General Equilibrium (without Production) or Exchange (Chapter 31)

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Events in one market have effects on other markets (spillovers)
Demand for $x$ depends upon prices of complements, substitutes; income
Supply of $x$ depends upon factor prices
Previously, we’ve taken these as given—doing partial equilibrium analysis
But it’s important to understand interdependence of markets—general equilibrium analysis

Partial equilibrium analysis says that competitive markets yield efficient outcomes—is this still true in general equilibrium?
Our approach:

- Simple environment—the *entire* economy
  - 2 kinds of goods
  - 2 people
- Focus on exchange
  - Abstract away from production of new goods
  - Give people endowments
  - Specify preferences
  - Allow them to trade
- Make predictions about behavior of utility-maximizers
- Evaluate welfare
Endowment Economy

- Consumers A and B; goods 1 and 2
- Endowments: $\omega^A = (\omega^A_1, \omega^A_2)$ and $\omega^B = (\omega^B_1, \omega^B_2)$
- Example: $\omega^A = (6, 4)$ and $\omega^B = (2, 2)$
- This means total endowment of good 1 is $\omega^A_1 + \omega^B_1 = 6 + 2 = 8$ and of good 2 is $\omega^A_2 + \omega^B_2 = 4 + 2 = 6$
Allocations

- Endowment represents where people start, but through trade, their allocations may change.
- General allocation or consumption: \( x^A = (x^A_1, x^A_2) \) and \( x^B = (x^B_1, x^B_2) \)
- \((x^A, x^B)\) is feasible if it uses at most the aggregate endowment:
  \[
  x^A_1 + x^B_1 \leq \omega^A_1 + \omega^B_1 \quad \text{and} \quad x^A_2 + x^B_2 \leq \omega^A_2 + \omega^B_2
  \]
- Helpful graphical tool: Edgeworth Box
- Allows us to simply depict all feasible allocations.
The Endowment Allocation

The endowment allocation

$\omega^A = (6,4)$

$\omega^B = (2,2)$
Edgeworth Box

Feasible Reallocations

\[ x_1^B \]
\[ x_1 \]
\[ x_2^B \]
\[ x_2 \]

\[ \omega_1^A + \omega_1^B \]
How do we think about equilibrium?

- In partial equilibrium analysis:
  - Treat each good separately
  - Find $p$ and $q$ that equate supply and demand

- But this is general equilibrium analysis: where do supply and demand come from?

- $A$ and $B$ can trade with each other

- For everything to be balanced, the amount that $A$ gives up has to equal amount that $B$ receives (for each good, and vice versa)

- In other words $Supply = Demand$ for each good

- This will determine prices for each good

- How do we find supply and demand curves?

- Go back to utility maximization problem

- Need to specify preferences to do this
Utility maximization

- Preference are given
- Given prices for each good, endowment bundle serves as income
- Can write down budget constraint

\[ p_1 x_1 + p_2 x_2 \leq p_1 \omega_1 + p_2 \omega_2 \]

- Solve utility maximization problem
- Gives you optimal allocation, as a function of price ratio
- \( x_1^* - \omega_1 > 0 \) means person demands more of good 1
- \( x_1^* - \omega_1 < 0 \) means person is willing to supply good 1
- Key question: what prices will make it so that A demand exactly as much of each good as B supplies?
Preferences of A

Adding Preferences to the Box

For consumer A.
Preferences of B

Adding Preferences to the Box

For consumer B.

More preferred

$\omega_B$
Preferences of B

Adding Preferences to the Box

For consumer B.

$\omega_1^B$

$O_B$

$\omega_2^B$

$X_1^B$

$X_2^B$

More preferred
Putting both of them together

Edgeworth’s Box

\[ x^A_1, x^A_2, x^B_1, x^B_2 \]

\[ \omega^A_1, \omega^A_2, \omega^B_1, \omega^B_2 \]
Given a particular allocation, a *Pareto-improving* allocation improves the welfare of at least one consumer *without reducing the welfare of another*.

How do we depict Pareto-improving allocations in the Edgeworth box?
Edgeworth’s Box

The Edgeworth Box is a diagram used in economic theory to illustrate the possibilities of trade between two individuals or groups (A and B). It shows the combinations of two goods that each individual can consume, with the axes representing the quantities of goods 1 and 2. The box is bounded by the indifference curves of the two individuals, indicating the combinations of goods that provide the same level of utility. The indifference curves for A and B are labeled as $\omega_1^A$, $\omega_1^B$, $\omega_2^A$, and $\omega_2^B$. Points inside the box represent feasible consumption bundles, while those outside are unattainable.
Pareto-improving allocations

The set of Pareto-improving allocations

Pareto-Improvements
Pareto-optimal allocations

- An allocation is *Pareto-optimal* if it is *feasible* and there is other feasible allocation that is a Pareto-improvement over it.
- In other words, there is no way to make anyone better off without making someone worse off.
- The set of all Pareto-optimal allocations is called the contract curve.
A Pareto-Optimal Allocation

Pareto-Optimality

- Both A and B are worse off
- B is strictly better off but A is strictly worse off
- A is strictly better off but B is strictly worse off
- Both A and B are worse off
Pareto-Optimal Allocations

Pareto-Optimality

All the allocations marked by a \( \bullet \) are Pareto-optimal.

The contract curve
Pareto-optimal Allocations

- From the figures, we can see that an allocation at which the indifference curves of the two consumers are tangent must be Pareto-optimal.
- Tangency implies they have the same slope.
- What is the slope of an indifference curve? The Marginal rate of substitution (MRS)!
- Condition for Pareto-optimality:

\[
MRS^A = \frac{\frac{\partial u^A(x_1^A, x_2^A)}{\partial x_1^A}}{\frac{\partial u^A(x_1^A, x_2^A)}{\partial x_2^A}} = \frac{\frac{\partial u^B(x_1^B, x_2^B)}{\partial x_1^B}}{\frac{\partial u^B(x_1^B, x_2^B)}{\partial x_2^B}} = MRS^B
\]

- We also require feasibility:

\[
x_1^A + x_1^B = \omega_1^A + \omega_1^B \quad \text{and} \quad x_2^A + x_2^B = \omega_2^A + \omega_2^B
\]
Example

Identifying Pareto-optimal allocations

- Recall total endoments: \( \omega_1^A + \omega_1^B = 6 + 2 = 8 \) and \( \omega_2^A + \omega_2^B = 4 + 2 = 6 \)
- Let \( u^A(x_1^A, x_2^A) = \ln(x_1^A) + 2 \ln(x_2^A) \) and \( u^B(x_1^B, x_2^B) = \ln(x_1^B) + 2 \ln(x_2^B) \).
- MRS of consumer A:
  \[
  MRS^A = \frac{1}{x_1^A} = \frac{x_2^A}{2x_1^A}
  \]
- Similarly,
  \[
  MRS^B = \frac{1}{x_1^B} = \frac{x_2^B}{2x_1^B}
  \]
- So a Pareto-optimum is a feasible allocation for which
  \[
  \frac{x_2^A}{2x_1^A} = \frac{x_2^B}{2x_1^B}
  \]
Which of these allocations is Pareto Optimal?

A) \((x_A^1, x_A^2, x_B^1, x_B^2) = (4, 2, 6, 3)\)

B) \((x_A^1, x_A^2, x_B^1, x_B^2) = (4, 1, 4, 5)\)

C) \((x_A^1, x_A^2, x_B^1, x_B^2) = (4, 4, 5, 1)\)

D) \((x_A^1, x_A^2, x_B^1, x_B^2) = (4, 3, 4, 3)\)

E) \((x_A^1, x_A^2, x_B^1, x_B^2) = (6, 4.5, 2, 1.5)\)
Which of these allocations is Pareto Optimal?

A) \((x_1^A, x_2^A, x_1^B, x_2^B) = (4, 2, 6, 3)\)

B) \((x_1^A, x_2^A, x_1^B, x_2^B) = (4, 1, 4, 5)\)

C) \((x_1^A, x_2^A, x_1^B, x_2^B) = (4, 4, 5, 1)\)

D) \((x_1^A, x_2^A, x_1^B, x_2^B) = (4, 3, 4, 5)\)

E) \((x_1^A, x_2^A, x_1^B, x_2^B) = (6, 4.5, 2, 1.5)\)

Let’s look at why...
Identifying Pareto-optimal allocations

- Recall that a Pareto-optimum is a feasible allocation for which

\[
\frac{x_2^A}{2x_1^A} = \frac{x_2^B}{2x_1^B}
\]

- In other words, A and B need to have the same \( x_1 : x_2 \) ratio, which makes sense, because they have identical preferences.
- Clicker option A meets the tangency condition, but is not feasible.
- B is feasible, but does not meet the tangency condition.
- C satisfies neither condition.
- However, both D and E satisfy both conditions, so they both are Pareto-optimal.
Example

Finding all Pareto-optimal allocations (deriving the contract curve)

- We can simplify tangency condition to:
  \[
  \frac{x_2^A}{x_1^A} = \frac{x_2^B}{x_1^B}
  \]

- Recall endowment/feasibility requirement:
  \[
  x_1^A + x_1^B = 8 \quad \text{and} \quad x_2^A + x_2^B = 6
  \]

- Re-write tangency condition, substituting \(x_1^B = 8 - x_1^A\) and \(x_2^B = 6 - x_2^A\):
  \[
  \frac{x_2^A}{x_1^A} = \frac{6 - x_2^A}{8 - x_1^A}
  \]
  or
  \[
  x_2^A = \frac{3}{4} x_1^A
  \]

- This is the equation of the contract curve.
- In this case, it’s just the diagonal of the rectangle.
Core Allocations

How can we narrow down our prediction about the outcome resulting from trade?

- We’ve derived the contract curve: $x_2^A = \frac{3}{4}x_1^A$
- Note that clicker options D and E are both on this curve.
- However, Pareto Optimal, but given her endowment, $(\omega^B = (2, 2))$, Person B would never agree to trade to allocation $E = (6, 4.5, 2, 1.5)$.
- We need to restrict attention to Pareto-optimal allocations that are Pareto-improvements over the initial endowment.
- These allocations are called core allocations, or the core—the set of all PO allocations that are welfare improving for both consumers relative to their own endowments
- An allocation will be in the core if it is feasible and it is not blocked by any consumer
Core Allocations

The Core

Pareto-optimal trades blocked by B

Pareto-optimal trades blocked by A
The Core

Pareto-optimal trades not blocked by A or B are the core.

Core Allocations
A competitive equilibrium are prices \((p_1, p_2)\) and allocations \((x^A, x^B)\) such that

1. Given the prices \((p_1, p_2)\), the allocation \((x^A, x^B)\) solves each consumer’s utility maximization problem. That is, \(x^A\) is the solution of the consumer A’s utility maximization problem, taking \((p_1, p_2)\) as given, and similarly for \(x^B\).

\[
\max_{(x^A_1, x^A_2)} u^A(x^A_1, x^A_2)
\]

s.t.

\[
p_1 x^A_1 + p_2 x^A_2 = p_1 \omega^A_1 + p_2 \omega^A_2
\]

2. Market clearing conditions: the total demand of good 1 is equal to the total endowment (supply) of good 1 and similarly for good 2.

\[
x^A_1 + x^B_1 \leq \omega^A_1 + \omega^B_1 \quad \text{and} \quad x^A_2 + x^B_2 \leq \omega^A_2 + \omega^B_2
\]
A’s Utility Maximization

Trade in Competitive Markets

For consumer A.

\[ p_1 x_1^A + p_2 x_2^A = p_1 \omega_1^A + p_2 \omega_2^A \]
B’s Utility Maximization

Trade in Competitive Markets

For consumer B.

\[ p_1 x_1^B + p_2 x_2^B = p_1 \omega_1^B + p_2 \omega_2^B \]
A’s Utility Maximization

Trade in Competitive Markets

Budget constraint for consumer A

\[ x_1^A, x_2^A \]

\[ \omega_1^A, \omega_2^A \]

\[ x_1^B, x_2^B \]

\[ \omega_1^B, \omega_2^B \]

\[ O_A, O_B \]
Market Clearing

Trade in Competitive Markets

So \( x_1^A + x_1^B = \omega_1^A + \omega_1^B \)
First Welfare Theorem: Given that consumers preferences are well-behaved, trading in perfectly competitive markets implements a Pareto-optimal allocation of the economy's endowment. That is, $CE \Rightarrow PO$.

Second Welfare Theorem: any Pareto-optimal allocation (i.e. any point on the contract curve) can be achieved by trading in competitive markets provided that endowments are first appropriately rearranged amongst the consumers. That is, $PO \Rightarrow CE$. 
First Welfare Theorem

First Fundamental Theorem

Implemented by competitive trading from the endowment $\omega$. 
Second Fundamental Theorem

Can this allocation be implemented by competitive trading from \( \omega \)? No.
But this allocation is implemented by competitive trading from $\theta$. 

Second Fundamental Theorem