Name ____________________

Final Exam, Economics 210A, December 2011

There are 6 questions. Answer as many as you can... Good luck!

Question 1. A firm has a production function

$$F(x_1, x_2) = \left(\sqrt{x_1} + \sqrt{x_2}\right)^2 .$$

It is a price taker in the factor markets.

A) Is this production function homogeneous? If so, of what degree?

B) Is this production function concave? Prove your answer.

C) Define the elasticity of substitution between two factors and calculate the elasticity of substitution between x_1 and x_2 for the firm in this problem.

D) Where the prices of the two inputs are p_1 and p_2, find the amount of each factor that would be used to produce one unit of output in the cheapest possible way.

E) Where the prices of the two inputs are p_1 and p_2, find the amount of each factor that would be used to produce y units of output in the cheapest possible way.

F) Find the cost function for producing this good.

Question 2. A consumer has utility function

$$U(x_1, x_2) = \left(x_1^{-1} + x_2^{-1}\right)^{-1}$$

defined over the set $\{(x_1, x_2) | x_1 > 0, x_2 > 0\}$.

A) Is this utility function strictly increasing in both goods at all points in its domain? (Prove your answer.)

B) Find this consumer's Marshallian demand functions for goods 1 and 2.

C) Find this consumer's indirect utility function.

D) Verify that Roy's identity applies.

Question 3. A consumer has utility function

$$V(x_1, x_2, x_3) = x_3^a U(x_1, x_2)^{1-a}$$

where $0 < a < 1$ and

$$U(x_1, x_2) = \left(x_1^{-1} + x_2^{-1}\right)^{-1}.$$

A) Solve for the Marshallian demand functions for goods 1, 2, and 3, using a two step procedure in which you first find the highest utility that a consumer can achieve if he spends a total amount of money m on goods 2 and 3. (Hint: use the answer from Question 2 to help you.) Now if the consumer has total income M, find the best way to divide his expenditure, spending $M - m$ on good 1 and m on goods 2 and 3.

B Write down the indirect utility function for this consumer.

Question 4.

A) State the weak axiom of revealed preference.

B) Prove that a utility-maximizing consumer who has strictly monotonic and strictly convex preferences will necessarily satisfy the weak axiom of revealed preference.

C) Show that a utility-maximizing consumer with weakly convex preferences might violate the weak axiom of revealed preference.

Question 5. An economy has two consumers and two goods. Consumer A has the utility function

$$U(x_1^A, x_2^A) = \alpha \ln x_1^A + (1 - \alpha) \ln x_2^A$$

and an initial endowment of ω_1^A units of good 1 and no good 2. Consumer B has the utility function

$$U(x_1^B, x_2^B) = \beta \ln x_1^B + (1 - \beta) \ln x_2^B$$

and an initial endowment of ω_2^B units of good 2 and no good 1.

A) Let good 2 be the *numeraire* with a price of 1, and denote the price of good 1 by p. Solve for the demand for good 1 by each consumer as a function of p.

B) Write an equation for excess demand for good 1 as a function of price p of good 1 when good 2 is the *numeraire*. At what price or prices p is this excess demand for good 1 equal to zero.

C) If the price of good 1 is the price you found in Part B, at what price will excess demand for good 2 be zero?

D) Is there a competitive equilibrium in which the price of good 2 is 3? If so, what must the price of good 1 be for there to be a competitive equilibrium with the price of good 2 equal to 3?

E) Find the quantities of good 1 and good 2 consumed by person A in competitive equilibrium. Find the quantities of good 1 and good 2 consumed by person B in competitive equilibrium.

F) Suppose that person B's initial endowment of of good 2 is increased by one unit. What is the effect on the equilibrium consumption of good 1 by person B? What is the effect on the equilibrium consumption of good 2 by person B?

Question 6. A pure exchange economy has 1000 people and one consumption good. There are two possible states of the world. If State A occurs, everyone will have an initial endowment of 10 units of the consumption good. If State B occurs, each person will have an initial endowment of 20 units of the consumption good. Each person is an expected utility maximizer, with von Neumann-Morgenstern (Bernoullian) expected utility $\pi^i \ln c_a^i + (1 - \pi^i) \ln c_B^i$

where π^i is person i's subjective probability that Event A happens and $1-\pi^i$ is i's subjective probability that B happens, and where c_A^i and c_B^i are i's consumption contingent on these events.

Before it is known which state will occur, persons can buy or sell either of two kinds of securities. A type A security will pay one unit of the consumption good if State A occurs and nothing if state B occurs. A type B security will pay one unit of the consumption good if state B occurs and nothing if state A occurs. The price of type A securities is p_A per unit and the price of type B securities is p_B per unit. Let c_A^i and c_B^i be i's consumption in events A and B, respectively. The difference between the value of i's consumption in event A and i's endowment in event A is equal to the number of type A securities that i purchases if this difference is positive and sells if this difference is negative. Similarly for type B securities. For each person, the total value of securities purchased must be equal to the total value of securities sold. Thus $p_A(c_A^i - 10) + p_B(c_B^i - 20) = 0$ for all i.

A) Let the type B security be the numeraire. Find the demand of person i for type A securities as a function of p_A and of i's subjective probability π^i.

B) If everybody has the same subjective probability for $\pi^i = \pi$ for event A, what is the competitive equilibrium price for type A securities?

C) Solve for the competitive price for type A securities where different people have different subjective probabilities for event A.

D) Suppose that $\pi^i \neq \pi^j$, what is the ratio of person i's competitive equilibrium consumption in event A to that of person j in event A?