TFP during a Credit Crunch∗

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Abstract

The financial crisis of 2008 was followed by sharp contractions in aggregate output and employment and an unusual increase in aggregate total factor productivity (TFP). This paper attempts to explain these facts by modeling the creation and destruction of jobs in the presence of heterogeneity in firm productivity and frictional credit and labor markets. The aggregate level of TFP is determined by both the underlying distribution of firm productivity and the structures of the credit and labor markets. Adverse shocks to credit markets destroy the least productive jobs and slow job creation, thus raising aggregate TFP and unemployment, and reducing output.

Key words: Aggregation; Productivity; Search; Job creation and destruction; Financial frictions; Business cycles
JEL codes: J64, E44, E32

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1 Introduction

The financial crisis that led to the worst recession since the Great Depression began with an unexpected, drastic, shock to credit markets. Indeed, a salient feature of the crisis, emphasized in Caballero and Kurlat (2009), concerns the element of surprise in the event, not factored into expectations, that lead to sharp declines in bank lending and commercial paper issuance, along with spikes in interest rate spreads.1 Another peculiar feature of this recession has been the increase in measures of aggregate productivity concurrent with strong contractions in output and employment. Fernald and Matoba (2009) report a 3.3% increase in total factor productivity (TFP) over the period from 2008:Q4 to 2009:Q2, compared to an average quarterly change of 1.4% since 2001, while Cociuba et al. (2009) document an unusually strong increase in labor productivity.2 In those few quarters the unemployment rate increased from 6.2% to 10%, driven by both slow job creation and waves of layoffs which had reached nearly 2,500,000 per month in February 2009.

This confounds real-business-cycles theory, which is predicated on movements in TFP driving the cyclical fluctuations in aggregate quantities and declining during recessions. Moreover, the credit crunch was associated with job losses at establishments that survived the event, not by bankrupt and exiting employers. In fact, aggregate job losses from closing establishments, measured by the B.L.S.’s Business Employment Dynamics (BED) survey display no change in trend during the crisis. This paper provides a qualitatively and quantitatively consistent explanation of this set of observations on the Great Recession in which the sorting of heterogeneous firms following an unexpected shock to credit markets plays a central role.

The theory models endogenous job creation and destruction in the presence of search frictions on credit and labor markets, and shows how the former are both functions of the state of credit markets. Firms are modeled as the joint venture between an entrepreneur and a creditor, require a unit of labor to produce a good, and creditors bear the costs of operating the firm when not producing. The duration of search for both sides of the credit market determines the financial cost to creating a new firm, and affects the interest paid to creditors on loans at producing firms. Producing firms randomly draw new job productivities, such that job losses are generated when the worker-firm pair is no longer viable in the manner of Mortensen and Pissarides (1994), with the exception that the structure of the credit market enters the determination of the job destruction threshold. That is, tighter credit markets results in larger flow transfers from entrepreneurs to creditors at existing

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1 According to Caballero and Kurlat (2009), “the surprise was in the distress of many parts of the financial system, even those very distant from the subprime market itself, including all structured products, commercial paper, and interbank lending.” Certain asset markets simply froze, with few trades, while commercial banks doubled their holdings of cash assets. In contrast, Bernanke and Lown (1991), in discussing the role of financial factors in the 1990–1991 recession, argue that a credit crunch did not occur. They see a drop in credit demand as playing a large role in the slowdown in credit market activity.

2 Cociuba et al. (2009) document an increase in labor productivity, based on hours reported in the CPS and unadjusted for capacity utilization, which is stronger than in past recessions. This empirical evidence is discussed in detail in Section 5.4. See also Wilson (2010).
jobs, such that some jobs may no longer be viable and are terminated. The destruction of a job, however, does not imply the destruction and exit of a firm. Frictional credit markets imply a positive economic value of a firm to creditors and entrepreneurs even without an employed worker. By aggregating over producing micro units, the aggregate production function for this economy depends on the underlying distribution of shocks and the structure of both markets. In particular, the level of aggregate productivity is increasing in the total costs of the financial sector as the latter determine the least productive active production unit, or job, in the economy.

The model is used to describe the effects of a breakdown in financial intermediation on aggregate activity, modeled as a sudden increase in lenders’ screening costs. This shock causes a decline in the amount of credit available for matching relative to the amount of projects looking for financing, or an increase in the tightness of the credit market. The disruption, unanticipated by agents, reduces the effectiveness of the financial sector as a whole, causing an increase in the costs of financial intermediation and interest rate spreads to spike. This increase in the real costs of financial intermediation, which Bernanke (1983) argues are the consequences of financial crises, raises the opportunity cost of existing job matches, thus reducing the surplus in all employment relationships. This sorts units with the lowest productivities out of production, by destroying the associated jobs, and slows down the creation of new jobs by all firms, continuing and new. Modeling on the events of the fall of 2008, this credit crunch leads to declines in aggregate output and employment, due to changes in job creation and destruction, along with an increase in aggregate productivity of the magnitudes observed during the crisis. The recession destroys the least productive jobs in the economy, creating a “cleansing” effect consistent with the evidence in Solon et al. (1994). The destruction of the least productive jobs and their associated wages drives a composition bias that can increase the average wage in spite of the downward pressure on individual wages that the deteriorating labor market places on workers’ outside option. This is consistent with the moderate rise in real wages between September 2008 and July 2009, another peculiar feature of the Great Recession. Finally, it is important to stress that the variation in aggregate quantities arises from variations in the fraction of producing firms through the creation and destruction of jobs, not the population of firms. This is consistent with the aforementioned evidence from the BED

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3The peculiar aspect of the credit crunch was the element of surprise: a new, negative, event which is both difficult to evaluate, and to which the responses are uncertain. For example, Chari et al. (2010) document the large variety of policies that were tried and that did not appear to “unfreeze” the market. Moreover, the reasons for these markets having somewhat recovered is unknown, arguing further for the unanticipated nature of the financial shock. In this sense, the event did not feature in agent’s expectations. See also Caballero and Kurlat (2009). The experiment compares two steady states, comparing the equilibrium before the crises to the trough of the recession. As discussed in detail in Section 4, the speed of adjustment in this type of economy is sufficiently quick for this to be a good approximation.

4This effect is slightly different than the cleansing effect in the work of Caballero and Hammour (1994) in which the least productive plants exit during a recession. Moreover, changes in aggregate productivity are driven by a destruction margin in this model rather than constraints on the creation margin of new, high-productivity, plants as occurs in their work.
which reveals that the large increase in job losses during the crisis is attributable to contractions at continuing establishments. There has been little change in the amount of job losses due to closing establishments.\textsuperscript{5}

The model also predicts an increase in the variance of the cross-sectional distribution of firm productivity during a credit crunch that, in the research of Bloom (2009), can be interpreted as an increase in uncertainty or risk. However, this variation is the endogenous response of the economy to a shock that does not affect the underlying distribution of idiosyncratic productivity from which firms make their draws. Nonetheless, this shock can be perceived as a volatility shock associated with declines in output and employment.

The presence of idiosyncratic uncertainty allows for the existence of a well-defined aggregate production function, a result similar to Lagos (2006) and present in a different context in Campbell (1998) and Gilchrist and Williams (2000).\textsuperscript{6} In particular, when the decision to break up the firm is endogenized and idiosyncratic productivity is drawn from a Pareto distribution, the aggregate production function is Cobb-Douglas in aggregate capital and employment, in spite of being Leontief at the level of the firm. There arises a well-defined region over the realized idiosyncratic productivity of non-production, in which firms neither desire to employ labor nor exit the economy. This region of inactivity is increasing in the degree of financial market imperfections, and corresponds to a measure of capacity utilization. The Solow residual, or aggregate TFP measured from this Cobb-Douglas aggregate production function, is shown to decline during a credit crunch and to need a correction for capacity utilization. Thus the model is also consistent with the findings of Fernald and Matoba (2009) regarding the Solow residual and adjusted TFP in the U.S. during the quarters that followed the Fall of 2008.

In a broader sense, the model implies that movements in observed aggregate productivity can originate from shocks to credit markets, specifically, but in general from frictional markets that affect the sorting of heterogeneous establishments. This calls into question how to correctly identify unanticipated movements in aggregate productivity as technology shocks and their role in accounting for business cycle fluctuations, reminiscent of the distinction between productivity and technology in Basu and Fernald (2002), and the results Chari et al. (2007) that disaggregated models with frictions can imply a different mapping between the parameters of the aggregate production technology and observables. Basu and Fernald (2002), for example, find that non-technological factors play an important role in short-run fluctuations in aggregate productivity, as occurs in the

\textsuperscript{5}Bresnahan and Raff (1991) document changes in the use of automotive industry plants during the Great Depression. They find evidence that output per worker did not decline at continuing plants, and that mothballing of assembly plants rather than outright closure was frequent. Hamermesh (1989) shows that plant-level labor adjustments occur in large jumps, while Bresnahan and Ramey (1994) show that nearly all the variance in aggregate production is due to margins that involve the number of people employed, not the average weekly hours of employed persons.

\textsuperscript{6}In addition, there will be a relationship between the variance of idiosyncratic productivity and aggregate productivity that also appears in Solow’s (1960) vintage capital model, and which plays a crucial role in the propagation of Gilchrist and Williams’ (2000) putty–clay investment economy.
context of the proposed model. Moreover, as there are any number of possible sources of business cycle fluctuations, the model does not imply that aggregate productivity must be countercyclical. A shock that produces a 1% decline in the mean of the distribution of idiosyncratic technology leads to the usual declines in employment and output, and a drop in aggregate productivity, obtained in real business cycle models. The cleansing effect only limits the decline in aggregate productivity, and it is also shown that a large shock to technology cannot explain the salient features of the Great Recession outlined above. Finally, this paper contributes to recent advances in incorporating firm-level heterogeneity in macroeconomic models, and in particular in conjunction with financial frictions as in Khan and Thomas (2010). In the latter, even temporary shocks to the financial sector cause lasting recessions as firms need time to accumulate assets to relax their collateral constraints. In the meantime, production shifts to larger, less productive firms, causing a drop, and not a rise, in aggregate TFP.

The rest of the paper is organized as follows. Section 2 develops the main model and discusses the relationship between financial costs and job creation and destruction. Section 3 derives the aggregate production function and examines its relationship with the state of the credit market. Section 4 explores the effects of a shock to financial markets. Section 5 allows for the endogenous destruction of a firm and derives a Cobb-Douglas aggregate production function. It also develops the relationship between capacity utilization, the Solow residual and utilization-adjusted TFP to credit market frictions and offers some empirical support for the predictions and implications of the model.

2 A model of credit markets, job creation and destruction

The market frictions regularly considered in financial markets are similar to those encountered in the labor market: moral hazard, heterogeneity and specificity in long term relationships. These frictions argue for modeling the financial market as a matching market, and the matching function is a convenient modeling device. This approach is not entirely novel. Den Haan et al. (2003) introduce matching in the credit market following arguments in Jaffee and Stiglitz (1990) that the market for credit is better conceived as a customer market in which borrowers form a single relationship with a lender. Lenders tend to specialize in particular sectors or regions, bringing

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7The identification of technology shocks, and their effects on aggregate quantities, has been studied in Evans (1992), Gali (1999), Basu et al. (2006), and Francis and Ramey (2005), among many others. Basu and Fernald (2002) also find that non-technological factors are not important for long-run productivity growth, which is not inconsistent with the present model.

8See also Veracierto (2002), Clementi and Palazzo (2010). Samaniego (2010) finds a positive relationship between the rate of industry technical change and rates of entry and exit. Although the current paper is not concerned with the effects of financial frictions on growth, the model could be extended in that direction in future research. See also Hsieh and Klenow (2009) on the effects of factor misallocation for aggregate TFP. For implications of heterogeneous households in macroeconomic models, see Ayagari (1994) and the survey by Heathcoate et al. (2009).
specificity to the relationships that are formed. In 2004, Wasmer and Weil studied an economy with matching in the labor and credit markets, with the view to analyzing a financial accelerator of productivity shocks that arises from the interaction of frictions in both markets.\footnote{There are significant departures in the current model from Wasmer and Weil (2004). They consider a representative agent environment with exogenous job destruction and firm destruction, making no distinction between firm and job destruction, i.e. firms are hit by death shocks that dissolve all relationships, both with the worker and the creditor. Moreover, aggregation is not an issue in their work: the market structures do not affect aggregate productivity.}

The implications of the matching approach have empirical foundations. For instance, firms tend to concentrate their lending from one source (Petersen and Rajan, 1994) and the benefits to borrowers increase with the length of the relationship with banks (Petersen and Rajan, 1994, Berger and Udell, 1995). Relationship lending appear to even be present in private equity (Prowse 1998).

2.1 Credit markets and the creation of firms

There are three types of agents in this economy: a continuum of entrepreneurs with no capital, a continuum of banks with no ability to produce, and a mass 1 of workers with no capital and no ability to start a business. The timing of events is as follows. All agents discount time with the economy risk free rate $r$. Entrepreneurs initially need to find a “banker” in order to start a business. This search process costs $e$ units of effort per unit of time. Search is successful with probability $p$. Once a relationship in the credit market is established, a firm is created as the joint venture between the banker and the entrepreneur, the rents from which will be determined below. The banker finances all operating costs when the firm is not producing, which for the moment include vacancy posting or recruiting costs $\gamma$ to attract a worker for the firm. Later we will include the cost of renting physical capital when production at the firm level will be Leontief in labor and capital. This search process on the labor market is successful with probability $q$. The firm is then able to produce and sell in the goods market, which generates the flow profit to the entrepreneur $y(x) - w(x) - \rho(x)$, where $x$ is a random shock to the productivity of the firm, $y(x)$ is the flow output assumed to be linear, $y(x) = x$, $w(x)$ is the wage and $\rho(x)$ the flow repayment to the bank. $x$ is specific to each firm and drawn at Poisson rate $\lambda$ from a distribution $G(x)$, with density $g(x)$. In addition, firms are subject to exogenous destruction shocks with Poisson parameter $\delta$.

The steady state asset values of the entrepreneurs are denoted by $E_j$, with $j \in \{c, l, g\}$ for the state in which the entrepreneur finds him/herself, standing, respectively, for the credit, labor and goods markets. Free entry on the credit market is assumed, that is $E_c \equiv 0$. We therefore have the
following Bellman equations:

\[
\begin{align*}
    r_{E_c} &= 0 = -e + pE_l \\
    r_{E_l} &= 0 + q \int \max [E_g(x) - E_l, 0] dG(x) \\
    r_{E_g}(x) &= x - w(x) - \rho(x) + \lambda \int \max [E_g(z), E_l, 0] dG(z) - (\lambda + \delta)E_g(x)
\end{align*}
\]

Equation (2) implies that firms may decide not to form a match with a worker upon meeting given the value of the idiosyncratic productivity draw.\(^\text{10}\) When a new productivity is drawn by a producing firm, equation (3) states that entrepreneurs may prefer the value of prospecting on the labor market for a new worker rather than continuing the existing relationship. If the later occurs, the economy experiences a destruction of a job at a continuing establishment.

Symmetrically, the bank’s asset values are denoted by \(B_j\), \(j \in \{c, l, g\}\), for each of the stages. Free entry is also assumed on the banker’s side of the market, implying that \(B_c = 0\). Let \(\kappa\) be the screening cost per unit of time of banks in the first stage, and let \(\hat{\rho}\) be the Poisson rate at which a bank finds an entrepreneur to be financed. We have

\[
\begin{align*}
    r_{B_c} &= 0 = -\kappa + \hat{\rho}E_l \\
    r_{B_l} &= -\gamma + q \int \max [B_g(x) - B_l, 0] dG(x) \\
    r_{B_g}(x) &= \rho(x) + \lambda \int \max [B_g(z), B_l, 0] dG(z) - (\lambda + \delta)B_g(x).
\end{align*}
\]

The matching rates \(p\) and \(\hat{\rho}\) are made mutually consistent by the existence of a matching function \(M_c(B, E)\) where \(B\) and \(E\) are, respectively, the number of bankers and of entrepreneurs in stage \(c\). This function is assumed to have constant returns to scale. Hence, denoting by \(\phi\) the ratio \(E/B\), a reflection of the tension on the credit market that will be called credit-market tightness, we have

\[
\begin{align*}
    p &= \frac{M_c(B, E)}{E} = p(\phi) \text{ with } p'(\phi) < 0 \\
    \hat{\rho} &= \frac{M_c(B, E)}{B} = \phi p(\phi) \text{ with } \hat{\rho}'(\phi) > 0.
\end{align*}
\]

After contact, the bank and the entrepreneur enter a long term relationship and engage in bargaining over the rents generated by the creation of the firm, \(F_l = E_l + B_l\). The flow transfer to the banker during production, \(\rho(x)\), is the solution to \(\arg \max (E_l)^{1-\beta} (B_l)^\beta\), where \(\beta \in (0, 1)\) is the bargaining power of the bank relative to the entrepreneur. This flow return, as will be discussed

\(^{10}\)In contrast to Caballero and Hammour (1994) or Mortensen and Pissarides (1994), new producing units will not necessarily embody the most productive technology at inception. This would certainly be important were growth the concern of the model, but it is not for the arguments developed here. See Lentz and Mortensen (2008) for a model of growth through product innovation and its relation to worker reallocation.
below, is related to the rate of return on loans made by bankers to entrepreneurs when the firm is not producing.

The first order condition to the bargaining problem yields a sharing rule, \((1 - \beta)B_l = \beta E_l\), which, in combination with (1) and (4), yields an equilibrium value of \(\phi\) denoted by \(\phi^*\):

\[
\phi^* = \frac{1 - \beta \kappa e}{\beta e}.
\]

Matching in the labor market is denoted by \(M_l(\mathcal{V}, u)\) where \(u\) is the rate of unemployment and the total number of unemployed workers since the labor force is normalized to 1. \(\mathcal{V}\) is the number of “vacancies,” that is the number of firms in stage \(l\). The function is also assumed to be constant returns to scale and the rate at which firms fill vacancies is a function of the ratio \(\mathcal{V}/u \equiv \theta\), or tightness of the labor market. We have

\[
q(\theta) = \frac{M_l(\mathcal{V}, u)}{\mathcal{V}} \text{ with } q'(\theta) < 0.
\]

Once an entrepreneur-bank pair is formed, they jointly operate as a firm on the labor and goods markets. We therefore describe the asset value of being on either market for a firm before turning to workers:

\[
\begin{align*}
RF_l &= -\gamma + q(\theta) \int \max [F_g(x) - F_l, 0] dG(x) \quad (7) \\
rF_g(x) &= x - w(x) + \lambda \int \max [F_g(z), F_l, 0] dG(z) - (\lambda + \delta)F_g(x). \quad (8)
\end{align*}
\]

During the recruiting stage, \(F_l\), a firm incurs the flow cost \(\gamma\), financed by the banker, and it generates the flow profit \(x - w(x)\) during the production stage, \(F_g(x)\).

The total financial costs related to creating a firm, following the assumption of free entry on credit markets, can be summarized as \(\frac{e}{\rho(\phi)} + \frac{\kappa}{\phi p(\phi)} \equiv C(\phi)\), a convex function of credit market tightness. Petrosky-Nadeau and Wasmer (2010) derive a Hosios condition for the credit market in which financial costs, \(C(\phi)\), are minimized when the creditor’s bargaining weight, \(\beta\), and the elasticity of the credit matching function with respect to bankers, \(\epsilon\), are equalized.\(^{11}\) A rise in these costs can originate from several sources: an increase in the prospecting costs \(\kappa\) and \(e\), a breakdown in the efficiency of the matching function \(M_C(\mathcal{B}, \mathcal{E})\), or deviations from the credit-market Hosios condition.

Workers are either unemployed, earning the flow value of non-employment \(b\), or employed at the wage \(w(x)\). Unemployed workers face a job finding hazard \(\theta q(\theta)\) and employed workers are

\(^{11}\)Away from this condition, the distortions caused by credit markets increase, as does the sensitivity of the economy to perturbations. To be precise, Petrosky-Nadeau and Wasmer (2010) show that the elasticity of labor market tightness to productivity shocks is increasing in total financial costs, \(C(\phi)\), and that this elasticity is minimized at the credit-market Hosios condition.
Since Bellman for the worker-firm pair’s surplus is

\begin{align}
 rU &= b + \theta q(\theta) \int \max [W(x) - U, 0] dG(x) \\
 rW(x) &= w(x) + \lambda \int \max [W(z) - U, 0] dG(z) - (\lambda + \delta) [W(x) - U],
\end{align}

where $U$ is the value of unemployment and $W(x)$ the value of employment at a firm with productivity $x$. Workers and firms split the surplus of their relationship, defined as $S(x) = F_g(x) - F_l + W(x) - U$, under a generalized Nash Bargaining process. This involves choosing a wage $w(x)$ that satisfies $\arg \max (F_g(x) - F_l)^{1-\alpha} (W(x) - U)^{\alpha}$, where $\alpha \in (0, 1)$ is the worker’s bargaining weight. The first-order condition for this problem implies the sharing rule $(1 - \alpha) (W(x) - U) = \alpha (F_g(x) - F_l).^{12}$

## 2.2 Job creation and job destruction under frictional credit markets

Combining (7), (8), (9) and (10) yields a Bellman equation for the surplus of a worker-firm pair:

\[(r + \lambda + \delta)S(x) = x + \lambda \int \max [S(z), 0] dG(z) - (r + \delta)F_l - rU.\]

Since $S'(x) = \frac{1}{r + \lambda + \delta} > 0$, there exists a reservation strategy such that if $x < \bar{\pi}$, where $\bar{\pi}$ is such that $S(\bar{\pi}) = 0$, the match is dissolved. That is to say, if the job draws $x < \bar{\pi}$, there is no value to the relationship for either party. Thus, $\bar{\pi}$ defines a job destruction threshold. Using this result, the Bellman for the worker-firm pair’s surplus is

\[(r + \lambda + \delta)S(x) = x - (r + \delta)F_l - rU + \lambda \int_{\bar{\pi}} S(z) dG(z),\]

which, evaluated at $\bar{\pi}$, yields what can be called a job destruction (JD) condition:

\[(r + \delta)C(\phi) + rU - \bar{\pi} = \lambda \int_{\bar{\pi}} S(z) dG(z),\]

where we have used the fact that the value of a firm in the recruiting stage, by (1) and (4), is equal to total financial costs, $F_l = C(\phi)$. For a job to remain viable, its expected future value must at least equal the values of the firm’s and worker’s outside options net of the match’s current production. Increases in $C(\phi)$ render all existing matches less profitable, and some no longer viable. In the canonical Mortensen-Pissarides model, $C(\phi) = 0$, such that financial costs raise the lowest viable job productivity relative to that benchmark. Inserting a solution $S(x) = \frac{x - \bar{\pi}}{r + \lambda + \delta}$ into the previous

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\[^{12}\text{It will be useful to write this sharing rule as } (1 - \alpha)S(x) = F_g(x) - F_l \text{ and } \alpha S(x) = W(x) - U.\]
equation yields \((r + \delta)C(\phi) + rU - \bar{x} = \frac{\lambda}{r + \lambda + \delta} \int_{\bar{x}}^{z}(z - \bar{x})dG(z)\).

Finally, using the existence of the job destruction threshold, a job creation (JC) condition is obtained by rearranging (7) as \((\frac{1 - \alpha}{r + \lambda + \delta}) \int_{\bar{x}}^{z}(z - \bar{x})dG(z) = \frac{rC(\phi) + \gamma}{q(\theta)}\), where we use the sharing rule \((1 - \alpha)S(x) = F_1(x) - F_2\). This condition states that the expected benefit from a job to the firm must equal the average creation cost, which not only depends on the unit recruiting cost, \(\gamma\), and the average duration of the recruiting spell, but also the total financial costs \(C(\phi)\). In the canonical Mortensen-Pissarides model, entry by firms on the labor market is unhindered by frictional credit markets and the value of the recruiting stage is driven to 0. The presence of financial imperfections implies \(C(\phi) > 0\), a positive lower bound to the value of a vacancy to a firm that restricts firms’ labor-market entry and job creation.

**Proposition 1** - There exists a unique equilibrium for this economy defined by a pair \((\bar{x}, \theta)\) that solve the job creation and job destruction conditions:

\[
\frac{(1 - \alpha)}{r + \lambda + \delta} \int_{\bar{x}}^{z}(z - \bar{x})dG(z) - \frac{\gamma + rC(\phi)}{q(\theta)} = 0
\]

\[
\bar{x} - \left( b + \frac{\alpha}{1 - \alpha} \theta (\gamma + rC(\phi)) + (r + \delta)C(\phi) \right) + \frac{\lambda}{r + \lambda + \delta} \int_{\bar{x}}^{z}(z - \bar{x})dG(z) = 0
\]

Proof - See Appendix A. Figure 1 plots the job creation and destruction conditions in \((\bar{x}, \theta)\) space, along with the equilibrium effects of an increase in total financial costs, \(C(\phi)\). A rise in \(C(\phi)\) shifts the job-creation condition downward and decreases its slope. For a given separation productivity, the increase in the cost of creating a firm means that the average total cost to creating a job is no longer covered by the product of the job relationship. Thus, fewer firms desire to create jobs, reducing the vacancy-to-unemployment ratio such that the job filling hazard \(q(\theta)\) increases. This, in turn, lowers the average search duration and firms’ labor recruiting costs. Finally, lower average job creation costs push the lowest viable job productivity down along the job destruction curve.

The same increase in financial costs, \(C(\phi)\), shifts the job destruction curve downward and slightly decreases its slope. For a given labor market tightness, this increases the level of the lowest viable job productivity as the latter must compensate for the rise in the value of the firm’s outside option. By the same token, this leads to a decline in labor market tightness along the job creation curve. The net effect is, first, an unambiguous decline in equilibrium labor market

\footnote{The derivation of the job destruction condition involved, as intermediary steps, combining \(\int_{\bar{x}}^{z}S(z)dG(z) = \frac{\gamma + rC(\phi)}{(1 - \alpha)q(\theta)}\) with (9) to write \(rU = b + \frac{\alpha}{1 - \alpha} \theta (\gamma + rC(\phi))\). The job creation condition has negative slope, and the job destruction condition a positive slope. See Appendix A for details.}
tightness. Second, movements of the JD curve dominate the effect from the JC curve for the job destruction threshold when \( \eta > \frac{\alpha r \theta}{ar\theta + (1-\alpha)(r+\delta)} \), where \( \eta \equiv -\frac{\partial q(\theta)}{\partial \theta} \frac{\theta}{q(\theta)} \) is the elasticity of the job filling rate with respect to labor market tightness, such that we have the following proposition.

**Proposition 2** - If \( \eta > \frac{\alpha r \theta}{ar\theta + (1-\alpha)(r+\delta)} \), where \( \eta \) is the elasticity of the job filling rate with respect to labor market tightness, increases in total financial costs, \( C(\phi) \), cause a rise in the job destruction threshold, \( x \), and a decline in labor market tightness \( \theta \).

**Proof** - See Appendix A.

### 2.3 The financial relationship and the efficiency of job separation

The job destruction condition was written so as to be efficient from the points of view of the worker and the firm. The latter, however, is comprised of two agents and it may be the case that the job separation threshold is not efficient for either the entrepreneur or the creditor individually. That is to say, although \( F_g(\bar{\lambda}) - F_l = 0 \), it is not necessarily the case that \( B_g(\bar{\lambda}) - B_l = 0 \) and \( E_g(\bar{\lambda}) - E_l = 0 \).\(^{14}\)

The entrepreneur’s asset value of the production stage (3) has slope \( E_g'(x) = \frac{(1-\beta)(1-\alpha)}{\lambda + r + \delta} \) from

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\(^{14}\)Section IV.B in Wasmer and Weil (2004) considers a case in which firms are still hit by a shock destroying both the credit and labor relationships, and in addition draw new values of productivity. They consider a destruction margin for the firm-creditor pair, under the assumption of a fixed wage. This is significantly different from the baseline model in this paper which keeps the destruction of the firm-creditor pair constant, allowing for endogenous wages and job destruction. Because of the fixed wage and the inability to endogenously separate from the worker in Wasmer and Weil, their model has a notion of financial fragility in their extension of Section IV.B. That is, the bank injects funds to cover the wage when productivity is below the wage even if the surplus to the creditor is negative. This does note take place in this model: the wage adjusts until separation with the worker. Job separation is efficient for all parties and, due to the specificity of the creditor-entrepreneur relationship, their joint surplus remains positive. There is no financial fragility.
the fact that a fraction \((1 - \beta)\) of the surplus from the creation of the firm accrues to the entrepreneur, and that a producing firm retains a fraction \((1 - \alpha)\) of the profit flow from labor. Recall that there is free entry on credit markets, such that \(E_l = \frac{\epsilon}{p(\Phi^*)}\), and call \(x^*\) a candidate job separation threshold efficient for an entrepreneur defined as \(E_l(x^*) - E_l = x^* \frac{(1-\beta)(1-\alpha)}{\lambda + r + \delta} - \frac{\epsilon}{p(\Phi^*)} = 0\). Since the slope of the banker’s asset value of the production stage is given by \(B_g(x) = \frac{\beta (1-\alpha)}{\lambda + r + \delta}\) and following the same arguments, a candidate job separation threshold for the banker \(\hat{x}\) is defined by \(B_g(\hat{x}) = B_l = \hat{x} \frac{\beta (1-\alpha)}{\lambda + r + \delta} - \frac{\kappa}{\phi p(\Phi^*)} = 0\). We now need to determine under what conditions \(x^* = \hat{x}\) by combining the two candidate thresholds conditions as \(\frac{\hat{x}}{x^*} = \frac{1 - \beta}{\beta} \frac{\kappa}{\phi e}\). Since free entry on the credit market yielded an equilibrium credit market tightness \(\phi^* = \frac{1 - \beta}{\beta} \frac{\kappa}{\phi e}\), it will always be the case that \(\frac{\hat{x}}{x^*} = 1\). It is straightforward to show in a final step that \(\overline{\phi} = \hat{x}\). Therefore the job separation threshold \(\overline{\phi}\) is efficient not only from the firm’s, but also the entrepreneur’s and banker’s points of view.

3 Financial markets, unemployment and aggregate productivity

The equilibrium rate of unemployment is determined by equating the steady state flows in and out of unemployment. Unemployed workers find a job at the rate \(\theta q(\theta) [1 - G(\overline{\phi})]\), as those meetings drawing a productivity below \(\overline{\phi}\) do not form a match, and workers can exit employment either because the firm is destroyed, at rate \(\delta\), or the job draws a productivity below the reservation value \(\overline{\phi}\). The latter implies an endogenous job destruction rate \(\lambda \int \overline{\phi} dG(x)\) such that the equilibrium rate of unemployment is given by

\[
u = \frac{\delta + \lambda G(\overline{\phi})}{\theta q(\theta) [1 - G(\overline{\phi})] + \delta + \lambda G(\overline{\phi})} \tag{13}\]

By Proposition 2, flows out of unemployment are reduced when the total financial cost, \(C(\phi)\), increases, as the lower equilibrium market tightness reduces the meeting hazard rate \(\theta q(\theta)\), and the increased threshold \(\overline{\phi}\) reduces the fraction \([1 - G(\overline{\phi})]\) of viable initial contacts. The rise in the lowest viable job productivity itself increases the equilibrium rate of endogenous job destruction \(\lambda G(\overline{\phi})\). Thus greater financial intermediation costs reduce job creation and increase job destruction, resulting in an unambiguous increase in the equilibrium rate of unemployment.

Figure 2 provides a Beveridge curve representation of (13). The equilibrium effects of a rise in total financial costs appear as a change in the slope of the ray representing equilibrium labor market tightness in the \((\nu, u)\) plane, and a change in the slope of the Beveridge curve. Greater financial costs are represented by an array with lesser slope \(\theta_1 < \theta_0\), leading to a movement down the Beveridge curve with fewer vacancies and greater unemployment. As credit markets also affect job destruction, the Beveridge curve tilts upwards, further increasing unemployment, but also
increasing equilibrium job vacancies. This raises the possibility that, under some configurations of the parameter space, credit market shocks lead to a positive co-movement of unemployment and vacancies. We will return to this point below.

Only firms with productivity greater than the threshold $\bar{x}$, determined by frictions on labor and credit markets, are producing goods. That implies that the average productivity of firms actually producing is given by the expected productivity conditional on surviving: $\frac{\int x dG(x)}{1 - G(x)}$. Before going further, it is useful to express the cross-sectional dispersion in productivity of producing firms with the distribution $H(x) = \frac{G(x) - G(\bar{x})}{1 - G(\bar{x})}$ and density $h(x) = \frac{g(x)}{1 - G(x)}$, where the threshold $x$ is the lower bound of the support. The cross-section of firms depends on the marginal job, similar to the marginal entrant in Melitz (2003), and the latter has the property $\int x dH(x) = \frac{\int x dG(x)}{1 - G(\bar{x})}$ which we use to express aggregate output:

$$Y = (1 - u) \int x dH(x) \Rightarrow Y = AN,$$

where $N = (1 - u)$ and $A \equiv \int x dH(x)$. It appears clearly from this aggregate production function that frictions on credit markets affect both level of aggregate productivity and employment, and hence output. Increases in financial cost in the economy, $C(\phi)$, force the lowest productivity firms to cease production, and the sorting results in an increase in aggregate productivity as $\frac{\partial A}{\partial C(\phi)} = \frac{\partial A}{\partial x} \frac{\partial x}{\partial C(\phi)} > 0.15$

**Proposition 3** - An increase in total financial costs in the economy, $C(\phi)$, leads to an increase in aggregate unemployment and an increase in aggregate productivity.

$15$This follows from $\frac{\partial A}{\partial x} = \frac{\partial (1 - u)}{\partial x} \frac{\partial x}{\partial C(\phi)} > 0$ and $\frac{\partial x}{\partial C(\phi)} > 0$. Aggregate productivity will depend on the labor market in the same manner as in Lagos (2006).
Financial frictions, and possibly financial shocks, have real effects in this model by altering the flows in and out of aggregate employment and, through the sorting of firms, affecting the aggregate level of productivity. The effect on aggregate output depends on whether the productivity or employment change dominates. In the example below, the latter will dominate and the equilibrium effect of a rise to total costs on financial markets will be to reduce aggregate output.

4 TFP during a Credit Crunch

We study the aggregate effects of a “credit crunch” as the sudden increase in creditor’s screening cost $\kappa$. This shock to the ability of creditors to gather and evaluate information, causes an increase in credit market tightness and the costs of financial intermediation. We describe first the choice of the form for the underlying distribution of idiosyncratic shocks and then calibrate the model on a set of observations for the U.S. economy prior to the crisis.

4.1 Functional form for the underlying distribution of productivity shocks

Several papers incorporating heterogeneity in firm productivity argue that the data on the cross section of firms present a fat right tail, and are well described by a Pareto distribution.\(^{16}\) Examples include Melitz (2003) in international trade and Houthakker (1955), Jones (2005) and Lagos (2006) in the aggregation literature. We follow this tradition for the distribution of idiosyncratic productivity shocks, $G(x)$, and assume

$$G(x) = \begin{cases} 0, & \text{if } x < m \\ 1 - \left(\frac{m}{x}\right)^{\mu}, & \text{if } x \geq m, \end{cases}$$

where $m$ and $\mu$ are scale and curvature parameters, respectively. Pareto distributions have convenient expressions for the mean, $E(x) = \frac{\mu m}{\mu - 1}$ for $\mu > 1$, and variance, $\text{var}(x) = \frac{m^2 \mu}{(\mu - 1)^2 (\mu - 2)}$ for $\mu > 2$, and imply that the cross-sectional distribution of idiosyncratic productivity shocks, $H(x)$, is also a Pareto distribution:

$$H(x) = \begin{cases} 0, & \text{if } x < \bar{x} \\ 1 - \left(\frac{\bar{x}}{x}\right)^{\mu}, & \text{if } x \geq \bar{x}, \end{cases}$$

with the same curvature parameter $\mu$ and where the job destruction threshold, $\bar{x}$, determines the scale of the distribution. Aggregate productivity is then given by $A = \frac{\mu \bar{x}}{\mu - 1}$ and the variance of the cross section of firm productivity is $\text{var}_H(x) = \frac{\bar{x}^\mu}{(\mu - 1)(\mu - 2)}$. Thus, under a Pareto distribution\(^{16}\)

\(^{16}\)The empirical literature on firm size finds support for either a Pareto distribution or concludes that it is difficult to distinguish between a Pareto and a log-normal distribution (Axtell, 2001).
for the underlying idiosyncratic productivity shocks, an increase in total financial costs, $C(\phi)$, increases the level of aggregate productivity and the dispersion in firm productivity. Changes in the dispersion of the cross-section of firm productivity, here, are the endogenous response of the economy to a credit market shock. Moreover, the aggregate level of productivity is increasing the dispersion of idiosyncratic productivity shocks, a result that also arises in vintage capital models of Solow (1960) and plays an important role in the propagation properties of putty-clay economies, as shown by Gilchrist and Williams (2000).

### 4.2 Parametrization and calibration

The model is calibrated to a set of observations for the U.S. economy in the fall of 2008, matching a set of targets for the labor and credit markets. We begin with the parameters for which we fix a value given previous studies or supported by empirical counterparts.

We consider a quarter as the basic unit of time. The rate of exogenous firm destruction is set to $\delta = 0.01$, corresponding to the average quarterly business failure rate for the United States. Regarding the underlying distribution of productivity shocks, $G(x)$, the curvature and scale parameters are set to $\mu = 3$ and $m = \frac{2}{3}$, respectively. This implies a mean idiosyncratic productivity of $E(x) = 1$ and variance of $\text{var}(x) = 1/3$. The choice of the curvature parameter will also imply, when firm exit is endogenized, an elasticity of the aggregate production function to capital of $1/3$, as is typically assumed for Cobb-Douglas specifications. The matching technology for the labor market is assumed to be a Cobb-Douglas, $M_L(y, u) = \chi_L y^{1-\eta} u^{\eta}$, with $\chi_L > 0$ and $0 < \eta < 1$. The elasticity $\eta = 0.72$ is chosen according to estimates by Shimer (2005). We use the labor-market Hosios condition to set the bargaining power of the worker in the wage determination to $\alpha = 0.72$. The value of non-employment activities, $b$, is set to match an average replacement rate of 0.54 for the United States (OECD, 2006), resulting $b = 0.32$.

The remaining labor market parameters are the unit vacancy cost of $\gamma$, the level parameter in the labor-matching function $\chi_L$, and the Poisson arrival rate of new idiosyncratic productivity draws, $\lambda$. Silva and Toledo (2009) estimate the average costs of recruiting a worker to be approximately 5% of the total wage bill. As a first target for the calibration, we aim for $\frac{\gamma q(\theta) + B \kappa / \phi p(\phi)}{W_N} = 5\%$. Second, we target an unemployment rate 6.03% as it stood in Fall quarter of 2008. Third, we target a quarterly job separation rate of $\lambda G(\bar{r}) + \delta = 4.5\%$, consistent with the evidence reported in the survey by Davis et al. (2006).

The credit-market calibration requires choosing parameters for the credit-matching function, assumed to be of the form $M_c(B, e) = \chi_C e^{1-e} B^e$, with $\chi_C > 0$ and $0 < e < 1$, the costs of prospecting on credit markets, $\kappa$ and $e$, and the bargaining weight $\beta$. First, we assume symmetry in the cost of prospecting on the credit market, $\kappa = e$. Second, we derive the rate of return of loans to firms and its spread over the economy’s risk-free rate to use as a calibration target. This rate, $R$,
as in Wasmer and Weil (2004), equalizes the expected discounted value of the each loan, \( \frac{\gamma}{R + q(x)} \), to the expected discounted repayment on the loan \( \frac{q(x)dG(x)}{R + \delta + \lambda G(x)} \). Thus, we can express the spread over the economy’s risk-free rate, which is increasing in \( C(\phi) \), as

\[
R - r = \int x \rho(x)dG(x) \frac{q(\theta)}{\gamma} - (r + \delta + \lambda G(x)).
\]

The risk-free rate is set to \( r = 0.01 \) which implies a 4% annualized return, or approximately the real rate on a Aaa corporate bond during the Fall of 2008. We set the elasticity of the credit matching function to \( \varepsilon = 0.5 \) and obtain the remaining parameter by targeting a 155 basis point spread \( R - r \). The later corresponds to the spread between Baa and Aaa corporate bonds at the start of the crisis. Table 1 presents the parameter values that used in the baseline experiment.

### 4.3 Quantitative results: Aggregate effects of a credit crunch

By November 2008 the yield on Baa rated debt had reached 9.21%, while it was 6.12% for Aaa bonds. The spread averaged 270 basis points over the period from October 2008 to July 2009, peaking at 309 basis points. We examine the effects negative shock to the screening cost \( \kappa \) that provokes a 80% increase in the spread, and report the ensuing changes in aggregate output, productivity and unemployment. The analysis considers a comparison of steady states as transitional dynamics in this class of models are rapid. That is, in the standard labor search of matching framework, 68% of the deviation of employment from its steady state is wiped out within a month (Shimer, 2010).

\[\text{Table 1: Summary of baseline parameter values}\]

<table>
<thead>
<tr>
<th>Scale ( m )</th>
<th>3</th>
<th>Poisson arrival ( \lambda )</th>
<th>0.24</th>
</tr>
</thead>
<tbody>
<tr>
<td>Curv. ( \mu )</td>
<td>2/3</td>
<td>Exog. exit shock ( \delta )</td>
<td>0.01</td>
</tr>
</tbody>
</table>

**Credit matching:**
- Level \( \kappa_c \): 0.61
- Elasticity \( \varepsilon \): 0.5
- Bank barg. weight \( \beta \): 0.02

**Labor matching:**
- Level \( \kappa_l \): 0.66
- Elasticity \( \eta \): 0.72
- Worker barg. weight \( \alpha \): 0.72

**Credit -market search:**
- Bank cost \( \kappa \): 0.003
- Entrep. cost \( e \): 0.003

**Labor-market search:**
- Vacancy cost \( \gamma \): 0.13
- Unemp. value \( b \): 0.32

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17 The average repayment is defined as: \( \int x \rho(x)dG(x) = \beta \int [x - w(x)]dG(x) + (1 - \beta)\frac{\gamma}{q(\theta)} [r + \delta + \lambda G(x)] \), where the wage is \( w(x) = \alpha [x + \theta \gamma - r(1 - \theta) + \delta]C(\phi)] + (1 - \alpha)b \). See Appendix A for details.

18 Levin et al., (2004) argue that the bond spread is a good indicator of the spread between frictional external financing and frictionless financing, which is the notion adopted here.
This is very quick, and the quantitative experiment here considers a time horizon of nine months. Therefore a comparison of steady states is appropriate unless the friction introduced in the credit market slows down the speed of convergence dramatically. This speed of convergence is governed by the margins of adjustment for employment: job creation and job destruction. The friction in the credit market could slow the adjustments of job creation through the margin of firm entry. However, this mechanism is quantitatively small as entry replaces a quarterly firm exit rate of only 1%. The vast majority of job creation is taking place at incumbent firms. The rate of job creation before the crisis is comparable to the steady state in the model by Shimer. While it drops following the credit shock, the new job finding rate is 25% lower, the second adjustment margin in fact accelerates the transition to a new steady state. That is, Shimer’s model has a fixed job destruction rate. Endogenizing job destruction accelerates the speed of adjustment as firms are not constrained in the use of this margin of employment adjustment. The economy can adjust more quickly to its new aggregate employment level. In fact, as we will see, 52% of the change in aggregate employment in the experiment is due to the change in job destruction. For these reasons, the speed of convergence is not likely to be slower than the case measured in Shimer. Therefore, comparing two steady states seems a reasonable exercise.\textsuperscript{19} The results are reported in Table 2, which also reports, for comparison, the change in measured aggregate output, productivity and unemployment between the onset of the crisis, during the third quarter of 2008, and the second quarter of 2009, the first record of a positive quarter-to-quarter change in aggregate output.

A credit crunch reduces the surplus in all jobs in the economy, causing first an increase in the job destruction threshold from 0.70 to 0.72, and resulting in a 2.65% increase in aggregate productivity. Thus, the economy destroys the least productive jobs during a credit crunch induced recession, which Solon et al. (1994) document as occurring during most business-cycle downturns. By comparison, U.S. aggregate productivity rose by 3.35% during the period of interest. The rate

\textsuperscript{19}See also the discussion in Rogerson et al. (2008).
of job destruction rises from 0.045 to 0.06, and the job finding rate, $\theta q(\theta)[1 - G(x)]$, declines from 0.69 to 0.52 as the incentive to create jobs is smaller and fewer initial contacts are successful. This leads to an increase in the rate of unemployment from 6.03% to 10.2%, or a 69% increase comparable to the actual 60% increase in the U.S. unemployment rate. In addition, the average duration of unemployment increases from approximately 17 to 23 weeks, compared to an average duration of 25 weeks in July 2009, up from 18 weeks in the fall of 2008.\textsuperscript{20} Using the decomposition of changes in the rate of unemployment accruing to variations in the rates of job creation and job destruction in Fujita and Ramey (2009), 52% of the increase in unemployment during the credit crunch is due to the rise in the job destruction rate. Finally, the conjunction of the change in employment and productivity causes a 2% decline in aggregate output, similar to the change observed in the data. Thus, the model does a good job of replicating the positive comovement of employment and production following the credit crunch while also generating the increase in aggregate productivity. The results are very robust to assuming that the underlying distribution of idiosyncratic technology is normal, as reported in the next row of Table 2.\textsuperscript{21}

The earlier discussion found an ambiguous general equilibrium effect of rising financial costs on aggregate job vacancies. In the numerical credit-crunch exercise, vacancies do decline, and by 14.61%. Indicators of job vacancies, such as the Conference Board’s Help-wanted Index, declined by a quarter between the Fall of 2008 and the Summer of 2009. Since that date, employment has not followed the rise in the index of job vacancies, and this break in the Beveridge relationship remains unexplained.

The last column of Table 2 reports that real average hourly earnings, obtained from the BLS, increased by 1% between September 2008 and July 2009, an observation that is difficult to reconcile with representative agent models of the business cycle. In the current model, the individual wage rule, $w(x)$, implies that individual wages should decline during a credit-market-induced downturn. This is principally due to the deterioration of the labor market which weakens a worker’s outside option in wage negotiations. However, as the credit crunch destroys the least productive jobs, it is possible for the average wage in the economy $W_t = \int w(x) dH(x)$ to rise. This is indeed what occurs in the model. The average wage increases by 0.95%. However, if the economy is subject to a small credit-market shock, the composition effect does not dominate and there is a decline in the average wage. For example, a shock that increases the spread by 20% causes a 0.25% decline in the average wage while output declines by 0.44%, unemployment increases by 11.86% and aggregate productivity by 0.34%.

This quantitative exercise raises several important issues, first concerning the sources of business cycles and second the notion of changes in an economy’s capacity utilization and employment

\textsuperscript{20}Data obtained from the BLS through the Federal Reserve Bank of Saint-Louis, series UEMPMEAN.

\textsuperscript{21}The calibration of the model assuming a normal distribution for $G(x)$ follows the same procedure. Parameter values are reported in the Appendix.
as a margin of adjustment. Indeed, variations in aggregate activity in this model arise from changes in the mass of producing firms, not the population of firms through entry and exit, the latter remaining constant at the exogenous rate $\delta$. The aggregate production function derived considered only active firms, such that the model’s measure of aggregate productivity corresponds to a capacity-utilization-corrected measure similar to that constructed by Basu et al. (2006) and used in Fernald and Matoba (2009). We discuss the first here, devoting a separate section to the second.

### 4.4 The nature of shocks and the sources of business cycles

Shocks to credit markets generate positive co-movement between employment and output characteristic of business cycles, without any change in the underlying distribution of technology. However, a residual of output and employment would indicate a movement in aggregate productivity that is both unanticipated and countercyclical. Thus, movements in aggregate productivity can be unrelated to shocks to technology, here the properties of the distribution $G(x)$, recalling the fundamental distinction between aggregate technology and productivity in Basu and Fernald (2002).

More broadly, the model highlights the challenge of identifying the sources of business cycle fluctuations. For one, as shocks to credit markets move aggregate productivity and generate business cycle comovement in quantities, this questions further the relative importance of technology to other shocks in accounting for business cycle fluctuations. As the model suggests, however, their identification is not trivial and depends on the model’s specific assumptions on heterogeneity and aggregation. For example, schemes to identify technology shocks based on long-run restrictions for their impact on productivity (e.g., Gali, 1999) would not identify technology shocks in economies with market distortions or where aggregation is not straightforward, an issue also discussed in Francis and Ramey (2005). At the same time, the model provides a potential mechanism for breaking the link between aggregate employment and labor productivity over the business cycle, an increasingly salient feature of the data according to Gali and van Rens (2010).

The model’s concept of a shock to aggregate technology is an innovation to the distribution $G(x)$, which in the results presented in the last rows of Table 2 is assumed normal in order to separate mean and variance. The effects of a 1% negative shock to the mean of technology on output and wages are similar to the canonical real business cycle model (e.g., King and Rebelo, 1999), and the increase in the unemployment rate is typical for this class of search models (e.g., Shimer, 2005). Moreover, the cleansing effect of recessions causes a decline in aggregate productivity that is less than the shock to technology. One could conceive that the latest recession was induced by an unusually large technology shock. This is investigated in the last row of Table 2 in which the shock to the mean of technology $G(x)$ is such that aggregate output declines by 3.56%. Clearly this is not a likely candidate explanation: aggregate productivity drops by 2.77%, unemployment increases by only 12.6% and the average wage declines by 2.13%. Moreover, the spike in interest spreads
that was a salient feature of the recession does not occur; the increase is a negligible 0.37%.

It has also been suggested that shocks to the variance of the cross section of firm productivity are important sources of business-cycle fluctuations (Bloom, 2009). The assumption of a Pareto distribution implies that the variance of the cross-sectional dispersion of producing firms’ productivity, \( \text{var}_H(x) = \frac{\mu^2}{(\mu - 1)(\mu - 2)} \), is increasing in the cut-off productivity, \( \bar{x} \). The baseline calibration results in \( \bar{x} = 0.70 \) and, hence, a variance \( \text{var}_H(x) = 0.37 \). After the credit crunch, this variance increases by 5.4% to \( \text{var}_H(x) = 0.39 \). Thus the dispersion in firm productivity is countercyclical in this case, consistent with evidence in Bloom (2009) and Bachmann et al. (2009). This increase in dispersion, however, does not arise from a change in the properties of the distribution from which firm-specific productivity draws are made; rather, it comes from the cross section of producing firms \( H(x) \). This brings up a slight distinction between the risk from the underlying technology draws, \( G(x) \), and the risk from the macroeconomic environment measured by the dispersion in \( H(x) \). The latter evolves as the endogenous response of the economy to shocks and, significantly, not from firms’ patterns of entry and exit, but the waves of job destruction in response to the shock.

In the model developed in Bloom (2009), the presence of uncertainty yields a region of inaction in hiring and investment space due to the presence of non-convex adjustment costs. Firms only hire and invest when business conditions are sufficiently good, and fire when conditions are sufficiently bad. Increases in uncertainty expand this region of inaction. Following a shock to uncertainty, aggregate productivity growth falls because the rate of hiring and investment reduces the rate of reallocation from low- to high-productivity firms. Thus, an uncertainty cannot alone explain the macroeconomic events following the credit crunch in which aggregate productivity increases and job losses play an important role in the decline of employment.

5 Endogenous firm destruction, aggregation and capacity utilization

In this section we endogenize the break up of firms, or the exit rate \( \delta \). To do so, we introduce a cost to operating a firm, even when idle/recruiting, that is a function \( \tilde{\gamma}(x) \) of the idiosyncratic productivity \( x \), with the property \( \tilde{\gamma}'(x) < 0 \). This function proxies for the efficiency in the use of other factors in production that are not explicitly modeled.\textsuperscript{22}

We will show that a reservation strategy exists for the entrepreneur–banker pair such that a firm is endogenously dissolved if \( x < \bar{x} \), where the threshold \( \bar{x} \) is defined by a condition on the value of the firm at the recruiting stage, \( F_i(\bar{x}) = 0 \). This threshold productivity for firm destruction is also shown to be below that of job destruction, such that for realizations of idiosyncratic productivity

\textsuperscript{22}An alternative would be to introduce correlation in the shocks conditional on a jump. This would lead to low draws being more costly as they tend to reoccur for a prolonged period of time.
in the range \([x,x]\), a firm remains idle in the sense of being inactive productive capacity. We show that with Pareto idiosyncratic productivity, the aggregate production function is Cobb-Douglas in aggregate capital and labor. Moreover, the Solow residual of the aggregate production function will depend on the least productive firm in the economy, whereas effective, or utilization-adjusted, productivity depends on the least productive job. The gap between the two is shown to be increasing in the degree of credit-market imperfections, such that a credit crunch should induce a decline in the Solow residual and an increase in corrected TFP, consistent with the empirical findings of Fernald and Matoba (2009) for the U.S. following the financial crisis.

5.1 A margin of firm destruction

We begin with the modified Bellman equations for both agents on credit markets, maintaining the assumption of free entry:

\[
\begin{align*}
  rE_c &= 0 = -e + p(\phi) \int \max[E_l(x), 0] \, dG(x) \\
  rB_c &= 0 = -\kappa + \phi p(\phi) \int \max[B_l(x), 0] \, dG(x),
\end{align*}
\]

which now reflect the decision to form a match upon meeting, given the productivity draw. Since the entrepreneur-bank pair share the surplus, \(E_l(x) + B_l(x) = F_l(x)\), under Nash bargaining the sharing rule is \((1 - \beta)B_l(x) = \beta E_l(x)\) and it remains the case that equilibrium credit market tightness is given by \(\phi^* = \frac{1 - \beta \kappa}{\beta e}\). Combining (15) and (16) links the expected value of a new firm to total financial costs as

\[
  C(\phi) = \int \max[F_l(x), 0] \, dG(x).
\]

Finally, the values of the recruiting and production stage are now

\[
\begin{align*}
  rF_l(x) &= -\tilde{\gamma}(x) + q(\theta) \int \max[F_g(z), F_l(z), 0] \, dG(z) \\
  &\quad + \lambda \int \max[F_l(z), 0] \, dG(z) - (\lambda + q(\theta)) F_l(x) \\
  rF_g(x) &= x - w(x) + \lambda \int \max[F_g(z), F_l(z), 0] \, dG(z) - \lambda F_g(x).
\end{align*}
\]

Equation (18) assumes, first, that firms draw a new productivity when meeting a worker that will apply even if a match does not form and, second, that firms can draw new values during the recruiting/idle stage.\(^{23}\) The asset equation of the production stage (19) has the same interpretation

\(^{23}\) An alternate assumption would be that firms do not draw a new productivity upon meeting a worker. All the results of this section still hold, as detailed in a supplemental appendix available upon request, with the exception of the aggregation of production being Cobb-Douglas. The reason is that the fraction of idle and producing firms under
ensures that $F_l(x) < 0$, where $\bar{x}$ is defined by $F_l(\bar{x}) = 0$. Thus, $\bar{x}$ defines a firm destruction threshold below which the relationship between creditor and borrower cannot be sustained and is dissolved. For the sake of simplicity, from here on the cost function, $\tilde{\gamma}(x)$, will take the form $\tilde{\gamma}(x) = \max[\gamma - \omega x, 0]$, where $0 < \omega < 1$.  

To ensure the existence of an equilibrium with employment, the value of the production stage must be greater than the value of the recruiting stage for productivities greater than the job destruction threshold, $F_g(x) - F_l(x) = 0$. Since the slope of the production-stage value is $F'_g(x) = \frac{1-\alpha}{r+\lambda} > 0$, it is necessary that $\frac{(1-\alpha)}{r+\lambda} > \frac{\omega}{r+\lambda+q(\theta)}$ such that $F'_g(x) > F'_l(x) > 0$. Naturally, the same condition ensures that $\bar{x} < x$.

Evaluating the value of the firm at the destruction margin $\bar{x}$, and using the expected value of the firm $\int_x \bar{F}_l(z) dG(z) = C(\phi)$ defined in (17), yields a firm destruction condition,  

$$\tilde{\gamma}(\bar{x}) = [\lambda + q(\theta)]C(\phi) + q(\theta) \int_{\bar{x}} [F_g(z) - F_l(z)] dG(z),$$

that ties the destruction threshold, $\bar{x}$, directly to conditions on credit and labor markets. By the first component on the right-hand side, the threshold is decreasing in the financial costs involved in setting-up a new firm as those costs raise the opportunity cost of an existing relationship between an entrepreneur and a bank. In other words, the option value of a firm increases with the degree of credit market imperfection, or the costs associated with its formation, such that larger flow losses are sustainable before dissolving the firm. However, an increase in financial costs will reduce labor market tightness and change the equilibrium job destruction threshold $x$. This affects the second component, the firm’s expected surplus from hiring a worker. The net effect on firm exit will be examined below.

We can now solve for the firm’s value function in each stage. During the recruiting stage, this takes the form  

$$F_l(x) = \frac{\omega(x - \bar{x})}{r + \lambda + q(\theta)}$$

and, using the definition of the job destruction condition, the value of the production stage can be calculated as:

$$F_g(x) = \frac{\omega(x - \bar{x})}{r + \lambda + q(\theta)}$$

as earlier.

Since the slope of the value of the recruiting stage is  

$$F'_l(x) = \frac{-\tilde{\gamma}'(x)}{r + \lambda + q(\theta)} > 0 \forall x,$$

for some $x < \bar{x}$ we have that $F_l(x) < 0$, where $\bar{x}$ is defined by $F_l(\bar{x}) = 0$. Thus, $\bar{x}$ defines a firm destruction threshold below which the relationship between creditor and borrower cannot be sustained and is dissolved. For the sake of simplicity, from here on the cost function, $\tilde{\gamma}(x)$, will take the form $\tilde{\gamma}(x) = \max[\gamma - \omega x, 0]$, where $0 < \omega < 1$.  

To ensure the existence of an equilibrium with employment, the value of the production stage must be greater than the value of the recruiting stage for productivities greater than the job destruction threshold, $F_g(x) - F_l(x) = 0$. Since the slope of the production-stage value is $F'_g(x) = \frac{1-\alpha}{r+\lambda} > 0$, it is necessary that $\frac{(1-\alpha)}{r+\lambda} > \frac{\omega}{r+\lambda+q(\theta)}$ such that $F'_g(x) > F'_l(x) > 0$. Naturally, the same condition ensures that $\bar{x} < x$.

Evaluating the value of the firm at the destruction margin $\bar{x}$, and using the expected value of the firm $\int_x \bar{F}_l(z) dG(z) = C(\phi)$ defined in (17), yields a firm destruction condition,  

$$\tilde{\gamma}(\bar{x}) = [\lambda + q(\theta)]C(\phi) + q(\theta) \int_{\bar{x}} [F_g(z) - F_l(z)] dG(z),$$

that ties the destruction threshold, $\bar{x}$, directly to conditions on credit and labor markets. By the first component on the right-hand side, the threshold is decreasing in the financial costs involved in setting-up a new firm as those costs raise the opportunity cost of an existing relationship between an entrepreneur and a bank. In other words, the option value of a firm increases with the degree of credit market imperfection, or the costs associated with its formation, such that larger flow losses are sustainable before dissolving the firm. However, an increase in financial costs will reduce labor market tightness and change the equilibrium job destruction threshold $x$. This affects the second component, the firm’s expected surplus from hiring a worker. The net effect on firm exit will be examined below.

We can now solve for the firm’s value function in each stage. During the recruiting stage, this takes the form  

$$F_l(x) = \frac{\omega(x - \bar{x})}{r + \lambda + q(\theta)}$$

and, using the definition of the job destruction condition, the value of the production stage can be calculated as:

$$F_g(x) = \frac{\omega(x - \bar{x})}{r + \lambda + q(\theta)}$$

---

\(^{24}\)Throughout, we will assume that the job destruction threshold is below the value of productivity for which $\tilde{\gamma}(x) = 0$. That is, $x < \bar{x}$ where $\bar{x} = \frac{\omega}{r+\lambda}$. For any positive $C(\phi)$, $\bar{x}$ will always be greater than the firm-destruction threshold $\bar{x}$. The appendix details all the derivations pertaining to this section.
Figure 3: Firm asset values and the inactive region.

written as \( F_g(x) = F_l(x) + \frac{1 - \alpha}{r + \lambda} (x - x) \), or

\[
F_g(x) = \frac{\omega(x - x)}{r + \lambda + q(\theta)} + \frac{1 - \alpha}{r + \lambda} (x - x).
\]

Figure 3 plots the firm asset values for each stage as a function of idiosyncratic productivity. The region between the thresholds \( x \) and \( x \) holds the mass of non-producing or idle firms, which is an endogenous quantity shown below to be increasing in the degree of financial market imperfection.

## 5.2 Job creation, destruction and equilibrium

We can now turn to the determination of job creation, job destruction and the equilibrium. In order to derive the extended model’s job destruction condition, we write the surplus of a worker-firm pair with productivity draw \( x \) as

\[
(r + \lambda) S(x) = x + \tilde{\gamma}(x) - rU + \lambda \int \max [S(z), 0] dG(z) - (1 - \alpha)q(\theta) \int \max [S(z), 0] dG(z) - q(\theta)C(\phi) + q(\theta)F_l(x).
\]

This surplus has slope

\[
S'(x) = \frac{1 + \tilde{\gamma}'(x) + q(\theta)F_l'(x)}{r + \lambda} > 0 \forall x,
\]

where \( F_l' = \frac{-\omega'(x)}{r + \lambda + q(\theta)} \). Since \( \tilde{\gamma}'(x) = -\omega \), the slope is equal to \( S'(x) = \frac{1}{r + \lambda} - \frac{\omega}{r + \lambda + q(\theta)} \) and is positive everywhere.\(^{25}\) Thus, there exists an \( x < x \) for which \( S(x) < 0 \) such that \( x \), defined by \( S(x) = 0 \), does indeed define a job destruction threshold. Evaluating the labor surplus at the job destruction threshold, \( \bar{x} \), and using the definitions of the firm asset values and destruction condition,

\(^{25}\)The slope of the surplus is \( S'(x) = \frac{1}{r + \lambda} \) for \( x > \bar{x} \).
we obtain the job destruction condition:
\[
b + \frac{\omega(x - \bar{x})(r + \lambda)}{r + \lambda + q(\theta)} - x = |\lambda - \alpha \theta q(\theta)| \int_x S(z) dG(z).
\]

This condition has a similar interpretation as earlier except the that outside option of the firm, here the second term on the left-hand side, is the value of the idle stage evaluated at \(x\).

The job creation condition, found by combining (17) and (18) with the definition of the job and firm destruction margins, still equates the average cost of filling a job vacancy to the expected benefit from a worker:
\[
\frac{rC(\phi) + \Gamma}{q(\theta)} = (1 - \alpha) \int_x S(z) dG(z),
\]

where \(r \equiv [r + (\lambda + q(\theta)) G(x)] / [1 - G(x)]\) is an adjusted discount factor, and \(\Gamma \equiv \int_x \gamma(x) dG(x) / [1 - G(x)]\) are average costs during the idle stage. We thus have the following proposition.

**Proposition 4** - Under the conditions that \(\frac{(1 - \alpha)}{r + \lambda} > \frac{\omega}{r + \lambda + q(\theta)}\) and \(\lambda > \alpha \theta q(\theta)\), there exists a unique equilibrium for this economy defined by the triplet \((\bar{x}, \bar{x}, \theta)\) that solves the firm destruction, job creation and job destruction conditions:

\[
\gamma(\bar{x}) - [\lambda + q(\theta)] C(\phi) - q(\theta) (1 - \alpha) \int_x S(z) dG(z) = 0 \quad (20)
\]

\[
(1 - \alpha) \int_x S(z) dG(z) - \frac{rC(\phi) + \Gamma}{q(\theta)} = 0 \quad (21)
\]

\[
x - b - \frac{\omega(x - \bar{x})(r + \lambda)}{r + \lambda + q(\theta)} + [\lambda - \alpha \theta q(\theta)] \int_x S(z) dG(z) = 0. \quad (22)
\]

Proof - See Appendix C

### 5.2.1 The equilibrium effect of an increase in \(C(\phi)\) on firm and job destruction

In order to understand the equilibrium effect of an increase in financial costs, \(C(\phi)\), on the firm destruction threshold, we first combine the firm destruction and job creation conditions, equation (20) and (21), respectively, to obtain: \(\omega \int_x (z - \bar{x}) dG(z) = [r + \lambda + q(\theta)] C(\phi)\). Differentiating with respect to \(C(\phi)\), we obtain the effect on the firm destruction threshold \(\bar{x}\),

\[
\omega \int_x dG(z) \frac{\partial \bar{x}}{\partial C(\phi)} = \eta \frac{q(\theta)}{\theta} C(\phi) \frac{\partial \theta}{\partial C(\phi)} - [r + \lambda + q(\theta)]
\]

which, provided labor market tightness is decreasing in \(C(\phi)\), is negative. In that case, a negative shock to financial markets results in the lowering of the firm destruction threshold. Since the cost
of entering and creating a firm is dearer following such an event, entrepreneurs and bankers are willing to sustain greater losses to keep the firm in place.\textsuperscript{26}

The Appendix establishes the conditions under which equilibrium labor market tightness is decreasing, and the job destruction threshold increasing in total financial costs $C(\phi)$ in this extension. When that is the case, the distance between the firm destruction $x$ and job destruction $x$ thresholds is increasing in $C(\phi)$. These conditions hold for a parametrization that follows the approach used in Section 4, as illustrated by Figure 4 which plots the equilibrium effect of increasing the efficiency of the matching function on credit markets on the equilibrium values of the destruction thresholds and labor market tightness.

### 5.3 The aggregate production function

If each firm operates with a unit of capital and a unit of labor, and the cost paid by the banker in the recruiting stage covers the rental of capital equipment which must be in place before a worker is hired, then the presence of a margin of endogenous firm destruction allows for a Leontief firm-level production technology to aggregate to a nonlinear aggregate production function relating the aggregate stock of capital, employment and output. In particular, if idiosyncratic productivity is drawn from a Pareto distribution, the aggregate production function is of the Cobb-Douglas form.

Under such an assumption about the firm-level production function, the aggregate stock of capital is simply the mass of firms in existence $K \equiv \mathcal{V} + N$, where, once again, $\mathcal{V}$ is the mass of idle firms and $N$ of producing firms. The cross-sectional dispersion in firm productivity, both idle and producing, is given by the distribution $T(x) = [G(x) - G(\underline{x})]/[1 - G(\underline{x})]$, where the firm destruction threshold $\underline{x}$ is the lower bound of the support. Thus, the fraction of producing firms is $\frac{N}{N + \mathcal{V}} = 1 - T(\underline{x})$ and the fraction of idle firms $\frac{\mathcal{V}}{N + \mathcal{V}} = T(\underline{x})$. As a result, the relationship between aggregate employment and the aggregate stock of capital can be expressed as $N = [1 - T(\underline{x})]K$. Note also that $[1 - T(\underline{x})]$ can be interpreted as the economy’s capacity utilization rate.

\textsuperscript{26}Kehrig (2011) provides some evidence that less productive firms are more likely to survive recessions.
Assuming idiosyncratic productivity is drawn from the earlier Pareto distribution, the cross section of all firms in existence, \( T(x) \), is also Pareto with scale parameter \( \bar{x} \) and curvature \( \mu \). Hence, the fraction of producing firms is given by \( 1 - T(x) = (x/\bar{x})^\mu \), and aggregate employment, \( N = (x/\bar{x})^\mu K \), can be rearranged to relate the job and firm destruction thresholds through the aggregate capital–labor ratio and the curvature parameter, \( \mu: x = (K/N)^{1/\mu} \bar{x} \). Aggregate output remains given by \( Y = AN \), where \( A = \mu x \). Inserting the expression for employment, \( N = (x/\bar{x})^\mu K \), we have \( Y = \frac{\mu}{\mu - 1} x^{1-\mu} \bar{x}^\mu K \), which, with the previous expression for the threshold \( x \), yields the aggregate production function

\[
Y = \bar{A}N^{1-v}K^v,
\]

(23)

where \( \bar{A} \equiv \frac{\mu x}{\mu - 1} \) and \( v = 1/\mu \), \( 0 < v < 1 \). This function is homogeneous of degree 1 in labor and capital and presents all the usual properties of the Cobb-Douglas production function. Lagos (2006) derives in a similar fashion a Cobb-Douglas production function by assuming a fixed cost of production such that some firms, though with a worker, prefer not to produce. The existence of a Cobb-Douglas aggregate production function has a fundamentally different source here: the presence of credit market imperfections.

Assuming one considers the aggregate production function to be given by (23), the Solow residual constructed with data on aggregate output, employment and capital that would correspond to \( \bar{A} \). This measure underestimates the productivity of producing firms by a factor \( A/\bar{A} = x/\bar{x} > 1 \). Moreover, this miss-measurement is increasing in the degree of credit market imperfection and is exacerbated in a credit market induced recession. A related implication is that a researcher constructing TFP as the the Solow residual, \( \bar{A} \), would measure a decline in productivity during a credit crunch, while the utilization-adjusted measure of TFP, \( A \), increases. This is exactly the evidence reported for the Great Recession in the U.S. by Fernald and Matoba (2009).

### 5.4 Empirical evidence

Figure 5 depicts the stylized facts regarding the Great Recession outlined at the outset, plotting the evolution of aggregate GDP, employment, hours and productivity from the first quarter of 2008, where productivity corresponds to business output per hour computed by the BLS. During the first quarters of 2008, labor productivity changed very little while employment, hours and output began to decline. From the credit crunch in the Fall of 2008 on, however, the adjustment in aggregate hours occurs mostly at the extensive margin as the contraction in employment accelerates and average weekly hours stabilize. At the same time, aggregate labor productivity begins to increase rapidly, and continues to do so for several quarters.

Labor productivity is one element of the unusual pattern of aggregate productivity during the Great Recession, as argued in Wilson (2010). However, the model yields precise predictions re-
Figure 5: Employment, hours, productivity and output during the Great Recession

garding measures of the Solow residual and utilization-adjusted TFP. These have followed each other quite closely during past recessions and expansions, as shown in Figure 6 which plots the data for both measures constructed by Fernald and Matoba (2009) over the period 1980:1 to 2009:3, with the shaded areas representing recessions. The divergence after mid-2008 is striking when compared to the pattern over previous decades. The model proposed here would explain this disconnect as the effects of the credit crunch, which should have also induced a decline the utilization rate. The second panel of Figure 6, which plots the credit spread and the rate of utilization during the last three decades, constructed by Fernald and Matoba (2009) following the methodology of Basu et al. (2006), shows a strong decline in the rate of utilization that almost perfectly mirrors the increase in the credit spread.27 Moreover, this relationship was not present during past business cycles. The correlation between utilization and the credit spread is -0.27 over the entire sample period, yet significantly different at the end points: -0.17 during the first 10 years and -0.55 during the last. This suggests that past recessions find their sources in shocks to other markets or technology. One exception is a small spike in the credit spread during the 1990–1991 recession but no large departure between both measures of aggregate productivity. By and large, Bernanke and Lown (1991) have argued that this particular recession was not due to a credit crunch. Rather, the state of credit market should be attributed to changes in credit demand.

The Jobs Openings and Labor Turnover Over Survey (JOLTS) and Business Employment Dynamics (BED) data collected by the BLS allow us to examine specific aspects of the labor market during the Great Recession. The top left panel of Figure 7 uses data from JOLTS to plot the number of layoffs and hirings in the U.S. between 2001 and 2010. Especially compared to the other recession covered by the JOLTS data, the Great Recession saw an immense rise in the number of layoffs, from about 1 800 000 per month in the early Summer of 2008 to nearly 2 500 000 by

\[27\] I thank Susanto Basu for bringing this to my attention. The credit spread here corresponds to the difference between the rates on Baa and Aaa corporate bonds.
February 2009. Hiring has dropped steadily since December 2007 and remains anemic. In comparison, the 2001 recession saw no particular spike in layoffs and a decline in hirings, a pattern commented upon by Hall (2005).\(^{28}\)

Gross job gains and losses in the private sector, plotted in the top right panel of Figure 7 paint a similar picture of rapidly rising job losses and declining job gains during the latest recession. In addition, the BED survey provides a more detailed picture of the sources of job gains and losses since the inception of the program. The bottom panels plot the components of private-sector gross job gains and losses at expanding and contracting establishments, and at opening and closing establishments since 1999. This distinction clearly shows that the rise in gross job losses during the Credit Crunch is due to job losses at contracting establishments, not closing establishments. In fact, the number of jobs lost at closing establishments has remained remarkably constant during the recession and not significantly different from the preceding expansion. This is consistent with the model developed here in which job losses occur at continuing establishments following the credit crunch in Section 4. The decline in job gains is due to a slowdown in both gains at existing establishments and a lack of new openings.

Finally, the extended model implies that firms reduce their workforce before breaking up or exiting, echoing Bresnahan and Ramey’s (1994) notion that firms use reductions in their workforce

\(^{28}\)See also Elsby et al. (2010) for a detailed discussion of the labor market during the Great Recession.
as a short-run adjustment margin. The destruction of a firm occurs when an entrepreneur and creditor dissolve their relationship, an event well proxied in the data by a default on debt or a filing for bankruptcy. Moreover, if the model is taken to represent large firms with multiple workers, the least productive jobs within a firm should be destroyed first, and average labor productivity of the remaining workers should increase as a result.

We verify this conjecture by looking at the growth rate of employment and the evolution of labor productivity at publicly listed firms having filed for bankruptcy over the period 1979 to 2004. The left panel of Figure 8 reports a contraction in the workforce beginning a year and a half before filing for bankruptcy, whereas the right panel shows that labor productivity increases even while the workforce is contracting. This suggests that firms are indeed terminating the least productive jobs in a bid to avoid bankruptcy.

---

29Bankruptcy filing dates are contained in the Bankruptcy Research Database compiled by Lynn PoLucki at UCLA. The data set is merged with firm level data from Compustat, yielding 751 separate cases. Firm employment growth rates were obtained from Compustat as data item 29 (employment) and using footnotes 34 and 35. Labor productivity is obtained as sales (data item 12) over employment.

30Although the model does not explicitly consider the decision to liquidate physical capital, the data also show the rate at which firms sell their capital equipment remains constant during the entire period up to the filing, reinforcing the view that labor is used as the margin of adjustment.
Figure 8: Employment and labor productivity in the run up to a bankruptcy. Data: Compustat and BRD, average of 751 cases.

6 Conclusion

Departures of aggregate productivity from technology have been recognized as sources of measurement error in accounting for business cycle fluctuations. Correcting for variable capital utilization, for example, has uncovered effects of technology shocks on aggregate hours worked that are contrary to the predictions of the canonical real business-cycle model (e.g. Basu et al., 2006). Similarly, the increases in various measures of aggregate productivity during the most recent recession pose a challenge for the real business cycle framework. This paper proposes that the latter can be explained as the consequences of the sorting of heterogeneous firms in production following a credit crunch. By modeling frictions on the credit market of a process of search, we can determine which firms produce with a hired worker, those that remain idle and those that are dissolved, according to their idiosyncratic productivity and the degree of frictions on financial markets. Worsening conditions on credit markets cause all employment relationship to be less profitable and the least productive jobs to be terminated. Significantly, the destruction of a job is not necessarily associated with the destruction of the firm, leading to an increase in aggregate productivity. The model replicates the main observations on output, employment and aggregate productivity during the Great Recession, and a composition bias in the employed work force causes a slight increase in the aggregate wage, as in the data. We also show that the increase in job losses in the U.S. following the credit crunch occurred at continuing, and not closing, establishments, as in the model.

The model presents some important omissions. For one, all labor adjustments occur at the extensive margin, and extending the model to allow for adjustment at the intensive margin would likely help for some quantitative results, such as the magnitude of the decline in output seen in the data. Second, the model abstracts from the role a firm’s net worth can have on the cost of external financing. However, the results would be similar as the change in employment at existing and continuing firms is due to the increase in the opportunity cost of credit. This would still occur in model with firm saving and external financing. Finally, indicators of job vacancies, such
as the Conference Board’s Help-wanted Index, declined by a quarter between the Fall of 2008 and the Summer of 2009. Under the parametrization for the quantitative exercise, the model also produces a decline in vacancies. Since that date, however, job vacancies have returned to their pre-crisis levels while employment has not recovered. The model implies that an increase in financial intermediation costs cause an outward shift in the Beveridge curve that would also be consistent with these observations, provided the shock to financial markets is very persistent. To that effect, Hatzius et al. (2010) argue that poor financial conditions continued to drag on even after spreads returned closer to pre-crisis level. Future work could focus on the transitional dynamics of the model with a view of exploring this possibility.

Finally, the model could provide a good framework for other questions in the area of business cycles which representative agent models cannot treat. For instance, the macroeconomic disasters literature considers rare events which cause large drops in output and earnings as a means of reconciling the real business cycle framework with second moments for asset prices. Changing beliefs in the likelihood of such events cause large swings in asset prices but not quantities (Barro, 2006, Gourio, 2010). These disasters are usually thought to be a significant destruction of the aggregate stock of capital and, slightly more problematic, coincident large declines in aggregate productivity. An alternative to the capital destruction view would be rare financial disasters, which have occurred twice in the last one hundred years, that cause large employment and earnings losses.

References


A model of credit and job destruction

The first section of this appendix details the derivations in the main text and provides the proofs for the propositions. These include the existence of a unique equilibrium on the labor market and the necessary conditions for the direction of the equilibrium effects of a change in financial costs on the equilibrium job destruction threshold.

A.1 Worker-firm surplus

Using equations (8) and (10), the steps to deriving the worker-firm surplus $S(x) = F_g(x) - F_l + W(x) - U$ are

$$rS(x) = y(x) - w(x) + \lambda \int \max [F_g(z) - F_l, 0] dG(z) - \lambda [F_g(x) - F_l] - \delta F_g(x)$$

$$w(x) + \lambda \int \max [W(z) - U, 0] dG(z) - \lambda [W(x) - U]$$

$$-rF_l - rU$$

$$rS(x) = y(x) + \lambda \int \max [S(z), 0] dG(z) - (\lambda + \delta) S(x) - (r + \delta) F_l - rU$$

$$(r + \lambda + \delta) S(x) = x + \lambda \int \max [S(z), 0] dG(z) - (r + \delta) F_l - rU.$$
to 0 as \( \bar{x} \) tends to the upper bound of its support. Second, from the JD condition, labor market tightness tends to infinity when as \( \bar{x} \) tends to the upper bound of it’s support, guaranteeing a unique equilibrium if the JC curve is above the JD at the lower bound of the support for \( x \). This will be the case for such distributions as the normal, and places a restriction on the scale parameter of a Pareto distribution.

### A.3 Equilibrium effects of financial costs \( C(\phi) \) on the job destruction threshold \( \bar{x} \)

Differentiate the job creation equation (11) with respect to \( C(\phi) \),

\[
-\left( \frac{1 - \alpha}{r + \lambda + \delta} \right) \int_{\bar{x}} dG(x) \frac{\partial x}{\partial C(\phi)} = \frac{r}{q(\theta)} - \frac{\gamma + rC(\phi)}{q(\theta)^2} q'(\theta) \frac{\partial \theta}{\partial C(\phi)},
\]

to obtain

\[
\frac{\partial \theta}{\partial C(\phi)} = \frac{r}{\gamma + rC(\phi)} q(\theta) + \frac{(1 - \alpha)q(\theta)^2}{(r + \lambda + \delta)(\gamma + rC(\phi))q'(\theta)} \frac{\partial x}{\partial C(\phi)}, \tag{A.1}
\]

Now differentiate the job destruction equation (12) with respect to \( C(\phi) \):

\[
0 = \frac{\partial x}{\partial C(\phi)} - \frac{\alpha \theta r}{1 - \alpha} - (r + \delta) - \frac{\alpha}{1 - \alpha} (\gamma + rC(\phi)) \frac{\partial \theta}{\partial C(\phi)} - \frac{\lambda \int_{\bar{x}} dG(x)}{r + \lambda + \delta} \frac{\partial x}{\partial C(\phi)}
\]

\[
\frac{\alpha \theta r}{1 - \alpha} + r + \delta \right] = \frac{\partial x}{\partial C(\phi)} \left[ 1 - \frac{\lambda \int_{\bar{x}} dG(x)}{r + \lambda + \delta} \right] - \frac{\alpha}{1 - \alpha} (\gamma + rC(\phi)) \frac{\partial \theta}{\partial C(\phi)}. \tag{A.2}
\]

Substitute (A.1) into (A.2),

\[
\left[ \frac{\alpha \theta r}{1 - \alpha} + r + \delta \right] + \frac{\alpha}{1 - \alpha} \frac{rq(\theta)}{q'(\theta)} = \frac{\partial x}{\partial C(\phi)} \left[ 1 - \frac{\lambda \int_{\bar{x}} dG(x)}{r + \lambda + \delta} \right] - \frac{\alpha q(\theta)^2}{(r + \lambda + \delta)q'(\theta)} \frac{\partial x}{\partial C(\phi)}
\]

\[
\frac{\alpha r \theta}{1 - \alpha} \left[ 1 + \frac{q(\theta)}{\theta q'(\theta)} \right] + r + \delta \tag{A.3}
\]

\[
= \left[ 1 - \frac{\lambda q'(\theta) + \alpha q(\theta)'^2}{(r + \lambda + \delta)q'(\theta)} \int_{\bar{x}} dG(x) \right] \frac{\partial x}{\partial C(\phi)}
\]

\[
\frac{\alpha r \theta}{1 - \alpha} \left[ 1 - \frac{1}{\eta} \right] + r + \delta = \left[ 1 - \frac{\eta \lambda - \alpha \theta q(\theta)}{\eta (r + \lambda + \delta)} \int_{\bar{x}} dG(x) \right] \frac{\partial x}{\partial C(\phi)},
\]

where \( \eta = -\frac{\partial q(\theta)}{\partial \theta} \frac{\theta}{q(\theta)} \). The left-hand side is positive if \( \eta > \frac{\alpha r \theta}{[\alpha r \theta + (1 - \alpha)(\eta + \lambda)]} \). Since \( \frac{\eta \lambda - \alpha \theta q(\theta)}{\eta (r + \lambda + \delta)} < 1 \), the coefficient on \( \frac{\partial x}{\partial C(\phi)} \) on the right hand side is always positive such that a condition for \( \frac{\partial x}{\partial C(\phi)} > 0 \) is \( \eta > \frac{\alpha r \theta}{[\alpha r \theta + (1 - \alpha)(\eta + \lambda)]} \).
### A.4 Repayment to creditors

The repayment to bankers is determined by $\rho(x) = \arg\max_{\rho(x)} (B_l - B_c)^\beta (E_l - E_c)^{1-\beta}$. The first-order condition from a log transformation is $\frac{\beta}{B_l - B_c} \frac{\partial B_l}{\partial \rho(x)} + \frac{1-\beta}{E_l - E_c} \frac{\partial E_l}{\partial \rho(x)} = 0$. It is straightforward to show that $\frac{\partial E_l}{\partial \rho(x)}$ such that, with the free-entry condition on the credit market, \( \left( \frac{\beta}{B_l - B_c} - \frac{1-\beta}{E_l - E_c} \right) \frac{\partial B_l}{\partial \rho(x)} = 0 \) which yields the sharing rule \((1-\beta)B_l = \beta E_l\), or $B_l = \beta F_l$. From equations (5) and (7), we then have that

\[
-\gamma + q(\theta) \int_\mathcal{X} [B_g(x) - B_l] dG(x) = \beta \left( -\gamma + q(\theta) \int_\mathcal{X} [F_g(x) - F_l] dG(x) \right)
\]

\[
\int_\mathcal{X} B_g(x) dG(x) = (1-\beta) \frac{\gamma q}{q(\theta)} + \beta \int_\mathcal{X} F_g(x) dG(x),
\]

from which we derive the expressions for the average repayment, $\int_\mathcal{X} \rho(x) dG(x)$. To do so, substitute the definitions of the production-stage asset values into equation (A.3),

\[
\int_\mathcal{X} \left( \rho(x) + \lambda \int_\mathcal{X} \max [B_g(z), B_l, 0] dG(z) - (\lambda + \delta) B_g(x) \right) dG(x) = (1-\beta) \frac{\gamma q}{q(\theta)}
\]

\[
+ \beta \int_\mathcal{X} \left( x - w(x) + \lambda \int_\mathcal{X} \max [F_g(z), F_l, 0] dG(z) - (\lambda + \delta) F_g(x) \right) dG(x),
\]

and rearrange to isolate the average repayment:

\[
\int_\mathcal{X} \rho(x) dG(x) = (1-\beta) \frac{\gamma q}{q(\theta)}
\]

\[
+ \beta \int_\mathcal{X} (x - w(x)) dG(x)
\]

\[
+ (\lambda + \delta) \int_\mathcal{X} (B_g(x) - \beta F_g(x)) dG(x)
\]

\[
- \lambda \int_\mathcal{X} \left( \int_\mathcal{X} \max [B_g(z), B_l, 0] dG(z) - \beta \int_\mathcal{X} \max [F_g(z), F_l, 0] dG(z) \right) dG(x).
\]

Using the fact that Nash bargaining implied that $\int_\mathcal{X} B_g(x) dG(x) - \beta \int_\mathcal{X} F_g(x) dG(x) = (1-\beta) \frac{\gamma q}{q(\theta)}$ yields

\[
\int_\mathcal{X} \rho(x) dG(x) = \beta \int_\mathcal{X} (x - w(x)) dG(x) + (1-\beta) \frac{\gamma q}{q(\theta)}
\]

\[
+ (\lambda + \delta) (1-\beta) \frac{\gamma q}{q(\theta)} - \lambda \int_\mathcal{X} \left( (1-\beta) \frac{\gamma q}{q(\theta)} \right) dG(x),
\]

which simplifies to

\[
\int_\mathcal{X} \rho(x) dG(x) = \beta \int_\mathcal{X} [x - w(x)] dG(x) + (1-\beta) \frac{\gamma q}{q(\theta)} [r + \delta + \lambda G(\mathcal{X})],
\]

37
**A.5 Individual wage rule \( w(x) \)**

The wage is the outcome of Nash bargaining over the surplus of the worker–firm pair \( S(x) = F_g(x) - F_l + W(x) - U \), which yields the sharing rule, \( (1 - \alpha) (W(x) - U) = \alpha (F_g(x) - F_l) \). To obtain the wage rule, expand on the previous equality.

\[
(1 - \alpha) \left( w(x) + \lambda \int_x [W(z) - U] dG(z) - (\lambda + \delta) [W(x) - U] - rU \right) =
\]

\[
\alpha \left( x - w(x) + \lambda \int_x [F_g(z) - F_l] dG(z) - \lambda [F_g(x) - F_l] - \delta F_g(x) - rF_l \right),
\]

and make use, once again, of the sharing rule \( (1 - \alpha) (W(x) - U) = \alpha (F_g(x) - F_l) \) to obtain

\[
(1 - \alpha) (w(x) - \delta [W(x) - U] - rU) = \alpha (x - w(x) - \delta F_g(x) - rF_l),
\]

which can be rearranged to have the wage on the left-hand side as

\[
w(x) = \alpha (x - \delta F_g(x) - rF_l) + (1 - \alpha) (rU + \delta [W(x) - U]).
\]

Replacing \( rU \) with \( b + \frac{\alpha}{1-\alpha} \theta \gamma \) yields the wage rule:

\[
w(x) = \alpha [x + \gamma \theta + C(\phi) (r(1 - \theta) - \delta)] + (1 - \alpha) b.
\]

**B Data sources and parameter values assuming a normal distribution \( G(x) \)**

Corporate bond rates are Moody’s Seasoned Corporate Bond Yields, aggregate TFP and utilization rate series were obtained from Fernald and Matoba (2009), available from the Federal Reserve Bank of San Francisco, labor productivity corresponds to business output per hour from the BLS series PRS84006092. Real GDP was obtained from the Federal Reserve Bank of Saint Louis as series GDPC1, converted to per capita units using the civilian population over 16 years series CNP16OV. Employment corresponds to all employees in private industries, series USPRIV. The unemployment rate corresponds to the civilian unemployment rate, persons 16 years and older, series UNRATE. Average duration of unemployment spells is the series UEMPMEAN. The JOLTS hiring series used is JTS10000000HIL, total private, and the layoffs and discharges numbers are from series JTS10000000LDL. The BED data are private sector gross job gains and job losses, seasonally adjusted.
# Table 3: Parameter values - Normal distribution

<table>
<thead>
<tr>
<th>Technology:</th>
<th></th>
<th>Credit matching:</th>
<th></th>
<th>Labor matching:</th>
<th></th>
<th>Labor-market search:</th>
<th></th>
<th>Labor-market search:</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>mean $X$</td>
<td>1</td>
<td>level $\chi_C$</td>
<td>0.44*</td>
<td>level $\chi_L$</td>
<td>0.61*</td>
<td>Bank cost $\kappa$</td>
<td>0.01*</td>
<td>vacancy cost $\gamma$</td>
<td>0.23*</td>
</tr>
<tr>
<td>variance $\sigma_X^2$</td>
<td>0.22*</td>
<td>elasticity $\varepsilon$</td>
<td>0.57*</td>
<td>elasticity $\eta$</td>
<td>0.72</td>
<td>Entrep. cost $e$</td>
<td>0.01*</td>
<td>unemp. value $b$</td>
<td>0.40*</td>
</tr>
</tbody>
</table>

*: Outcome of calibration.

## C Endogenous firm destruction

This section details the solutions and proofs when the destruction of the firm is endogenized. It first details the derivation of the worker-firm surplus and the job creation and destruction conditions. The effects of an increase in financial costs, $C(\phi)$, on the equilibrium is detailed last.

### C.1 Worker-firm surplus under endogenous firm death

In order to derive the surplus for the worker-firm pair, we combine equations (18) and (19) with the definition of said surplus:
\[ rS(x) = x - w(x) + \lambda \int \max [F_g(z), F_I(z), 0] dG(z) - \lambda F_g(x) \]

\[ + \tilde{\gamma}(x) - q(\theta) \int \max [F_g(z), F_I(z), 0] dG(z) - \lambda \int \max [F_I(z), 0] dG(z) + (\lambda + q(\theta)) F_I(x) \]

\[ + w(x) + \lambda \int \max [W(z) - U, 0] dG(z) - \lambda \int [W(x) - U] - rU \]

\[ rS(x) = x + \tilde{\gamma}(x) - rU - \lambda \int [W(x) - U + F_g(x) - F_I(x)] + q(\theta) F_I(x) \]

\[ + \lambda \int \max [F_g(z) - F_I(z) + W(z) - U, 0] dG(z) \]

\[ - q(\theta) \int \max [F_g(z), F_I(z), 0] dG(z) \]

\[ rS(x) = x + \tilde{\gamma}(x) - rU - 2S(x) + 2\lambda \int [S(z), 0] dG(z) \]

\[ + q(\theta) F_I(x) - q(\theta) \int \max [F_g(z), F_I(z), 0] dG(z) \]

\[ rS(x) = x + \tilde{\gamma}(x) - rU + \lambda S(x) + \lambda \int \max [S(z), 0] dG(z) - q(\theta) C(\phi) + q(\theta) F_I(x). \]

\section{C.2 Job creation condition under endogenous firm death}

The job creation condition is derived from the expected value of a firm under free entry on credit markets:

\[ rC(\phi) = r \int \mathcal{F}_I(x) dG(x). \]

Rearrange equation (18) as

\[ rF_I(x) = -\tilde{\gamma}(x) + q(\theta) \int [F_g(z) - F_I(z)] dG(z) + (\lambda + q(\theta)) \int F_I(z) dG(z) - (\lambda + q(\theta)) F_I(x) \]

\[ = -\tilde{\gamma}(x) + q(\theta)(1 - \alpha) \int S(z) dG(z) + (\lambda + q(\theta)) [C(\phi) - F_I(x)] \]

and combine with the free entry condition to obtain

\[ rC(\phi) = \int [S(z) dG(z) + (\lambda + q(\theta))(C(\phi) - F_I(x))] dG(x) \]

\[ = -\int S(z) dG(x) + [1 - G(\lambda)] q(\theta)(1 - \alpha) \int S(z) dG(z) + (\lambda + q(\theta)) \int C(\phi) dG(x) \]

\[ + (\lambda + q(\theta)) \int F_I(x) dG(x) \]
\[ [r + (\lambda + q(\theta))G(\phi)]C(\phi) = -\int_x \tilde{y}(x)dG(x) + [1 - G(\phi)]q(\theta)(1-\alpha) \int_x S(z)dG(z), \]

which, rearranging terms, is

\[ [r + (\lambda + q(\theta))G(\phi)]C(\phi) + \int_x \tilde{y}(x)dG(x) = [1 - G(\phi)]q(\theta)(1-\alpha) \int_x S(z)dG(z). \]

Call \( r = \frac{[r+(\lambda+q(\theta))G(\phi)]}{1-G(\phi)} \) and \( \Gamma = \frac{\int_x \tilde{y}(x)dG(x)}{1-G(\phi)} \) such that the job creation condition is

\[ \frac{rC(\phi) + \Gamma}{q(\theta)} = (1-\alpha) \int_x S(z)dG(z). \]

### C.3 Job destruction condition under endogenous firm death

Recall that the worker-firm surplus is

\[ (r + \lambda)S(x) = x + \tilde{y}(x) - rU + [\lambda - q(\theta)(1-\alpha)] \int_x S(z)dG(z) - q(\theta)C(\phi) + q(\theta)F_i(x), \]

which, evaluated at \( x \) yields

\[ 0 = x + \tilde{y}(x) - rU + [\lambda - q(\theta)(1-\alpha)] \int_x S(z)dG(z) - q(\theta) [C(\phi) - F(x)]. \]

Since a solution for the value of the idle stage is \( F_i(x) = \frac{\omega(x-x)}{r+\lambda+q(\theta)} \), we have

\[ 0 = x + \tilde{y}(x) - rU + [\lambda - q(\theta)(1-\alpha)] \int_x S(z)dG(z) - q(\theta) \left[ C(\phi) - \frac{\omega(x-x)}{r+\lambda+q(\theta)} \right]. \]

Finally, to substitute out the value of unemployment, \( U \), express the value of unemployment as

\[ rU = b + \theta q(\theta)\alpha \int_x S(z)dG(z), \]

and substitute the expected value of the labor surplus using the job creation condition such that

\[ rU = b + \frac{\alpha}{1-\alpha} \theta (rC(\phi) + \Gamma). \]

This results in an expression for the job destruction condition:

\[ 0 = x + \tilde{y}(x) - \left( b + \frac{\alpha}{1-\alpha} \theta (rC(\phi) + \Gamma) + q(\theta) \left[ C(\phi) - \frac{\omega(x-x)}{r+\lambda+q(\theta)} \right] \right) \]

\[ + [\lambda - q(\theta)(1-\alpha)] \int_x S(z)dG(z). \]
If the firm and job destruction conditions are combined, we have a simpler expression,

\[0 = x - b - \frac{\omega(x - x)(r + \lambda)}{r + \lambda + q(\theta)} - \frac{\alpha \theta}{1 - \alpha} (rC(\phi) + \Gamma) + \lambda S_x \int_x (z - x) dG(z),\]

and using the job creation condition, we have

\[0 = x - b - \frac{\omega(x - x)(r + \lambda)}{r + \lambda + q(\theta)} + [\lambda - \alpha f(\theta)] S_x \int_x (z - x) dG(z).\]

### C.4 Conditions for existence of equilibrium, endogenous firm death

The triplet \((x, x, \theta)\) is determine by a system of three nonlinear equations,

\[
\tilde{\gamma}(\tilde{x}) - [\lambda + q(\theta)]C(\phi) = q(\theta)(1 - \alpha) \int_x S(z) dG(z).
\]

\[b + \frac{\omega(x - x)(r + \lambda)}{r + \lambda + q(\theta)} - x = [\lambda - \alpha f(\theta)] \int_x S(z) dG(z).
\]

\[
\frac{rC(\phi) + \Gamma}{q(\theta)} = (1 - \alpha) \int_x S(z) dG(z),
\]

where \(r \equiv [r + (\lambda + q(\theta))G(\tilde{x})] / [1 - G(\tilde{x})]\) is an adjusted discount factor and \(\Gamma \equiv \int_x \tilde{\gamma}(x) dG(x) / [1 - G(\tilde{x})]\).

Combine the firm destruction and job creation conditions to express the firm-destruction condition as a function of labor market tightness,

\[
\omega \int_x (z - x) dG(z) - [r + \lambda + q(\theta)] C(\phi) = 0,
\]

which is plotted in Figure 9 in \((x, \theta)\) space. Differentiating, we have that

\[
\frac{dx}{\theta} = -\frac{\omega \int_x dG(z)}{q'(\theta)} > 0.
\]

Next, for a given firm destruction threshold, the slope of the job creation condition in \((x, \theta)\) is:

\[
\frac{\partial r}{\partial \theta} C(\phi) d\theta + \frac{\partial \Gamma}{\partial \theta} d\theta = q'(\theta)(1 - \alpha) S_x \int_x (z - x) dG(z) d\theta + q(\theta)(1 - \alpha) \frac{\partial S_x}{\partial \theta} \int_x (z - x) dG(z) d\theta
\]

\[= -q(\theta)(1 - \alpha) S_x \int_x dG(z) dx
\]

\[
\frac{d\theta}{dx} = \frac{q(\theta)(1 - \alpha) S_x \int_x dG(z)}{\left(q'(\theta) S_x + q(\theta) \frac{\partial S_x}{\partial \theta}\right)} \left(1 - \alpha\right) \int_x (z - x) dG(z) - \frac{\partial r}{\partial \theta} C(\phi) - \frac{\partial \Gamma}{\partial \theta}.
\]

Recall that \(S_x = \frac{1}{r + \lambda} - \frac{\omega}{r + \lambda + q(\theta)}\), such that \(\frac{\partial S_x}{\partial \theta} = \frac{\omega}{(r + \lambda + q(\theta))^2} q'(\theta)\) and \(\left(q'(\theta) S_x + q(\theta) \frac{\partial S_x}{\partial \theta}\right) = q'(\theta) \left(\frac{1}{r + \lambda} - \frac{[r + \lambda] \omega}{[r + \lambda + q(\theta)]^2}\right)\). Further, \(\frac{\partial r}{\partial \theta} = \frac{q'(\theta) G(\tilde{x})}{[1 - G(\tilde{x})]} + \left(\frac{(\lambda + q(\theta))}{[1 - G(\tilde{x})]} + \frac{(r + (\lambda + q(\theta)) G(\tilde{x}))}{[1 - G(\tilde{x})]^2}\right) g(\tilde{x}) \frac{\partial \tilde{r}}{\partial \theta} + \frac{\partial \Gamma}{\partial \theta} = 42
we have that as is clear from rearranging it as

\[
\int_{\gamma(x)} \frac{\partial x}{\partial \theta} \, dG(z). \quad \text{Substituting into the previous expression, we obtain}
\]

\[
\frac{d\theta}{dx} = \frac{q(\theta)(1 - \alpha)S_x \int dG(z)}{\frac{G(\theta)}{1 - G(\theta)} C(\phi) - \left( \frac{1}{r + \lambda} - \frac{\omega}{[r + \lambda + q(\theta)]^2} \right) (1 - \alpha) \int (z - x) dG(z) - \Theta}
\]

where we define \( \Theta \equiv \frac{\partial \Gamma}{\partial \theta} + \left( \frac{\lambda + q(\theta)}{1 - G(\phi)} + \frac{r + (\lambda + q(\theta)) G(\phi)}{[1 - G(\phi)]^2} \right) g(\phi) \frac{\partial x}{\partial \theta} > 0. \) Since the denominator

\[
\left( \frac{G(\phi)}{1 - G(\phi)} C(\phi) - \left( \frac{1}{r + \lambda} - \frac{\omega}{[r + \lambda + q(\theta)]^2} \right) (1 - \alpha) \int (z - x) dG(z) - \Theta \right) < 0
\]

as is clear from rearranging it as

\[
-\frac{[1 - G(\phi)] \omega q(\theta)^2 (1 - \alpha)}{[r + \lambda + q(\theta)]^2} \int (z - x) dG(z) < \left[ r + \lambda G(\phi) \right] C(\phi) + \int \tilde{\gamma}(x) dG(x)
\]

\[
+ q(\theta) [1 - G(\phi)] \Theta
\]

we have that \( \frac{d\theta}{dx} < 0. \)

Now consider the slope of the job destruction condition:

\[
\left( 1 - \frac{\omega (r + \lambda)}{r + \lambda + q(\theta)} - [\lambda - \alpha f(\theta)] S_x \int dG(z) \right) dx
\]

\[
= \left[ \alpha f'(\theta) S_x - [\lambda - \alpha f(\theta)] \frac{\partial S_x}{\partial \theta} \right] \int (z - x) dG(z) d\theta - \frac{\omega (x - z)(r + \lambda)}{[r + \lambda + q(\theta)]^2} q'(\theta) d\theta
\]
\[(r + \lambda G(x) + \alpha f(\theta)(1 - G(x)))S_x dx\]

\[= \left[\alpha f'(\theta)S_x - [\lambda - \alpha f(\theta)]\frac{\partial S_x}{\partial \theta}\right] \int_x (z - x) dG(z) d\theta + \eta \frac{q(\theta) \omega(x - \bar{x})(r + \lambda)}{\theta [r + \lambda + q(\theta)]^2} d\theta.\]

The left-hand side is always positive. A sufficient condition for the right-hand side to be positive is \([\lambda - \alpha f(\theta)] > 0\), under which the slope of the job destruction curve is positive, \(\frac{d\theta}{dx} > 0\). If \([\lambda - \alpha f(\theta)] < 0\), then the condition for \(\frac{d\theta}{dx} > 0\) is

\[[\lambda - \alpha f(\theta)] \frac{\partial S_x}{\partial \theta} \int_x (z - x) dG(z) < \alpha f'(\theta)S_x \int_x (z - x) dG(z) + \eta \frac{q(\theta) \omega(x - \bar{x})(r + \lambda)}{\theta [r + \lambda + q(\theta)]^2}\]

### D Equilibrium effect of financial cost \(C(\phi)\) - Endogenous firm destruction

This section provides the details to obtaining the results in Section 5.2.1 and Figure 4. The model is calibrated following the same procedure as in the first section, with the exception that the quarterly rate of firm destruction of 1% enters the set of targets with the introduction of the cost parameter \(\omega\). In other words, we have that \(\lambda G(x) = 1\%\) and the rate of job destruction is \(\lambda G(x) = 3.5\%\).

<table>
<thead>
<tr>
<th>Table 4: Parameter values - extension with endogenous firm destruction</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Technology:</strong></td>
</tr>
<tr>
<td>mean (\bar{X})</td>
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<tr>
<td>variance (\alpha^2_x)</td>
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<tr>
<td><strong>Credit matching:</strong></td>
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<td>Entrep. cost (e)</td>
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