Midterm Exam, Econ 210, Fall 2009

Answer Question 1 and any 3 of the other questions.

Question 1. Mary Granola consumes only two goods and her utility function is

$$U(x_1, x_2) = (\min\{2x_1 + x_2, x_1 + 2x_2\})^2 .$$

a) Draw some indifference curves for Mary.

b) Is Mary's utility function concave? Is it quasi-concave?

c) Is Mary's utility function homogeneous? Is it homothetic?

d) Find Mary's Marshallian demands for the two goods. (Be sure to account for corner solutions and note that at certain prices her demand is not single-valued.)

e) Find Mary's indirect utility function. (Be sure to show this function for price-income situations that lead to corner solutions as well as interior solutions.)

f) Verify that Roy's identity holds for Mary.

g) Find Mary's expenditure function. (Hint: You can ease the task of finding the expenditure function by making use of the fact that $v(p, e(p, u)) = u$

h) Find Mary's Hicksian demand functions.

Question 2. Calculate the directional derivative of

$$f(x, y) = xy^2 + x^3 y$$

at the point $(1, -1)$ in the direction $(1/\sqrt{10}, 3/\sqrt{10})$.

Question 3. Harry consumes two goods and his indirect utility function is given by

$$v(p, m) = (m + p_1 + 2p_2)\, p_1^{-1/2} p_2^{-1/2}$$

for all price and income combinations at which he demands positive amounts of both goods.

a) Find Harry's Marshallian demand function for each of the two goods. At what price-income combinations does he buy positive amounts of both goods?

b) Find Harry's expenditure function. (Hint: You can ease the task of finding the expenditure function by making use of the fact that $v(p, e(p, u)) = u$)

Question 4. Gary's utility function u is defined for all positive values of x_1 and x_2 and is given by

$$u(x_1, x_2) = \left(a_1 x_1^{b_1} + a_2 x_2^{b_2}\right)^c .$$

For what values of the parameters (a_1, a_2, b_1, b_2, c) is Gary's utility function

a) Homothetic

b) Concave

c) Quasi-concave

d) Convex

Question 5. Prove or disprove each of the following:

a) Let f and g be convex functions whose domain is a convex subset D of $\Re^n$. Where h is the function defined so that $h(x) = f(x) + g(x)$, h is a convex function.

b)Let f and g be quasi-concave functions whose domain is a convex subset D of $\Re^n$. Where h is the function defined so that $h(x) = \min\{f(x), g(x)\}$, h is a quasi-concave function.