Bargaining with Periodic Participation Costs

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Abstract

We study a bargaining game in which a player needs to pay a fixed cost in the beginning of every period t, if he wants to stay in the game in period t + 1, in case a deal has not been reached by the end of t. Whether a player pays this cost is his private information. Every efficient payoff vector can be approximated in equilibrium. When costs of delay vanish, the value of every symmetric stationary equilibrium converges to zero. Applications of our model include bargaining through representatives, trading off bargaining and other activities, and collusion in auctions.

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1 Introduction

Bargaining may take time, and how long it will take may be unknown in advance. The ability to bargain for as long as necessary is therefore important. Acquiring this ability may be costly, and possessing it may be a player's private information. In this paper we study a 2-player game that is designed to focus on these aspects of non-cooperative bargaining.

Our game is as follows. Two players need to divide a pie. In the beginning of every period there is an initial stage in which each player chooses whether or not to take a costly action ("invest"); the cost of this action is $c > 0$. This choice is unobservable to his opponent. Once these choices are made, bargaining takes place in the second stage of the period, in the following fashion: a proposer is selected at random by Nature, with each player being equally likely to be selected, and then the proposer offers a split of the pie. If the offer is accepted by the responder then it is implemented and the game ends; otherwise, the game moves one period forward, but a player who did not take the costly action drops out of the game. If only one player moves to the next period, he receives the entire pie. If both players move to the next period, the above story repeats itself.

Here are two applications (or interpretations) of our model; a further application, the presentation of which requires some technical preliminaries, will be given in Subsection 2.1.

Bargaining through representatives. Consider the case where the bargainer is not the economic agent, but a representative who bargains on his behalf; e.g., a lawyer. The agent needs to pay the representative in advance if he wants the latter to keep working on the case and sitting at the negotiation table tomorrow, in case a deal has not been reached today. For such applications, one can interpret our model in the following alternative way: conditional on not taking the costly action and not reaching a deal in the present period, a player does not leave the game, but rather stays in the game while *losing his voice* at the negotiation table. For example, a defendant does not walk away from the court if his representing lawyer drops the case; the trial goes on, but the defendant's position is so weak, that it is possible to impose upon him a low payoff.

Trading off bargaining and other activities. Consider the case where each player faces an infinite stream of *tasks*, one in every period. In each period, the player can either take care of his task, or be engaged in bargaining, but he cannot do both. The periodic tasks must be performed. Therefore, if a player wishes to sit down at the negotiation table and bargain, he needs to make sure that the periodic tasks are taken care of, as long as he is busy bargaining. He can do that at a cost c per task, and payment should be made one period in advance.

The players share a common discount factor, δ. Thus, there are two dimensions to time preferences in our model: the cost c (the *participation cost*) and discounting. Our focus is on the case where the frictions are low; namely, when (δ, c) is close to $(1, 0)$. Some of our results require $\delta < 1$ and some allow $\delta \leq 1$.

Since our game is recursive, we focus on perfect Bayesian equilibrium in stationary strategies. Our first result is an "anything goes" result: every Pareto efficient payoff vector can be approximated in a stationary equilibrium. That is, for every Pareto efficient payoff vector u and every $\epsilon > 0$ there exists a stationary equilibrium whose payoff vector is ϵ-close to u, provided that (δ, c) is sufficiently close to $(1, 0)$. This result holds both for $\delta < 1$ and for $\delta = 1$.

The approximating strategy is not symmetric, even if the approximated payoff vector is $(\frac{1}{2}, \frac{1}{2})$. The equilibrium that we construct is as follows. Suppose that the target payoff vector is (u_1, u_2), with $u_2 \geq u_1$. Take $x \in (\frac{1}{2}, 1)$, arbitrarily close to u_2.[1] We let each player invest with certainty in the beginning of each period, and always

[1] It is possible to take $x = u_2$, unless $u_2 \in \{\frac{1}{2}, 1\}$.

demand x for himself whenever he is called by Nature to be the proposer. We let player 2 play aggressively, and always reject the opponent's offer (on the equilibrium path) while player 1 follows a compromising strategy, under which he always accepts the opponent's offer (on the path). Thus, with probability one player 2's proposal is implemented, and he ends up with a pie slice of size x; consequently, when (δ, c) is sufficiently close to $(1, 0)$ his payoff is approximately x, and that of player 1 is approximately $1 - x$.[2] For $u_1 \geq u_2$, the analogous construction, under which player 1 is aggressive, delivers the desired result. In this way, the entire Pareto frontier can be approximated.

The fact that the approximating strategy is not symmetric is inevitable, since when $\delta < 1$ the combination of stationarity and symmetry has the following implication: when (δ, c) converges $(1, 0)$, the value of every symmetric stationary equilibrium converges to zero. The issue with symmetric stationary strategies is that they lead to a coordination problem. In a symmetric stationary equilibrium each player invests randomly in the beginning of every period, and conditional on the event "both players invest" there is disagreement and the game moves to the next period. The investment probability converges to one as (δ, c) converges $(1, 0)$. Therefore, when $(\delta, c) \sim (1, 0)$ and under symmetric and stationary strategies, our game becomes a war of attrition in which the entire social surplus is destroyed.

If $\delta = 1$ then agreement is possible in equilibrium conditional on "both players invest." This allows us to construct a symmetric stationary equilibrium in which such agreement occurs, and, consequently, there is no war of attrition. The payoffs in this equilibrium converge to $(\frac{1}{2}, \frac{1}{2})$ as $c \to 0$. This equilibrium, however, is problematic, in the following sense: in this equilibrium, a player's conditional-on-being-the-proposer payoff is independent of whether he invested or not. We think this is unappealing,

[2] $x > \frac{1}{2}$ is necessary for making player 2 willing to reject player 1's offer and trigger another bargaining period. $x < 1$, on the other hand, stems from player 1's incentive constraint: $x = 1$ would mean that player 1 receives a zero share of the pie, in which case investment is suboptimal for him, no matter how small c is.

since, intuitively, being a proposer-who-invested is the best bargaining position. More generally, we believe that no matter whether a player is in the position of the proposer or the responder, investment should affect his conditional expected utility positively. We formalize this idea as an equilibrium refinement, *investment-monotonicity*. Under this refinement our "destructive War of Attrition" result continues to hold even if the players are perfectly patient; namely, even if $\delta = 1$.

Though efficiency cannot be obtained in a symmetric stationary equilibrium if $\delta < 1$ or if investment-monotonicity is imposed, it can be obtained in an *ex ante symmetric equilibrium*—an equilibrium which is symmetric from the ex ante point of view, but not conditional on every history. We construct an efficient ex ante symmetric equilibrium on the basis of the construction that underlies our "anything goes" result. Specifically, the equilibrium is as follows. Both players invest in the first period and Nature's first-period role-assignment is regarded as an assignment of who will play the aggressive strategy and who will play the compromising strategy, starting from the second period onwards. So, for example, if Nature selects player 1 to be the proposer in the first period, then player 1 is being assigned the compromising role: he makes the equilibrium offer, which player 2 rejects, and starting from the second period onwards an asymmetric equilibrium is played in which player 1 is the compromising player and player 2 is the aggressive player. When δ is sufficiently large and c is sufficiently small, this strategy can be sustained in equilibrium, for example for $x = \frac{2}{3}$ (i.e., each player demands—and the aggressive player eventually obtains—$\frac{2}{3}$ of the pie). Clearly, this strategy is ex ante symmetric, though it is not symmetric conditional on any non-initial history.[3]

When one sets the participation cost to zero, our game becomes, practically, a game of complete information—a symmetrized Rubinstein-game.[4] Namely, a Rubinstein-type bargaining game in which the proposer is selected randomly (with probability $\frac{1}{2}$) in the beginning of every period, rather than in an alternating fashion.

[3]This is the only result of our paper that involves a non-stationary strategy.

[4]If investment is costless, then clearly it (weakly) dominates not investing.

This game, which has been studied by Binmore (1987), has a unique subgame perfect equilibrium.[5] This equilibrium is symmetric and stationary, and it involves an immediate agreement. A player's expected utility in this equilibrium is $\frac{1}{2}$. Thus, our results imply that this complete information bargaining game is not robust to the addition of arbitrarily small (and private) participation costs. Specifically, this non-robustness manifests itself in two ways. First, the set of equilibrium payoffs that can be approximated in the incomplete information game includes the entire frontier, whereas in the complete information game $(\frac{1}{2}, \frac{1}{2})$ is the unique equilibrium payoff vector. Second, under the restriction to symmetric and stationary strategies, every equilibrium payoff vector converges to $(0, 0)$—not to $(\frac{1}{2}, \frac{1}{2})$—as (δ, c) converges to $(1, 0)$.

The rest of the paper is organized as follows. In Subsection 1.1 we review relevant literature. In Section 2 we formally describe our model; following this description we provide, in Subsection 2.1, another application (or interpretation) of the model. In Section 3 we construct (asymmetric) stationary equilibria that approximate any point on the Pareto frontier when (δ, c) is sufficiently close to $(1, 0)$. In Section 4 we consider symmetric stationary equilibria. In this section we establish, through a sequence of intermediate results, that when $\delta < 1$ the combination of symmetry and stationarity implies a war of attrition in which the entire social surplus is destroyed. We also show that when $\delta = 1$ exactly two payoff vectors are attainable in a symmetric stationary equilibrium as $c \to 0$: $(\frac{1}{2}, \frac{1}{2})$ and $(0, 0)$. However, we show that $(\frac{1}{2}, \frac{1}{2})$ is excluded if investment-monotonicity is imposed. In Section 5 we construct an ex ante symmetric equilibrium that achieves efficiency as $(\delta, c) \to (1, 0)$. In Section 6 we discuss the connection to Binmore's (1987) complete information game, and in Section 7 we conclude.

[5]Binmore's model, like Rubinstein's, assumes $\delta \in (0, 1)$.

1.1 Related literature

Modeling costs of delay in extensive-form bargaining games as fixed costs goes back to Rubinstein (1982). Though his model is stated in terms of abstract preferences, fixed costs are one of the two leading applications of time preferences in his model (the other being discounting). In Rubinstein's game with fixed costs the cost is unavoidable: in case of a failure to reach an agreement, each player pays the cost of moving to a new bargaining period. A slightly different role of fixed costs was considered by Perry (1986). In Perry's model, the cost is associated with *making an offer*. Hence, if player i makes an offer which is rejected by j—in which case play moves to the next period and the roles alternate—only i pays a cost but j does not. A follow-up paper on Perry's work is by Cramton (1991), which combines (similar to our specification) both fixed costs and discounting in the players' utility functions. In Cramton's model, like in Rubinstein's, a player pays the fixed bargaining cost in every period, no matter whether he was the proposer or responder in that period. The fixed participation cost in our model is different in that it is paid for each period that passes (like in Rubinstein's and Cramton's models), but, contrary to these models, it must be paid in advance. More importantly, whether or not a player pays the cost is his private information.

Another group of related papers consists of those that study bargaining models in which the players compete for a favorable position in the bargaining process. For example, Board and Zwiebel (2012) study a bargaining game in which two players compete, in every period, for the right to make a proposal regarding the split of a pie. The competition for the proposer's position is through a first-price auction, the players have budget constraints, and the horizon is finite. Ali (2015) studies a similar game, in a framework that allows for both finite and infinite horizon, in which each of n players compete in every period for the right to make a proposal through an all-pay or first-price auction. Yildirim (2007, 2010) studies a model in which the players compete for the proposer's position not through an auction, but by exerting effort that influences the selection of the proposer through a contest success function. An earlier related paper is by Evans (1997), who studies a coalitional bargaining game in which, at each stage, the players compete for the right to make a proposal. In all these papers the players compete for being the proposer, which is a favorable position in the extensive form. In our game, the costly investments guarantee another kind of favorable position—the ability to continue bargaining in case it is necessary; to put it differently, the favorable position is to be the "last one standing".

2 Model

Let G be the following game. Two players, 1 and 2, need to divide a pie of size one. The players share a common discount factor, δ. In the first period of the game, $t = 1$, both players are *active*. In every period t in which both players are active, play is as follows. First, each player privately decides whether or not to take a costly action, *invest*. If a player invests he pays a cost, $c > 0$, upfront. Not investing has no cost. The investment decisions are not publicly observable. After these decisions have been made, Nature selects a *proposer* and a *responder* with equal probabilities. The proposer offers a split of the pie, (x_1, x_2), where x_i is i's proposed share and $x_1 + x_2 = 1$.[6] If the responder accepts the offer, the game ends. If he rejects, then the game moves to period $t + 1$, but a player who did not invest in the beginning of period t drops out of the game and does not reach period $t + 1$. If neither player invested in the beginning of t then the game ends and nobody receives anything. If only one player invested then he is the sole active player at $t + 1$, and he obtains the entire pie then. If both reach $t + 1$, then both are active at $t + 1$.

The payoff of player i from agreeing on (x_1, x_2) in the first period is x_i if he did not invest and $x_i - c$ if he did. His payoff from agreeing on (x_1, x_2) at $t+1$ (for $t \geq 1$),

[6] We will often write an offer as $(x, 1 - x)$, with the understanding that the proposer asks x for himself.

evaluated in the beginning of the game, is:

$$\delta^t x_i - \sum_{i=0}^{\tau-1} \delta^i c,$$

where $\tau \in \{t, t+1\}$. The value $\tau = t$ corresponds to the case where he did not invest in the last period, and $\tau = t+1$ corresponds to the case where he did invest in that period. The utility from perpetual disagreement is $-\sum_{i=0}^{\infty} \delta^i c$ if $\delta < 1$, and is $-\infty$ if $\delta = 1$.

Let $t+1$ be a period in which both players are active, $t \geq 1$. A *history leading to* $t+1$ is a list $((i_1, x_1), \cdots, (i_t, x_t))$, meaning that in period $k \in \{1, \cdots, t\}$ the proposer was player i_k, he demanded x_k for himself, and the opponent rejected his offer. The set of those histories is denoted H_{t+1}. The initial history is $\emptyset$, and we set $H_1 \equiv \{\emptyset\}$. The set of histories is $H \equiv \cup_t H_t$.

A *strategy* for i is a triplet of functions, $\sigma^i = (I^i, f^i, g^i)$. The function $I^i \colon H \to [0,1]$ prescribes the investment probability in the beginning of every period in which both players are active, as a function of the history leading to this period. The function $f^i \colon H \times \{0,1\} \to [0,1]$ assigns an offer as a function of the history and the player's investment decision; a player uses this component of the strategy whenever he is called by Nature to be the proposer. The function $g^i \colon H \times \{0,1\} \times [0,1] \to \{Accept, Reject\}$ assigns a response to the opponent's offer as a function of the history, the player's investment decision, and the opponent's offer; a player uses this component of the strategy whenever he is called by Nature to be the responder. A pair of strategies is denoted by $\sigma = (\sigma^1, \sigma^2)$.

A *system of beliefs* for player i is a function $\mu^i \colon H \to [0,1]$. Given a history leading to $t+1$, h_{t+1}, the number $\mu^i(h_{t+1})$ is the probability that i attaches to the event that j invested in the beginning of $t+1$. A pair of belief systems is denoted by $\mu = (\mu^1, \mu^2)$.

The pair (σ, μ) is a *perfect Bayesian equilibrium* (PBE) if:

1. For every i and every history h after which player i is active, each component

of σ^i assigns an optimal action for i, given σ^j and the beliefs μ^i.

2. The beliefs μ obey Bayes' rule whenever possible.

Throughout the paper, "equilibrium" means PBE.[7] We say that the strategy σ is sustainable in equilibrium if there is μ such that (σ, μ) is an equilibrium.

Given a strategy σ and a history $h \in H$, let $Pr_\sigma(h)$ be the probability of h under σ. Given a history h and a strategy σ, let $\sigma|_h$ be the conditioning of σ on h. Namely, this is the continuation strategy that σ induces in the subgame that starts with h. An equilibrium (σ, μ) is *stationary* if $\sigma|_h$ is independent of h on $H(\sigma) \equiv \{h \in H : Pr_\sigma(h) > 0\}$. Namely, an equilibrium is stationary if play is independent of history as long as no one has deviated.

An equilibrium (σ, μ) is *symmetric* if for every $h \in H(\sigma)$:

- $I^1(h) = I^2(h)$,
- If $I^1(h) = I^2(h) > 0$ then $f^1((h, 1)) = f^2((h, 1))$,
- If $I^1(h) = I^2(h) < 1$ then $f^1((h, 0)) = f^2((h, 0))$,
- If $I^1(h) = I^2(h) = 0$ then $g^1((h, 0, f^2((h, 0)))) = g^2((h, 0, f^1((h, 0))))$,
- If $I^1(h) = I^2(h) = 1$ then $g^1((h, 1, f^2((h, 1)))) = g^2((h, 1, f^1((h, 1))))$, and
- If $I^1(h) = I^2(h) \in (0, 1)$ then $g^1((h, d, o)) = g^2((h, d, o))$ for every investment decision $d \in \{0, 1\}$ and opponent-offer $o \in \{f^i(h, 0), f^i(h, 1)\}$.

That is, an equilibrium is symmetric if the players' strategies coincide on the path. We allow off-path play to be non-symmetric. Under this definition, a symmetric path of play which is supported by asymmetric off-path threats is legitimately called "symmetric equilibrium" (even though σ^1 and σ^2 need not coincide on the entire game

[7] In Section 6 we consider a complete information version of our game. There (and only there) the solution concept is subgame perfect equilibrium.

tree). An equilibrium which is not symmetric is called *asymmetric*.

In a symmetric stationary equilibrium, the expected utility of a player is the same, no matter after which on-path history it is computed, and no matter who is the player is question; we call this utility the *value* of the equilibrium. For a stationary equilibrium which is asymmetric, there is a value for each player and these values may differ. A symmetric stationary equilibrium is *pooling* if a single offer is made on its path, otherwise it is *separating*.

Given (δ, c), we denote by $E(\delta, c)$ the set of payoff vectors that are attainable in stationary equilibria, and by $E^s(\delta, c)$ the set of payoff vectors that are attainable in symmetric stationary equilibria. Also, we denote by Δ the set of Pareto efficient payoff vectors (i.e., the unit simplex).

Given a history h let h' be identical to h except that each i_k is replaced by $3 - i_k$. Namely, h' is the same as h except that the roles of the players are interchanged. An equilibrium is *ex ante symmetric* if $Pr_\sigma(h) = Pr_\sigma(h')$ for all h. We abuse terminology a little, as follows: given an ex ante symmetric equilibrium, we use the term "value" to refer to a player's ex ante expected utility in this equilibrium.

2.1 Bargaining for "infinity plus one" periods

Having the formalities spelled out, we can now present one more application of our model. Setting $\delta = 1$ makes our model suitable for describing situations in which the players can make an unbounded number of moves within a finite time frame. Specifically, consider the following bribing-in-auctions game.[8] Two bidders are about to participate in a common value auction for a good whose value is one. Before the auction they can try to bribe each other: when player i gets to make a bribe offer, he offers player j a money transfer b, which will be payed in exchange for j's abstention from the auction. Acceptance of the transfer means that the briber will become the sole bidder in the auction and obtain the good for free, hence the resulting utility

[8] This example is inspired by the bribing-in-auctions model of Esö and Schummer (2004).

vector will be $(u_i, u_j) = (1-b, b)$. Each round of pre-auction communication entails a cost $c > 0$ for each player, which can be thought of as the disutility from engaging in an illegal activity (namely, collusion). Note that there aren't infinitely many periods here, in the sense of the passage of time, but rather a fixed duration within which infinitely many "costly-action-rounds" may take place. This time frame, whose order type is $\omega + 1$, is due to Aumann and Hart (2003), who studied 2-player games that are preceded by infinitely many rounds of communication.

3 Anything goes: every Pareto efficient payoff vector can be approximated in a stationary equilibrium

Consider the following strategy. Fix an $x \in (\frac{1}{2}, 1)$. A player always invests with certainty, and demands x whenever he is called to make an offer. Player 1 accepts player 2's offer if and only if it gives him at least $1-x$.[9] Player 2 accepts an offer if and only if it gives him at least δ. The prescribed rejections are supported by the following beliefs: whenever a player sees an unexpected offer that the strategy instructs him to reject, he adopts the belief that the proposer did not invest, hence rejecting the offer is optimal, as it guarantees the entire pie in the next period. If a player does not invest (which is a deviation) he accepts any offer of the opponent. If player 1 did not invest and he is selected to be the proposer, he demands $1 - \delta$ for himself. If player 2 did not invest and he is selected to be the proposer, he demands x for himself. Denote this strategy by $\sigma(x, 2)$. Similarly, let $\sigma(x, 1)$ denote the analogous strategy, where player 1 is the aggressive player who always rejects the x-offer and player 2 always accepts it.

[9] In equilibrium a player "cannot refuse" an offer that gives him more than δ. Since we are interested in equilibrium for large enough δ's (and small enough c's), we may assume that $1 - x \leq \delta$.

Proposition 1. *Fix $x \in (\frac{1}{2}, 1)$ and $i \in \{1, 2\}$. There exist $\delta(x) \in (0, 1)$ and $c(x) > 0$ such that the following holds: $\sigma(x, i)$ is sustainable in a stationary equilibrium, provided that $\delta \in (\delta(x), 1]$ and $c < c(x)$.*

Proof. Fix an $x \in (\frac{1}{2}, 1)$. Without loss of generality, we suppose that $i = 2$; namely, player 2 is the aggressive one. We consider $\delta = 1$; the same arguments establish equilibrium existence for δ sufficiently close to one.

Let v_i be player i's value from the abovementioned strategies. For player 1 we have $-c + \frac{1}{2}(1 - x) + \frac{1}{2}v_1 = v_1 \Rightarrow v_1 = 1 - x - 2c$. In particular, $1 - x > v_1$, so accepting player 2's offer is better than rejecting it and triggering the next period.

The following condition guarantees that investing is better than not investing: $\frac{1-x}{2} \leq v_1$, or:

$$2c \leq \frac{1 - x}{2}. \tag{1}$$

Given that player 1 invested, demanding the equilibrium demand is optimal for him, since $(1 - \delta) \leq \delta v_1$ clearly holds for $\delta = 1$. Given that he did not invest, demanding $1 - \delta$ is optimal for him, as player 2 will reject any greedier offer.

For player 2 the value satisfies $\frac{1}{2}x + \frac{1}{2}v_2 - c = v_2$, or $v_2 = x - 2c$. The following condition guarantees that investing is optimal: $\frac{1}{2}x + \frac{1}{2}(1 - x) = \frac{1}{2} \leq v_2$, or:

$$2c \leq x - \frac{1}{2}. \tag{2}$$

Since player 2's strategy prescribes him a rejection of $1 - x$, it needs to be the case that $1 - x \leq v_2$, or $1 - x \leq x - 2c$. This condition is satisfied if c is sufficiently small since $x > \frac{1}{2}$.

Given that player 2 invested, demanding the equilibrium demand is optimal for him, since $x \geq \max\{\delta v_2, 1 - \delta\}$ clearly holds for $\delta = 1$. It is also easy to see that given that he did not invest, demanding x is optimal for him.

Combining (1) and (2) we get:

$$2c \leq \min\{\frac{1-x}{2}, x - \frac{1}{2}\}.$$[10]

Clearly, the above requirement holds for all sufficiently small c's. □

It is worth noting that the strategies in the equilibrium of Proposition 1 are stationary in a stronger sense than the one of our stationarity definition. Specifically, the stationarity of these strategies is not restricted to the equilibrium path, but holds everywhere. Namely, the stationarity is like the one in the subgame perfect equilibrium of Rubinstein's game, where deviations that do lead to a termination of the game lead to a continuation game in which the deviation is ignored, hence this continuation game is equivalent to the entire super-game.

The equilibrium of Proposition 1 is different from that of Rubinstein's game in that that a responder is *not* indifferent between accepting and rejecting the equilibrium offer. To see this, consider first player 1. Since $1 - x > v_1$, accepting the offer is strictly better for him than rejecting it. Note that if player 2 cuts down his offer to some $y \in (v_1, 1 - x)$ it would be rejected by player 1, due to player 1's belief that player 2 did not invest; this belief makes it optimal for player 1 to reject this "above-value offer" and take the entire pie in the next period. Now consider player 2. He strictly prefers rejecting the equilibrium offer to accepting it, since $v_2 > 1 - x$. If player 1 were to change his demand such that player 2 would be offered an ϵ above the share that turns the above inequality to equality (i.e., player 1 would demand $\tilde{x} - \epsilon$, where $\tilde{x} = 1 - v_2$) player 2 would reject it on the basis of the belief that player 1 did not invest.

The following is our main result for this section; it is an immediate consequence of Proposition 1.

Theorem 1. *For every Pareto efficient payoff vector u and every $\epsilon > 0$ there exist $\delta(u, \epsilon) \in (0, 1]$ and $c(u, \epsilon) > 0$ such that the following holds provided that*

[10] For a general δ the corresponding condition is $2c \leq \min\{1 - x + \frac{x}{2}(2-\delta) - \frac{1}{2}(2-\delta)^2, x + \frac{\delta}{2} - 1\}$.

$\delta \in (\delta(u, \epsilon), 1]$ and $c < c(u, \epsilon)$: G has a stationary equilibrium whose payoff vector is ϵ-close to u.

The behavior of the compromising player (the one who accepts the equilibrium offer) is of very limited relevance to the equilibrium outcome, since the equilibrium outcome is that the aggressive player's proposal is implemented with probability one. Thus, as long as the compromising player does not make offers that the aggressive player "cannot refuse," the equilibrium outcome will not change. This implies the following corollary.

Corollary 1. *For every Pareto efficient payoff vector u and every $\epsilon > 0$ there exist $\delta(u, \epsilon) \in (0, 1]$ and $c(u, \epsilon) > 0$ such that the following holds provided that $\delta \in (\delta(u, \epsilon), 1]$ and $c < c(u, \epsilon)$: G has infinitely many non-stationary equilibria, each of which has a payoff vector ϵ-close to u.*

Finally, it is worth noting that Theorem 1 can be phrased in an alternative way, using limit terms.

Corollary 2. $lim_{\delta \to 1} lim_{c \to 0} E(\delta, c) = lim_{c \to 0} lim_{\delta \to 1} E(\delta, c) = \Delta$.

Proof. The proof of Proposition 1 established that $\lim_{c \to 0} \lim_{\delta \to 1} E(\delta, c) = \Delta$. We will now prove that nothing changes if we reverse the order of limits. That is, given a fixed $x \in (\frac{1}{2}, 1)$, we will set $c = 0$ and prove that $\sigma(x, i)$ is sustainable in equilibrium for all δ sufficiently close to one.

Like in the proof of Proposition 1, we consider, wlog, $i = 2$.

For player 1 the value satisfies $\frac{1}{2}(1 - x) + \frac{1}{2}\delta v_1 = v_1$, hence $1 - x = (2 - \delta)v_1$. Since $1 - x > \delta v_1$, accepting player 2's offer is better than rejecting it. The fact that investment is better than not investing follows from $\frac{1-x}{2} + \frac{1}{2}(1 - \delta) \leq v_1 = \frac{1-x}{2-\delta}$, which holds for all δ sufficiently close to one.

For player 2 the value satisfies $\frac{1}{2}x + \frac{1}{2}\delta v_2 = v_2$, hence $x = (2 - \delta)v_2$. Rejecting $1 - x$ is optimal since $\delta v_2 = \frac{\delta x}{2-\delta} \geq 1 - x$ holds for δ's sufficiently close to one. Investment in optimal since $\frac{1}{2} \leq v_2 = \frac{x}{2-\delta}$, which holds for δ's sufficiently close to one.

The analogous arguments from the proof of Proposition 1 establish that it is non-profitable to deviate and demand something different from the equilibrium demand.

□

4 Symmetric stationary equilibria

Whereas every Pareto efficient payoff vector can be approximated by some non-symmetric strategy, imposing symmetry has far reaching implications in our model. Specifically, under symmetric stationary strategies, only arbitrarily small payoffs can be achieved in equilibrium, when the costs of delay vanish.

Theorem 2. *For every $\epsilon > 0$ there exist $\delta(\epsilon) \in (0,1)$ and $c(\epsilon) > 0$ such that the following holds provided that $\delta \in (\delta(\epsilon), 1)$ and $c < c(\epsilon)$: the value of every symmetric stationary equilibrium of G is at most ϵ.*

As opposed to Theorem 1, Theorem 2 does not hold in the no-discounting case, but requires that δ be strictly less than one. The combination of Theorems 1 and 2 (under the assumption $\delta < 1$) can be viewed as an extreme form of an equity-efficiency trade off: with stationary strategies every efficient payoff vector can be approximated, but with the addition of symmetry—which can be viewed as a form of an equality requirement (for example, it precludes the possibility that one player be tough and the other be cooperative)—the attainable payoffs converge to zero.

Proving Theorem 2 requires a relatively extensive analysis. In the following two subsections we study pooling and separating (symmetric, stationary) equilibria. On the basis of this analysis, we prove Theorem 2 in subsection 4.3.

When $\delta < 1$ a certain type of pooling equilibria exist; we describe the conditions under which they exist in Corollary 4 below. Existence of separating equilibria when $\delta < 1$ is an open problem. In the last subsection, 4.4, we consider the case $\delta = 1$, for which we are able to establish the existence of separating equilibria. We also characterize the set of payoff vectors that are attainable in such equilibria when $\delta = 1$ and $c \to 0$. There are precisely two such vectors: $(\frac{1}{2}, \frac{1}{2})$ and $(0, 0)$. This shows, in particular, that the restriction $\delta < 1$ cannot be dispensed with in Theorem 2.

4.1 Pooling equilibria

In a symmetric pooling stationary equilibrium the same offer is made in every period, no matter who is the proposer and no matter whether he invested or not. We call the symmetric stationary pooling equilibrium in which the common offer is $(d, 1-d)$ the *d-equilibrium*.

Fix a $d \in [0, 1]$. Consider a discount factor and a participation cost (δ, c), such that a d-equilibrium exists. Let p denote the investment probability in this equilibrium. Clearly, in equilibrium a responder who did not invest accepts the offer $1 - d$. Also, one can easily show that if $c < \frac{\delta}{2}$ then $p > 0$.[11] Thus, a responder-who-invested rejects the offer; otherwise, the offer would be accepted for sure, which would imply that investment cannot occur in equilibrium, in contradiction to $p > 0$. Additionally, $p = 1$ is impossible, since it implies perpetual disagreement. Hence, it must be that a player is indifferent between investing and not investing. The expected utility from not investing is $\frac{1}{2}(1-p)d + \frac{1}{2}(1-d)$ and that of investing is $-c + \frac{1}{2}\{p\delta V + (1-p)d\} + \frac{1}{2}\delta V$, where V is the equilibrium's value. Indifference implies that they are equal, hence:

$$V = \frac{2c + 1 - d}{\delta(1 + p)}. \tag{3}$$

Since $V = \frac{1}{2}(1 - p)d + \frac{1}{2}(1 - d)$, we get that the investment probability solves the following equation:

$$\delta d p^2 - \delta(1 - d)p + [4c + 2(1 - d) - \delta] = 0. \tag{4}$$

Since (4) is quadratic in p it may have two (real-valued) solutions. Let $p(\delta, c, d)$ denote the minimal solution (when a solution exists) when the parameters are (δ, c, d). For

[11] We provide the argument in the proof of Proposition 3 below.

(δ, c, d) such that (4) does not have a (real-valued) solution, set $p(\delta, c, d) \equiv 0$.

Proposition 2. *Fix a $d \in [0, 1]$. Then $p(\delta, c, d) \to 1$ as $(\delta, c) \to (1, 0)$.*

Proof. Substituting $(\delta, c) = (1, 0)$ in (4) gives $dp^2 - (1-d)p + 1 - 2d = 0$, the solutions to which are:

$$p = \frac{1 - d \pm (3d - 1)}{2d}.$$

That is, the solutions are $p = \frac{-2d+1}{d}$ and $p = 1$. Thus, we have that $p(\delta, c, d) \sim \frac{-2d+1}{d}$ or $p(\delta, c, d) \sim 1$ for (δ, c) sufficiently close to $(1, 0)$. We argue that $p(\delta, c, d) \sim 1$ is the only possibility.

Suppose first that $p(\delta, c, d) \sim 0$. This can only happen if $d = \frac{1}{2}$. It follows from the application of the Implicit Function Theorem to equation (4) that $\frac{\partial p}{\partial \delta} > 0$ and $\frac{\partial p}{\partial c} < 0$, which implies that $p(c, \delta, d) < 0$ for (δ, c) sufficiently close to $(1, 0)$. This is impossible.

Next, consider $p(\delta, c, d) \sim \frac{-2d+1}{d} \in (0, 1)$. In this case, $d \in (\frac{1}{3}, \frac{1}{2})$. Then it follows from (3) that for (δ, c) sufficiently close to $(1, 0)$ the value of the equilibrium is approximately d. However, if $d < \frac{1}{2}$ then a responder-who-invested will not reject the offer $1 - d$ in order to obtain a continuation value of approximately d. Therefore, as argued, $p(\delta, c, d) \sim 1$ is the only possibility. Thus, if a sequence of the form $\{p(\delta, c, d)\}$ converges to a limit, this limit must be one. Convergence to a limit is guaranteed by the fact that such a sequence belongs to $[0, 1]$—a compact set. $\square$

It is worthwhile noting that the order of limits is irrelevant in Proposition 2. We will utilize this fact later, in subsection 4.3.[12]

Proposition 3 below shows that the (δ, c)-space is partitioned into two regions. In one, d-equilibrium exists, is unique, and is trivial. For the other, the proposition only

[12] It does not matter which of δ or c converges faster: when $(\delta, c) \sim (1, 0)$ the solutions to (4) are (approximately) $p = \frac{-2d+1}{d}$ and $p = 1$.

specifies necessary conditions for equilibrium existence. We will make use of them in the proof of Theorem 2, and to characterize d-equilibrium for $d = 1$.

Proposition 3. *If $c \geq \frac{\delta}{2}$, then there exists a unique d-equilibrium. In this equilibrium $d = 1$; the players do not invest and the proposer demands (and obtains) the entire pie. If $c < \frac{\delta}{2}$ then if a d-equilibrium exists then (I)-(III) are satisfied, where:*

(I) $d \in [1 - \delta, 1]$;

(II) Equation (4) has a solution (i.e., p) in $(0, 1)$,

(III) The aforementioned solution p satisfies $\frac{2c+1-d}{\delta(1+p)} \geq 1 - d$.

Proof. Consider first the case $c \geq \frac{\delta}{2}$. It is easy to check that the following symmetric profile is an equilibrium: a player does not invest, demands the entire pie when he is the proposer, accepts any offer if he is a responder who did not invest, and accepts an offer if and only if it is above δ in case he did invest (i.e., after he deviated at the investment stage). It remains to show that there is no other d-equilibrium in this case.

Assume by contradiction that such an equilibrium exists. Let p be the investment probability in this equilibrium. If $p = 0$, then the equilibrium is the one we just described above. Suppose, then, that $p > 0$. We argue that $p < 1$ must hold. To see this, assume by contradiction that $p = 1$. If the responder accepted the d-offer in equilibrium, then there would be no reason to invest: no matter whether the proposer i invested, j would accept the d-offer. If, on the other hand, the responder rejected the d-offer in equilibrium, then the outcome of the equilibrium would be perpetual disagreement, which is impossible. Therefore, $p < 1$. In particular, $p \in (0, 1)$; both investment and non-investment occur on the path.

Next, we argue that in a period in which both players invested, there is disagreement. To see this, assume by contradiction that conditional on this event there is agreement; that is, a responder-who-invested accepts the d-offer. But since the responder who did not invest also accepts it, it follows that the d-offer is accepted for sure, and this, in turn, implies that there is no reason to invest, in contradiction to $p > 0$.

Since a responder-who-invested rejects the equilibrium offer, and since a proposer-who-did-not-invest can secure the payoff $(1-p)$ by demanding the entire pie, it follows that $d = 1$.

Since $p \in (0, 1)$, a player is indifferent between investing and not. Combining this indifference with our findings that $d = 1$ and that investment by both players implies disagreement, we obtain:

$$-c + \frac{1}{2}[p\delta V + (1-p)] + \frac{1}{2}\delta V = \frac{1-p}{2},$$

where V is the value of the equilibrium. Therefore $V = \frac{2c}{\delta(1+p)}$. Since the value equals $\frac{1-p}{2}$ (the expected utility from not investing), it follows that $p^2 = 1 - \frac{4c}{\delta}$. Therefore $\delta > 4c$. Combining this with $c \geq \frac{\delta}{2}$ we conclude that $\delta > 2\delta$—a contradiction.

Now consider $c < \frac{\delta}{2}$. Let p denote the investment probability. By the argument from above, $p < 1$. Also, it must be that $p > 0$: $p = 0$ implies the existence of a profitable deviation—to invest, demand the entire pie, and reject anything short of the entire pie (this deviation is profitable since it brings $-c + \frac{1}{2} + \frac{\delta}{2} > \frac{1}{2}$).

Next, we argue that if both players invest in the beginning of a period, then there is disagreement in that period. To see this, assume by contradiction that if both players invest, then there is agreement. Since a responder-who-invested agrees to the common demand d, everybody agrees to it. But then there is no reason to invest: no matter whether player i invested or not, player j will accept i's offer if i is selected to be the proposer; and player i—regardless of his investment decision—accepts j's offer if j is called to be the proposer. Therefore investment is sub-optimal, in contradiction to $p > 0$.

The combination of $p \in (0, 1)$ and the fact that investment by both players leads to disagreement implies (4). Also, $d < 1 - \delta$ is impossible, since it is equivalent to $1 - d > \delta$, which is impossible in equilibrium: the proposer can slightly increase his demand, and the responder would still agree.

Finally, a responder-who-invested needs to reject the offer, so $1-d$ must be weakly below the value of the equilibrium; i.e., (III) holds. Thus, existence of a d-equilibrium implies (I)-(III). ☐

Based on Proposition 3, we easily obtain the following result, which is an important building block in the proof of Theorem 2.

Corollary 3. *Let $d < 1$. Then there exist $\delta(d) < 1$ and $c(d) > 0$ such that if $\delta \in (\delta(d), 1]$ and $c < c(d)$, then a d-equilibrium does not exist.*

Proof. Rearranging condition (III) we get $2c \geq (1-d)[\delta(1+p)-1]$. By Proposition 2, $p \to 1$ as $(\delta, c) \to (1, 0)$. Therefore $0 \geq 1-d$. ☐

Note that, in contrast to $d < 1$, the extreme demand $d = 1$ can be supported in equilibrium when (δ, c) is close to $(1, 0)$.

Corollary 4. *Let $d = 1$. Then a d-equilibrium exists if and only if:*

$$1 - \delta^2 \leq \frac{4c}{\delta} < 1. \tag{5}$$

Proof. Recall that conditions (I)-(III) from Proposition 3 are necessary for a d-equilibrium. Conditions (I) and (III) are satisfied when $d = 1$, and equation (4) has a solution in $(0, 1)$ (i.e., condition (II) is satisfied) if and only if $4c < \delta$. The solution is $p = \sqrt{1 - \frac{4c}{\delta}}$. Also, a weak proposer should find it optimal to stick to the equilibrium and demand the entire pie, rather than offer the responder δ, which the responder "cannot refuse." Therefore it must be that $1 - \delta \leq 1 - p$, or:

$$\sqrt{1 - \frac{4c}{\delta}} \leq \delta,$$

or $1 - \delta^2 \leq \frac{4c}{\delta}$. Therefore (5) is necessary for 1-equilibrium.

Now suppose that (5) holds. Consider the following symmetric stationary profile. On the path, behavior is as follows: a player invests with probability $p = \sqrt{1 - \frac{4c}{\delta}}$, demands the entire pie if he is the proposer, and rejects the opponent's offer if he invested; a responder who did not invest accepts the opponent's offer. Off the path, behavior is as follows: any offer different from zero is interpreted as a signal that the opponent did not invest, and is therefore accepted by a strong responder if and only if it gives the responder at least δ. In the beginning of every subgame that follows a deviation, the deviation is ignored, and play is as in the first period.

We argue that this is an equilibrium. It is optimal for a weak proposer to stick to the course of actions described above—demanding the entire pie is weakly better than offering δ, hence demanding the entire pie is optimal. If a strong proposer demands $x < 1$, his payoff is $(1-p)x + p\delta V < (1-p) + p\delta V = V$, where V is the value. Hence, demanding the entire pie is optimal. Now consider a proposer who did not invest. Since a demand $x \in (1-\delta, 1)$ is accepted if and only if the opponent is weak and a demand $x < 1-\delta$ is accepted for sure, the only candidates for a best-response are the demands $1-\delta$ and 1. The inequality $1-\delta \leq 1-p$ guarantees the optimality of the latter. It is easy to see that the prescribed responses are optimal. □

It is worthwhile emphasizing that Corollary 4 is not a limit result. That said, if one focuses on the existence of 1-equilibrium as $\delta \to 1$ and $c \to 0$, then the order of limits is important for (5) to be valid (i.e., first $\delta \to 1$ and then $c \to 0$) .

4.2 Separating equilibria

The following result is the restricted-to-separating-equilibria version of Theorem 2.

Proposition 4. *For every $\epsilon > 0$ there exist $\delta(\epsilon) \in (0,1)$ and $c(\epsilon) > 0$ such that the following holds provided that $\delta \in (\delta(\epsilon), 1)$ and $c < c(\epsilon)$: the value of every symmetric stationary separating equilibrium of G is at most ϵ.*

To prove it, we make use of the following lemmas.

Lemma 1. *If $\delta < 1$ then the following holds in every symmetric stationary separating equilibrium: if both players invest in the beginning of a period, then there is* *disagreement in this period. Moreover, if a player who did not invest is called to be the proposer, then he demands the entire pie.*

Proof. Assume by contradiction that if both players invest, then there is agreement. Let p denote the common investment probability. Since the equilibrium is separating, $p \in (0, 1)$.

Let s and w denote the pie-shares demanded by a weak (non-investing) and strong (investing) proposer. Since a player can secure the payoff $(1 - p) > 0$ by demanding the entire pie, $w > 0$.[13] We argue that the strong responder rejects the w-offer. Otherwise, (i.e., if he accepts it), then the w-offer is accepted for sure—namely, by both types of responders. However, the s-offer is also accepted by both types: it is clearly accepted by the weak responder, and per our assumption it is accepted by the strong responder. Thus, each offer is accepted with certainty, which is clearly impossible in equilibrium (a proposer will choose the offer that maximizes his share of the pie). Therefore, the strong responder rejects the w-offer.

So, the weak proposer's payoff is $(1-p)w$. Since the weak proposer can guarantee the payoff $(1-p)$, it follows that $w = 1$. Incentive compatibility for the weak proposer implies $(1-p) \geq s$ (since, per our assumption, the strong responder agrees if the proposer asks for s, the s-offer is accepted for sure if it is made). Incentive compatibility for the strong proposer implies $s \geq (1 - p) + p\psi$, where ψ is the payoff he receives by making the weak proposal to a strong responder. Therefore, $(1-p) \geq s \geq (1-p)+p\psi$. Since $p \in (0, 1)$, it is enough to prove that $\psi > 0$ in order to establish a contradiction. This is indeed the case because the player can, for example, avoid investing in the beginning of next-period's subgame and then demand $1-\delta$ if he is selected by Nature to be the proposer. Therefore, $\psi \geq \frac{\delta(1-\delta)}{2}$. The first part of the lemmas is therefore proved: if both players invest, there is disagreement.

Now consider player i who did not invest, and who is called by Nature to be the

[13] If a player demands the entire pie and the opponent is weak, the opponent agrees to this proposal. The probability of the opponent being weak is $(1 - p)$.

proposer. He knows that no matter what equilibrium offer he will make—the s-offer or the w-offer—it will be rejected by the responder j, if the latter invested. Therefore, his payoff is bounded by $(1-p)z$, for $z \in \{s, w\}$. Since he can guarantee $(1-p)$, it follows that $w = 1$. □

Lemma 2. *If $\delta < 1$ then the following holds in every symmetric stationary separating equilibrium: a responder-who-invested rejects the offer of a proposer who did not invest.*

Proof. Let w be the demand of a proposer who did not invest. By Lemma 1, $w = 1$. If the strong responder agrees to this demand, then it is accepted for sure. Hence, a proposing player will necessarily demand the entire pie, in contradiction to the assumption that the equilibrium is separating. □

Equipped with Lemmas 1 and 2, we can turn to the proof of Proposition 4.

Proof of Proposition 4: Consider a symmetric stationary separating equilibrium. Let $(s, 1-s)$ and $(w, 1-w)$ be the offers that are made by the investing (strong) and non-investing (weak) proposer, where $s \neq w$. By the lemmas, a strong responder rejects both offers.

Let p denote the equilibrium investment probability. A weak proposer's payoff is $(1-p)$. Since he can secure the payoff $(1-p)$ by demanding the entire pie, $w = 1$. The indifference condition between investing and not investing is:

$$\frac{p(1-s)}{2} + \frac{1-p}{2} = -c + \frac{1}{2}[p\delta V + (1-p)s] + \frac{1}{2}\delta V, \tag{6}$$

where V is the value of the equilibrium. Therefore $\frac{p(1-s)}{2} + c = (\frac{1+p}{2})\delta V + \frac{1-p}{2}(s-1)$, or $p(1-s) + 2c = (1+p)\delta V + (p-1)(1-s)$. Therefore,

$$(1-s) + 2c = (1+p)\delta V. \tag{7}$$

Since a responder-who-invested rejects the s-offer, it must be that $(1 - s) \leq \delta V$. Therefore, $2c \geq p\delta V$. Let $p = p(\delta, c)$. Consider a sequence (δ, c) that converges to $(1, 0)$. If $p \nrightarrow 0$ as $(\delta, c) \to (1, 0)$, we are done. Suppose, then, that $p \to 0$. Then it follows from the LHS of (6) that V converges to $\frac{1}{2}$. However, we argue that there is a deviation that gives a higher payoff. For example, investing and then rejecting anything short of the entire pie gives a payoff which is weakly greater than $-c + \frac{1}{2}(1-p) + \frac{1}{2}(1-p)\delta$; and the latter expression converges to 1 as $(\delta, c) \to (1, 0)$. $\square$

The following is analogous to Proposition 2 from the previous subsection.

Proposition 5. *Let $\{e(\delta, c)\}$ be a sequence of symmetric stationary separating equilibria of G, indexed by (δ, c). Let $p(\delta, c)$ be the investment probability in $e(\delta, c)$. Then, $p(\delta, c) \to 1$ as $(\delta, c) \to (1, 0)$.*

Proof. The value of each equilibrium in the sequence is given by the counterpart of the LHS of (6), and we know (from Proposition 4) that the LHS of (6) converges to zero as $(\delta, c) \to (1, 0)$; hence, it must be that $p \to 1$. $\square$

4.3 Proof of Theorem 2

Equipped with the analysis from Subsections 4.1-4.2, we can turn to Theorem 2's proof.

Proof of Theorem 2: By Corollary 3, it is enough to prove the theorem for pooling equilibrium where $d = 1$, and separating equilibria. The case of $d = 1$ follows from (3). The case of separating equilibria is covered by Proposition 4. $\square$

The essence of the proof (of Theorem 2) boils down to two facts about symmetric stationary equilibria: (a) when both players invest there is disagreement, and (b) the investment probability converges to one as $(\delta, c) \to (1, 0)$. Fact (a) is proved in the body of Proposition 3 for pooling equilibria and in Lemma 1 for separating equilibria. Fact (b) is proved in Propositions 2 and 5.

Finally, like Theorem 1, Theorem 2 can be phrased in limit terms.

Corollary 5. $lim_{\delta\to 1} lim_{c\to 0} E^s(\delta, c) = lim_{c\to 0} lim_{\delta\to 1} E^s(\delta, c) = \{(0, 0)\}$.

Proof. As we proved in Corollary 3, the only pooling equilibrium that "survives in the limit" is the d-equilibrium where $d = 1$. Corollary 3 builds on Proposition 2, and in Proposition 2 the order of limits does not matter. The value of the d-equilibrium for $d = 1$ converges to zero, as seen in (3). As for separating equilibria, it is clear from the proof of Proposition 4 that the order of limits is irrelevant for the payoffs. □

4.4 Symmetric stationary separating equilibria when $\delta = 1$

Constructing a symmetric stationary separating equilibrium requires taking care of incentive compatibility (IC) constraints, that ensure that a strong (weak) proposer does not want to mimic a weak (strong) proposer. Of the two IC's, the one that pertains to the strong proposer is more demanding, for the following reason: if a proposer who invested pretends to be a non-investing proposer by making the latter's offer, then in case this offer is rejected play moves to a continuation game in which this deviation is common knowledge. The proposer should suffer a sufficiently low payoff after such a deviation, in order to make sure that it is not profitable. Constructing such a punishment (and such equilibria) for the case $\delta < 1$ remains, at present, an open problem. Below we show that this can be done for $\delta = 1$, provided that $c \leq \frac{1}{2}$. The reason is that under these parameter values G has an asymmetric equilibrium in which one player's payoff is zero; this equilibrium can be utilized to deliver a sufficiently deterrent punishment.

In what follows we describe the punishment equilibrium. Then we utilize it in our last result of this section, which is a characterization of the set of payoffs that are attainable in symmetric stationary separating equilibria when $\delta = 1$ and $c \to 0$.

4.4.1 The (off-path) punishment phase

For $c \leq \frac{1}{2}$ take a number $z \in [2c, 1]$ and consider the following stationary profile. Player 1 never invests, always offers $(z, 1-z)$, and accepts any offer from player 2; following his own investment (which is a deviation) player 1 behaves in the same way as above: he offers $(z, 1-z)$ if he is selected to be the proposer and accepts any offer if he is selected to be the responder. Player 2 invests with certainty in the beginning of every period, demands the entire pie whenever he is called to make an offer, and rejects any offer of player 1. After not investing (which is a deviation) player 2 behaves in the following way: if selected to be the responder he accepts any offer, and if selected to be the proposer he demands the entire pie. Denote this profile by σ_2^z. Let σ_1^z be the profile where the roles of the players are reversed (i.e., player 1 is the "dictator").

Lemma 3. *Suppose that $\delta = 1$ and $c \leq \frac{1}{2}$. Then for every $z \in [2c, 1]$ and $i \in \{1, 2\}$, σ_i^z is sustainable in a stationary equilibrium.*

Proof. Without loss of generality, we consider σ_2^z. Player 1 obtains zero payoff, and his strategy is clearly a best-response against player 2's strategy. As for player 2, we let him believe that player 1 did not invest after having seen player 1's offer (be it the prescribed $(z, 1-z)$ or a deviation). Thus, we only need to verify that investing is optimal for player 2. The expected utility form not investing is $\frac{1}{2}(1-z) + \frac{1}{2}$ and the expected utility from investing is $1 - c$. Thus, $z \geq 2c$ implies that investing is optimal. □

4.4.2 Equilibrium existence and payoff characterization

Theorem 3. *Let $\delta = 1$. Let $\{e(c)\}$ be a sequence of symmetric stationary separating equilibria, indexed by c. Let $V(c)$ be $e(c)$'s value, and let $V^* = lim_{c \to 0} V(c)$. Then $V^* \in \{0, \frac{1}{2}\}$. Moreover, each of $\{0, \frac{1}{2}\}$ can be achieved as a limit of such a sequence $\{V(c)\}$.*

Proof. We first construct two equilibria, one whose value converges to $V^* = 0$ as $c \to 0$ and one whose value converges to $V^* = \frac{1}{2}$ as $c \to 0$. After that we show that any other limit-value of such an equilibrium must be either zero or half.

Step 1: An equilibrium with $V^* = 0$. Consider the following profile. A weak proposer demands the entire pie, a strong proposer demands s, and the investment probability is p. Restrictions on s and p will be specified shortly. A responder-who-invested rejects both equilibrium-offers. In case the proposer is still in the game after he demanded something different from s and greater than c an absorbing punishment phase begins, in which the deviator's payoff is zero and that of the punishing player is $1 - c$. If the proposer deviates and asks for himself a pie share smaller than c, then the responder accepts (this is an offer he "cannot refuse"). This behavior of the strong responder—to reject any off-path offer unless it is an offer he can't refuse—is supported by the belief that the proposer did not invest.

The indifference condition is as in the proof of Proposition 4 and equation (7), consequently, applies; when $\delta = 1$ this equation is solved by:

$$V = \frac{1 - s + 2c}{1 + p}.$$

The following conditions guarantee that this is an equilibrium:

$$1 - p \geq c, \tag{8}$$

$$V \geq 1 - s, \tag{9}$$

and

$$pV + (1 - p)s \geq \max\{c, 1 - p\}. \tag{10}$$

Condition (8) guarantees that demanding the entire pie is optimal for the weak proposer,[14] (9) guarantees that rejecting the equilibrium offer of the strong proposer is

[14] If he asks for $x \leq c$, then his payoff will be $x \leq c \leq 1 - p$; and if he asks for $x > c$ his demand

optimal for the strong responder, and (10) guarantees that demanding s is optimal for the strong proposer.[15]

Combining (8) and the solution for V, we see that (10) becomes:

$$p(\frac{1-s+2c}{1+p}) \geq (1-p)(1-s),$$

or,

$$2pc \geq (1-s)(1-p^2-p). \tag{11}$$

Thus, for a given c we need to find s and p such that (8), (9), and (11) hold.

Take

$$s = p = 1-c.$$

Then (8) is satisfied and (11) is satisfied for all c small enough. It only remains to verify (9): $\frac{1-s+2c}{1+p} = \frac{1-s+2c}{1+s} \geq 1-s$, or $1-s+2c \geq 1-s^2$. This becomes $2c \geq s(1-s)$. Since $s = 1-c$, this becomes $2c \geq (1-c)c$, which clearly holds.

Step 2: An equilibrium with $V^* = \frac{1}{2}$. Consider the following profile. A weak proposer demands the entire pie, a strong proposer demands $s = 2c$, and the investment probability is $p = 1-2c$. A strong responder accepts $1-s$ and rejects the offer of the weak proposer. The response to off-equilibrium offers is the same as in the equilibrium from Step 1: the strong responder accepts such offers if he is given at least $1-c$, and he rejects them otherwise. Any deviation triggers the punishment phase in which the deviator's payoff is zero and that of the punishing player is $1-c$. The indifference condition is:

will be accepted if and only if the responder is weak, implying the expected utility $(1-p)x \leq 1-p$. Demanding the entire pie gives the expected utility $1-p$.

[15] He can guarantee c by asking for c; if he asks for more, then this demand will be accepted if and only if the responder is weak. Hence such a demand cannot yield an expected utility greater than $1-p$.

$$\frac{p(1-s)}{2} + \frac{1-p}{2} = -c + \frac{1}{2}s + \frac{1}{2}[p(1-s) + (1-p)],$$

which is indeed satisfied for $s = 2c$. We argue that there is no profitable deviation. To prove this, let us first consider a deviation that involves investment, i.e., a player invests, but demands a pie share different from s if he is called to be the proposer.[16] The off-path demand, call it s', is accepted by the opponent if and only if (1) the opponent is weak or (2) $s' \leq c$. Since rejection of this offer means that the deviator's continuation payoff is zero, it follows that for $s' \leq c$ the resulting payoff is $s' \leq c < 1-p$, where the strict inequality follows from the fact that $p = 1-2c$, and for $s' > c$ the payoff is $(1-p)s' \leq 1-p$. Therefore, the best deviation that involves investment is to invest and then demand the entire pie. The expected utility from this deviation is $-c + \frac{1}{2}[p(1-s) + (1-p)] + \frac{1-p}{2}$, hence the non-profitability of the deviation is equivalent to $-c + \frac{p(1-s)}{2} + 1 - p \leq \frac{p(1-s)}{2} + \frac{1-p}{2}$, implying $p \geq 1-2c$.

Now consider a deviation that involves no investment. Denote again the demand by s'. Suppose first that $s' \neq s$. From the arguments given above we know that $s' \leq c$ cannot be profitable since it gives $s' \leq c < 1-p$. Secondly, $s' \in (c, 1)$ is suboptimal since it gives $(1-p)s' < 1-p$. If $s' = s$ then the demand is accepted with certainty. Hence non-profitability is equivalent to $1-p \geq s = 2c$, or $p \leq 1-2c$.

Note that the value of this equilibrium (either side of the indifference condition) converges to $\frac{1}{2}$ as $c \to 0$.

Step 3: Non-existence of other value-limit. Consider a symmetric stationary separating equilibrium. If conditional on both players investing there is disagreement (i.e., the analog of Lemma 1 holds), then the arguments from the proof of Theorem 2 go through. Hence, the value in the limit is zero. Consider, on the other hand, the possibility that conditional on both players investing there is agreement. Let s

[16] The response policy of such a strategy must coincide with the proposed-equilibrium strategy: rejection of s implies a zero continuation payoff, and any offer different from s is interpreted as a signal that the opponent did not invest.

denote the amount that the strong proposer demands for himself, and let p denote the investment probability. Then the indifference condition is the same as in Step 2, hence $s = 2c$. Note that as $c \to 0$ the LHS of the indifference condition converges to half. □

It is worth noting that the punishment phase that is utilized in Theorem 3 was not needed in our previous results that concerned equilibrium existence—namely Proposition 1 and Corollary 4. In both of these results, once a proposer demands a pie-share that he is not supposed to demand in equilibrium, and this demand is rejected, which triggers a continuation game in which the deviation is common knowledge, the deviation is ignored. Effectively, the post-deviation continuation game is equivalent to the entire super-game. In the case of separating equilibria, such deviations cannot be ignored—ignoring them will distort incentives and make the equilibrium unravel. Specifically, when the strong proposer is suppose to demand $s < 1$ and the weak proposer demands the entire pie, ignoring the deviation implies that the strong proposer be better off asking for the entire pie, hence the equilibrium breaks down.

4.4.3 A refinement

As seen in the analysis above, when one moves from the case $\delta < 1$ to $\delta = 1$, the set payoff vectors that can be obtained in a symmetric stationary equilibrium changes discontinuously: $(\frac{1}{2}, \frac{1}{2})$ is added to $(0, 0)$. However, the equilibrium that achieves $(\frac{1}{2}, \frac{1}{2})$—the one that is constructed in Step 2 of the proof of Theorem 3—is unintuitive in the following sense: conditional on being selected to be the proposer, a player's payoff does not depend on whether he invested or not. In particular, investing has no advantage conditional on being the proposer. Intuitively, we would expect investment to influence positively conditional payoffs.

We formalize this requirement as follows. Given a stationary symmetric equilibrium, denote by $V(Prop|+)$ the payoff conditional on being a proposer who invested, denote by $V(Prop|-)$ the payoff conditional on being a proposer who did not invest, and by $V(Res|+)$ and $V(Res|-)$ the corresponding payoffs to the responder. Then, an equilibrium is *investment-monotonic* if $V(\iota|+) > V(\iota|-)$ for both $\iota \in \{Prop, Res\}$. Investment-monotonic equilibria exist. For example, the equilibrium which is constructed in Step 1 in Theorem 3's proof is such an equilibrium. The following result shows that under this refinement, our "destructive War of Attrition" result, namely Theorem 2, extends to the case $\delta = 1$.

Proposition 6. *Let $\delta = 1$. Let $\{e(c)\}$ be a sequence of investment-monotonic symmetric stationary separating equilibria, indexed by c. Let $V(c)$ be $e(c)$'s value. Then $lim_{c\to 0}V(c) = 0$.*

Proof. Consider a symmetric stationary separating equilibrium. If conditional on both players investing there is disagreement (i.e., the analog of Lemma 1 holds), then the arguments from the proof of Theorem 2 go through. Hence, the value in the limit is zero. Consider, on the other hand, the possibility that conditional on both players investing there is agreement. Let s denote the amount that the strong proposer demands for himself, and let p denote the investment probability. Then the indifference condition is the same as in Step 2 of Theorem 3's proof, hence $s = 2c$. This offer is accepted by both types of the responder, because if it were rejected by a strong responder, then it would be better to demands the entire pie (like a weak proposer does). Therefore, $V(Prop|+) = 2c$. The incentive constrains that are described in Step 2 are also in effect here, hence $p = 1 - 2c$. Therefore, $V(Prop|-) = 1 - p = 2c$. Therefore, $V(Prop|+) = V(Prop|-)$—in contradiction to investment-monotonicity. □

5 An efficient ex ante symmetric equilibrium

Symmetric stationary equilibria imply complete waste of the social surplus if $\delta < 1$ or if investment-monotonicity is imposed. Nevertheless, an efficient outcome can be approximated under a weaker notion of symmetry, provided that one is willing to compromise on stationarity. In this section we construct an equilibrium that approximates the payoff vector $(\frac{1}{2}, \frac{1}{2})$ when $(\delta, c) \to (1, 0)$, which is symmetric in the ex ante sense, but not conditional on every history. The compromise on stationarity is minimal, in the sense that play is stationary conditional on any non-initial history, but not ex ante.

The construction is based on the equilibria that we described in Proposition 1; specifically, the ex ante symmetric equilibrium implements a uniform lottery over $\{\sigma(\frac{2}{3}, 1), \sigma(\frac{2}{3}, 2)\}$.

Theorem 4. *There exist $\delta^* < 1$ and $c^* > 0$ such that the following holds provided that $\delta \in (\delta^*, 1]$ and $c < c^*$: G has an ex ante symmetric equilibrium. The value of this equilibrium is:*

$$-c + \frac{\delta}{2} \cdot \frac{1 - 4c}{2 - \delta}.$$

In particular, the value converges to $\frac{1}{2}$ as $(\delta, c) \to (1, 0)$.

Proof. Consider the following profile. In the first period both players invest with certainty. Denote by i the player who is selected by Nature to be the first period's proposer. After i has been selected, play is according to $\sigma(\frac{2}{3}, j)$. In particular, when i makes his offer it is rejected by j and the game moves to the next period, where the continuation play is as specified in the equilibrium of Proposition 1.

It follows from Proposition 1 that conditional on being at the post-investment stage of the first period, continuation play forms a equilibrium. Note that playing the above strategy is, in effect, starting to play the equilibrium of Proposition 1 in the second period of the game, with each player being equally likely to be in the weak/strong role. The value of this is $\delta\frac{1}{2}(v_1 + v_2)$, where $(v_1 + v_2)$ is the sum of values of the equilibrium of Proposition 1. It is not hard to check that $(v_1 + v_2) = \frac{1-4c}{2-\delta}$. Since the players also invest in the first period, the overall value from the above strategy is $-c + \frac{\delta}{2} \cdot \frac{1-4c}{2-\delta}$, which converges to $\frac{1}{2}$ as $(\delta, c) \to (1, 0)$. It remains to show that the utility from deviating and not investing in the first period is strictly below $\frac{1}{2}$ when (δ, c) is sufficiently close to $(1, 0)$.

Consider then a player who does not invest in the first period. Conditional on being the responder, he accepts the offer $1 - x$ (which, on the equilibrium's path, he is supposed to reject). Conditional on making an offer he can obtain no more than $1 - \delta$. Hence, the overall payoff from the deviation is bounded by $\frac{1}{2}(1-x) + \frac{1}{2}(1-\delta)$, which converges to $\frac{1}{2}(1-x)$, which is approximately $\frac{1}{6}$ when δ is sufficiently close to one, since $x = \frac{2}{3}$. □

The structure of the equilibrium in Theorem 4 is similar to that of the *randomized dictatorship* mechanism. Under randomized dictatorship, a fair coin flip determines which player will be the "dictator" and obtain his maximum possible payoff, while the other player will receive nothing.[17] The equilibrium in Theorem 3 is less extreme, in the sense that the dictator's payoff is not maximal and that of the non-dictator is not minimal. Rather, the ratio between their payoffs is approximately two when $(\delta, c) \sim (1, 0)$.

One can think of the equilibrium in Theorem 4 as telling the following story: it is important to establish, early on in the negotiation process, a reputation for toughness. In this equilibrium, the player who enjoys the privilege of making the first rejection signals that continuation play will be an asymmetric equilibrium that favors him.

6 Small participation costs make a big difference

Consider the case where $\delta \in (0, 1)$ and the participation cost is set to zero; namely, $c \equiv 0$. Zero cost implies that investing (weakly) dominates non-investing. The resulting game is, practically, a game of complete information, which is a symmetrized

[17]This idea/mechanism has been studied extensively in the context of cooperative bargaining. See, for example, Sobel (1981), Myerson (1984), Anbarci (1998), and Rachmilevitch (2016).

version of Rubinstein's game.[18] This random-proposer bargaining game has been studied by Binmore (1987), who demonstrated that it has a unique subgame perfect equilibrium (SPE). This equilibrium is stationary and symmetric. It involves an immediate agreement and each player's equilibrium expected payoff is $\frac{1}{2}$. This result is independent of δ. However, to ease the comparison with our results, we write δ explicitly and denote by $B(\delta)$ the set of SPE payoff vectors in Binmore's game when the discount factor is δ. That is, $B(\delta) = \{(\frac{1}{2}, \frac{1}{2})\}$ for all $\delta \in (0,1)$. Let $B^* = \lim_{\delta\to 1} B(\delta) = \{(\frac{1}{2}, \frac{1}{2})\}$.

Our analysis reveals that the complete information game is not robust to the addition of arbitrarily small participation costs (whose payment is private information). Mathematically, this non-robustness is expressed in the form of discontinuity, and it has two manifestations.

First, from Corollary 2 we have:

$$\lim_{\delta\to 1}\lim_{c\to 0} E(\delta, c) = \lim_{c\to 0}\lim_{\delta\to 1} E(\delta, c) = \Delta \neq B^*.$$

Second, from Corollary 5 we have:

$$\lim_{\delta\to 1}\lim_{c\to 0} E^s(\delta, c) = \lim_{c\to 0}\lim_{\delta\to 1} E^s(\delta, c) = \{(0,0)\} \neq B^*.$$

This second type of discontinuity is particularly interesting, since the jump is from the origin to the Pareto frontier.

It is essential for the above non-robustness that paying the participation cost is a player's private information. To highlight the importance of this private information, we end the paper by studying a complete-information version of our game. In this version of the game—hereafter the *complete information G*—the players' investment choices are publicly observable, hence the proposer can condition his offer (and the responder can condition his response) on the pair of investment choices. Our solution

[18] Alternatively, we can make investment mandatory, which would not change the analysis significantly.

concept for this game is SPE.

The complete information G has an equilibrium akin to the one from Binmore's game. In this equilibrium, each player invests with certainty in every period, and the remainder of the strategy (i.e., the proposals and accept/reject policy it prescribes) is like in Binmore's game. Specifically, the equilibrium is stationary, there is immediate agreement, the equilibrium's proposal makes the responder indifferent between accepting and rejecting it, and each player's expected payoff is $\frac{1}{2}$. We call this equilibrium the *Binmore equilibrium.*

We do not have a characterization of all the SPEs of the complete information G. However, we show, under suitable conditions on (δ, c), that (1) the Binmore equilibrium is the unique symmetric SPE, and (2) it is also the unique stationary SPE. This means that the fact that the costs in our original game G are private information is crucial for our "anything goes" result (Theorem 1) and for our "destructive War of Attrition" result (Theorem 2).

Proposition 7. *Suppose that $\delta \in (0,1)$ and that $c < \frac{\delta}{2}$. Then the complete information G has a unique symmetric SPE. It is the Binmore equilibrium.*

Proof. Consider the strategy under which each player invests with certainty in the beginning of every period, the proposer always proposes $(1-\delta(-c+\frac{1}{2}), \delta(-c+\frac{1}{2}))$, and the responder always accepts anything that gives him at least as in this offer, and nothing less. It is easy to verify (by applying the one-shot deviation principle) that $c < \frac{\delta}{2}$ guarantees that this is a SPE; it is obviously symmetric and stationary. We will now prove that this is the unique symmetric SPE.

Since $\frac{\delta}{2} > c$, it follows that in every SPE every player invests with a strictly positive probability in the beginning of every period (the argument given in the proof of Proposition 3 is also valid here). Let V_t denote the equilibrium's value in the subgame that starts in period t (the value may be time-dependent because we do no impose stationarity; it cannot, however, depend on a player's identity, because of the equilibrium's symmetry). We argue that $V_t < \frac{1}{2}$. Otherwise, if $V_t \geq \frac{1}{2}$, then the total value for both players is $2V_t = 1$, which is inconsistent with the fact that each player invests with a strictly positive probability.

Since $V_t < \frac{1}{2}$ for every t, it follows that there is an agreement in the beginning of every subgame. To see this, assume by contradiction that there is disagreement on the path, in some period t. This means that the responder invested in the beginning of the period, and the proposer also invested.[19] Then the proposer's payoff is $\delta V_{t+1} < \frac{1}{2}$. We argue that he has a profitable deviation: if he offers the responder $\delta V_{t+1} + \epsilon$ for an arbitrarily small ϵ, the responder must accept (because of subgame perfection) and for a sufficiently small ϵ the proposer will obtain more than half the pie. Therefore, as argued, there is agreement in every subgame.

To complete the proof, it is enough to show that each player invests with certainty in the beginning of every period. Assume, then, by contradiction, that there is a period t in which each player invests with probability $p \in (0, 1)$. It is easy to check that the utility from not investing in the beginning of t is $\frac{1}{2}[p(1-\delta) + (1-p)] = \frac{1-p\delta}{2}$. To compute the utility from investing, we invoke the just-proved fact, that on the equilibrium path there is agreement in every period. In particular, in the specific period that we consider, t, a proposer-who-invested proposes to the responder δV_{t+1}, and the latter accepts. Therefore, the utility from investing is:

$$-c + \frac{1}{2}[p\delta V_{t+1} + (1-p)\delta] + \frac{1}{2}[p(1 - \delta V_{t+1}) + (1-p)] = -c + \frac{1}{2} + \frac{(1-p)\delta}{2}.$$

Indifference between investing and not implies $-c + \frac{1}{2} + \frac{(1-p)\delta}{2} = \frac{1-p\delta}{2}$, and so $\frac{\delta}{2} = c$—in contradiction to $\frac{\delta}{2} > c$.

□

Proposition 8. *There exist a discount factor $\delta^{**} < 1$ and a cost $c^{**} > 0$, such that the following holds: if $\delta \in (\delta^{**}, 1)$ and $c < c^{**}$ then the complete information G has a unique stationary SPE. It is the Binmore equilibrium.*

[19] Had the proposer not invested, he would have offered δ to the responder, which would be accepted.

Proof. Fix an arbitrary stationary SPE. We will prove that it must be the Binmore equilibrium (provided that (δ, c) is sufficiently close to $(1, 0)$). Let p_i denote player i's investment probability in this equilibrium. By the argument from Proposition 3/Proposition 7, $p_i > 0$ for both i.

Case 1: $p_i \in (0,1)$ for some i. Wlog, suppose that $i = 1$. Namely, player 1 is indifferent between investing and not. His utility from not investing is $\frac{1}{2}[p_2(1-\delta) + (1-p_2)] = \frac{1-p_2\delta}{2}$. To calculate his utility from investing, we consider two possibilities separately.

Case 1.1: When both players invest, there is disagreement. In this case player 1's utility from investing is $-c + \frac{1}{2}[p_2\delta V_1 + (1-p_2)\delta] + \frac{1}{2}[p_2\delta V_1 + (1-p_2)] = -c + p_2\delta V_1 + (\frac{1-p_2}{2})(1+\delta)$, where V_1 is player 1's equilibrium's value.[20] This expression is equal to V_1, which, in turn, is equal to $\frac{1-p_2\delta}{2}$. Therefore:

$$-c + p_2\delta(\frac{1-p_2\delta}{2}) + (\frac{1-p_2}{2})(1+\delta) = \frac{1-p_2\delta}{2}.$$

Considering $c = 0$ and simplifying this equation yields:

$$\delta^2 p_2^2 + p_2(1-\delta) - \delta = 0,$$

whose solution is $p_2 = \frac{-(1-\delta)+\sqrt{1-2\delta+\delta^2+4\delta^3}}{2}$. Therefore, as $\delta \to 1$ we have that $p_2 \to 1$ and therefore $V_1 \to 0$.

Note that p_2 is also in $(0,1)$ and therefore—by the same arguments as above–it follows that $V_2 \to 0$. This, however, is impossible: when the proposer i proposes V_j the responder will accept, which means that i's equilibrium's value is bounded from below by a number which is approximately $\frac{1}{2}(1-V_i) \sim \frac{1}{2}$.

Case 1.2: The investing players disagree. In this case, subgame perfection implies that the investing proposer i proposes δV_j to the investing responder, and i's utility from investing is therefore given by $-c + \frac{1}{2}[p_j\delta V_i + (1-p_j)\delta] + \frac{1}{2}[p_j(1-\delta V_i) + (1-p_j)] =$

[20] The value may depend on the player's identity (because we do not impose symmetry) but not on the subgame (because we do assume stationarity).

$-c+\frac{p_j}{2}+(\frac{1-p_j}{2})(1+\delta)$. For $i=1$, equating this utility to the utility of not investing, $\frac{1-p_2\delta}{2}$, yields $\delta=2c$, which is impossible when (δ,c) is sufficiently close to $(1,0)$.

We conclude that Case 1 is impossible.

Case 2: $p_1=p_2=1$. In this case there must be immediate agreement in equilibrium. Let $(y-c,1-y-c)$ be the equilibrium's expected utilities. Player 1 offers player 2 $\delta(1-y-c)$ and player 2 offers player 1 $\delta(y-c)$, since in equilibrium each player is indifferent between accepting his opponent's offer and rejecting it. Let us look at player 2's indifference condition:

$$\delta(1-y-c)=\delta\{-c+\frac{1}{2}\delta(1-y-c)+\frac{1}{2}[1-\delta(y-c)]\}.$$

Simplifying this gives $y=\frac{1}{2}$. Therefore, the equilibrium is Binmore's equilibrium. □

7 Conclusion

Costly investments that are made prior to the beginning of negotiations to improve one's bargaining position are ubiquitous. People prepare for negotiations in various ways, and preparations are almost always costly.[21] We studied a model where such an investment can be made in the beginning of every period, and the favorable bargaining position that it brings about is the ability to stay in the game if this turns out to be necessary. The idea that a player may need to pay some cost every period in order to stay in the game seems to be applicable to other settings and problems, not only bargaining.

Our results imply that one of the most basic complete information bargaining

[21] Thompson (2013) labels pre-negotiation preparations as "the magic bullet" and she argues (in Thompson, 2009) that about 80 per-cent of a negotiator's effort should be spent in preparations, which involve activities such as detailed analysis of all issues under consideration, prioritization of issues, fact-finding, perspective taking, identifying all the alternative course of actions, taking into account all contingencies, cooking up an opening proposal, setting aspirations etc. These preparations take time (hence should be done in advance), money, and energy.

games—the “symmetrized Rubinstein game”—is not robust to the addition of arbitrarily small private-information-participation-costs, as these change the equilibria set discontinuously. Moreover, under the restriction to symmetric and stationary strategies, the change is dramatic: only negligible payoffs can be obtained in the presence of these costs, whereas in the costless model there is a unique equilibrium whose outcome is efficient. Small participation costs, therefore, make a big difference.

One may consider an alternative cost structure where each player decides, in the beginning of the game, for how may periods he will be active, and the amount he pays upfront is increasing in the “activity periods” he buys. This structure is significantly different from ours, and studying its implications remains a topic for future research. It could also be argued that in bargaining games such as the one we studied, a player-who-invested wants to reveal the fact that he did so. This, too, is more than a simple extension of our model, hence we leave it for future research.

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