

# Review of Bossaerts, The Paradox of Asset Pricing

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## Abstract

By "the paradox of asset pricing" Peter Bossaerts refers to his contention that, despite its apparent generality and sophistication, the theory of finance has largely been a failure empirically. Bossaerts reviews the major areas of finance: theory, empirical methods, empirical results and experiments. The explanatory variables for average asset returns suggested by theory—market beta and consumption beta—predict returns less successfully than variables for which the theoretical basis is weak. This reviewer agrees with Bossaerts' assessment.

Bossaerts proposes weakening the hypothesis of market efficiency from full rational expectations to efficient learning: agents update priors optimally—that is, according to Bayes' rule—but start with possibly biased priors. He develops ingenious empirical implications of this specification. For example, he shows that under efficient learning, inverse asset returns are fair games going backward in time (under some additional assumptions). Empirical implementation of these tests appear promising.

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# 1 Introduction

Five or ten years from now the book under review may well have dropped from sight, lost among the myriad of books and papers that propose novel research programs that turn out not to succeed empirically.<sup>1</sup> However, another possibility—that this book will be seen as having a major impact on empirical study of financial markets and as generating valuable insights about security prices—seems to me to be more likely. I believe that Bossaerts’ book is essential reading for economists who are serious about empirical study of security prices. More important, analysts need to develop the empirical methods Bossaerts has outlined and determine the extent of their applicability.

According to Bossaerts, the “paradox of asset pricing” refers to the fact that, despite the simplicity and appeal of the assumptions that financial theory is based on (not to mention its undisputed elegance and mathematical sophistication), the theory turns out to be largely a failure empirically. This fact is well known among financial economists, although Bossaerts is one of the first to state the point so baldly and place it front and center in his analysis.

## 2 Theory Overview

Before proposing a (partial) resolution to the paradox of asset pricing, Bossaerts presents a comprehensive overview of the theory of security pricing (Chapter 1). Although I will point out a few errors below, for the most part this discussion is masterful. Bossaerts begins with a derivation of the Euler equation, the first-order

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<sup>1</sup>Peter Bossaerts, *The Paradox of Asset Pricing*, Princeton University Press, Princeton, 2002. Pp. xiii, 170.

condition of a dynamic stochastic portfolio optimization problem. This optimization is initially presented in a very general setting; Bossaerts then demonstrates the simplification that results when competitive conditions are assumed. Taking this route is unusual but very worthwhile.

In an equilibrium setting the stochastic Euler equation translates into the equilibrium condition that the risk premium on any security (the difference between the expected return on that security and the risk-free return) is proportional to the covariance between the security's return and a stochastic discount factor. In representative-agent models the stochastic discount factor is the representative agent's marginal rate of substitution. Asset pricing models are differentiated according to how they specialize the specification of the stochastic discount factor. The Capital Asset Pricing Model identifies it with the return on the market portfolio. Having the stochastic discount factor be observable is a major point in favor of CAPM, although Roll (1977) reminds us that CAPM is testable only in conjunction with a commitment to a particular measure of the market portfolio.

For Bossaerts a disadvantage of CAPM is that it is a two-date model. It is not entirely clear exactly what Bossaerts means by this: there is no shortage of papers interpreting CAPM as applying period-by-period in a multirate setting (Myers and Turnbull (1977), for example). Cochrane (2001) gives a derivation of CAPM in continuous time with logarithmic utility. Perhaps what Bossaerts means is that the assumptions that imply CAPM in a two-date setting—quadratic utility, for example—do not deliver CAPM in a multirate setting, a fact which is not widely known. However, a variant of CAPM supplied by Rubinstein (1976) does apply in a multirate setting if agents have logarithmic utility. Bossaerts makes much use of Rubinstein's model in the book under review. Under log utility the stochastic discount factor is the reciprocal of the return on the market portfolio, which is essentially the same thing

as CAPM (in fact, the simplest derivation of CAPM under log utility in continuous time consists of Rubinstein’s model plus the observation that the reciprocal of the return on the market portfolio can be represented locally by a linear function of the return on the market portfolio).

### **3 Empirical Tests**

The discussion of theory in Chapter 1 is followed by two chapters presenting the theoretical basis for empirical tests of security pricing theory (Chapter 2) and the outcome of these tests (Chapter 3). Bossaerts reviews the attempts of Fama and Macbeth (1973) and Fama and French (1988) to test CAPM by constructing empirical measures of the beta coefficients of individual securities and determining whether these measures are correlated with risk premia, as CAPM requires. As is well known, the outcome of such tests has not been favorable to CAPM. Then Bossaerts discusses the equity premium puzzle of Mehra and Prescott (1985), which is now generally regarded as Exhibit 1 in the case against the empirical correctness of consumption-based security pricing, at least in its simplest form.

To counter the widespread belief—at least prior to appearance of the Hansen-Jagannathan paper (1991)—that Mehra-Prescott’s results depend critically on their adoption of a two-state framework, Bossaerts then reviews the Hansen-Jagannathan bounds on the volatility of the stochastic discount factor. This study duplicated Mehra-Prescott’s conclusion—that empirically accurate values for the equity premium and risk-free return can be generated under consumption-based security pricing models only if agents’ risk aversion is specified to be unrealistically high—under the assumption that security returns are lognormal. As noted below, there are some errors in Bossaerts’ discussion here, but these are minor and do not interfere with

the main line of Bossaerts' argument. Readers not familiar with the equity premium puzzle (if there are any left) might want to supplement Bossaerts' discussion of this material with that of Campbell, Lo and MacKinlay (1996), which is excellent. See Kocherlakota (1996) for the comprehensive review of the literature.

Bossaerts' Chapter 1 discussion of existence of equilibrium in models of financial markets is not as clear as one would like. He asserts that under standard assumptions the problem comes up only when markets are incomplete, because in that setting individuals cannot insure all possible risks. The reader is left to wonder what insurability of all risks has to do with existence of equilibrium. In fact, in the standard two-date model there is no difficulty with existence (again, under standard assumptions) whether or not markets are complete. This is easy to see: one has only to redefine the objects of choice as securities rather than contingent claims. This redefinition (plus the observation that concavity is preserved under the redefinition) implies that the equilibrium model is formally identical to a setting with complete markets. Therefore existence of equilibrium is proved using standard methods. Problems come up only in the multirate setting: in that case changes in security prices may reduce the dimension of the payoff space as securities become redundant. The induced discontinuities in demand functions imply that fixed-point theorems do not apply, possibly resulting in nonexistence of equilibrium.

Bossaerts' discussion of the equity premium puzzle of Mehra and Prescott (1985) is also unsatisfactory. First, Bossaerts states that for reasonable parameter values Lucas's (1978) model predicts price volatility lower than that we see in real-world markets. Bossaerts situates this as a manifestation of the equity premium puzzle. The opposite is the case. As usually stated, the equity premium puzzle consists in the fact that expected excess returns on equity are as high as they are, given return volatility (and assuming reasonably low levels of risk aversion). Turning this

around, the equity premium puzzle may be viewed as consisting of the fact that return volatility is as low as it is given the equity premium, just the opposite of Bossaerts' assertion.

The observation that existing models underpredict price and return volatility came earlier (LeRoy and Porter (1981), Shiller (1981) <sup>2</sup>). That this result has no obvious connection to the equity premium puzzle is indicated by the fact that the authors just cited assumed constant discount factors, reflecting risk neutrality. Indeed, it may be precisely the assumption of constant discount factors that led to the underprediction of volatility in the variance-bounds literature: at least in some settings, increases in risk aversion increase volatility (LaCivita and LeRoy (1981), Grossman and Shiller (1981)).

However, there is a line arguing the point that Bossaerts attributes to Mehra and Prescott: that real-world price volatility is excessive relative to the predictions of models that incorporate risk aversion. Rietz (1988) and Salyer (1998) (see Cecchetti, Lam and Mark (1993) for related discussion) showed that a three-state model incorporating a low-probability crash state can explain the equity premium in models incorporating moderate degrees of risk aversion (the two-state model of Mehra-Prescott and the lognormal model of Hansen-Jagannathan (1991) required relative risk aversion on the order of 10 to explain the equity premium puzzle, as Bossaerts explains). However, Salyer showed that if a crash-state model is calibrated to explain the equity premium puzzle, price volatility is underpredicted by a considerable margin. Bossaerts does not discuss this work.

Problems with Bossaerts' discussion of the equity premium puzzle continue in his analysis of how the equity premium and the risk-free return depend on the assumed

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<sup>2</sup>These papers apply a different standard in defining overprediction or underprediction of price volatility: for LeRoy-Porter and Shiller dividend volatility provides the benchmark; for Mehra-Prescott it is the equity premium, as just observed. Therefore they are not strictly comparable.

level of risk aversion. His discussion of this topic in Chapter 2 is oversimplified, and is difficult to relate to the parallel discussion of the material in Chapter 3 in the context of the Hansen-Jagannathan bounds on the volatility of the marginal rate of substitution (which is correct).

It is clear from Bossaerts discussion of the Hansen-Jagannathan bounds in Chapter 3, but not from his discussion of the equity premium puzzle in Chapter 2, that for realistic parameter values the risk-free return depends on the coefficient of relative risk aversion in a non-monotone fashion. To see why this is, recall that the reciprocal of the risk-free return equals the expectation of the marginal rate of substitution. The latter can be written as the expectation of the growth rate of consumption exponentiated by the coefficient of relative risk aversion, plus a term reflecting Jensen's inequality. The former is a decreasing function of the risk aversion parameter, while the latter is an increasing function. For realistic parameter values the former predominates under low to moderate values of the risk aversion parameter, while the latter predominates for very high values. Therefore the risk-free return rises with risk aversion, then falls. For very high risk aversion both the equity premium and the risk-free return can be explained.

This non-monotone effect of the risk aversion parameter on the risk-free return is clear in Hansen-Jagannathan (1991), discussed by Bossaerts in Chapter 3 (see the diagram on p. 85). Moderate levels of risk aversion, combined with positive consumption growth rates, imply that expected future consumption is heavily discounted relative to current consumption, compared to risk neutrality. This results in an increased risk-free return, as observed by Weil (1988), who first noted this "risk-free rate" puzzle. However, under extremely high risk aversion, the Jensen's inequality term dominates, and a constant relative risk aversion utility function approaches a utility function that identifies utility with the minimum consumption level.

If (in a finite-event parametrization) the lowest possible consumption growth rate is negative, as is reasonable, the utility function will equate utility with next-period consumption, resulting in an interest rate of -100% asymptotically. Appreciating this nonmonotonicity in the effect of risk aversion on the risk-free return is essential in understanding why extremely high risk aversion resolves both the equity premium puzzle and the risk-free return puzzle. Bossaerts does not makes this clear.

Of course, most analysts would argue against the idea that relative risk aversion on the order of 10 or 20 is a plausible specification. Such utility functions imply, unrealistically, that agents would reject lotteries with a high probability of a large gain combined with a small probability of a small loss.

## 4 Experimental Evidence

Not many writers are as familiar both with finance theory and financial econometrics as Bossaerts (Campbell, Lo and MacKinlay, and Cochrane, come to mind, and only a very few others). In Chapter 4 Bossaerts goes on the discuss another area of evidence on security pricing to which he and his colleagues at Caltech (notably Charles Plott) have made major contributions: experimental evidence. The appeal of experiments in testing security pricing theory is evident: as usual with experiments, the experimenter can control the environment and replicate the experiments. Here again Bossaerts' discussion is very impressive.

The major prediction of general consumption-based security pricing is that the state prices implied by security prices are ordered inversely with aggregate consumption. In the case of CAPM, the prediction is that the market portfolio has maximal Sharpe ratio. The record is mixed, but Bossaerts concludes that on the whole the experimental evidence on these predictions of theory is more favorable to the theory

than the econometric evidence from real-world financial markets. This outcome is striking, especially in light of the fact that, as Bossaerts emphasizes, the subjects in the experiments were not given enough information to enable them to implement theory that they (might have) learned in class. For example, even if subjects were familiar with the CAPM model, in the experiments Bossaerts reports they have no way to implement its predictions. This is so because subjects were not told the composition of the market portfolio, and there was no (obvious) way for them to infer its composition from the experimental data.

By way of introducing the discussion that is to follow, Bossaerts observes that the element of received security pricing theory that is least supported in the experiments is the assumption that agents have unbiased priors. For example, when experimental markets supported the conclusions from theory, they generally did so only after an extended period of what Bossaerts calls “price discovery”, as would be the case if agents had mistaken priors.

Here it must be observed that the outcomes of the experiments Bossaerts reports do not support this conclusion ambiguously. For example, on Bossaerts’ account there is strong evidence that experimental subjects misprice securities because they do not understand the nature of randomness: they seem to assume that an outcome that was underrepresented in the past will be overrepresented in the future, so that when the past and future are averaged, observed frequencies will be close to population probabilities. If so, problems will occur with updating of priors just as much as with the priors themselves. However this may be, Bossaerts’ conclusion that the experimental evidence gives stronger support to security pricing theory than the econometric evidence is very interesting.

## 5 “Efficient Learning Models”

The continuation of Bossaerts’ book gives his ideas about how to resolve the “paradox of asset pricing”. As valuable as Bossaerts’ discussions of the theory, empirical evidence and experimental evidence are, this last part of the book is by far the best: Bossaerts provides a concrete suggestion for reformulating the theory being tested. As observed in the introduction, this suggestion is well worth further investigation.

For Bossaerts, the “Efficient Markets Model” is at the heart of empirical tests of security pricing theory. Bossaerts observes that received definitions of market efficiency are unsatisfactory, a point that has been made many times before (LeRoy (1976), (1989), for example). For Bossaerts the problem is the ambiguity of the requirement that in efficient markets prices “fully reflect” available information. However, it is easy to reverse-engineer a definition of market efficiency by looking at the tests actually conducted and seeing what outcomes are interpreted as supporting or contradicting market efficiency.

By examining the empirical tests that are actually conducted and their interpretation, Bossaerts lists three critical ingredients in the definition of market efficiency: (1) correct priors, (2) rational expectations in the sense of Radner (1972), so that agents are assumed to know the prices that will obtain at each event, and (3) stationarity. For some reason Bossaerts does not mention risk-neutrality; many empirical investigations of market efficiency in fact tested a compound null hypothesis that includes risk neutrality.

Rational expectations in the sense of Lucas (1978) combines (1) and (2) above, and usually also (3). However, nothing about security pricing theory requires the assumption of correct priors, as Radner’s (1972) model clearly indicates. Bossaerts proposes that the model being tested be weakened to relax the assumption of cor-

rect priors: instead of testing capital market efficiency as defined above, Bossaerts proposes that we identify security pricing theory with Efficiently Learning Markets, which involves relaxing the assumption that agents have correct priors. The agents being modeled are assumed to form posterior distributions by correctly updating (that is, updating according to Bayes' Theorem) priors that are possibly incorrect, and to price securities accordingly.

This proposal is very appealing. Lucas's formulation of rational expectations entails the complete elimination of genuinely new events: agents are assumed always to be able to assimilate apparently new events into preexisting patterns, and to do so correctly. Learning models, on the other hand, envision settings in which genuine innovation sometimes occurs, and these innovations are initially not interpreted correctly. However, in Efficiently Learning Markets, agents do update priors correctly, so that there will eventually be a convergence to rational expectations in the sense of Lucas if no further innovations occur.

Learning is not as clean as rational expectations in the sense of Lucas: in formal models there is a clear distinction between priors and updating, but the same is not true in reality. Consider the example of initial public offerings of stock, which Bossaerts discusses at length. Ritter (1991), among many others, established that IPOs spectacularly underperformed stocks generally, contradicting market efficiency. However, Bossaerts shows that the evidence (most of it, at least) is consistent with the hypothesis that agents are correctly updating biased priors, so that security pricing theory is corroborated under the ELM interpretation. As a matter of interpretation, there are no clear grounds for determining whether a new event occurred, for which agents are permitted to have arbitrary priors, or whether their beliefs about a purportedly new event must in fact be modeled as reflecting correct updating of a prior belief.

Of course, if agents' prior beliefs differ from the relevant objective probabilities, econometric tests will reject the martingale model, implying that capital markets are inefficient. The same outcome can occur if markets are efficient but the economist has a biased sample. For example, if realizations in which security returns take on particular values are overrepresented or underrepresented in the sample, statistical tests will lead to a conclusion that subjective and objective probabilities differ. This is a particularly important problem in financial economics: mutual funds that do badly are more likely to shut down than those that do well. If these are just deleted from sample, upward bias in estimated mutual fund returns will result. In such cases capital market efficiency as usually defined is likely to be rejected even if it is true.

Bossaerts proposes an ingenious econometric test of the hypothesis that agents price securities correctly in light of beliefs that display efficient learning, but not necessarily unbiased priors. The test will be valid even in the presence of selection biases of the type described in the preceding paragraph.

## 6 An Example

It is easiest to present this test in the context of an example. Suppose that some mechanism generates two successive realizations of a random variable that takes on values Up and Down. Initially, assume that Up occurs with probability  $1/2$  on the first draw. On the second draw Up occurs with probability  $2/3$  if Up occurred on the first draw, and with probability  $1/2$  if Down occurred on the first draw. A security has payoff  $V$ , which equals 1 if the number of Up draws is even (that is, 0 or 2), and zero otherwise. This model is illustrated in Figure 1, where the entries at the nodes of the tree represent the prices of the security and the entries in the boxes (top row) equal the transition probabilities.

Capital market efficiency entails assuming that agents (act as if they) know these probabilities and payoffs. If investors are risk-neutral and do not discount the future, the price of the security at date 0 equals the probability that  $V$  equals 1, which is  $7/12$ . At date 1 the price of the security is  $2/3$  if Up occurred at date 0,  $1/2$  otherwise. These prices are a martingale (that is, the prices at dates 0 and 1 equal the conditional expectation of the prices at the following date; equivalently, the expected net rate of return is zero).

The problems discussed above are easily analyzed in this setting. First, suppose that investors have a biased prior on  $V$ . Specifically, let the objective probability that  $V$  equals 1 be  $\pi$ , maintaining the assumption that the subjective probability of this event is  $7/12$ . If (for simplicity) we maintain the date-1 transition probabilities specified above, this is equivalent to assuming that the date-0 objective probability of Up is  $p$ , where  $p$  and  $\pi$  are related by  $p = 6\pi - 3$  (which, of course, admits  $p = 1/2$  and  $\pi = 7/12$ , so that bias is zero, as a special case).

In the above example, the assumption that investors update their beliefs about  $V$  correctly at date 1 even if  $\pi$  differs from  $7/12$  (that is, their subjective probability that Up will occur with probability  $2/3$  ( $1/3$ ) at date 2 conditional on Up (Down) occurring at date 1 agrees with the objective probability) means that the model satisfies Bossaerts' definition of efficient learning.

It happens that in this case we have the remarkable result that the conditional expectation of the inverse return equals 1 even if  $\pi$  does not equal  $7/12$ . To see this, observe that Bayes' Theorem implies that the probability that the state at date 1 is Up conditional on  $V = 1$  equals  $4 - 2\pi$ . The conditional probability of Down equals  $2\pi - 3$ . The inverse returns  $p_0/p_1$  equal  $3\pi/2$  if Up and  $2\pi$  if down. These have a weighted average equal to 1. In the special case  $\pi = 7/12$ , the inverse returns are  $7/8$  if Up and  $7/6$  if Down occurs, and the conditional probability of Up equals  $4/7$ .

The expectation of these returns is again 1.

Note that if the analyst were not confident that he had a representative sample, he could simply discard all the  $V = 0$  realizations and test the hypothesis of efficient learning using the observations for which  $V = 1$ .

Bossaerts presents empirical tests based on the foregoing result. For the most part the outcomes of these tests are more favorable to the hypothesis of Efficiently Learning Markets than to Efficient Capital Markets. That is, according to Bossaerts failure of tests of capital market efficiency can usually be attributed to biased priors. For example, in the case of initial public offerings, the data are broadly (although not entirely) consistent with efficient learning in the presence of wildly overoptimistic priors.

Bossaerts' new methods for testing his model require some assumptions that were satisfied in the example above, although they were not stated formally here. The range of applicability of these assumptions is not clear, at least to this reviewer. One wonders whether these methods can be applied to the study of the equity premium puzzle. Some analysts (such as Brown, Goetzmann and Ross (1995)) attribute the equity premium puzzle in US data to survivorship bias, arguing that the US is atypical in having had continuously functioning equity and bond markets throughout the twentieth century. Can Bossaerts' results be used to construct estimates of the equity premium that allow for this survivorship bias?

## 7 Conclusion

On his own account Bossaerts has presented only a few preliminary results; application of these methods to the equity premium puzzle and other problems remains for the future. But Bossaerts is surely correct that this line of research is very promising.

One hopes that he will continue work along these lines and that others will follow his lead.