

# Bubbles and the Intertemporal Government Budget Constraint

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October 10, 2004

## Abstract

Recent years have seen a protracted debate on the "fiscal theory of the price level". This doctrine is based on the intertemporal government budget constraint, which says that the real value of the government debt equals the discounted value of future government surpluses. It is observed that the intertemporal government budget constraint consists of the proposition that government debt management defines a portfolio strategy that has no bubble. Therefore the intertemporal government budget constraint is satisfied in models in which bubbles can be ruled out, and it fails in settings in which bubbles can occur in equilibrium.

## 1 Introduction

Recent years have seen a protracted—and in places confused—debate on the “fiscal theory of the price level” (Eric Leeper [6], Michael Woodford [12], John H. Cochrane [3], Narayana Kocherlakota and Christopher Phelan [5], for example). The analysis is based on the “intertemporal government budget constraint” (hereafter IGBC), which says that the real value of government liabilities equals the discounted value of government surpluses. Cochrane, in contrast to such writers as Thomas Sargent and Neil Wallace [9], expressed the opinion that the IGBC is correctly interpreted as an equilibrium condition rather than as a budget constraint: instead of restricting government fiscal and monetary policy, it should be seen as determining the price level.<sup>1</sup> Since the IGBC is derived by substituting the single-period government

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<sup>1</sup>This observation reflects a tendency on the part of many analysts to view particular equations as determining particular variables, so that discussion centered on whether one should think of the price level as determined by a quantity equation or, instead, by the IGBC. Such loose treatment rarely extends to formal modeling, where analysts generally recognize that the question is whether some endogenous variables are determined in upper recursive blocks, so that they can be taken as

budget constraint—which, in the simplest version, says that the government’s budget deficit equals the value of newly-issued government debt—into itself recursively and taking a limit, it is not clear why Cochrane resisted characterizing the IGBC as a budget constraint.

We provide a reason here, although it may not be what Cochrane had in mind.

## 2 The Intertemporal Budget Constraint

Begin with the single-period government budget constraint, given by

$$s_t = b_{t-1}(1 + r_t) - b_t, \tag{1}$$

$t = 1, 2, \dots$ , where  $b_t$  denotes the value of outstanding government bonds at date  $t$ ,  $s_t$  is the government primary surplus (the primary surplus equals the budget surplus as conventionally measured plus interest on the outstanding debt), and  $r_t$  is the real return on bonds from  $t - 1$  to  $t$ . This version of the budget constraint suppresses nominal prices, money and the seigniorage revenue associated with money creation. By recursive substitution, the budget constraint (1) can be written as

$$b_0 = \sum_{i=1}^n \prod_{j=1}^i (1 + r_j)^{-1} s_i + \prod_{j=1}^n (1 + r_j)^{-1} b_n. \tag{2}$$

Letting  $n$  go to infinity and “applying the usual transversality condition”

$$\lim_{n \rightarrow \infty} \prod_{j=1}^n (1 + r_j)^{-1} b_n = 0, \tag{3}$$

in Cochrane’s phrase, there results the IGBC,

$$b_0 = \sum_{i=1}^n \prod_{j=1}^i (1 + r_j)^{-1} s_i. \tag{4}$$

Justification for (3) is never provided in the macroeconomics literature, yet it is the imposition of this condition that justifies characterizing (4) as an equilibrium condition rather than a budget constraint.

From the point of view of the private sector, the transversality condition (3) says that the portfolio strategy of private-sector investors collectively in lending to the government has no bubble, so the value of the debt equals the discounted value of future primary surpluses. A large literature has analyzed theoretical conditions under which portfolio strategies can or must have bubbles in equilibrium; to date

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given in some equations. This consideration motivated Woodford’s distinction between “Ricardian” and “non-Ricardian” fiscal policies.

this literature has not been connected with the discussion of the IGBC and the fiscal theory of the price level. This note brings these two literatures together.

Since we are interested in the IGBC itself, rather than in its application in the fiscal theory of the nominal price level, we can work with nonmonetary general equilibrium models throughout. Further, the issues of interest do not involve uncertainty, so it is appropriate to restrict attention to deterministic models.

### 3 Overlapping Generations Models

Following the papers of Neil Wallace [11] and Jean Tirole [10], overlapping generations models have become the standard vehicles for analyzing bubbles. The reason is that, because the number of both goods and agents is infinite, equilibrium allocations in overlapping generations models can be Pareto-suboptimal even in the absence of externalities and the like. Therefore the positive wealth increment to owners of securities (in positive net supply) implied by existence of bubbles does not necessarily contradict the feasibility of the equilibrium allocation. In contrast, when the equilibrium allocation is Pareto optimal the wealth increment that would be implied by a bubble on a security in positive net supply would contradict Walras' Law, so the bubble cannot exist (see Santos and Woodford [8], Huang and Werner [4] and LeRoy [7] for further discussion).

These ideas are readily applied to the IGBC. Beginning with a standard deterministic overlapping generations model without production, one can add a government which runs surpluses and deficits. In the simplest case, which is sufficient for the present purpose, surpluses are generated by lump-sum taxation accompanied by loans to the private sector, while deficits are associated with lump-sum transfers to private agents accompanied by borrowing from the private sector. If the government ran deficits in the past it has positive indebtedness to the private sector, while if it ran surpluses in the past it is a net holder of the liabilities of the private sector.

If equilibrium interest rates are positive, the aggregate endowment has finite value. It follows both that the equilibrium allocation is Pareto optimal and that bubbles cannot exist on the government's portfolio strategy. If they did exist, private agents' wealth (equal to the value of the debt plus the present value of future endowments) would not equal the present value of consumption, a violation of Walras' Law. Existence of a bubble on the government debt would imply that the government debt would increase to the point where generations could no longer transfer it from generation to generation. Absence of a bubble on the government debt implies that the IGBC is necessarily satisfied.

In contrast, if the endowment pattern is such that equilibrium interest rates under the allocation in the bubble-free equilibrium are negative, then Walras' Law fails and the endowment allocation is not Pareto optimal.<sup>2</sup> In that case there exists a

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<sup>2</sup>For simplicity we are ignoring the boundary case, in which the endowment has infinite value,

continuum of equilibrium paths. On all but one of these paths the IGBC is not satisfied due to the existence of a bubble on the equilibrium portfolio strategy.

Most simply, one can imagine a situation in which the private sector holds a strictly positive amount of government debt, and the government implements a balanced budget forever in the future. This setting, of course, is exactly that used in monetary economics to demonstrate how money can be valued despite being intrinsically useless (Wallace [11]).

## 4 Conclusion

It follows from this analysis that the virtually universal practice of assuming the automatic validity of the IGBC in the analysis of fiscal policy is, at best, very loose. If one wants the IGBC, a better procedure would involve explicitly adopting assumptions that rule out bubbles. One way to do this is to state explicitly the trading restrictions that are necessary in any model involving an infinite future if Ponzi schemes are to be avoided. Depending on what trading restrictions are invoked to eliminate Ponzi schemes, bubbles may or may not also be ruled out (Huang and Werner [4]). Alternatively, the endowment pattern and technology can be restricted so that equilibrium interest rates turn out to be positive. This course has the advantage of agreeing with the empirical evidence, at least if the analysis of Abel et al. [1] is accepted. I have observed elsewhere (LeRoy [7]), however, that ruling out bubbles by invoking the Pareto optimality of equilibrium is putting the neoclassical/rational expectations paradigm to very hard use.

If one is unwilling to make any of assumptions that rule out bubbles, one must allow for the possibility of bubbles on the equilibrium portfolio strategy of private agents. The indeterminacy of equilibrium that results when bubbles are admitted renders it difficult to reach definite conclusions. In particular, the justification for the fiscal theory of the price level would be very much in question, given its dependence on the IGBC. However, it seems better to face these problems explicitly than to avoid them by directly invoking the IGBC.

## References

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but the equilibrium is Pareto optimal (by the Cass [2] criterion).

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