Problem 1

Arne, Barack, and Christy are four regressions away from completing their doctoral dissertations, and they have four days in which to finish. If one of them spends \( h_t \) hours running regressions on day \( t \), her utility that day is given by \( u_t = -h_t \). The production function for regressions is \( r_t = \sqrt{h_t} \). Completing the dissertation provides no utility—it is merely a binding constraint! So expected utility is simply the appropriately-discounted sum of effort costs. Assume \( \delta = 1 \) throughout.

a) How many regressions does Arne, a TC with \( \hat{\beta} = \beta = 1 \), run each day?

b) What about Barack, who is naive about his present-bias (\( \hat{\beta} = 1 \), but \( \beta = \frac{1}{2} \))? Also, on day one, how many regressions does he plan to run each day?

c) What about Christy, who is like Barack, but sophisticated about her present-bias? Discuss your approach to working out your answer.

Problem 2

Nina lives three periods. In period 0 she is born with $2 and can save any amount, \( s \), of this wealth in an illiquid asset (a two-period CD) that yields \((1 + \pi_s)s\) in period 2, but cannot access this \( s \) in period 1. In period 1, she chooses to consume \( c_1 \leq 2 - s \). Because she faces no uncertainty and earns no interest on period 1 savings, Nina will save in period zero whatever she plans not to consume in period 1. However, in period 1 Nina also has the option of borrowing against his period 2 savings and consuming this. If she borrows \((1 + \pi_b)b\) from future spending, she gets to spend \( b \) more now. Assume throughout that \( \pi_b > \pi_s > 0 \), except where told to assume otherwise.

Nina has per-period instantaneous utility \( u_t = \ln(c_t) \) each of periods 1 and 2 (she doesn’t consume in period 0). She faces no uncertainty and no borrowing or spending constraints except \( s, b \geq 0 \) and that she is not allowed to die in debt.

a) Suppose that Nina doesn’t discount between periods, so her choice problem is

\[
\max_{b, s} \ln(2 - s + b) + \ln((1 + \pi_s)s - (1 + \pi_b)b).
\]

Solve for Nina’s choice of \( s \) and \( b \) (one of these will be a corner solution—you can assert this without doing the algebra) as a function of \( \pi_s \) and \( \pi_b \). State the implied consumption levels, \( c_1 \) and \( c_2 \) and interpret.

b) Suppose Nina has present-biased preferences with \( \beta = \frac{1}{2} \) (and \( \delta = 1 \)) and she is fully naive. Solve again for \( s \) and \( b \), etc. and interpret, comparing to TC Nina from part a).
c) Now suppose Nina has $\beta = \frac{1}{2}$, but is sophisticated. Redo as much of the previous analysis as is possible, choosing a handful of specific plausible values of $\pi_s$ and $\pi_b$ (instead of finding the general solution) and conjecturing, if necessary.

d) Now consider the case where $\pi_b = \infty$, and let $\pi_s < 0$ (!!). Now clearly there might be a motive for Nina to save in period 1, even at no interest. If Nina does not discount at all between periods (and thus is TC), her maximization problem is:

$$\max_{s_0, s_1} \ln(2 - s_0 - s_1) + \ln(s_1 + (1 + \pi_s)s_0),$$

where $s_0$ is her illiquid savings from period 0 and $s_1$ is her unspent money from period 1. Find her $s_0, s_1$ and the resulting $c_1$ and $c_2$, giving an intuition for this answer.

e) Do the same for the naive present-biased Nina ($\beta = \frac{1}{2}$).

f) Try to do the same for the sophisticated PB-Nina.

**Problem 3**

Firm 1 and firm 2 each choose to produce $q_i \in [0, 1]$ at zero marginal cost, to sell in a joint market where the market price is determined by $p = 1 - Q$, where $Q = q_1 + q_2$. Suppose each has identical social preferences, with firm $i$’s utility being a convex combination of the profits of the two firms, with $\rho$ being the weight put on the profits of the other firm.

a) Suppose that $\rho = 0$. Solve for the Cournot outcome.

b) Now solve for the Stackelberg outcome (where firm 1 is the leader).

c) Repeat part a) for non-zero $\rho$. Considering the case where $\rho \in (0, \frac{1}{2})$, how much will each produce. How do quantities and profits depend on $\rho$? Interpret these results and compare them to the classical case from part a).

d) What happens as $\rho \nearrow 1$? Interpret.

e) What happens as $\rho \searrow -\infty$? Interpret.

f) Now consider the Stackelberg case for $\rho \in (0, \frac{1}{2})$. Discuss prices, profits, limits as $\rho \nearrow \frac{1}{2}$ and compare to the Cournot case.

**Problem 4**

Suppose that players $A$ and $B$ have Charness/Rabin distributional preferences parameterized by $\rho_a, \rho_b, \sigma_a, \sigma_b \in (-\infty, \frac{1}{2}]$, where $\rho_i \geq \sigma_i$ for $i \in \{a, b\}$.

a) Briefly interpret the four parameters and discuss the how we can reasonably interpret the preferences for various possible ranges of $\sigma$ and $\rho$.

b) Propose at least one set of values that seem improbable and one set of values that are not *a priori* improbable, but are not supported by the experimental evidence (in the papers that we mentioned in class).
c) Consider a situation with payoff uncertainty: suppose the monetary payoffs for \( A \) and \( B \) are \((\pi_a, \pi_b)\), \( \pi_a > \pi_b \), with probability \( q \) and \((\pi'_a, \pi'_b)\), \( \pi'_a < \pi'_b \), with probability \( 1 - q \). Which is the correct expression for \( A \)’s expected utility:

\[
q[(1 - \rho_a)\pi_a + \rho_a \pi_b] + (1 - q)[(1 - \sigma_a)\pi'_a + \sigma_a \pi'_b],
\]

or

\[
(1 - \rho_a)[q\pi_a + (1 - q)\pi'_a] + \rho_a[q\pi_b + (1 - q)\pi'_b],
\]

(for the case that \( q\pi_a + (1 - q)\pi'_a \geq q\pi_b + (1 - q)\pi'_b \), otherwise with \( \sigma \) instead of \( \rho \). Interpret the two alternatives.

d) Consider the game tree depicted below, where the numbers in a parentheses at the terminal nodes represent the material payoffs \((\pi_a, \pi_b)\). Ignoring cases of indifference, find the SPNE of this game as a function of the parameters \((\rho_a, \sigma_a, \rho_b, \sigma_b)\).

![Game Tree Diagram]

Figure 1: Game from Problem 4. Monetary payoffs in parentheses.

e) Discuss at least one possible value of \( p \) for which the SPNE seems inadequate or unrealistic.