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What is Oligopoly?

- Oligopoly is a kind of market structure, like monopoly or perfect competition

- An oligopolistic industry is an industry consisting of a few firms (duopoly = two firms)

- Example industries: auto, operating systems, mp3/music players, airlines
Questions

How can we analyze an oligopolistic industry?

- How are the market prices and quantities determined?
- How does this impact welfare?
- How do we think about competition among oligopolists?
- Why might firms want to collude (form a cartel)?
- How can a cartel be sustained?
We use Game Theory to Study Oligopoly

- With PC and monopoly market structures, we analyze a firm making an individual decision
- PC: very many firms, one firm’s actions do not impact others
- Monopoly: only one firm, no one else to impact
- However, oligopoly: each firm’s $p$, $q$ decisions affect competitor’s profits
- Strategic interaction/interdependence $\implies$ apply game theory
Oligopoly models

Considerations:

- Do firms compete on price or quantity?
- Do firms act sequentially (leader/followers) or simultaneously (equilibrium)
- Stackelberg models: quantity leadership
- Cournot equilibrium models: simultaneous choice quantity competition
- Bertrand equilibrium models: simultaneous choice price competition
Oligopoly

Today:

- Cournot model
- Compare to PC, monopoly

Next time:

- Stackelberg model
- Bertrand model
- Cartels
Example: comparing market structures

- The basics:
  - Inverse demand: \( p = a - Q \) (where \( Q \) is total quantity)
  - Marginal cost: \( c \) (no fixed cost)

- First establish baseline predictions about outcomes + welfare
  - Perfect Competition \( (P = MC) \)
  - Monopoly \( (MR = MC) \)

- Then examine Cournot model
  - Duopoly (two firms)
  - More general oligopoly \( (N \text{ firms}) \)
Example: comparing market structures

Baseline predictions:

- **Baseline: Perfect competition ($p = MC$)**
  - $p = c$, $Q = a - c$ (individual $q_i \approx 0$)
  - $\Pi = 0$, $CS = \frac{1}{2}(a - c)^2$, $W = \frac{1}{2}(a - c)^2$

- **Baseline: Monopoly ($MR = MC$)**
  - $p = \frac{a+c}{2}$, $q = Q = \frac{a-c}{2}$
  - $\Pi = \frac{1}{4}(a - c)^2$, $CS = \frac{1}{8}(a - c)^2$, $W = \frac{3}{8}(a - c)^2$
Cournot Model of Duopoly

- Two firms compete in the same market
  - Simultaneously choose $q_i$
  - This determines total $Q$...
  - ...which determines price

- Each would love to be monopolist, but can’t control behavior of other

- Each firm’s choice affects competitor
  - Given competitor’s quantity, $q_j$, firm $i$ would choose $q_i$ to max profits.
  - But given $q_i$, firm $j$ might choose different $q'_j$ to maximize profits (so $q_i$ would change
Q: How do we make predictions about behavior?

A: Use notion of (Nash) equilibrium

- If firms keep adjusting their quantities in response to one another, where will they end up?
- At a point where each firm is maximizing profits given the behavior of the other
- \( q_i \) is the best response to \( q_j \) and \( q_j \) is the best response to \( q_i \)
- At this point, neither firm has any incentive to change its quantity
- System is in equilibrium

*Nash Equilibrium*: taking the behavior of others as given, each party is choosing an optimal response.
Finding Nash Equilibrium in the Cournot Model

- Suppose firm \( j \) chooses \( q_j \). What should firm \( i \) do?
- Choose \( q_i \) that maximizes profits
- Write down \( i \)'s profits, as a function of \( q_i, q_j \):
  \[
  \Pi_i(q_i, q_j) = pq_i - cq_i = (a - q_i - q_j - c)q_i
  \]
- First-order condition:
  \[
  \frac{\partial \Pi_i}{\partial q_i} = a - 2q_i - q_j - c = 0
  \]
- Solve for firm \( i \)'s reaction function (gives best response for each value of \( q_j \):
  \[
  q_i^*(q_j) = \frac{a - q_j - c}{2}
  \]
Finding Nash Equilibrium in the Cournot Model

- **reaction function:**
  
  \[ q_i^*(q_j) = \frac{a - q_j - c}{2} \]

- Because of symmetry, firm j’s reaction function is:
  
  \[ q_j^*(q_i) = \frac{a - q_i - c}{2} \]

- How to find equilibrium?
  - Both firms must be best responding to each other so
    
    \[ q_j = q_j^*(q_i) \text{ and } q_i = q_i^*(q_j) \]

- Also, by symmetry, \( q_i^* = q_j^* \)

  \[ q_i^* = q_j^* = \frac{a - q_i^* - c}{2} \]

- Solve:

  \[ q_i^* = \frac{a - c}{3} = q_j^* \]
Finding Nash Equilibrium in the Cournot Model

- Optimal quantities:
  \[ q_i^* = \frac{a - c}{3} = q_j^* \]

- So \( Q = q_i + q_j = \frac{2}{3}(a - c) \)

- and \( p = a - Q = \frac{a + 2c}{3} \)

- Calculate welfare
  - \( CS = \frac{1}{2}[a - \frac{a+2c}{3}][\frac{2}{3}(a - c)] = \frac{2}{9}(a - c)^2 \)
  - \( \pi_i = (p - c)q_i = [\frac{a+2c}{3} - c]\frac{a-c}{3} = \frac{(a-c)^2}{9} \)
  - \( W = CS + \Pi = \frac{2}{9}(a - c)^2 + 2 \times \frac{(a-c)^2}{9} = \frac{4}{9}(a - c)^2 \)

Behavior and welfare lie between PC and monopoly
Now suppose that there are \( N \) Cournot competitors

- Write down \( i \)'s profits, as a function of \( q_1, \ldots, q_N \):

\[
\Pi_i(q_1, \ldots, q_N) = (p - c)q_i = (a - (q_i - Q_{-i} - c))q_i,
\]

where \( Q_{-i} \) is the sum of all the \( N - 1 \) competitors quantities

- First-order condition:

\[
\frac{\partial \Pi_i}{\partial q_i} = a - 2q_i - Q_{-i} - c = 0
\]

- Firm \( i \)'s reaction function:

\[
q_i^*(Q_{-i}) = \frac{a - Q_{-i} - c}{2}
\]

- Because of symmetry, every firm has same reaction function and behavior, so \( q_1^* = q_2^* = \cdots = q_i^* = \cdots = q_N^* \)

- This means \( Q_{-i} = (N - 1)q_i^* \), so \( q_i^* = \frac{a - (N - 1)q_i^* - c}{2} \)

- Solve: \( q_i^* = \frac{a - c}{N + 1} \) and \( Q^* = \frac{N}{N + 1}(a - c) \)