' costs of capital is related to Botosan [1997] and Botosan and Plumlee [2002] (who find mixed evidence of an association between measures of voluntary disclosure and firms' implied cost of capital depending on disclosure proxies); our work on information transfers relates to the empirical work on information transfers of, e.g., Foster [1981], Dietrick and Ohlsen and Dietrich [1985], Baginski [1987], and Han, Wild and Ramesh [1989] (all of whom document that information transfers occur for firms with correlated returns).

The paper proceeds as follows. The next section, Section 2, reviews the univariate risk-neutral model of Dye [1985] and Jung and Kwon [1988] when the firm's CF, as well as the firm's manager's private estimate of those future cash flow (which she may disclose to or withhold from investors), are both normally distributed. Section 3 takes the univariate model of section 2 and extends it to a model with risk-averse investors. Sections 4-6 are the heart of the paper. Section 4 contains an analysis of a manager's disclosure decisions and related pricing effects in a multi-firm model with risk-averse investors. Section 5 studies information transfers, i.e., the impact of a firm's disclosures on investors' perceptions of other firms, in the context of the model of Section 4. Section 6 studies how the sometimes-disclosing firm's actions influence both its and other firms' costs of capital, also in the context of Section 4. Section 7 summarizes our findings, and also suggests some possible proposals for future research.

2 The Univariate Model of Voluntary Disclosure with Risk-Neutral Investors

As a baseline for what follows, it is useful to start the formal discussion by reviewing the single firm voluntary disclosure model of Dye [1985] and Jung and Kwon [1988] under risk-neutral pricing in the case where the firm's cash

tions between the then contemporary theoretical and empirical papers on disclosure.

flows are normally distributed.

There is a firm whose CF $\tilde{x}$ are ex ante normally distributed with mean m and variance σ_x^2, henceforth written as $\tilde{x}\tilde{\ }N(m, \sigma_x^2)$. Before a market for the firm's shares opens, with probability $p \in (0, 1)$ the manager of the firm privately receives an imperfect, but unbiased, normally distributed estimate of the realization of $\tilde{x}$. Specifically, the estimate is the realization v of the random variable $\tilde{v}$ where $\tilde{v}|_x = x + \tilde{\varepsilon}$, and $\tilde{\varepsilon}\tilde{\ }N(0, \sigma_\varepsilon^2)$ is a noise term independent of all other random variables with variance σ_ε^2. We denote the prior variance of $\tilde{v}$ by σ_v^2, and we let $G(\cdot)$ and $g(\cdot)$ respectively denote the cumulative distribution function (cdf) and density of $\tilde{v}$.

When the manager of the firm receives estimate v, either the manager discloses the estimate or else she discloses nothing. As is typical in this literature, we assume: all disclosures are confined to be truthful; the manager cannot partially disclose v; the manager cannot credibly disclose that she did not receive information; the manager makes no disclosure when she does not receive v (so she has a disclosure decision to make only when she receives information). We posit that the manager's goal is to maximize the market value of her firm, so in the event the manager receives v, she discloses v to investors only if disclosure produces a higher market price for the firm than does no disclosure.

All investors are posited to be homogeneously informed about the distribution of $\tilde{x}$ at all points in time, so there is no need to index information sets by individual investors. We let Ω denote the public information set associated with the firm. Ω consists of v when the manager discloses the estimate v, and it consists of the fact of the manager's nondisclosure when the manager makes no disclosure. That is, either $\Omega = \{v\}$ or $\Omega = \{\text{no disclosure}\}$.

We denote the price of the firm if its manager discloses v by $P^d(v)$. We suppose that risk-neutral pricing prevails. Since one way of writing the conditional expectation $E[\tilde{x}|v]$ is $m + \frac{\sigma_x^2}{\sigma_v} \times \frac{v-m}{\sigma_v}$, it follows that:

$$P^d(v) = m + \frac{\sigma_x^2}{\sigma_v} \times \frac{v - m}{\sigma_v}. \tag{1}$$

Clearly, $P^d(v)$ is increasing in v. If the manager makes no disclosure, the firm's price, call it P^{nd}, will be a constant. As a consequence, the manager's preferred disclosure policy is described by some cutoff v^c, with the manager disclosing v

when received if and only if $v \geq v^c$.[5] The "no disclosure" price of the firm will depend on what this cutoff is; to reflect that, we write $P^{nd}(v^c)$.

We now calculate $P^{nd}(v^c)$, following Jung and Kwon [1988]. Since the ex ante probability the manager makes no disclosure is $1 - p + pG(v^c)$, an easy application of Bayes' Rule allows us to conclude that the conditional probability the manager is informed (resp., uninformed) given that the manager makes no disclosure equals: Pr(manager is informed|no disclosure)= $\frac{pG(v^c)}{1-p+pG(v^c)}$ (resp., Pr(manager is uninformed|no disclosure)= $\frac{1-p}{1-p+pG(v^c)}$). If investors knew the reason for the manager's nondisclosure was that she withheld information (resp., she was uninformed), investors would infer that the firm's CF have expected value $E[\tilde{x}|\tilde{v} < v^c]$ (res., $E[\tilde{x}] = m$). Combining the preceding, it follows that investors who observed the manager make no disclosure and who believed she was using the cutoff v^c would assess the firm's expected CF, and hence the firm's "no disclosure" market value, to be:

$$P^{nd}(v^c) = \frac{(1-p)m + pG(v^c)E[\tilde{x}|\tilde{v} < v^c]}{1 - p + pG(v^c)}. \tag{2}$$

In light of (1) and (2), as well as in view of the manager's assumed interest in choosing a disclosure policy so as to maximize the price of her firm, an equilibrium cutoff is defined as follows.

Definition 2 *A disclosure cutoff v^c is an equilibrium cutoff if it satisfies*

$$P^d(v^c) = P^{nd}(v^c). \tag{3}$$

We denote the equilibrium cutoff in the risk-neutral model of this section by v^{RNc}.

In the following, it is often more convenient to describe the manager's disclosure policy not in terms of a "regular" cutoff v^c, but rather in terms of the "standardized" cutoff z^c defined in terms of a regular cutoff by:

$$z^c = \frac{v^c - m}{\sigma_v}. \tag{4}$$

If we convert a regular cutoff into its standardized value, and at the same time we convert the realized estimate v into its standardized counterpart, $z = \frac{v-m}{\sigma_v}$, we

[5] It makes no difference in what follows whether the disclosure set includes or excludes the point $v_i = v_i^c$ in what follows; we include it for the sake of specificity.

can rewrite the equilibrium equation (3) in terms of the density $\phi(\cdot)$ and the cdf $\Phi(\cdot)$ of a standard normal random variable. Performing these transformations, we show in the appendix (in Lemma 12) that the equilibrium equation (3) is algebraically equivalent to the equation:

$$z^c = f(z^c), \tag{5}$$

where

$$f(z^c) \equiv -\frac{p\phi(z^c)}{1-p+p\Phi(z^c)}. \tag{6}$$

This transformation converts a model with arbitrary initial prior mean and variance, and arbitrary initial precision of the manager's estimate, into a model where the disclosing firm's CF are standard normal and where the precision of the manager's estimate is infinite (i.e., the estimate perfectly predicts/reveals the firm's CF). In the transformed model, $f(z^c)$ constitutes investors' perception of the expected value of the firm's CF, conditional on the manager making no disclosure and using the cutoff z^c to define her disclosure policy. Equation (5) establishes that the standardized equilibrium cutoff is a fixed point of the function $f(\cdot)$.

Of course, once we identify a standardized cutoff z^c that solves (5), which we denote by z^{RNc}, we can recover the corresponding "regular" equilibrium cutoff v^{RNc} that solves (3) by taking the inverse of (4), i.e., by setting

$$v^{RNc} = \sigma_v \times z^{RNc} + m. \tag{7}$$

In view of this ability to convert from the standardized cutoff to the "regular" cutoff and vice versa, throughout the following, as context dictates, we describe disclosure cutoffs either in terms of their regular or standardized values.

It is easy to show that z^{RNc} exists and is unique, i.e., that equation (5) has one and only one solution. Five properties of z^{RNc} will be important in what follows: 1. it is strictly decreasing in p; 2 it is negative; 3. it attains $\min_{z^c\in\mathbb{R}} f(z^c)$; 4. it is independent of all parameters of the model other than p, and 5. $z^c - f(z^c) < 0$ if $z^c < z^{RNc}$ and $z^c - f(z^c) > 0$ if $z^c > z^{RNc}$.[6] The first property is a specific example of the general fact that the firm's manager

[6] This assertion is proven in Lemma 20 in the Appendix.

discloses a broader set of the private information she receives as the probability risk-neutral investors believe she has received information increases. Since standard normal distributions are symmetric around 0, the second property implies that, conditional on receiving information, the probability the manager will disclose her private information exceeds 1/2. The third property was discovered by Acharya et al [2011]; it is sometimes referred to as the "minimum" property of the equilibrium cutoff. The fourth property is clear, since the only parameter of the model on which the function $f(\cdot)$ depends is p. This fourth property implies that the equilibrium probability of disclosure is independent of both the precision of the estimate $\tilde{v}$ the manager of the firm receives and also the prior variance of $\tilde{x}$. While the independence of the equilibrium probability of disclosure from either the precision of the estimate $\tilde{v}$ or the variance σ_x^2 does not seem to us to be empirically reasonable, we shall show below that this fourth property is key to the more empirically supported comparative statics we obtain in the next section where investors are posited to be risk-averse.[7] The fifth property is a generalization of the third property.[8] This fifth property is key to the variance-increasing feature of the manager's nondisclosure mentioned in the Introduction.[9]

3 The Univariate Model of Voluntary Disclosure with Risk-Averse Investors

In this section, we extend the model in the previous section so as to incorporate risk-averse investors.

The model begins as does the previous model, with the manager of the firm receiving the realization of the unbiased estimate $\tilde{v}$ of $\tilde{x}$ with probability p, which she may disclose to or withhold from investors. We let $E[\tilde{x}|\Omega]$ and $var(\tilde{x}|\Omega)$

[7] See Corollary 4 below.

[8] This follows because in order for z^{RNc} to be the global minimizer of $f(\cdot)$, as property (3) asserts, z^{RNc} must of course also be the local minimizer of $f(\cdot)$, which requires that $f'(z^c) < 0$ for all z^c close to, but below z^{RNc}, and that $f'(z^c) > 0$ for all z^c close to, but above z^{RNc}. Since it is easy to confirm that $f'(z^c) = -f(z^c) \times (z^c - f(z^c))$ and since $-f(z^c)$ is positive for all z^c, it follows that $z^c - f(z^c)$ must be negative for z^c close to but below z^{RNc} and $z^c - f(z^c)$ must be positive for z^c close to but above z^{RNc}. Property 5 obviously generalizes these last two local conditions.

[9] See Theorems 7 and 11, along with footnote 18.

denote investors' perceptions of the mean and variance of $\tilde{x}$ conditional on the public information set Ω.

There are now K homogenously informed risk-averse investors $k = 1, 2, ..., K$. Investor k has CARA preferences $-e^{-\gamma_k c_k}$, where c_k denotes the investor's end of period consumption and $\gamma_k > 0$ is investor $k's$ risk aversion parameter. These investors' risk aversion parameters need not all be the same. As usual, we define the aggregate risk aversion parameter γ by: $\frac{1}{\gamma} = \Sigma_{k=1}^{K} \frac{1}{\gamma_k}$. Each investor k has to choose what fraction of the firm to purchase, with the remainder of the investor's wealth invested in a risk-free, zero interest bond. If investor k starts with initial wealth W_k and purchases fraction θ_k of the firm when the price of the whole firm is P and the firm's CF turn out to have realized value x, investor k consumes $c_k = W_k + \theta_k(x - P)$ at the end of the period. Thus, given public information set Ω, investor $k's$ portfolio problem consists of choosing θ_k to maximize

$$E[-e^{-\gamma_k(W_k + \theta_k(\tilde{x} - P))}|\Omega]. \tag{8}$$

All investors are assumed to behave competitively on the securities market, i.e., they are price takers. The manager of the firm does not participate on the securities market.

If the manager of the firm discloses estimate v, we appeal to standard results in the asset pricing literature to conclude, first, that investor $k's$ expected utility in (8) is given by

$$-e^{-\gamma_k(W_k + \theta_k(E[\tilde{x}|v] - P) - .5\gamma_k \theta_k^2 var(\tilde{x}|v))}; \tag{9}$$

second, that investor $k's$ asset demand function for the firm's shares is given by $\theta_k(P|v) = \frac{E[\tilde{x}|v] - P}{\gamma_k var(\tilde{x}|v)}$; and third, by the market-clearing requirement $\Sigma_k \theta_k(P|v) = 1$, the equilibrium price for the firm is given by $P^d(v) = E[\tilde{x}|v] - \gamma var(\tilde{x}|v)$. Standard Bayesian updating yields:

$$\tilde{x}|_{\tilde{v}=v}\tilde{\ } N(m + \frac{\sigma_x^2}{\sigma_v} \times \frac{v - m}{\sigma_v}, \alpha\sigma_x^2), \text{ where } \alpha \equiv \frac{\sigma_\varepsilon^2}{\sigma_v^2}. \tag{10}$$

We note for future reference that this Bayesian updating implies that the manager's disclosure is "variance-reducing," i.e., that investors' perceptions of the variance of the manager's firm's CF conditional on her disclosure of her estimate is lower than investors' prior (beginning of period) perceptions of this variance, as (obviously):

$$\alpha\sigma_x^2 < \sigma_x^2. \tag{11}$$

We conclude that the equilibrium price of the firm in the event its manager discloses v can be expressed as $P^d(v) = m + \frac{\sigma_x^2}{\sigma_v} \times \frac{v-m}{\sigma_v} - \gamma\alpha\sigma_x^2$, or, employing the standardized notation $z = \frac{v-m}{\sigma_v}$, as:

$$P^d(z) = m + \frac{\sigma_x^2}{\sigma_v} \times z - \gamma\alpha\sigma_x^2. \tag{12}$$

Here, $\gamma\alpha\sigma_{x_i}^2$ is the risk-premium to which the disclosing firm is subject.

Clearly, the price $P^d(z)$ is increasing in z . Thus, as in the risk-neutral case, the manager's preferred disclosure policy is described by some cutoff z^c, and the "no disclosure" price, which we write as $P^{nd}(z^c)$, will depend on what this cutoff is. Also analogous to the risk-neutral case, an equilibrium cutoff satisfies $P^d(z^c) = P^{nd}(z^c)$ (which we sometimes write alternatively in terms of the regular cutoff v^c).

We next investigate how the firm's equilibrium "no disclosure" price is determined given that the manager of the firm makes no disclosure and given investors believe the manager uses disclosure cutoff v^c. To do that, we pick an investor, say k, and evaluate investor $k's$ expected utility (8) from purchasing fraction θ_k of the firm's shares when the price of (all of) the firm is P and the firm's manager makes no disclosure. We show in the appendix[10] that investor $k's$ expected utility under these circumstances can be expressed as:

$$-\frac{e^{-\gamma_k(W_k+\theta_k(m-P)-.5\gamma_k\theta_k^2\sigma_x^2)}}{1-p+p\Phi(\frac{v^c-m}{\sigma_v})} \times \left(1-p+p\Phi(\frac{v^c-m}{\sigma_v}+\gamma_k\theta_k\frac{\sigma_x^2}{\sigma_v})\right). \tag{13}$$

To explain how this expression is derived, we note that, when the manager of the firm makes no disclosure, investors do not know why the manager made no disclosure. They know that, with probability $\frac{1-p}{1-p+p\Phi(\frac{v^c-m}{\sigma_v})}$, the reason the manager did not make a disclosure is that the manager did not receive information. If that is the case, investor k, by buying fraction θ_k of the firm shares, anticipates receiving expected utility $E[-e^{-\gamma_k(W_k+\theta_k(\tilde{x}-P))}]$ from doing so, where

[10] Under the heading "Proof of Expression 29." (The proof is presented for the case where investors can purchase a porfolio of risky securities, rather than just a single risky security.)

the expectation over the firm's CF $\tilde{x}$ here is based on investors' (common) initial prior beliefs. Investors also know that with probability $\frac{p\Phi(\frac{v^c-m}{\sigma_v})}{1-p+p\Phi(\frac{v^c-m}{\sigma_v})}$, the reason the manager made no disclosure was because she was withholding an estimate that was below the threshold v^c. If that was the case, then investor k, by buying fraction θ_k of the firm shares, anticipates receiving expected utility $E[-e^{-\gamma_k(W_k+\theta_k(\tilde{x}-P))}|\tilde{v} < v^c]$.[11] Expression (13) combines the two preceding observations. It calculates the expected utility to investor k of this "lottery over lotteries" that arises because the investor cannot be sure of which of: not receiving information or deliberately withholding bad information, explains why the manager made no disclosure.[12]

Notice that the expression (13) for investor $k's$ portfolio-and-price contingent expected utility conditional on no disclosure is the same as the expression for his expected utility were it calculated based just on his initial prior beliefs regarding the distribution of the firm's CF, apart from the adjustment for the factor $\frac{1-p+p\Phi(\frac{v^c-m}{\sigma_v}+\gamma_k\theta_k\frac{\sigma_x^2}{\sigma_v})}{1-p+p\Phi(\frac{v^c-m}{\sigma_v})}$.

Then, by going through the essentially the same steps as we did above in the case where the manager of the firm disclosed her estimate (first, deriving an asset demand function for each investor, and then, determining the equilibrium price of a security by imposing the "market clearing" requirement $\Sigma_k\theta_k(P^{nd}|$no disclosure, $v^c) = 1$), we obtain the equilibrium "no disclosure" price for the firm given its manager made no disclosure and given that investors believe the manager uses a disclosure policy defined by the cutoff v^c to be:[13]

$$P^{nd}(v^c) = m - \gamma\sigma_x^2 - \frac{p\phi(\frac{v^c-m}{\sigma_v} + \frac{\gamma\sigma_x^2}{\sigma_v})\frac{\sigma_x^2}{\sigma_v}}{1-p+p\Phi(\frac{v^c-m}{\sigma_v} + \frac{\gamma\sigma_x^2}{\sigma_v})}. \tag{14}$$

This is a natural price: had the manager of the firm never received (and hence never disclosed) any information, then its equilibrium market price would be

[11] Obtaining a closed form expression for this last expected utility entails knowing how to compute the moment-generating function of a truncated normal random variable, something we believe was developed originally in Tallis [1961] and which was exploited in accounting first by Jorgensen and Kirschenheiter [2003].

[12] Since (13) is a special case of the corresponding multivariate results (29) below, we do not provide a separate proof of (13) in the appendix.

[13] We do not prove this univariate result in the Appendix. Rather, we prove the multivariate counterpart of it, (31), from which expression (14) immediately follows.

given by:

$$P^{\text{noinfo}} \equiv m - \gamma\sigma_x^2.$$

So, when the manager does sometimes receive information, the price of the firm given the manager makes no disclosure is almost the same as the preceding "no information" price, apart from the adjustment for the last term $\frac{p\phi(\frac{v^c-m}{\sigma_v}+\frac{\gamma\sigma_x^2}{\sigma_v})\frac{\sigma_x^2}{\sigma_v}}{1-p+p\Phi(\frac{v^c-m}{\sigma_v}+\frac{\gamma\sigma_x^2}{\sigma_v})}$.

A standardized equilibrium cutoff for the model of this section, denoted by z^{RAc} ("RA" for "risk-averse"), is defined as the obvious analogue to corresponding definition of the risk-neutral model in the previous section: as that cutoff that satisfies:

$$P^d(z^{RAc}) = P^{nd}(z^{RAc}). \tag{15}$$

Similar to how algebraic manipulation of equilibrium equation (3) led to its reformulation as equilibrium equation (5), algebraic manipulation of equilibrium equation (15) leads to:[14]

$$z^{RAc} + \frac{\gamma\sigma_x^2}{\sigma_v} = f(z^{RAc} + \frac{\gamma\sigma_x^2}{\sigma_v}). \tag{16}$$

Notice that this last equation shows that the sum $z^{RAc} + \frac{\gamma\sigma_x^2}{\sigma_v}$ is a fixed point of the function $f(\cdot)$. Since we know from the risk-neutral section above that z^{RNc} is the unique fixed point of $f(\cdot)$, it must follow that:

Theorem 3 *The equilibrium cutoff when there is a single firm and investors are risk-averse exists, is unique, and it is given by:*

$$z^{RAc} = z^{RNc} - \frac{\gamma\sigma_x^2}{\sigma_v}.$$

According to this theorem, to find the standardized equilibrium cutoff of the risk-averse model all one has to do is take the standardized equilibrium cutoff of the univariate risk-neutral model and subtract $\frac{\gamma\sigma_x^2}{\sigma_v}$ from it.

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[14] Just note that, written out, equation (15) equals:

$$m + \frac{\sigma_x^2}{\sigma_v} \times z^c - \gamma\alpha\sigma_x^2 = m - \gamma\sigma_x^2 + f(z^c + \frac{\gamma\sigma_x^2}{\sigma_v})\frac{\sigma_x^2}{\sigma_v}.$$

Then, moving the term $\gamma\sigma_x^2$ to the LHS and recognizing that $1 - \alpha = \frac{\sigma_x^2}{\sigma_v^2}$, and cancelling common terms from both sides of the resulting equation yields (16).

The tight connection displayed between z^{RAc} and z^{RNc} in Theorem 3, combined with the fact noted in the risk-neutral section above that z^{RNc} is strictly decreasing in p but is otherwise independent of all other parameters of the model, immediately yields the following comparative statics results.

Corollary 4 *In the model of this section, under an equilibrium disclosure policy, the equilibrium probability the manager discloses her information, conditional on receiving information, is:*

(a) strictly increasing in p;

(b) strictly increasing in σ_x^2;

(c) strictly decreasing in σ_ε^2;

(d) strictly increasing in γ.

All of these results are intuitive. Part (a) asserts that if it becomes more probable the manager received information, then the manager must disclose her information, when she receives it, more often. This extends one of the fundamental results of Dye [1985] and Jung and Kwon [1988] to the case of risk-averse investors. Part (b) can be interpreted as implying that voluntary disclosures are a substitute for other information sources, because investors' prior beliefs about the variance of a firm's CF will vary inversely with other, pre-voluntary disclosure, information investors receive about the firm's CF. The assertion in part (b) is supported empirically: see, e.g., Breuer, Hombah, and Muller [2016]. We recall from the previous section that the conclusion of part (b) does *not* hold when investors are risk-neutral. Part (b) also implies that the equilibrium "regular" cutoff, defined in terms of the equilibrium standardized cutoff via the transformation (7) as $v^{RAc} \equiv m + \sigma_v \times z^{RAc}$, is strictly decreasing in σ_x^2. This is consistent with Jung and Kwon's [1988] Proposition 3 that asserts that when one distribution of CF is replaced by another which second-order stochastically dominates the first, the disclosure threshold increases.[15] Part (c)

[15] We thank the referee for suggesting we make this connection to Jung and Kwon[1988].

To verify the claim here, first note that the assertion in part (b) is equivalent to the statement that the standardized cutoff z^{RAc} is strictly decreasing in σ_x^2. Recalling the tranformation (7), it follows that $\frac{\partial v^{RAc}}{\partial \sigma_x^2} = \sigma_v \times \frac{\partial \ z^{RAc}}{\partial \sigma_x^2} + z^{RAc} \times \frac{\partial \sigma_v}{\partial \sigma_x^2}$. Since z^{RAc} is negative and $\frac{\partial \sigma_v}{\partial \sigma_x^2}$ is positive, it follows that both of the components of $\frac{\partial v^{RAc}}{\partial \sigma_x^2}$ are negative and thus $\frac{\partial v^{RAc}}{\partial \sigma_x^2}$ is negative.

asserts that if the manager of the firm knows more - i.e., receives a more precise estimate - then she is inclined to disclose the information she receives more often. Part (d) indicates that increases in investors' risk-aversion also induce the firm's manager to disclose her information more often. We shall return to provide more intuition for this result later in this section.

Next, notice that the firm equilibrium no disclosure price (14) evaluated at the equilibrium cutoff z^{RAc} can be written in the following simple way:[16]

$$P^{nd}(z^{RAc}) = m - \gamma\sigma_x^2 + z^{RNc} \times \frac{\sigma_x^2}{\sigma_v}. \tag{17}$$

$\gamma\sigma_x^2 - z^{RNc} \times \frac{\sigma_x^2}{\sigma_v}$ is the equilibrium risk-premiun in this model (recall that z^{RNc} is negative). Thus, the equilibrium no disclosure price evaluated at the equilibrium cutoff when investors are risk-averse can be written entirely in terms of the *risk-neutral* equilibrium cutoff z^{RNc}. Recalling that z^{RNc} is independent of all parameters of the model other than p, and that z^{RNc} is strictly decreasing in p, we can immediately obtain the following additional comparative statics.

Corollary 5 *When investors are risk-averse and the manager of firm i adopts an equilibrium disclosure policy, the equilibrium "no disclosure" price of the firm when the manager makes no disclosure is:*

(a) strictly decreasing in p;

(b) strictly decreasing in σ_x^2;

(c) strictly increasing in σ_ε^2;

(d) strictly decreasing in γ.

While the comparative statics on display here are intuitive, we defer discussion of them until we get to the multivariate model of the next section, where the price-related comparative statics are similar but richer.

We now turn to consider how investors' beliefs about the riskiness of the firm's CF evolve from their initial priors upon witnessing its manager make no disclosure. We let $var(\tilde{x}|nd, z^c)$ denote investors' beliefs about the variance of

[16] This follows because when the last term in (14), $-\frac{p\phi(\frac{v^c-m}{\sigma_v}+\frac{\gamma\sigma_x^2}{\sigma_v})\frac{\sigma_x^2}{\sigma_v}}{1-p+p\Phi(\frac{v^c-m}{\sigma_v}+\frac{\gamma\sigma_x^2}{\sigma_v})}$, is evaluated at z^{RAc}, the result is $f(z^{RNc})$, and then recall that z^{RNc} is the fixed point of $f(\cdot)$.

the firm's CF conditional on no disclosure by its manager and use of the cutoff z^c. In the appendix we show that:[17]

$$var(\tilde{x}|nd, z^c) = \eta(z^c) \times \sigma_x^2, \tag{18}$$

where:

$$\eta(z^c) \equiv 1 + \frac{\sigma_x^2}{\sigma_v^2} \times f(z^c) \times (z^c - f(z^c)). \tag{19}$$

(19) shows that investors' posteriors regarding the variance of the firm's CF conditional on observing no disclosure by the manager is their unconditional (or prior) perceptions of this variance scaled up (or scaled down) by the factor $\eta(z^c)$, which depends on what cutoff disclosure policy z^c the manager adopts. If the manager adopts the cutoff policy $z^c = z^{RNc}$, it is immediate from inspecting (19) that $\eta(z^{RNc}) = 1$, since z^{RNc} is the unique fixed point of $f(\cdot)$. Thus, we can conclude: *if investors are risk-neutral, then nondisclosure is "variance-neutral," i.e., $var(\tilde{x}|nd, z^{RNc}) = \sigma_x^2$.* Alternatively put, when investors are indifferent toward risk and the manager adopts an equilibrium disclosure policy mindful of that indifference, the equilibrium disclosure policy has the property that investors' perceptions about the riskiness of the firm remain unchanged upon observing the manager's nondisclosure.[18]

This result regarding risk-neutral investors is a useful reference point in trying to evaluate investors' interpretation of the manager's nondisclosure when investors are risk-averse. To analyze the situation with risk-averse investors, we begin with the following lemma.

Lemma 6 *$var(\tilde{x}|nd, z^c)$ is larger than σ_x^2 when $z^c < z^{RNc}$, and is smaller than σ_x^2 when $z^c > z^{RNc}$.*

[17] This assertion is a special case of Lemma 13 below; the proof of that lemma may be found in the Appendix.

[18] For those readers familiar with Acharya et al [2011], some further insight into why this result obtains follows from their "minimum" principle. As was noted at the end of the previous section, that principle states, in the notation of our model: $z_i^{RNc} = \arg\min_{z_i^c} f(z_i^c)$. Of course, this immediately implies that $f'(z_i^{RNc}) = 0$. Since the derivative $f'(z_i^c)$ can be written as: $f'(z_i^c) = -f'(z_i^c) \times (z_i^c - f'(z_i^c))$, it follows that the scale factor $\eta(z_i^c)$ can be written as: $\eta(z_i^c) \equiv 1 - \frac{\sigma_{x_i}^2}{\sigma_{v_i}^2} \times f'(z_i^c)$. Thus, when $z_i^c = z_i^{RNc}$, we see: $\eta(z_i^{RNc}) = 1 - \frac{\sigma_{x_i}^2}{\sigma_{v_i}^2} \times f'(z_i^{RNc}) = 1$, which renders investors' posterior (conditional on no disclosure) perceptions of the variance of the firm's future cash flows the same as their priors.

The proof of this lemma follows almost immediately from point 5 at the end of the last section, when combined with Theorem 3 and the functional form of $\eta(z^c)$ in (19). Specifically, since point 5 asserts that $z^c - f(z^c) < 0$ whenever $z^c < z^{RNc}$ and Theorem 3 proves that $z^{RAc} < z^{RNc}$, it follows since $f(\cdot) < 0$, that $f(z^{RAc})(z^{RAc} - f(z^{RAc}))$ is positive, and hence $\eta(z^{RAc}) > 1$.

Combining Lemma 6 with Theorem 3, we conclude that:

Theorem 7 *If investors are risk-averse and the manager of the firm employs the equilibrium cutoff for such investors, then nondisclosure is variance-increasing, i.e., investors' perceptions of the variance of the firm CF conditional on no disclosure by the manager are higher than their prior (beginning of period) perceptions of this variance.*

This theorem is surprising because, by the so-called "law of total variance,"[19] for any unknown random variable with finite variance (such as a firm's CF), accumulating information about the random variable (such as learning that a firm's manager is not going to make a disclosure) must reduce the *expected* value of the variance of the random variable. That is, investors' learning about anything must, on average, be variance-reducing.

To explain this result, we start by observing that when the manager of the firm makes no disclosure, investors perceive the distribution of the firm's CF to be a mixture of two distributions, one based on the manager's estimate being drawn from a truncated-from-above by the cutoff v^c (and hence skewed left) normal distribution - applicable when she withholds an unfavorable estimate she receives from investors - and one based on the manager not having received information and hence being the symmetric normal distribution corresponding to investors' initial priors. These two components of the mixture distribution become more dissimilar from each other as the cutoff v^c (or equivalently, the standardized cutoff z^c) shifts farther and farther left. When the equilibrium cutoff is far enough left, the components of the mixture distribution are sufficiently dissimilar from each other that investors are more uncertain about the firm's CF after learning of the manager's nondisclosure than they were at the start of the period. In fact, as Lemma 6 reports, this variance-increasing effect

[19] See, e.g., Weiss [2005].

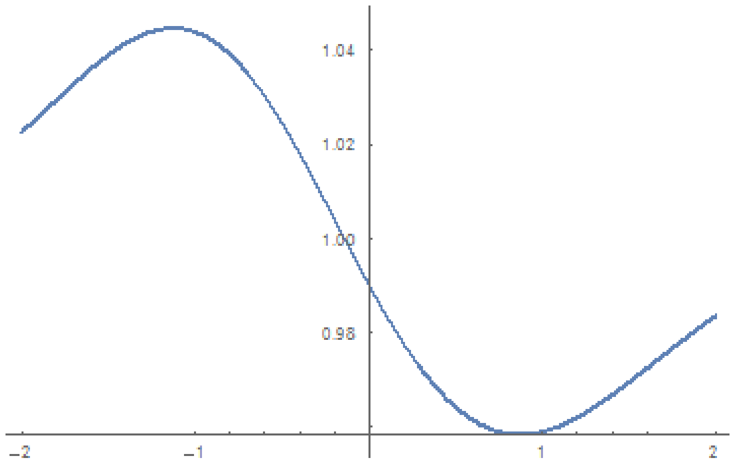
of nondisclosure obtains as long as the cutoff the manager uses is below the equilibrium cutoff for risk-neutral investors. Since it is clear why the equilibrium cutoff when investors are risk-averse must be to the left of the equilibrium risk-neutral cutoff (risk-averse investors penalize the firm for nondisclosure more than risk-neutral investors do because they have to be compensated more for the uncertainty nondisclosure imposes on them, and so the manager of the firm responds by disclosing information more often when facing risk-averse investors to reduce the incurrence of this penalty (as Corollary 5 (d) confirms)), it follows that nondisclosure must be variance-increasing in equilibrium when investors are risk-averse.

An obvious consequence of Theorem 7 is that:

$$var(\tilde{x}|nd, z^{RAc}) > \alpha\sigma_x^2.$$

Since the RHS of this last inequality is investors' perceptions of the variance of the firm's CF conditional on the manager's disclosure of her estimate (documented in (10) above), we conclude that nondisclosure by the manager is also *contemporaneously* variance-increasing relative to disclosure. Thus, at the time a manager is choosing whether to disclose her information, she must recognize and account for the fact that, if she decides not to disclose an estimate she possesses, she will be increasing investors' uncertainty about her firm's CF relative to their uncertainty had she disclosed her estimate. This fact suggests, and Corollary 4(d) confirms, that were investors to become more sensitive to risk, i.e., were the aggregate risk parameter γ to increase, then the manager of the firm optimally will respond by disclosing the information she receives more often.

It is important to emphasize that Theorem 7 is not a statistical result: it does not hold for all cutoff policies. It is guaranteed to hold only for equilibrium cutoffs. While this point is confirmed by Lemma 6, it may be helpful to see a visual display of this assertion. In the figure below, we display a plot of the posterior variance of investors' perceptions of the firm's CF conditional on no disclosure as a function of the cutoff z^c, for the case where $p = .3$, $\sigma_x^2 = 1$, $\sigma_v^2 = 2$. Note that there are regions of the cutoff where this posterior variance is below one, which is the prior variance of the firm's CF, as well as regions of the cutoff where the posterior variance is above one. This is consistent with Lemma 6.



Plot of Investors' Perceptions of Variance of Firm $i's$ Future Cash Flows Conditional on No Disclosure ($p = .3$, $\sigma^2_{x_i} = 1$, $\sigma^2_{v_i} = 2$)

At this point, we could also exhibit results concerning the firm's cost of capital. However, we defer presenting cost of capital results to Section 6, where we consider a sometimes-disclosing firm's cost of capital in the presence of other firms, both because the results in that section generalize any results that we could have presented in the univariate model of this section, and also because the general results we obtain concerning a firm's cost of capital are just as intuitive and easy to understand as the univariate results we could have displayed here.

We close this section by observing that, rather than developing the model "from the ground up" as we have in this section, by starting with investors' equilibrium asset demand functions, and then determining a firm's equilibrium disclosure-contingent prices via the market-clearing requirement given those investors' equilibrium asset demand functions, we could have started the study of voluntary disclosures with risk-averse investors simply by positing that the price of the firm is of "mean-variance" form: $P(\Omega) = E[\tilde{x}|\Omega] - \gamma var(\tilde{x}|\Omega)$. That is, we could have, as an alternative to our approach, begun by positing that, when the manager discloses estimate v, the price for the firm is set at $P(\{v\}) = E[\tilde{x}|v] - \gamma var(\tilde{x}|v)$, in place of (12), and that, when the manager makes no disclosure, the price for the firm is set at $P(\{\text{no disclosure}\}) = E[\tilde{x}|\text{no disclosure}] - \gamma var(\tilde{x}|\text{no disclosure})$, in place of (14). Here, we briefly assess how much of a change such an alternative, reduced form, pricing approach would have had on our analysis and conclusions.

The first thing to note is that, when the manager discloses her estimate, a mean-variance approach to pricing exactly coincides with our approach, since (12) *is* a mean-variance price. In contrast, when the manager makes no disclosure, our approach and a mean-variance pricing approach are not equivalent, since the "no disclosure" price (14) is not a mean-variance price. But, we now demonstrate that it is close to a mean-variance price. To see this, first observe that, written using a standardized cutoff, the "no disclosure" asset pricing equation (14) can be expressed as

$$P^{nd}(z^c) = m - \gamma\sigma_x^2 + f(z^c + \frac{\gamma\sigma_x^2}{\sigma_v})\frac{\sigma_x^2}{\sigma_v}. \tag{20}$$

In the accompanying footnote, we show that a first-order Taylor approximation of (20) is given by:[20]

$$P^{nd}(z^c) \approx m - \gamma\sigma_x^2 + f(z^c) \times (1 - \frac{\gamma\sigma_x^2}{\sigma_v}(z^c - f(z^c))) \times \frac{\sigma_x^2}{\sigma_v}. \tag{21}$$

Now, contrast this approximation with what a mean-variance pricing approach would yield when the manager makes no disclosure and investors posit that the manager adopted a disclosure policy defined by this same cutoff z^c. In the appendix,[21] we show that $E[\tilde{x}|nd, z^c]$, the expected value of the firm's CF conditional on no disclosure and use of the cutoff z^c, can be expressed as: $E[\tilde{x}|nd, z^c] = m + \frac{\sigma_x^2}{\sigma_v} \times f(z^c)$. From our work with conditional variances above, we already know $var(\tilde{x}|nd, z^c) = \sigma_x^2 \times (1 + \frac{\sigma_x^2}{\sigma_v^2} \times f(z^c) \times (z^c - f(z^c)))$. Combining these last two observations, we see that the "mean-variance" price of the firm conditional on the manager's nondisclosure and use of the cutoff z^c

[20] The derivative of f can be written as: $f'(z^c) = -f(z^c) \times (z^c - f(z^c))$. Hence, if we were to approximate $f(z^c + \frac{\gamma\sigma_x^2}{\sigma_v})$ in (20) by a first-order Taylor expansion around z^c, we would obtain

$$f(z^c + \frac{\gamma\sigma_x^2}{\sigma_v}) \approx f(z^c) + f'(z^c) \times \frac{\gamma\sigma_x^2}{\sigma_v} = f(z^c) - \frac{\gamma\sigma_x^2}{\sigma_v} \times f(z^c)(z^c - f(z^c)).$$

Substituting this into (20), we get (21).

[21] See (48) in the Appendix for the general expresion for the conditional mean $E[\tilde{x}|nd, v^c]$.

is given by:

$$\begin{aligned} P^{nd\text{meanvariance}}(z^c) &= E[\tilde{x}|nd, z^c] - \gamma \times var(\tilde{x}|nd, z^c) \\ &= m + \frac{\sigma_x^2}{\sigma_v} \times f(z^c) - \gamma \times \sigma_x^2 \times (1 + \frac{\sigma_x^2}{\sigma_v^2} \times f(z^c) \times (z^c - f(z^c))). \end{aligned}$$

By rearranging this last expression, we see that it is exactly the same as (21). Thus, the first-order Taylor approximation of $P^{nd}(z^c)$ is the mean-variance "no disclosure" price using this same cutoff.

In view of this relationship between mean-variance prices and the prices that arise under our approach, both when the manager discloses her information and when she makes no disclosure, one might expect to obtain similar results for a mean-variance pricing formulation of the model as for the formulation of the model we have adopted. In fact, many of the results in this paper are the same, regardless of whether we set up the model using this mean-variance pricing formulation or whether we use the formulation we started the paper with. For example, the variance-increasing effects of nondisclosure described above in Theorem 7 hold, without change, for the mean-variance pricing formulation of the model. For reasons of space, we shall not present the proof corresponding to this, or other parallel, mean-variance theorems, but as the paper progresses, we will remark on other instances where the stated results hold for both formulations of the model. This provides some evidence of both the robustness of our results to perturbations in assumptions and also to the possible advantages to a heuristic, reduced form, approach in some settings.[22]

4 The Multivariate Model of Voluntary Disclosure with Risk-Averse Investors

The model in this section expands the model of the previous section to incorporate a securities market on which n firms' shares can be traded. These n firms are indexed by $j = 1, ..., n$. For each $j = 1, .., n$, firm $j's$ CF are the realization x_j of the random variable $\tilde{x}_j$. These n random variables $\tilde{x}_j$, $j = 1, 2, ..., n$ are jointly normal, and they realize their values after the securities market closes.

[22] Hanson and Ladd [1991] present other findings that also suggest the reasonableness of mean-variance approximations when working with truncated normals.

We now call the firm whose manager's disclosure decisions are the focus of attention firm i. To keep the model tractable, we do not entertain the possibility that the managers of firms other than firm i privately receive any information or face any disclosure decisions during the period we study. Apart from the introduction of these other firms, the model parallels as closely as possible the model in the previous section.

The private estimate the manager of firm i sometimes receives is now denoted by v_i. This estimate is the realization of $\tilde{v}_i = \tilde{x}_i + \tilde{\varepsilon}_i$. The manager's disclosure decision is now described by a cutoff policy v_i^c (or its standardized equivalent z_i^c). After the manager has made her disclosure decision, a securities market opens on which all n firms' shares can be traded. On this securities market, investors exchange rights to the CF of the n firms. We let $\tilde{\mathbf{x}}$ denote the $nx1$ vector of random variables describing all n firms' CF, with $\tilde{x}_j$ being the jth component of this vector. We take the priors on $\tilde{\mathbf{x}}$ to have mean $\mathbf{m}$, with jth component $m_j \equiv E[\tilde{x}_j]$, and nxn covariance matrix $\mathbf{S}$, with its $(i,j)th$ component denoted either by $cov(\tilde{x}_i, \tilde{x}_j)$ or σ_{ij}. $cov(\tilde{\mathbf{x}},\tilde{v}_i)$ denotes the $nx1$ (column) vector of the covariances between $\tilde{x}_j$ and $\tilde{v}_i$.[23] The public information set Ω_i consists of $\{v_i\}$ if the manager of firm i discloses v_i and otherwise it consists of the fact that the manager of firm i made no disclosure. The notation $cov(\tilde{x}_i, \tilde{x}_j|\Omega_i)$ denotes investors' perceptions of the covariance between $\tilde{x}_i$ and $\tilde{x}_j$ conditional on information set Ω_i.

Investors are exactly the same as in the model of the previous section, except here each investor k with initial wealth W_k now has to choose what portfolio $\boldsymbol{\theta}_k$ (an $nx1$ vector with jth component θ_k^j denoting the fraction of firm j that investor k buys) of the n firms to purchase when the prices of the (whole) n firms are given by $\mathbf{P}$ (an $nx1$ vector with jth component P_j), with the remainder of the investor's wealth $W_k - \boldsymbol{\theta}_k'\mathbf{P}$ invested in a risk-free, zero interest bond. Given Ω_i, investor $k's$ portfolio problem consists of choosing the portfolio $\boldsymbol{\theta}_k$ to maximize

$$E[-e^{-\gamma_k(W_k+\boldsymbol{\theta}_k'(\tilde{\mathbf{x}}-\mathbf{P}))}|\Omega_i]. \tag{22}$$

[23]Since the error term $\tilde{\varepsilon}_i$ in $\tilde{v}_i$ is assumed to be independent of all other variables in the model for each $i = 1, 2, ..., n$, this vector of covariances is the same as the vector of covariances $cov(\tilde{\mathbf{x}},\tilde{x}_i)$.

Suppose the manager of firm i discloses v_i. Denote by $\mathbf{m}(v_i)$ and $\mathbf{S}(v_i)$ investors' updated perceptions of the mean (vector) and covariance matrix of $\tilde{\mathbf{x}}$. We appeal to standard results in the asset pricing literature to conclude, first, that if investor k chooses portfolio $\boldsymbol{\theta}_k$, his expected utility is given by

$$-e^{-\gamma_k(W_k+\boldsymbol{\theta}_k'(\mathbf{m}(v_i)-\mathbf{P})-.5\gamma_k\boldsymbol{\theta}_k'\mathbf{S}(v_i)\boldsymbol{\theta}_k)};$$

second, that investor $k's$ asset demand function for firms' shares is given by $\boldsymbol{\theta}_k(\mathbf{P}|v_i) = \frac{1}{\gamma_k} \times \mathbf{S}(v_i)^{-1}(\mathbf{m}(v_i)-\mathbf{P})$. Third, by the market-clearing requirement $\Sigma_{k=1}^K \boldsymbol{\theta}_k(\mathbf{P}^d(v_i)|v_i) = \mathbf{1}$, the equilibrium prices of the firms are given by

$$\mathbf{P}^d(v_i) = \mathbf{m}(v_i) - \frac{1}{\gamma} \times \mathbf{S}(v_i)\mathbf{1}, \tag{23}$$

where standard Bayesian updating yields:

$$\mathbf{m}(v_i) \equiv E[\tilde{\mathbf{x}}|v_i] = \mathbf{m}+cov(\tilde{\mathbf{x}},\tilde{v}_i) \times \frac{v_i - m_i}{\sigma_{v_i}^2} \tag{24}$$

and

$$\mathbf{S}(v_i) \equiv \mathbf{S} - \frac{1}{\sigma_{v_i}^2} \times cov(\tilde{\mathbf{x}},\tilde{v}_i) \times cov(\tilde{\mathbf{x}},\tilde{v}_i).' \tag{25}$$

Thus, for any $h,\, j \in \{1, 2, ..., n\}$, the posterior covariance matrix $\mathbf{S}(v_i)$ of $\tilde{\mathbf{x}}$ has $(h, j)^{th}$ element $\sigma_{hj} - \frac{\sigma_{hi}\sigma_{ij}}{\sigma_{v_i}^2}$.

Letting $\mathbf{1}$ be a column of n ones, and $\mathbf{1}_i$ be a column of n zeroes except for a "1" in the ith place, we note for future reference that an implication of (25) is that:

$$\mathbf{1}_i'\mathbf{S}(v_i)\mathbf{1} = \alpha \times \Sigma_{j=1}^n \sigma_{ij}, \text{ where } \alpha \equiv 1 - \frac{\sigma_{x_i}^2}{\sigma_{v_i}^2}. \tag{26}$$

The basis of this result is the simple statistical fact that observation of the manager's disclosure of the estimate $\tilde{v}_i = v_i$ shrinks investors' perceptions of both the variance of $\tilde{x}_i$ and the covariance between $\tilde{x}_i$ and $\tilde{x}_j$ by the constant α, for any $j = 1, 2, ..., n$ and any v_i; that is:

$$cov(\tilde{x}_i, \tilde{x}_j|v_i) = \alpha\sigma_{ij}. \tag{27}$$

We conclude that the equilibrium price of firm i in the event its manager discloses v_i is given by $P_i^d(v_i) = \mathbf{1}_i'\mathbf{P}^d(v_i) = m_i + \frac{\sigma_{x_i}^2}{\sigma_{v_i}} \times \frac{v_i - m_i}{\sigma_{v_i}} - \gamma\alpha\Sigma_{j=1}^n\sigma_{ij}$ or, employing the standardized value $z_i = \frac{v_i - m_i}{\sigma_{v_i}}$, as:

$$P_i^d(z_i) = m_i + \frac{\sigma_{x_i}^2}{\sigma_{v_i}} \times z_i - \gamma\alpha\Sigma_{j=1}^n\sigma_{ij}. \tag{28}$$

This price is the expected value of firm $i's$ CF conditional on the manager's disclosure of the standardized estimate z_i net of the risk-premium $\gamma\alpha\Sigma_{j=1}^n\sigma_{ij}$. Since $\alpha\Sigma_{j=1}^n\sigma_{ij} = cov(\tilde{x}_i, \Sigma_j\tilde{x}_j|v_i)$, this risk premium is simply the aggregate risk aversion parameter γ multiplied by the disclosure-contingent covariance between firm $i's$ CF and the market's aggregate cash flows.

As in the univariate model of the previous section, we conclude: the manager's preferred disclosure policy is described by some cutoff v_i^c; the "no disclosure" price $P_i^{nd}(v_i^c)$ will depend on this cutoff; and the equilibrium cutoff satisfies $P_i^d(v_i^c) = P_i^{nd}(v_i^c)$ (as above, we sometimes alternatively write a regular cutoff v_i^c in standardized form z_i^c).

We proceed now, also as in the univariate case, to investigate how firm $i's$ equilibrium "no disclosure" price is determined given that the manager of firm i makes no disclosure and given investors believe the manager uses disclosure cutoff v_i^c, by first evaluating investor $k's$ expected utility (22) from purchasing portfolio $\boldsymbol{\theta}_k$ of firms' shares when the price of (all of) the firms is $\mathbf{P}$ and firm $i's$ manager makes no disclosure. We show in the appendix[24] that investor $k's$ expected utility under these circumstances can be expressed as:

$$-\frac{e^{-\gamma_k(\boldsymbol{\theta}_k'(\mathbf{m}-\mathbf{P}^{nd})-.5\gamma_k\boldsymbol{\theta}_k'\mathbf{S}\boldsymbol{\theta}_k)}}{1-p+p\Phi(\frac{v_i^c-m_i}{\sigma_{v_i}})} \times \left(1-p+p\Phi(\frac{v_i^c-m_i}{\sigma_{v_i}}+\frac{\gamma_k s_k}{\sigma_{v_i}})\right), \qquad (29)$$

where

$$s_k = s_k(\boldsymbol{\theta}_k) \equiv \boldsymbol{\theta}_k' cov(\tilde{\mathbf{x}}, \tilde{v}_i) = \sum_{j=1}^n \theta_j^k \sigma_{ji}. \qquad (30)$$

The term s_k in (30) is the covariance between the estimate $\tilde{v}_i$ and investor $k's$ portfolio-contingent terminal cash flows $W_k + \Sigma_{j=1}^n\theta_j^k\tilde{x}_j - \Sigma_{j=1}^n\theta_j^k P_j$.

(29) is the natural mulitvariate counterpart of (13).

Next, we proceed just as we did in the previous section (recall (14)), to derive the equilibrium "no disclosure" price for firm i when investors believe the manager of firm i uses a disclosure policy defined by the cutoff v_i^c. We

[24] The proof appears under the heading "Proof of Expression 29" in the appendix.

obtain:[25]

$$P_l^{nd}(v_i^c) = m_l - \gamma \Sigma_{j=1}^n \sigma_{lj} - \frac{p\phi(\frac{v_i^c - m_i}{\sigma_{v_i}} + \frac{\gamma \sum_{j=1}^n \sigma_{ij}}{\sigma_{v_i}})\frac{\sigma_{li}}{\sigma_{v_i}}}{1 - p + p\Phi(\frac{v_i^c - m_i}{\sigma_{v_i}} + \frac{\gamma \sum_{j=1}^n \sigma_{ij}}{\sigma_{v_i}})}, \text{ for each } l = 1, 2, .., n. \tag{31}$$

These "no disclosure" prices are very natural prices: if we were studying a model where the manager of firm i never received an estimate of her firm's CF, so investors priced all firms based just on their initial priors, then the equilibrium price of any firm l would be given by:

$$P_l = m_l - \gamma \Sigma_{j=1}^n \sigma_{lj}.$$

Thus, no disclosure by the manager of firm i results in the same price for any firm l as in a model where no information were ever received or disclosed, apart for an adjustment for the last term $\frac{p\phi(\cdot)\frac{\sigma_{li}}{\sigma_{v_i}}}{1-p+p\Phi(\cdot)}$.

An equilibrium cutoff for this multivariate model with risk-averse investors, denoted as z_i^{MRAc} (*"MRA"* for "multi-firm risk-averse"), is now defined as the obvious analogue to the univariate model above: as that cutoff that satisfies $P_i^d(z_i^{MRAc}) = P_i^{nd}(z_i^{MRAc})$.

The next theorem characterizes the equilibrium cutoff for the multivariate risk-averse model in terms of the equilibrium cutoff for the univariate risk-neutral model.[26]

Theorem 8 *The standardized equilibrium cutoff of the risk-averse multivariate model of this section, z_i^{MRAc}, exists, is unique, and is given by:*

$$z_i^{MRAc} = z^{RNc} - \frac{\gamma \Sigma_{j=1}^n \sigma_{ij}}{\sigma_{v_i}}. \tag{32}$$

According to this theorem, to find the standardized equilibrium cutoff of the multivariate risk-averse model all one has to do is take the standardized equilibrium cutoff of the univariate risk-neutral model and subtract $\frac{\gamma \Sigma_{j=1}^n \sigma_{ij}}{\sigma_{v_i}}$ from it.

With this theorem and (31) in hand, and recalling that z_i^{RNc} is the unique fixed point of $f(\cdot)$, we can write the equilibrium "no disclosure" price P_l^{nd} of

[25] The proof appears under the heading "The Proof of Expression 31" in the appendix.
[26] The proof is in the appendix.

any firm $l = 1, 2, .., n$ when the manager of firm i adopts the equilibrium cutoff described in (32) as

$$P_l^{nd} = m_l - \gamma \Sigma_{j=1}^n \sigma_{lj} - C \times \sigma_{li}, \tag{33}$$

where $C \equiv \frac{-z^{RNc}}{\sigma_{v_i}}$. C is a positive constant that is common to all firms, and it depends only on the disclosure cutoff z^{RNc} and the standard deviation of the manager's private information σ_{v_i}. Thus, (33) shows that the equilibrium "no disclosure" price of each firm l is its "no information" price $m_l - \gamma \Sigma_{j=1}^n \sigma_{lj}$ adjusted for the term $C \times \ \sigma_{li}$, which is the product of a constant that is common across all firms and another term, the ex ante covariance between firm $l's$ and firm $i's$ future cash flows. This demonstrates, as was asserted in the Introduction, that these asset pricing equations yield a cross-equation restriction that is testable.

We now turn to examine comparative statics. Before stating any comparative statics, we introduce the following maintained assumption:

Assumption *Throughout the remainder of the paper, we shall restrict attention to the case where* $\Sigma_{j=1}^n \sigma_{ij} > 0$.

Adopting this assumption both avoids a proliferation of cases and confines attention to the most typical case, where the CF $\tilde{x}_i$ of firm i and the CF of the whole market $\Sigma_j \tilde{x}_j$ have positive covariance.

Comparative statics derived from the equilibrium include those in the following corollary.[27][28]

Corollary 9 *The probability in equilibrium that the manager of firm i discloses her information for the multivariate model with risk-averse investors of this section is:*

(a) strictly increasing in p;

(b) strictly decreasing in $\sigma_{\varepsilon_i}^2$;

(c) strictly increasing in γ;

[27] It should be noted that we do not present a comparative static regarding the effect of increasing $\sigma_{x_i}^2$ on the equilibrium probability the manager of firm i makes a disclosure. Such a comparative static is ambiguous in the multivariate model without knowing how large $\sigma_{\varepsilon_i}^2$ is relative to $\Sigma_{j \neq i} \sigma_{ij}$. It is easy to confirm that if $2\sigma_{\varepsilon_i}^2 + \sigma_{x_i}^2 - \Sigma_{j \neq i} \sigma_{ij}$ is positive (resp., negative), then the equilibrium probability of disclosure increases (resp., declines) in $\sigma_{x_i}^2$.

[28] These proofs are all straightforward, and so they are omitted.

(d) strictly increasing in σ_{ij} for any $j \neq i$.

These comparative statics are similar to the corresponding comparative statics of the univariate model reported in Corollary 4. Accordingly, we comment here only on the new comparative static in part (d). Part (d) establishes the result that as the covariance between firm $i's$ CF and any other firm's CF increases, voluntary disclosure occurs more often. While increases in such covariances increase the risk firm $i's$ shareholders are subject to whether or not firm $i's$ manager makes a disclosure, the increased risk shareholders of firm i are subject to attending the increase in any of these covariances is greater when the manager of the firm makes no disclosure both because, as we previously noted (recall (27) above), disclosures shrink covariances (relative to their prior values) and also because, as we shall show below (see Corollary 11 (b)), investors' perceptions of such covariances increase in absolute value attending nondisclosure (relative to their priors).

All of these comparative statics hold for the "mean-variance" pricing formulation of the model as well.

Next, we obtain the following additional testable implications concerning firms' equilibrium no disclosure prices:[29]

Corollary 10 *The equilibrium no disclosure price of any firm $l = 1, 2, ..., n$ in the multivariate model with risk-averse investors evaluated at the equilibrium cutoff, $P_l^{nd}(z_i^{MRAc})$, is:*

(a) strictly decreasing in p if σ_{li} is positive, and strictly increasing in p if σ_{li} is negative;

(b) strictly increasing in $\sigma^2_{\varepsilon_i}$ if σ_{li} is positive, and strictly decreasing in $\sigma^2_{\varepsilon_i}$ if σ_{li} is negative;

(c) strictly decreasing in γ;

(d) strictly decreasing in σ_{lj} for any j if $l \neq i$ (and if $l = i$, for any $j \neq i$).

All of these results are intuitive. Part (a) contains the new finding that as the probability p firm $i's$ manager gets information goes up, this generates an informational externality on other firms: it results in a decrease in the price of

[29] These results are easy to prove and so the proofs are omitted.

any firm l whose CF positively covary with firm $i's$ CF and an increase in the price of any firm l whose CF negatively covary with the CF of firm i. This is, of course, an example of the information transfer effects of (non)disclosure across firms, and is a topic we shall discuss further both in later parts of the present corollary and also in Section 5 below.

Part (b) considers the "no disclosure" price consequences of reducing the precision of the estimate the manager of firm i sometimes receives. As was already noted in the discussion of Corollary 9 (b), if the precision of the manager of firm $i's$ estimate decreases, then the disclosure of the estimate has a smaller impact on firm $i's$ price, and so this reduces the manager's propensity to disclose her information. This implies that, as far as firm $i's$ shareholders are concerned, nondisclosure by the manager of firm i is less of an adverse statement about the value of firm i, and so the no disclosure price of firm i increases. But, the same is also true for any firm $l \neq i$ whose CF positively covary with those of firm i, whereas the opposite is true for any firm whose CF negatively covary with firm $i's$ CF.

The intuition for part (c) is clear, and won't be further discussed. The intuition for part (d) is also clear: any increase in the covariance between firm $l's$ and any other firms' CF results in a larger risk premium associated with firm l, and hence a lower price, for that firm.

We next proceed to document how nondisclosure in equilibrium alters investors' uncertainty about firm $i's$ CF, as well as their beliefs about the covariance between firm $i's$ and firm $j's$ CF, for any $j = 1, 2, .., n$.

Theorem 11 *In equilibrium*:

(a) nondisclosure by firm i is variance-increasing, i.e., the nondisclosure by the manager of firm i increases investors' perceptions of the variance of firm i's CF relative to their initial prior beliefs about this variance;

(b) nondisclosure by firm i is covariance-increasing, i.e., the nondisclosure by the manager of firm i increases investors' perceptions of the absolute value of the covariance between firm $i's$ and firm $j's$ CF relative to their initial prior beliefs about this covariance, for any firm $j \neq i$.

Part (a) of this theorem is the multivariate counterpart of the corresponding result in the univariate case, Theorem 7, and requires no further discussion. Part (b) is new to the multivariate analysis of this section. To discuss it, we first expand on what we mean by covariance-increasing. We introduce the notation $cov(\tilde{x}_i, \tilde{x}_j|nd, z_i^c)$ for investors' perceptions of the covariance between $\tilde{x}_i$ and $\tilde{x}_j$ conditional on no disclosure by the manager of firm i when she uses the standardized cutoff z_i^c. Employing this notation, we can state part (b) more precisely as asserting that the following inequalities hold:

$$cov(\tilde{x}_i, \tilde{x}_j|nd, z_i^{MRAc}) > cov(\tilde{x}_i, \tilde{x}_j) \text{ if } cov(\tilde{x}_i, \tilde{x}_j) > 0 \text{ and} \tag{34}$$

$$cov(\tilde{x}_i, \tilde{x}_j|nd, z_i^{MRAc}) < cov(\tilde{x}_i, \tilde{x}_j) \text{ if } cov(\tilde{x}_i, \tilde{x}_j) < 0. \tag{35}$$

A rough explanation for the finding in Theorem 11 (b) is that it is the reverse of the result displayed in (27) above: conditioning on more information shrinks covariances toward zero, since if one conditions on enough information, the random variables become determined by the information being conditioned on (and hence have no covariance with anything). A more complete, but somewhat more technical, explanation for this finding relies on the following covariance-increasing counterpart to the univariate result in Lemma 6.[30]

Lemma 12 *Given a fixed, but arbitrary, cutoff v_i^c by the manager of firm i along with its standardized value $z_i^c = \frac{v_i^c - m_i}{\sigma_{vi}}$, investors' perceptions of covariance between firm i's and firm j's CF is given by:*

$$cov(\tilde{x}_i, \tilde{x}_j|nd, v_i^c) = \eta(z_i^c) \times cov(\tilde{x}_i, \tilde{x}_j), \tag{36}$$

where $\eta(z_i^c)$ is the same scaling factor originally defined in (19) above.

Combining this lemma with Lemma 6 and Theorem 8, we now see why the covariance-increasing property of nondisclosure in equilibrium, i.e., inequalities (34) and (35), must hold. Lemma 6 implies that the scale factor $\eta(z_i^c)$ is larger than 1 when $z_i^c < z^{RNc}$, and is smaller than 1 when $z_i^c > z^{RNc}$. Since we know by Theorem 8 that $z_i^{MRAc} < z^{RNc}$, it follows that $\eta(z^{MRAc}) > 1$, and so, appealing to the characterization of the conditional covariance $cov(\tilde{x}_i, \tilde{x}_j|nd, v_i^c)$ in Lemma 12, the covariance-increasing property of nondisclosure follows.

[30] The result is a special case of Lemma 13 that appears below.

Before concluding the discussion of Theorem 11, we note that both parts of this result also hold for mean-variance pricing models, exactly as stated. Here, by a "mean-variance pricing model," we mean that the price of firm i takes the following form:

$$P_i(\Omega_i) = E[\tilde{x}_i|\Omega_i] - \gamma \sum_j cov(\tilde{x}_i, \tilde{x}_j|\Omega_i). \tag{37}$$

Related, just as in the univariate risk-averse case above, one can show in the present multivariate risk-averse case that the first-order Taylor approximation of (31), written in terms of a standardized cutoff z_i^c, is exactly the same as the mean-variance price $P_i^{nd\text{meanvariance}}(z_i^c) = E[\tilde{x}_i|nd, z_i^c] - \gamma \times cov(\tilde{x}_i, \Sigma_j \tilde{x}_j|nd, z_i^c)$. Both are equal to

$$m_i - \gamma \Sigma_{j=1}^n \sigma_{lj} + \frac{\sigma_{x_i}^2}{\sigma_{v_i}} \times f(z_i^c) \times (1 - \frac{\gamma \Sigma_j \sigma_{ij}}{\sigma_{v_i}}(z_i^c - f(z_i^c))).$$

5 Information Transfers

In this section, we round out the analysis of "own" firm nondisclosure by considering its effects on investors' perceptions of the distribution of the CF of other firms, in the context of the model of Section 4. This analysis is motivated by one of the traditional concerns in accounting, that of "information transfer" between firms (discussed previously in the empirical literature by, for example, Olsen and Dietrich [1985], and in the theoretical literature by, for example, Dye [1990]), here extended to the informational impact on other firms arising from "own" firm's nondisclosure.

We start by extending our notation in the obvious fashion to any firms j and k as follows: we let $E[\tilde{x}_j|nd, v_i^c]$, $var(\tilde{x}_j|nd, v_i^c)$, and $cov(\tilde{x}_j, \tilde{x}_k|nd, v_i^c)$ respectively denote investors' perceptions of the mean and variance of firm $j's$ CF and the covariance between firm $j's$ and firm $k's$ CF conditional on no disclosure *by firm* i, when investors' believe the manager of firm i uses the disclosure policy defined by the cutoff v_i^c. Expanding the preceding notation to all n firms, we let $E[\tilde{\mathbf{x}}|nd, v_i^c]$ denote the column vector $(E[\tilde{x}_1|nd, v_i^c], E[\tilde{x}_2|nd, v_i^c], ..., E[\tilde{x}_n|nd, v_i^c])'$. We recall that we previously let $\mathbf{S}$ and $\mathbf{S}(v_i)$ denote the prior variance-covariance matrix of $\tilde{\mathbf{x}}$ and the variance-covariance matrix of $\tilde{\mathbf{x}}$ given disclosure of $\tilde{v}_i = v_i$

respectively. Analogously, we now let $\mathbf{S}(nd,z_i^c)$ denote investors' perceptions of the variance-covariance matrix of $\tilde{\mathbf{x}}$ given nondisclosure by the manager of firm i when investors perceive the manager uses disclosure cutoff z_i^c. Throughout this section, we assume that $\gamma > 0$, i.e., that investors are risk-averse.

The following lemma gives a complete description of investors' updated perceptions of all first and second moments of all firms' CF conditional on no disclosure by the manager of firm i.

Lemma 13 [31] *(a) Given any cutoff v_i^c (or its standardized value $z_i^c = \frac{v_i^c - m_i}{\sigma_{v_i}}$) used by the manager of firm i, then:*

(a)

$$E[\tilde{\mathbf{x}}|nd, v_i^c] = E[\tilde{\mathbf{x}}] + \frac{f(z_i^c)}{\sigma_{v_i}} \times cov(\tilde{\mathbf{x}}, \tilde{v}_i); \tag{38}$$

(b)

$$\mathbf{S}(nd,z_i^c) = \mathbf{S} + \frac{f(z_i^c)(z_i^c - f(z_i^c))}{\sigma_{v_i}^2} \times cov(\tilde{\mathbf{x}},\tilde{v}_i) \times cov(\tilde{\mathbf{x}},\tilde{v}_i)'. \tag{39}$$

Part (a) of Lemma 13 is the foundation for the following theorem.

Theorem 14 [32] *For any disclosure cutoff v_i^c by the manager of firm i - equilibrium or otherwise - nondisclosure by firm i reduces (resp., increases) investors' perceptions of the expected value of firm $j's$ cash flows when the unconditional covariance between firm $i's$ and firm $j's$ CF is positive (resp., negative).*

This result is intuitive: we previously documented that nondisclosure has a negative effect on investors' perceptions of the first moment of "own" firm's CF. If "own" firm's and other firms' CF positively covary, nondisclosure by "own" firm also adversely affects investors' perceptions of the expected CF of other firms too. Conversely, if two firms' CF negatively covary, then what investors perceive of as bad news regarding the expected CF of "own" firm will be perceived of as good news regarding the other firm's CF.

Next note that when the matrix in (39) is evaluated at its $(j,j)th$ component, we get:

$$var(\tilde{x}_j|nd, z_i^c) = var(\tilde{x}_j) + \frac{f(z_i^c)(z_i^c - f(z_i^c))}{\sigma_{v_i}^2} \times (cov(\tilde{x}_i, \tilde{x}_j))^2. \tag{40}$$

[31] For the proof of part (a), see 47 in the Appendix with $\tilde{x}_j$ replacing $\tilde{y}$. The proof of part (b) is in the Appendix.

[32] The proof is obvious given Lemma 13.

This equation is the foundation for the following theorem.

Theorem 15 [33] *When the unconditional covariance $cov(\tilde{x}_i, \tilde{x}_j)$ between firm i's and firm j's CF is nonzero:*

(a) investors' perceptions of the variance of firm j's cash flows conditional on no disclosure by firm i are higher (resp., lower) than their prior (unconditional) perceptions of that variance if and only if the standardized value of the cutoff z_i^c used by the manager of firm i is below (resp., above) z^{RNc};

(b) when the manager of firm i adopts the equilibrium disclosure policy z_i^{MRAc}, investors' perceptions of the variance of firm j's cash flows conditional on no disclosure by firm i are always strictly higher than their prior (unconditional) perceptions of that variance.

Theorem 15 (a) follows from point 5 at the end of Section 2, which implies that $f(z_i^c)(z_i^c - f(z_i^c))$ is positive or negative respectively depending on whether z_i^c is below or above z_i^{RNc}. Theorem 15 (b) then follows immediately from Theorem 15 (a) since we know $z_i^{MRAc} < z^{RNc}$ by Theorem 8. In part (b), we see that, when attention is confined to equilibrium disclosure policies, we get the strong conclusion that investors' perceptions of the variance of firm j's CF conditional on nondisclosure by firm i are always strictly higher than their prior perceptions of the variance firm $j's$ CF, as long as the unconditional covariance between the two firms' CF is nonzero.

Next note that when the matrix in (39) is evaluated at a general $(j,k)th$ component, we get:

$$cov(\tilde{x}_j, \tilde{x}_k | nd, v_i^c) = \sigma_{jk} + \frac{f(z_i^c)(z_i^c - f(z_i^c))}{\sigma_{v_i}^2} \times \sigma_{ij} \times \sigma_{ik}. \tag{41}$$

This last result, when combined with Theorem 8 immediately yields:

Theorem 16 *When the manager of firm i adopts the equilibrium disclosure policy z_i^{MRAc}, then for any firms j, $k \in \{1, 2.,,, n\}$:*

(a) when $\sigma_{ij} \times \sigma_{ik} > 0$, then investors' perceptions of the covariance between $\tilde{x}_j$ and $\tilde{x}_k$ conditional on nondisclosure by firm i are strictly larger than their prior perceptions of the covariance σ_{jk};

[33] The proof is also obvious given Lemma 13.

(b) when $\sigma_{ij} \times \sigma_{ik} < 0$, then investors' perceptions of the covariance between $\tilde{x}_j$ and $\tilde{x}_k$ conditional on nondisclosure by firm i is strictly smaller than their prior perceptions of the covariance σ_{jk}.

Theorem 16 generalizes each of Theorem 7, Theorem 11, and Theorem 15 above, and it allows us to generate additional results as well. For example, it can be applied to study the effects of nondisclosure on investors' perceptions of the distributions of firms' CF when the firms' future cash $\tilde{\mathbf{x}}$ have a factor structure, i.e., where the distribution of each $\tilde{x}_j$ is of the form $\tilde{x}_j = \tilde{\delta}_j + b_j \times \tilde{f}$, where $\tilde{f}$ is the common factor with variance σ_f^2 and $\tilde{\delta}_j$ is distributed $N(0, \sigma_{\tilde{\delta}_j}^2)$ and is independent of all other variables in the model, and where b_j is firm $j's$ (assumed known) factor sensitivity. With this specialization, then the unconditional covariance between $\tilde{x}_j$ and $\tilde{x}_k$ is $b_j b_k \sigma_f^2$ for any $j \neq k$. Hence, according to Theorem 16, equilibrium nondisclosure by firm i will result in investors' perceptions of the covariance between firm $j's$ and firm $k's$ CF increasing (resp., decreasing) from their prior values when b_j and b_k are of the same (resp., opposite) sign.

Adding together the conclusions of Corollary 11, part (b) of Theorem 15, Theorem 11, and Theorem 16, we now know that when investors are risk-averse, then in equilibrium, nondisclosure by the manager of firm i increases investors' perceptions of each of: 1. the variance of firm $i's$ own CF; 2. the absolute value of the covariance between firm $i's$ and any other firm's CF; 3. the variance of any other firm's CF, when the unconditional covariance between "own" firm's and other firms' CF is nonzero; and 4. the covariance between any firm $j's$ and firm $k's$ CF, for any two firms j, $k \in \{1, 2, ..., n\}$ provided the product $\sigma_{ij} \times \sigma_{ik}$ is positive.

6 Effects of Voluntary Disclosure on Firms' Costs of Capital

In this section, we derive several natural and intuitive results concerning the impact of voluntary disclosure on the cost of capital of both the sometimes-disclosing firm and other firms whose CF covary with the sometimes disclosing firm, in the context of the multivariate disclosure model of Section 4. In the following, there are two subsections, one which considers a firm's cost of capital on a pre-disclosure basis, and one which consider a firm's cost of capital on a post-disclosure basis.

6.1 Pre-disclosure Cost of Capital

In this subsection, we define the cost of capital for a firm l (not necessarily firm i) when the manager of firm i adopts cutoff z_i^c (or its "regular" cutoff equivalent v_i^c) as the expected value of its risk-premium, i.e., as the difference between the expected value of firm $l's$ CF calculated before the firm receives or discloses any information and its expected selling price.[34] That is, it is defined as:

$$CC_l(z_i^c) = m_l - \{\Pr(\text{no disclosure}) \times P_l^{nd}(z_i^c) + \Pr(\text{disclosure}) \times E[P^d(\tilde{v}_i)|\tilde{v}_i \geq v_i^c]\}.$$

In the preceding, Pr(no disclosure) and Pr(disclosure) respectively refer to the probability of no disclosure and the probability of disclosure by the manager of firm i, of course. In the appendix,[35] we show that given a set of exogenous parameter values - γ, p, σ_{ij} for all $i, j = 1, 2, ..., n$ and σ_{v_i} - if we evaluate firm $l's$ cost of capital when firm $i's$ manager adopts the unique equilibrium disclosure policy for those parameter values, then firm $l's$ cost of capital can be expressed as:

$$CC_l = \gamma \Sigma_{j=1}^n \sigma_{lj} - \frac{p\sigma_{li}}{\sigma_{v_i}} \times \left[H(z_i^{MRAc}) - H(z_i^{RNc})\right], \tag{42}$$

where $H(z) \equiv \phi(z) - (1 - \Phi(z))z$. Firm $l's$ "no information" cost of capital in the event its manager never receives (and perforce, never discloses) any information is given by $\gamma \Sigma_{j=1}^n \sigma_{lj}$. So, firm $l's$ cost of capital when firm $i's$ manager sometimes receives and optimally discloses information is just firm $l's$ "no information" cost of capital adjusted for the last term $\frac{p\sigma_{li}}{\sigma_{v_i}} \times \left[H(z_i^{MRAc}) - H(z_i^{RNc})\right]$ in (42). It is easy to show that $H(\cdot)$ is strictly decreasing and so since $z_i^{MRAc} < z_i^{RNc}$ when $\gamma \Sigma_{j=1}^n \sigma_{ij} > 0$, it follows that $\frac{p\sigma_{li}}{\sigma_{v_i}} \left[H(z_i^{MRAc}) - H(z_i^{RNc})\right]$ is positive when $\sigma_{li} > 0$ and is negative when $\sigma_{li} < 0$, and so firm $l's$ cost of capital is below (resp., above) its "no information" value when $\sigma_{li} > 0$ (resp., $\sigma_{li} < 0$). In particular, the disclosing firm's - firm $i's$ - cost of capital when its

[34] This is the definition of a firm's cost of capital used in, e.g., Cheynel [2013].
[35] The proof is presented as part of the proof of Corollary 17.

manager adopts an equilibrium disclosure policy is below its "no information" value since in the case $\sigma_{li} = \sigma_{ii} = var(\tilde{x}_i) > 0$.

The next corollary shows how firm $i's$ cost of capital changes as various parameters of the model change when the manager of firm i optimally adjusts her disclosure policy as the parameters change.

Corollary 17 *When the manager of firm i optimally adjusts her disclosure policy as exogenous parameters of the model change, then:*

(a) if p increases, then firm $i's$ cost of capital CC_i decreases;

(b) if σ_{ij} increases for any $j \neq i$, then firm $i's$ cost of capital CC_i increases;

(c) if the precision of the manager of firm $i's$ estimate increases, then firm $i's$ cost of capital CC_i decreases;

(d) if the aggregate risk-aversion parameter γ increases, then firm $i's$ cost of capital CC_i increases.

All of these comparative static results are intuitive and go in the expected direction. Also, all of these comparative statics hold for the mean-variance formulation of the model as well.

Next, we study how disclosures by the manager of firm i affects the cost of capital of any firm $l \neq i$.

Corollary 18 *When the manager of firm i optimally adjusts her disclosure policy as exogenous parameters of the model change, then for any firm $l \neq i$:*

(a) if p increases, then firm $l's$ cost of capital CC_l decreases if $\sigma_{il} > 0$ and increases if $\sigma_{il} < 0$;

(b) if σ_{ij} increases for $j \notin \{l, i\}$, then firm $l's$ cost of capital CC_l increases when $\sigma_{il} < 0$, and decreases when σ_{il} is sufficiently big;

(c) if $\sigma^2_{x_i}$ increases and $i \neq l$, then firm $l's$ cost of capital CC_l decreases when $\sigma_{il} > 0$ and increases when $\sigma_{il} < 0$;

(d) if the precision of the manager of firm $i's$ estimate decreases, then firm $l's$ cost of capital CC_l increases if $\sigma_{il} > 0$ and decreases if $\sigma_{il} < 0$;

(e) if the aggregate risk-aversion parameter γ increases, then firm $l's$ cost of capital CC_l increases if both $\sigma_{li} < 0$ and $\Sigma_{j=1}^{n} \sigma_{lj} > 0$.

For reasons of space, we discuss only two representative results here: parts (a) and (c). Regarding part (a), we know from Corollary 17 (a) that an increase in the probability p the manager of firm i receives information reduces firm $i's$ cost of capital, and so it is no surprise that an increase in p also reduces the cost of capital of any firm l whose CF positively covary with those of firm i. For firms whose CF negatively covary with firm $i's$ CF, we should not be surprised that an increase in p has the opposite effect on its cost of capital, because we previously saw in Corollary 10 (a) that the effect of an increase in p on the "no disclosure" price of firm i moves in the opposite direction from that of a firm whose CF negatively covary with those of firm i. Regarding part (c), we know from footnote 27 that if the prior variance $\sigma_{x_i}^2$ is big enough, then the manager of firm i discloses the information she receives more frequently as $\sigma_{x_i}^2$ increases. We would expect that if σ_{il} is positive, then this would in turn cause firm $l's$ cost of capital to decline, because firm l can then piggy-back on the more frequent disclosures by firm i. Also, we should expect the opposite effect when$\sigma_{il} < 0$.

Collectively, Corollaries 10, 17, and 18 above display a rich array of pre-disclosure cost of capital-related comparative statics.

6.2 Post-disclosure Cost of Capital

In this subsection, we study how voluntary disclosure affects the sometimes-disclosing firm's disclosure-contingent systematic risk β_i. To acknowledge that firm $i's$ systematic risk depends on what information Ω_i its manager releases, we write this systematic risk as one of: β_i^{nd} when the manager makes no disclosure; $\beta_i(v_i)$ (or $\beta_i(z_i)$) when firm $i's$ manager discloses v_i (or z_i); or $\beta_i(\Omega_i)$ when we wish to refer to its systematic risk without specifying whether the manager of firm i has or has not previously made a disclosure. In all cases, we define firm $i's$ systematic risk in the usual way as $\beta_i(\Omega_i) = \frac{cov(\tilde{R}_i, \tilde{R}_M|\Omega_i)}{var(\tilde{R}_M|\Omega_i)}$, where $\tilde{R}_i \equiv \frac{\tilde{x}_i}{P_i(\Omega_i)}$ and $\tilde{R}_m \equiv \frac{\tilde{x}_M}{P_M(\Omega_i)}$ are respectively the gross returns to investing in firm i and to investing in the whole market (here, $\tilde{x}_M \equiv \Sigma_l \tilde{x}_l$ is the market's aggregate CF, and P_M(no disclosure) $\equiv P_M^{nd} \equiv \Sigma_l P_l^{nd}$ (resp., $P_M(v_i) \equiv P_M^d(v_i) \equiv \Sigma_l P_l^d(v_i)$) is the aggregate market value of all firms when the manager of firm i makes no disclosure (resp., discloses estimate v_i).

In view of (18), (27), and (36), it is clear that $\beta_i^{nd} = \frac{1}{P_i^{nd} P_M^{nd}} \frac{cov(\tilde{x}_i, \Sigma_j \tilde{x}_j | nd, z_i^c)}{\frac{var(\tilde{x}_M)}{(P_M^{nd})^2}} = \frac{P_M^{nd}}{P_i^{nd}} \frac{\eta(z_i^{RAc}) \Sigma_j \sigma_{ij}}{var(\tilde{x}_M | nd, z_i^{RAc})}$ and $\beta_i(v_i) = \frac{P_M^d(v_i)}{P_i^d(v_i)} \frac{\alpha \Sigma_j \sigma_{ij}}{var(\tilde{x}_M | v_i)}$. For our main result below, we appeal to the following version of a "large economy" assumption: the ratio $D \equiv \frac{P_M(\Omega_i)}{var(\tilde{x}_M | \Omega_i)}$ is assumed to be independent of whether $\Omega_i = \{v_i\}$ or $\Omega_i = \{$no disclosure$\}$ for any v_i. This assumption amounts to assuming that the market is so big that the manager of firm $i's$ disclosure decision is irrelevant to the market-wide terms $var(\tilde{x}_M | \Omega_i)$ and $P_M(\Omega_i)$.

The main result of this section is:

Theorem 19 *Given the large economy assumption, investors' assessment of the systematic risk $\beta_i^d(v_i)$ of firm i for every estimate v_i its manager discloses in equilibrium is below investors' assessment of its systematic risk β_i^{nd} if the manager of firm i makes no disclosure.*

What this theorem shows is that, when measuring a firm's cost of capital in terms of its systematic risk, our model reaches a conclusion similar to that of Cheynel [2013]'s Proposition 5: in equilibrium, for those estimates firm $i's$ manager chooses to disclose, investors' perceptions of her firm's systematic risk is always lower than is investors' assessment of the firm's systematic risk when its manager makes no disclosure. The theorem should not be interpreted as implying that disclosure *always* results in lower perceived systematic risk than nondisclosure: the latter is true in general only for equilibrium disclosures. This theorem yields yet another testable implication of the model.

7 Summary of Findings and Possible Directions for Future Research

We have studied the economic consequences of the equilibrium voluntary disclosure decisions of the manager of a single firm when the shares of multiple firms exist and can be traded by risk-averse investors. With this model, we have developed new equilibrium asset pricing formulas in the case the manager makes no disclosure, and these new formulas impose testable cross-equation restrictions on firms' market values. We have shown that a manager's propensity to disclose the information she receives is increasing in each of: the probability she receives information, the precision of the information she receives, the aggregate risk-aversion of investors, and the ex ante uncertainty about the firm's CF. We have shown that the sometimes-disclosing firm's market value decreases more with nondisclosure if the covariance between its firm's CF and the CF of other firms increases, as the prior probability the disclosing firm receives information increases, as investors' aggregate risk-aversion increases, and as the precision of the information the sometimes-disclosing firm increases. We have shown that the sometimes-disclosing firm's cost of capital increases in intuitive ways, and that the sometimes-disclosing firm's disclosure policy also affects other firms' costs of capital, when the covariance between the CF of the sometimes-disclosing firm and the other firms is nonzero. We have shown that investors' learning that a manager is not going to make a disclosure is bad news for other firms whose CF positively covary with the nondisclosing firm's CF, but is good news for other firms whose CF negatively covary with the nondisclosing firm's CF. We have shown that in equilibrium nondisclosure is always variance-increasing for the nondisclosing firm, and further, as long as the covariance between two firms' (one sometimes-disclosing, one not) CF is nonzero, also variance-increasing for the other firm as well. Furthermore, we have shown nondisclosure in equilibrium is also always covariance-increasing, that is, nondisclosure increases investors' perceptions of the absolute value of the covariance between two firms' CF. We have also shown that, in equilibrium, disclosure of the manager's information always results in lower perceived systematic risk for her firm than does no disclosure in a "large economy" formulation of our model.

The present paper invites several additional research studies. As is evident from the summary of our results above, we have generated a host of comparative statics and other predictions. It would be natural to test many of these predictions. For example, are the variance-increasing and covariance-increasing implications of nondisclosure detectible empirically? Does nondisclosure by one firm result in the asserted information transfers to other firms? Are nondisclosing firms' systematic risks higher than those of disclosing firms? In addition to providing a theoretical foundation to additional empirical research, our study could also prompt a variety of additional theoretical investigations. For example, our results are predicated on the securities market operating competitively. It would be natural to ask how the results here would vary were some traders to recognize that their trades are large enough so as to influence firms' equilibrium market values. It would also be worthwhile to examine how our results can be extended to a model that incorporates multiple firms' managers making concurrent voluntary disclosure decisions. Further, while the distributional assumptions we make regarding firms' cash flows - multivariate normal - are conventional, in view of the surprising nature of our variance-increasing results, it also would be desirable to better understand how broadly our results extend beyond the class of normal distributions.

8 References

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9 Appendix: Proofs

Lemma 12 Equilibrium equation (3) is algebraically equivalent to equation (5).

Proof of Lemma

We start by computing $E[\tilde{y}|nd, v^c]$, the expected value of the random variable $\tilde{y}$ given the manager of firm i makes no disclosure and uses the cutoff v^c in deciding whether to disclose the estimate v of $\tilde{v}$ she sometimes receives. Here, $\tilde{y}$ is any random variable such that $(\tilde{x}, \tilde{y})$ is bivariate normal.

Analogous to the way we computed the no disclosure price of the firm conditional on the cutoff v^c in the text, we see that

$$E[\tilde{y}|nd, v^c] = \frac{(1-p)E[\tilde{y}] + pG(v^c)E[\tilde{y}|\tilde{v} < v^c]}{1-p+pG(v^c)}. \tag{43}$$

Writing $E[\tilde{x}|\tilde{v} < v^c]$ as $E[E[\tilde{x}|\tilde{v}]|\tilde{v} < v^c]$, it is clear that (43) is the same as $\frac{(1-p)E[\tilde{y}]+p\int^{v^c} E[\tilde{y}|v]g(v)dv}{1-p+pG(v^c)}$. Now, recall the elementary fact concerning bivariate normal random variables that, with

$$\beta_y \equiv \frac{cov(\tilde{y},\tilde{v})}{\sigma_v^2}, \tag{44}$$

we have $E[\tilde{y}|v] = E[\tilde{y}] + \beta_y(v-m)$. Hence:

$$\begin{aligned} E[\tilde{y}|nd, v^c] &= \frac{(1-p)E[\tilde{y}] + p\int^{v^c}(E[\tilde{y}] + \beta_y(v-m))g(v)dv}{1-p+pG(v^c)} \\ &= E[\tilde{y}] + \frac{p\beta_y \int^{v^c} vg(v)dv - p\beta_y mG(v^c)}{1-p+pG(v^c)}. \end{aligned} \tag{45}$$

Next, confirm that, since $\tilde{v}\,\tilde{}\,N(m,\sigma_v^2)$, we have

$$\int_{-\infty}^{v^c} vg(v)dv = mG(v^c) - \sigma_v^2 g(v^c), \tag{46}$$

and substitute the latter into (45) to conclude:

$$\begin{aligned} E[\tilde{y}|nd, v^c] &= E[\tilde{y}] - \frac{p\beta_y \sigma_v^2 g(v^c)}{1-p+pG(v^c)} \\ &= E[\tilde{y}] - \frac{p cov(\tilde{y},\tilde{v})g(v^c)}{1-p+pG(v^c)}. \end{aligned}$$

Recall from the text that ϕ and Φ denote the density and cdf of a standard normal random variable. With $z^c \equiv \frac{v^c-m}{\sigma_v}$, it is clear that $\sigma_v g(v^c) = \phi(z^c)$ and $G(v^c) = \Phi(z^c)$, so $E[\tilde{y}|nd, v^c] = E[\tilde{y}] - \frac{p\frac{cov(\tilde{y},\tilde{v})}{\sigma_v}\phi(z^c)}{1-p+p\Phi(z^c)}$, and thus, recalling the definition of $f(z^c)$ in (6):

$$E[\tilde{y}|nd, v^c] = E[\tilde{y}] + \frac{cov(\tilde{y},\tilde{v})}{\sigma_v} f(z^c). \tag{47}$$

In case $\tilde{y} \equiv \tilde{x}$, then $cov(\tilde{y},\tilde{v}) = \sigma_x^2$ and so we get:

$$E[\tilde{x}|nd, v^c] = m + \frac{\sigma_x^2}{\sigma_v} f(z^c). \tag{48}$$

Thus, since $cov(\tilde{x},\tilde{v}) = \sigma_x^2$ and $E[\tilde{x}] = E[\tilde{v}] = m$, we can write $E[\tilde{x}|v]$ as $E[\tilde{x}|v] = m + \frac{cov(\tilde{x},\tilde{v})}{\sigma_v^2} \times (v-m)$ or as $E[\tilde{x}|v] = m + \frac{\sigma_x^2}{\sigma_v} \times z$, when $z = \frac{v-m}{\sigma_v}$. Thus, in view of the preceding, equilibrium equation (3) in the text can be written as:

$$m + \frac{\sigma_x^2}{\sigma_v} \times z^c = m + \frac{\sigma_x^2}{\sigma_v} f(z^c),$$

which is obviously algebraically equivalent to (5). This proves the lemma.$\blacksquare$

Lemma 20 *For all $p \in [0,1]$, the point z^{RNc} defined by the solution to equation (5) in the text exists, is unique, is negative, and LHS(5)-RHS(5) is positive if $z^c > z^{RNc}$ and LHS(5)-RHS(5) is negative if $z^c < z^{RNc}$.*

Proof of Lemma 20 Define

$$\Gamma(z^c) \equiv z^c\Phi(z^c) + \phi(z^c), \tag{49}$$

and notice that

$$z^c(1-p) + p\Gamma(z^c) = z^c(1-p+p\Phi(z^c)) + p\phi(z^c), \tag{50}$$

and also notice that (50) has the same sign as LHS(5)-RHS(5) It is easy to check that the function $\Gamma(z^c)$: is continuous and strictly increasing, and $\lim_{z^c\to\infty}\Gamma(z^c) = \infty$. An application of L'Hospital's rule further shows that $\lim_{z^c\to-\infty}\Gamma(z^c) = 0$, so $\Gamma(z^c)$ is positive for all (finite) z^c. Hence, (50) is strictly continuously increasing on all of $\mathbb{R}$ and ranges over $-\infty$ to $+\infty$. Accordingly, as claimed, there is a unique value z^c, which we call z^{RNc}, such that the equation (5) has a solution. Clearly, z^{RNc} is negative. The preceding also shows that LHS(5)-RHS(5) is positive when $z^c > z^{RNc}$ and LHS(5)-RHS(5) is negative when $z^c < z^{RNc}$.■

From this point onward in the Appendix, the firm whose manager has a disclosure decision to make is alwarys referred to as firm i.

Proof of Lemma 13

Without loss of generality, we take $E[\tilde{x}_i] = m_i = 0$. Select any two firms k and j. It is easy to check that if we consider the trivariate normal random variable $(\tilde{x}_j, \tilde{x}_k, \tilde{v}_i)$, then the covariance between $\tilde{x}_j$ and $\tilde{x}_k$ conditional on $\tilde{v}_i$, which we denote by $\sigma_{jk|i}$, is given by $\sigma_{jk|i} = \sigma_{jk} - \frac{\sigma_{ji}\sigma_{ki}}{\sigma^2_{v_i}}$. We define $\beta_k \equiv \frac{\sigma_{ik}}{\sigma^2_{v_i}}$ and $\beta_j \equiv \frac{\sigma_{ij}}{\sigma^2_{v_i}}$. We know $E[\tilde{x}_j|v_i] = m_j + \beta_j v_i$ and $E[\tilde{x}_k|v_i] = m_k + \beta_k v_i$. Hence, using (46), $E[\tilde{x}_k|v_i]E[\tilde{x}_j|v_i] + \sigma_{jk|i} = E[\tilde{x}_k\tilde{x}_j|v_i]$, and since $m_i = 0$ implies $\tilde{v}_i\,\tilde{}\,N(0,\sigma^2_{v_i})$ which in turn implies

$$\int_{-\infty}^{v_i^c} v_i^2 g(v_i)dv_i = \sigma^2_{v_i}(G(v_i^c) - v_i^c g(v_i^c)),$$

we get $E[\tilde{x}_k\tilde{x}_j|nd, v_i^c] = \frac{(1-p)E[\tilde{x}_k\tilde{x}_j] + p\int^{v_i^c}(\sigma_{jk|i} + E[\tilde{x}_k|v_i]E[\tilde{x}_j|v_i])g(v_i)dv_i}{1-p+pG(v_i^c)}$, which in turn equals each of the expressions in the following bullet list:

- $\frac{(1-p)E[\tilde{x}_k\tilde{x}_j]+p\int^{v_i^c}\{\sigma_{jk|i}+(m_j+\beta_j v_i)(m_k+\beta_k v_i)\}g(v_i)dv_i}{1-p+pG(v_i^c)}$
- $\frac{(1-p)(\sigma_{kj}+m_k m_j)+p\{(\sigma_{jk|i}+m_j m_k)G(v_i^c)-\sigma_{v_i}^2(m_j\beta_k+m_k\beta_j)g(v_i^c)+\beta_j\beta_k\sigma_{v_i}^2(G(v_i^c)-v_i^c g(v_i^c))\}}{1-p+pG(v_i^c)}$
- $\frac{(1-p)(\sigma_{kj}+m_k m_j)+\frac{p}{\sigma_{v_i}^2}\{(\sigma_{jk}\sigma_{v_i}^2-\sigma_{ji}\sigma_{ki}+m_j m_k+\sigma_{ji}\sigma_{ki})G(v_i^c)-\sigma_{v_i}^2(m_j\beta_k+m_k\beta_j)g(v_i^c)-\beta_j\beta_k\sigma_{v_i}^2 v_i^c g(v_i^c)\}}{1-p+pG(v_i^c)}$
- $\frac{(1-p)(\sigma_{kj}+m_k m_j)+p\{(\sigma_{jk}+m_j m_k)G(v_i^c)-\sigma_{v_i}^2(m_j\beta_k+m_k\beta_j)g(v_i^c)+\frac{\sigma_{ij}\sigma_{ik}}{\sigma_{v_i}^2}v_i^c g(v_i^c)\}}{1-p+pG(v_i^c)}$
- $\sigma_{kj}+m_k m_j-\frac{p\{\sigma_{v_i}^2(m_j\beta_k+m_k\beta_j)g(v_i^c)+\frac{\sigma_{ij}\sigma_{ik}v_i^c}{\sigma_{v_i}^2}g(v_i^c)\}}{1-p+pG(v_i^c)}$
- $\sigma_{kj}+m_k m_j-\frac{p\{(m_j\sigma_{ik}+m_k\sigma_{ij})g(v_i^c)+\frac{\sigma_{ij}\sigma_{ik}}{\sigma_{v_i}}\frac{v_i^c}{\sigma_{v_i}}g(v_i^c)\}}{1-p+pG(v_i^c)}$
- $\sigma_{kj}+m_k m_j+(m_j\sigma_{ik}+m_k\sigma_{ij})\frac{f(z_i^c)}{\sigma_{v_i}}+\frac{\sigma_{ij}\sigma_{ik}}{\sigma_{v_i}^2}z_i^c f(z_i^c)$.

Using (47) to write $E[\tilde{x}_j|nd,v_i^c]=m_j+\frac{\sigma_{ij}}{\sigma_{v_i}}f(z_i^c)$ and $E[\tilde{x}_k|nd,v_i^c]=m_k+\frac{\sigma_{ik}}{\sigma_{v_i}}f(z_i^c)$, we conclude $cov(\tilde{x}_k,\tilde{x}_j|nd,v_i^c)=$

$$\begin{aligned}
& E[\tilde{x}_k\tilde{x}_j|nd,v_i^c]-E[\tilde{x}_k|nd,v_i^c]\times E[\tilde{x}_j|nd,v_i^c] \\
= \; & \sigma_{kj}+m_k m_j+(m_j\sigma_{ik}+m_k\sigma_{ij})\frac{f(z_i^c)}{\sigma_{v_i}}+\frac{\sigma_{ij}\sigma_{ik}}{\sigma_{v_i}^2}z_i^c f(z_i^c)-(m_j+\frac{\sigma_{ij}}{\sigma_{v_i}}f(z_i^c))(m_k+\frac{\sigma_{ik}}{\sigma_{v_i}}f(z_i^c)) \\
= \; & \sigma_{kj}+\frac{\sigma_{ij}\sigma_{ik}}{\sigma_{v_i}^2}z_i^c f(z_i^c)-\frac{\sigma_{ij}}{\sigma_{v_i}}f(z_i^c)\frac{\sigma_{kj}}{\sigma_{v_i}}f(z_i^c) \\
= \; & \sigma_{kj}+\frac{f(z_i^c)(z_i^c-f(z_i^c))}{\sigma_{v_i}^2}\times\sigma_{ij}\sigma_{ik}. \qquad (51)
\end{aligned}$$

If we specialize (51) to the case where $k=j=i$, we get (18). If we specialize (51) to the case where $k=i$ and $j\neq i$, we get (36). If we specialize (51) to the case where $k=j\neq i$, we get (40). If we consider the most general case, we get, expressed in matrix form, equation (39) in Lemma 13.■

Proof of Expression (29)

We recall that the moment generating function of a multivariate normal random vector with mean $\mathbf{n}$ and covariance matrix $\mathbf{C}$ is $\phi(\mathbf{t})=e^{\mathbf{t'n}+.5\mathbf{t'Ct}}$. Combining this result with (24) and (25), it follows that the expected utility of investor k who purchases fractions $\boldsymbol{\theta}_k$ of all n firms' securities when the market prices of those firms are $\mathbf{P}$ conditional on this investor knowing that $\tilde{v}_i=v_i$ is given by:

$$\begin{aligned}
E[-e^{-\gamma_k \boldsymbol{\theta}_k'(\tilde{\mathbf{x}}-\mathbf{P})}|v_i] &= -e^{\gamma_k \boldsymbol{\theta}_k' \mathbf{P}} \times E[e^{-\gamma_k \boldsymbol{\theta}_k' \tilde{\mathbf{x}}}|v_i] \\
&= -e^{-\gamma_k \boldsymbol{\theta}_k'(\mathbf{m}+\frac{v_i-m_i}{\sigma_{v_i}^2} cov(\tilde{\mathbf{x}},\tilde{v}_i))+.5\gamma_k^2 \boldsymbol{\theta}_k'(\mathbf{S}-\frac{1}{\sigma_{v_i}^2}\times cov(\tilde{\mathbf{x}},\tilde{v}_i)\times cov(\tilde{\mathbf{x}},\tilde{v}_i)')\boldsymbol{\theta}_k+\gamma_k \boldsymbol{\theta}_k' \mathbf{P}} \\
&= -e^{-v_i \frac{\gamma_k}{\sigma_{v_i}^2} \boldsymbol{\theta}_k' cov(\tilde{\mathbf{x}},\tilde{v}_i)} e^{-\gamma_k \boldsymbol{\theta}_k'(\mathbf{m}-\frac{m_i}{\sigma_{v_i}^2} cov(\tilde{\mathbf{x}},\tilde{v}_i))+.5\gamma_k^2 \boldsymbol{\theta}_k'(\mathbf{S}-\frac{1}{\sigma_{v_i}^2}\times cov(\tilde{\mathbf{x}},\tilde{v}_i)\times cov(\tilde{\mathbf{x}},\tilde{v}_i)')\boldsymbol{\theta}_k+\gamma_k \boldsymbol{\theta}_k' \mathbf{P}} \\
&= -e^{-v_i \frac{\gamma_k}{\sigma_{v_i}^2} s_k} e^{-\gamma_k \boldsymbol{\theta}_k' \mathbf{m}+\frac{\gamma_k m_i}{\sigma_{v_i}^2} s_k+.5\gamma_k^2 \boldsymbol{\theta}_k' \mathbf{S}\boldsymbol{\theta}_k-\frac{1}{\sigma_{v_i}^2}\times .5\gamma_k^2 s_k^2+\gamma_k \boldsymbol{\theta}_k' \mathbf{P}}
\end{aligned}$$

In the above, we have employed the notation (30).

If, rather than learning the exact realization v_i of $\tilde{v}_i$, investor k just learned that $\tilde{v}_i < v_i^c$, it would follow that his conditional expected utility (from purchasing fractions $\boldsymbol{\theta}_k$ of all n firms' securities when the market prices of those firms are $\mathbf{P}$) is given by:

$$\begin{aligned}
E[E[-e^{-\gamma_k \boldsymbol{\theta}_k'(\tilde{\mathbf{x}}-\mathbf{P})}|v_i]|\tilde{v}_i &< v_i^c] \\
&= -e^{-\gamma_k \boldsymbol{\theta}_k' \mathbf{m}+\frac{\gamma_k m_i}{\sigma_{v_i}^2} s_k+.5\gamma_k^2 \boldsymbol{\theta}_k' \mathbf{S}\boldsymbol{\theta}_k-\frac{1}{\sigma_{v_i}^2}\times .5\gamma_k^2 s_k^2+\gamma_k \boldsymbol{\theta}_k' \mathbf{P}} E[e^{-\tilde{v}_i \frac{\gamma_k}{\sigma_{v_i}^2} s_k}|\tilde{v}_i < v_i^c] \\
&= -e^{-\gamma_k \boldsymbol{\theta}_k' \mathbf{m}+\frac{\gamma_k m_i}{\sigma_{v_i}^2} s_k+.5\gamma_k^2 \boldsymbol{\theta}_k' \mathbf{S}\boldsymbol{\theta}_k-\frac{1}{\sigma_{v_i}^2}\times .5\gamma_k^2 s_k^2+\gamma_k \boldsymbol{\theta}_k' \mathbf{P}} \\
&\quad \times e^{-m_i \frac{\gamma_k}{\sigma_{v_i}^2} s_k+.5\sigma_{v_i}^2 (\frac{\gamma_k}{\sigma_{v_i}^2} s_k)^2} \times \frac{\Phi(\frac{v_i^c-m_i}{\sigma_{v_i}} + \frac{\gamma_k}{\sigma_{v_i}^2} s_k \sigma_{v_i})}{\Phi(\frac{v_i^c-m_i}{\sigma_{v_i}})} \\
&= -e^{-\gamma_k \boldsymbol{\theta}_k' \mathbf{m}+.5\gamma_k^2 \boldsymbol{\theta}_k' \mathbf{S}\boldsymbol{\theta}_k-\gamma_k \boldsymbol{\theta}_k' \mathbf{P}} \times \frac{\Phi(\frac{v_i^c-m_i}{\sigma_{v_i}} + \frac{\gamma_k s_k}{\sigma_{v_i}})}{\Phi(\frac{v_i^c-m_i}{\sigma_{v_i}})} \\
&= -e^{-\gamma_k (\boldsymbol{\theta}_k'(\mathbf{m}-\mathbf{P})-.5\gamma_k \boldsymbol{\theta}_k' \mathbf{S}\boldsymbol{\theta}_k)} \times \frac{\Phi(\frac{v_i^c-m_i}{\sigma_{v_i}} + \frac{\gamma_k s_k}{\sigma_{v_i}})}{\Phi(\frac{v_i^c-m_i}{\sigma_{v_i}})}. \qquad (52)
\end{aligned}$$

The preceding appeals to a special case of Tallis [1961], i.e., the fact that if $\tilde{w}\tilde{\ }N(m,\sigma_w^2)$ and w_0 is any constant, then

$$E[e^{t\tilde{w}}|\tilde{w} < w_0] = e^{tm+.5t^2\sigma_w^2} \times \frac{\Phi(\frac{w_0-m}{\sigma_w} - t\sigma_w)}{\Phi(\frac{w_0-m}{\sigma_w})}.$$

Next, suppose investor k only knows that firm i has made no disclosure and investor k believes that, in deciding whether to disclose his information, the manager of firm i uses the cutoff policy v_i^c. Then, as we discussed in the univariate section, the investor's assessment of the probability the manager of firm i received information, given she did not make any disclosure is $\frac{pG(v_i^c)}{1-p+pG(v_i^c)}$. Since the investor's expected utility (from purchasing fractions $\boldsymbol{\theta}_k$ of all n firms' securities when the market prices of those firms are $\mathbf{P}$) conditional on knowing $\tilde{v} < v^c$, is as specified in (52), and since the investor's expected utility (from purchasing fractions $\boldsymbol{\theta}_k$ of all n firms' securities when the market prices of those firms are $\mathbf{P}$) given the firm did not receive information is given by $-e^{-\gamma_k(\boldsymbol{\theta}_k'(\mathbf{m}-\mathbf{P})-.5\gamma_k\boldsymbol{\theta}_k'\mathbf{S}\boldsymbol{\theta}_k)}$, it follows that the investor's expected utility (from purchasing fractions $\boldsymbol{\theta}_k$ of all n firms' securities when the market prices of those firms are $\mathbf{P}$) conditional *only* on knowing that the manager did not make a disclosure is given by:

$$-\frac{pG(v_i^c)e^{-\gamma_k(\boldsymbol{\theta}_k'(\mathbf{m}-\mathbf{P})-.5\gamma_k\boldsymbol{\theta}_k'\mathbf{S}\boldsymbol{\theta}_k)}\frac{\Phi(\frac{v_i^c-m_i}{\sigma_{v_i}}+\frac{\gamma_k s_k}{\sigma_{v_i}})}{\Phi(\frac{v_i^c-m_i}{\sigma_{v_i}})}+(1-p)e^{-\gamma_k(\boldsymbol{\theta}_k'(\mathbf{m}-\mathbf{P})-.5\gamma_k\boldsymbol{\theta}_k'\mathbf{S}\boldsymbol{\theta}_k)}}{1-p+pG(v^c)}$$

$$= \quad -\frac{e^{-\gamma_k(\boldsymbol{\theta}_k'(\mathbf{m}-\mathbf{P})-.5\gamma_k\boldsymbol{\theta}_k'S\boldsymbol{\theta}_k)}}{1-p+pG(v^c)} \times \left(pG(v^c)\frac{\Phi(\frac{v_i^c-m_i}{\sigma_{v_i}}+\frac{\gamma_k s_k}{\sigma_{v_i}})}{\Phi(\frac{v_i^c-m_i}{\sigma_{v_i}})}+1-p\right).$$

Since $G(v_i^c)=\Phi(\frac{v_i^c-m_i}{\sigma_{v_i}})$, this last expression equals:

$$-\frac{e^{-\gamma_k(\boldsymbol{\theta}_k'(\mathbf{m}-\mathbf{P})-.5\gamma_k\boldsymbol{\theta}_k'\mathbf{S}\boldsymbol{\theta}_k)}}{1-p+pG(v^c)} \times \left(p\Phi(\frac{v_i^c-m_i}{\sigma_{v_i}}+\frac{\gamma_k s_k}{\sigma_{v_i}})+1-p\right).$$

This verifies expression **(29)** in the text.■

Proof of Expression (31)

We maximize investor $k's$ portfolio-and-price contingent expected utility conditional on the manager of firm $i's$ no disclosure and his perceptions of the manager of firm $i's$ use of the cutoff v_i^c, as it appears in (29) in the text. In the following it is helpful to simplify notation by temporarily suppressing reference to the argument of Φ, as well as the (same) argument of its derivative ϕ. This (common) argument is:

$$\frac{v_i^c-m_i}{\sigma_{v_i}}+\frac{\gamma_k s_k}{\sigma_{v_i}}. \tag{53}$$

Pick asset/firm $l \in \{1,2,...,n\}$. Investor $k's$ optimal fractional investment in firm l is described by the following first-order condition (after factoring out the common terms $-\frac{\gamma_k e^{-\gamma_k(\boldsymbol{\theta}_k'(\mathbf{m}-\mathbf{P})-.5\gamma_k\boldsymbol{\theta}_k'\mathbf{S}\boldsymbol{\theta}_k)}}{1-p+pG(v^c)}$):

$$0=\left(m_l-P_l-\gamma_k\Sigma_{j=1}^n\theta_k^j\sigma_{lj}\right)\times(p\Phi+1-p)-p\phi\frac{\sigma_{li}}{\sigma_{v_i}}. \tag{54}$$

We now conjecture that a feature of an equilibrium is that, for each investor k:

$$\boldsymbol{\theta}_k'=\left(\ \frac{\gamma}{\gamma_k},\quad \frac{\gamma}{\gamma_k},\quad \frac{\gamma}{\gamma_k},\qquad \frac{\gamma}{\gamma_k},\quad \frac{\gamma}{\gamma_k}\ \right). \tag{55}$$

(Once we identify the equilibrium prices for securities with the cutoff v_i^c, one can go back and confirm that with these prices and that cutoff, the conjecture is sustained.) With this conjecture, the first-order condition (54) for investor k may be rewritten:

$$P_l \times (p\Phi + 1 - p) = \left(m_l - \gamma\Sigma_{j=1}^n \sigma_{lj}\right) \times (p\Phi + 1 - p) - p\phi \frac{\sigma_{li}}{\sigma_{v_i}},$$

or as, now writing P_l as $P_l^{nd}(v_i^c)$ both to emphasize that we are identifying a "no disclosure" price and to emphasized the dependence of the price on the cutoff v_i^c :

$$P_l^{nd}(v_i^c) = m_l - \gamma\Sigma_{j=1}^n \sigma_{lj} - \frac{p\phi \frac{\sigma_{li}}{\sigma_{v_i}}}{p\Phi + 1 - p} \text{ for all } l = 1, 2, ..., n. \tag{56}$$

Notice that this price for firm l is the same regardless of which investor's first-order condition (54) is being examined (as no investor k-specific variables appear in the specification of these prices once one substitutes (55) into it).

This verifies expression (**31**) in the text.

The preceding shows that there exists an equilibrium with no disclosure prices given by (56) and investor portfolios as specified by (55). That these equilibrium portfolios in (55) are unique follows immediately by the argument used to prove the two-fund separation theorem (e.g., Huang and Litzenberger [1988]); the accompanying footnote contains the details.[36] Since the equilibrium no disclosure prices are uniquely determined by (56) given the equilibrium portfolios (55), we conclude the equilibrium no disclosure prices are unique, too.■

Proof of Theorem 8

[36] Mossin [1977] (chapter 8) shows that in a one-period model of a stock market where investors share identical beliefs about the probability distribution of firms' future cash flows, and all investors' utility functions belong to one of the CARA, logarithmic, or power utility function classes, then the equilibrium allocations that result from a stock market economy are always Pareto optimal. Mossin [1977] (chapter 6) further shows that a necessary condition for Pareto optimality of a stock market economy is that each investor's equilibrium portfolio be such that the investor owns the same fraction of every firm. It follows from this last result that every investor's end-of-period consumption is a linear function of all firms' aggregate cash flows $\sum_l x_l$; alternatively put, that every investor's equilibrium end-of-period consumption is a linear function of aggregate cash flows. Wilson [1968] showed that among all sharing rules in an economy where all individuals have CARA preferences, all Pareto optimal sharing rules are linear (in the aggregate risk to be shared), and optimal linear sharing rules entail that individuals share risk in proportion to their risk-tolerances. Thus, in order for the Mossin's first result (that stock market equilibrium allocations are Pareto optimal) to hold when investors have CARA preferences, Wilson's result requires that the stock market equilibrium portfolio allocations must be in proportion to investors' risk-tolerance; said differently, the equilibrium portfolio of every investor k is uniquely defined by (55) above.

At the equilibrium cutoff we require that firm $i's$ disclosure and no disclosure prices be equal:

$$P_i^{nd}(v_i^c) = P_i^d(v_i^c),$$

and so (using the notation $\sigma_{x_i}^2$ in place of σ_{ii}) :

$$m_i - \gamma\Sigma_{j=1}^n \sigma_{ij} - \frac{p\phi\frac{\sigma_{x_i}^2}{\sigma_{v_i}}}{p\Phi + 1 - p} = m_i + \frac{\sigma_{x_i}^2}{\sigma_{v_i}} \times \frac{v_i^c - m_i}{\sigma_{v_i}} - \gamma\alpha\Sigma_{j=1}^n \sigma_{ij}.$$

Using the conjecture (55), the (common) argument of both the density ϕ and the cdf Φ, originally specified in (53), and writing z_i^c in place of $\frac{v_i^c - m_i}{\sigma_{v_i}}$, is given by:

$$z_i^c + \frac{\gamma\Sigma_{j=1}^n \sigma_{ij}}{\sigma_{v_i}}. \tag{57}$$

Hence, the equilibrium condition for the cutoff can be expressed (now displaying the arguments of ϕ and Φ) as:

$$\frac{\sigma_{x_i}^2}{\sigma_{v_i}} \times z_i^c + \gamma(1-\alpha)\Sigma_{j=1}^n \sigma_{ij} = -\frac{p\phi(z_i^c + \frac{\gamma\Sigma_{j=1}^n \sigma_{ij}}{\sigma_{v_i}})\frac{\sigma_{x_i}^2}{\sigma_{v_i}}}{p\Phi(z_i^c + \frac{\gamma\Sigma_{j=1}^n \sigma_{ij}}{\sigma_{v_i}}) + 1 - p}$$

or as

$$\frac{\sigma_{x_i}^2}{\sigma_{v_i}} \times z_i^c + \gamma\frac{\sigma_{x_i}^2}{\sigma_{v_i}^2}\Sigma_{j=1}^n \sigma_{ij} = -\frac{p\phi(z_i^c + \frac{\gamma\Sigma_{j=1}^n \sigma_{ij}}{\sigma_{v_i}})\frac{\sigma_{x_i}^2}{\sigma_{v_i}}}{p\Phi(z_i^c + \frac{\gamma\Sigma_{j=1}^n \sigma_{ij}}{\sigma_{v_i}}) + 1 - p},$$

or as

$$z_i^c + \frac{\gamma\Sigma_{j=1}^n \sigma_{ij}}{\sigma_{v_i}} = -\frac{p\phi(z_i^c + \frac{\gamma\Sigma_{j=1}^n \sigma_{ij}}{\sigma_{v_i}})}{p\Phi(z_i^c + \frac{\gamma\Sigma_{j=1}^n \sigma_{ij}}{\sigma_{v_i}}) + 1 - p}.$$

Observe that this last equation is the same as:

$$z = f(z),$$

with $z = z_i^c + \frac{\gamma\Sigma_{j=1}^n \sigma_{ij}}{\sigma_{v_i}}$. Since we know that $f(\cdot)$ has as its unique fixed point the risk-neutral equilibrium cutoff z_i^{RNc}, it follows that the equilibrium cutoff in the risk-averse case, z_i^{RAc}, is also unique and must satisfy $z_i^{RAc} + \frac{\gamma\Sigma_{j=1}^n \sigma_{ij}}{\sigma_{v_i}} = z_i^{RNc}$.

This proves the theorem.■

Proof of Corollaries 17 and 18

We begin by developing an expression for $E[\tilde{P}_l|z_i^c]$, the expected price of any firm $l \in \{1, 2, ..., n\}$ when the manager of firm i uses any fixed, arbitrary disclosure cutoff v_i^c (or its associated standardized value $z_i^{c\cdot}$). It is easy to confirm that $\int_{v_i^c} v_i g(v_i)dv_i = (1-G(v_i^c))m_i+\sigma_{v_i}^2 g(v_i^c)$, so $E[\tilde{v}_i|\tilde{v}_i > v_i^c] = m_i+\frac{\sigma_{v_i}^2 g(v^c)}{1-G(v^c)} = m_i + \frac{\sigma_{v_i}\phi(z_i^c)}{1-\Phi(z_i^c)}$ and hence $E[\frac{\tilde{v}_i-m_i}{\sigma_{v_i}}|\tilde{v}_i \geq v_i^c] = \frac{\phi(z_i^c)}{1-\Phi(z_i^c)}$. Next, extracting the market price of firm l from (23), we see that the disclosure of v_i, or equivalently, $z_i = \frac{v_i-m_i}{\sigma_{v_i}}$, yields:

$$P_l^d(z_i) = m_l + \frac{\sigma_{li}}{\sigma_{v_i}} z_i - \gamma(\Sigma_{j=1}^n \sigma_{lj} - \frac{1}{\sigma_{v_i}^2}\Sigma_{j=1}^n \sigma_{li}\sigma_{ij}).$$

Hence:

$$E[P_l^d(\tilde{v}_i)|\tilde{v}_i \geq v_i^c] = m_l - \gamma\Sigma_{j=1}^n\sigma_{lj} + \frac{\sigma_{li}}{\sigma_{v_i}}\left(\frac{\phi(z_i^c)}{1-\Phi(z_i^c)} + \frac{\gamma\Sigma_{j=1}^n\sigma_{ij}}{\sigma_{v_i}}\right).$$

Also recall that from (31), we know that the "no disclosure" price for firm l when firm i uses the cutoff z_i^c can be expressed as: $P_l^{nd}(z_i^c) = m_l - \gamma\Sigma_{j=1}^n\sigma_{lj} + f(z_i^c + \frac{\gamma\Sigma_{j=1}^n\sigma_{ij}}{\sigma_{v_i}})\frac{\sigma_{li}}{\sigma_{v_i}}$. Since $1-p+p\Phi(z_i^c)$ (resp., $p(1-\Phi(z_i^c))$ is the probability of no disclosure (resp., disclosure) when the manager of firm i uses the cutoff z_i^c, it follows that the expected price of firm l is given by:

$$\begin{aligned} & E[\tilde{P}_l|z_i^c] && (58)\\ = \quad & m_l - \gamma\Sigma_{j=1}^n\sigma_{lj} + (1-p+p\Phi(z_i^c))f(z_i^c + \frac{\gamma\Sigma_{j=1}^n\sigma_{ij}}{\sigma_{v_i}})\frac{\sigma_{li}}{\sigma_{v_i}} + p(1-\Phi(z_i^c)) \times \frac{\sigma_{li}}{\sigma_{v_i}}\{\frac{\phi(z_i^c)}{1-\Phi(z_i^c)} + \frac{\gamma\Sigma_{j=1}^n\sigma_{ij}}{\sigma_{v_i}}\} \\ = \quad & m_l - \gamma\Sigma_{j=1}^n\sigma_{lj} + \frac{\sigma_{li}}{\sigma_{v_i}}\left((1-p+p\Phi(z_i^c))f(z_i^c + \frac{\gamma\Sigma_{j=1}^n\sigma_{ij}}{\sigma_{v_i}}) + p(\phi(z_i^c) + (1-\Phi(z_i^c))\frac{\gamma\Sigma_{j=1}^n\sigma_{ij}}{\sigma_{v_i}})\right) \end{aligned}$$

When the manager adopts the equilibrium cutoff for the risk-averse case, z_i^{RAc}, then we have:

$$
\begin{aligned}
& E[\tilde{P}_l|z_i^{RAc}] - (m_l - \gamma\Sigma_{j=1}^n \sigma_{lj}) \\
= \; & \frac{\sigma_{li}}{\sigma_{v_i}}\{(1-p+p\Phi(z_i^{RAc}))f(z_i^{RNc}) + p\phi(z_i^{RAc}) + p(1-\Phi(z_i^{RAc}))\frac{\gamma\Sigma_{j=1}^n \sigma_{ij}}{\sigma_{v_i}}\} \\
= \; & \frac{\sigma_{li}}{\sigma_{v_i}}\{(1-p+p\Phi(z_i^{RNc}) + p(\Phi(z_i^{RAc}) - \Phi(z_i^{RNc}))f(z_i^{RNc}) + p\phi(z_i^{RAc}) + p(1-\Phi(z_i^{RAc}))\frac{\gamma\Sigma_{j=1}^n \sigma_{ij}}{\sigma_{v_i}}\} \\
= \; & \frac{\sigma_{li}}{\sigma_{v_i}}\{p\phi(z_i^{RAc}) - p\phi(z_i^{RNc}) + p(\Phi(z_i^{RAc}) - 1 - (\Phi(z_i^{RNc}) - 1))f(z_i^{RNc}) + p(1-\Phi(z_i^{RAc}))\frac{\gamma\Sigma_{j=1}^n \sigma_{ij}}{\sigma_{v_i}}\} \\
= \; & \frac{\sigma_{li}}{\sigma_{v_i}}\{p\phi(z_i^{RAc}) - p\phi(z_i^{RNc}) + p(\Phi(z_i^{RAc}) - 1 - (\Phi(z_i^{RNc}) - 1))z_i^{RNc} + p(1-\Phi(z_i^{RAc}))\frac{\gamma\Sigma_{j=1}^n \sigma_{ij}}{\sigma_{v_i}}\} \\
= \; & \frac{\sigma_{li}}{\sigma_{v_i}}\{p\phi(z_i^{RAc}) - p\phi(z_i^{RNc}) + p(1-\Phi(z_i^{RNc}))z_i^{RNc} + p(1-\Phi(z_i^{RAc}))(\frac{\gamma\Sigma_{j=1}^n \sigma_{ij}}{\sigma_{v_i}} - z_i^{RNc}\} \\
= \; & \frac{\sigma_{li}}{\sigma_{v_i}}\{p\phi(z_i^{RAc}) - p\phi(z_i^{RNc}) + p(1-\Phi(z_i^{RNc}))z_i^{RNc} - p(1-\Phi(z_i^{RAc}))z_i^{RAc}\} \\
= \; & \frac{p\sigma_{li}}{\sigma_{v_i}}\{H(z_i^{RAc}) - H(z_i^{RNc})\} \\
= \; & \frac{p\sigma_{li}}{\sigma_{v_i}}\{H(z_i^{RNc} - \frac{\gamma\Sigma_{j=1}^n \sigma_{ij}}{\sigma_{v_i}}) - H(z_i^{RNc})\}
\end{aligned}
\tag{59}
$$

where, in this last line, we have introduced the function $H(z)$ defined by: $H(z) \equiv \phi(z) - (1-\Phi(z))z$. Hence, firm $l's$ cost of capital can be expressed as

$$
CC_l(z_i^c) = m_l - E[\tilde{P}_l|z_i^c] = \gamma\Sigma_{j=1}^n \sigma_{lj} - \frac{p\sigma_{il}}{\sigma_{v_i}}\left[H(z_i^{RAc}) - H(z_i^{RNc})\right],
$$

as asserted in (42).

Since $H'(z) = -(1-\Phi(z))$, and hence, $H(z)$ is everywhere strictly decreasing in z, it follows from $z_i^{RAc} < z_i^{RNc}$ that:

$$
H(z_i^{RAc}) - H(z_i^{RNc}) > 0, \tag{60}
$$

and hence firm $l's$ cost of capital when the manager of firm i optimally discloses the information she receives is below firm $l's$ "no information" cost of capital $\gamma\Sigma_{j=1}^n \sigma_{ij}$ when $\sigma_{il} > 0$, and is above firm $l's$ "no information" cost of capital when $\sigma_{il} < 0$.

In view of the inverse relationship between a firm's expected selling price and its cost of capital, and with (59) in hand, comparative statics involving firms' costs of capital are easy to obtain. For example, we now show that

$$
\frac{d}{dp}E[\tilde{P}_l]|_{z_i^c = z_i^{RAc}} \lesseqgtr 0 \text{ as } \sigma_{il} \lesseqgtr 0. \tag{61}
$$

In view of (59), we can write the expected price of firm l when firm $i's$ manager uses the unique equilibrium disclosure policy $z_i^{RAc}(p)$ as:

$$E[\tilde{P}_l] = m_l - \gamma\Sigma_{j=1}^n \sigma_{lj} + \frac{p\sigma_{li}}{\sigma_{v_i}}\left[H(z_i^{RNc} - \frac{\gamma\Sigma_{j=1}^n\sigma_{ij}}{\sigma_{v_i}}) - H(z_i^{RNc})\right]. \qquad (62)$$

Thus:

$$\begin{aligned}&\frac{d}{dp}E[\tilde{P}_l]|_{z_i^c=z_i^{RAc}}\\ = \;& \frac{p\sigma_{li}}{\sigma_{v_i}}\left[H'(z_i^{RNc} - \frac{\gamma\Sigma_{j=1}^n\sigma_{ij}}{\sigma_{v_i}}) - H'(z_i^{RNc}))\right]\frac{dz_i^{RNc}}{dp} + \frac{\sigma_{li}}{\sigma_{v_i}}[H(z_i^{RNc} - \frac{\gamma\Sigma_{j=1}^n\sigma_{ij}}{\sigma_{v_i}}) - H(z_i^{RNc})].\end{aligned}$$

It is easy to check that $H(\cdot)$ is convex and that $\frac{dz_i^{RNc}(p)}{dp}$ is negative. Hence, the sign of the first term in the preceding derivative, $\frac{p\sigma_{li}}{\sigma_{v_i}}\left[H'(z_i^{RNc} - \frac{\gamma\Sigma_{j=1}^n\sigma_{ij}}{\sigma_{v_i}}) - H'(z_i^{RNc}))\right]\frac{dz_i^{RNc}}{dp}$, is the same as the sign of σ_{li}. The same is true of of the sign of the second term, $\frac{\sigma_{li}}{\sigma_{v_i}}[H(z_i^{RNc} - \frac{\gamma\Sigma_{j=1}^n\sigma_{ij}}{\sigma_{v_i}}) - H(z_i^{RNc})]$. Hence, so is the sign of $\frac{d}{dp}E[\tilde{P}_i]|_{z_i^c=z_i^{RAc}}$. This proves (61).

We next prove

$$\frac{d}{d\sigma^2_{\varepsilon_i}}E[\tilde{P}_l]|_{z_i^c=z_i^{RAc}} \lesseqgtr 0 \text{ as } \sigma_{il} \gtreqless 0. \qquad (63)$$

Recalling that z_i^{RNc} is independent of all parameters of the model other than p, and using (62), we see that:

$$\begin{aligned}&\frac{d}{d\sigma^2_{\varepsilon_i}}E[\tilde{P}_l]|_{z_i^c=z_i^{RAc}}\\ = \;& -\frac{p\sigma_{li}}{\sigma^2_{v_i}}\frac{d\sigma^2_{v_i}}{d\sigma^2_{\varepsilon_i}}\left[H(z_i^{RNc} - \frac{\gamma\Sigma_{j=1}^n\sigma_{ij}}{\sigma_{v_i}}) - H(z_i^{RNc})\right] + \frac{p\sigma_{li}}{\sigma_{v_i}}H'(z_i^{RNc} - \frac{\gamma\Sigma_{j=1}^n\sigma_{ij}}{\sigma_{v_i}})\frac{\gamma\Sigma_{j=1}^n\sigma_{ij}}{\sigma^2_{v_i}}\frac{d\sigma^2_{v_i}}{d\sigma^2_{\varepsilon_i}}\\ = \;& \frac{p\sigma_{li}}{\sigma^2_{v_i}}\frac{d\sigma^2_{v_i}}{d\sigma^2_{\varepsilon_i}}\left(-(H(z_i^{RNc} - \frac{\gamma\Sigma_{j=1}^n\sigma_{ij}}{\sigma_{v_i}}) - H(z_i^{RNc})) + H'(z_i^{RNc} - \frac{\gamma\Sigma_{j=1}^n\sigma_{ij}}{\sigma_{v_i}})\frac{\gamma\Sigma_{j=1}^n\sigma_{ij}}{\sigma_{v_i}}\right).\end{aligned}$$

Since $H(\cdot)$ is decreasing and convex, the bracketed term in this last expression is negative. Hence, since $\frac{d\sigma^2_{v_i}}{d\sigma^2_{\varepsilon_i}} > 0$, the sign of $\frac{d}{d\sigma^2_{\varepsilon_i}}E[\tilde{P}_l]|_{z_i^c=z_i^{RAc}}$ is the opposite of the sign of σ_{li}. This proves (63).

The other comparative statics reported in Corollaries 17 and 18 can be proven similarly.■

Proof of Theorem 19 When the large economy assumption holds, we can write β_i^{nd} as $\beta_i^{nd} = D \times \frac{cov(\tilde{x}_i,\Sigma_j\tilde{x}_j|nd,z_i^c)}{P_i^{nd}} = D \times \frac{\eta(z_i^{RAc})\Sigma_j\sigma_{ij}}{P_i^{nd}}$ (where the last result follows from (18) and Lemma 12) and we can write $\beta_i^d(v_i)$ as $\beta_i^d(v_i) =$

$D\times\frac{cov(\tilde{x}_i,\Sigma_j\tilde{x}_j|v_i)}{P_i^d(v_i)}=D\times\frac{\alpha\Sigma_j\sigma_{ij}}{P_i^d(v_i)}$. By definition of the equilibrium cutoff, $P_i^d(v_i)>P_i^{nd}$ for every v_i such that $z_i=\frac{v_i-m_i}{\sigma_{v_i}}>z_i^{RAc}$, with this inequality replaced by an equality at that $v_i=v_i^{RAc}$ defined by $\frac{v_i^{RAc}-m_i}{\sigma_{v_i}}=z_i^{RAc}$. Since $P_i^d(v_i)$ is obviously increasing in v_i, it follows that $\beta_i^d(v_i)$ is strictly decreasing in v_i. Since $\alpha<1$ and $\eta(z_i^{RAc})>1$, it follows that $\beta_i^d(v_i^{RAc})=D\times\frac{\alpha\Sigma_j\sigma_{ij}}{P_i^d(v_i^{RAc})}=D\times\frac{\alpha\Sigma_j\sigma_{ij}}{P_i^{nd}}<D\times\frac{\eta(z_i^{RAc})\Sigma_j\sigma_{ij}}{P_i^{nd}}=\beta_i^{nd}$. Since, with probability 1, every disclosed value of v_i is such that $v_i>v_i^{RAc}$, it follows that with probability 1, for every v_i disclosed in equilibrium, we have $\beta_i^d(v_i)<\beta_i^d(v_i^{RAc})=\beta_i^{nd}$.$\blacksquare$