Equilibrium
Chapter 16
Firms are ‘price-takers’ in competitive markets, but how is the market price (and quantity) determined? **competitive equilibrium**

What happens to equilibrium price and quantity when either supply or demand changes? **comparative statics**

What are the effects of taxes and subsidies on prices and quantities?

What are the welfare effects of taxes and subsidies? **deadweight loss, tax incidence**
Taxes

- Quantity (or excise) tax
  - Effect on $p, q$
  - Subsidy
  - Incidence
  - Welfare effects

- Price tax: effect on $p, q$
Quantity Taxes

- Levied on each unit sold.
- E.g. gasoline tax: seller sets price at $2.05/gallon and gasoline tax is $0.35/gallon. Consumer must pay $p_d = 2.05 + 0.35 = 2.40$ dollars/gallon
- Seller gets $p_s = 2.05$
- Like any tax, this creates a wedge between what consumer pays and what producer receives
- The $0.35$ tax, collected by the govt., is the difference between the consumer price, $p_d$, and the producer price, $p_s$:

\[ p_d - p_s = 0.35 \]
Suppose gasoline tax is \( t \) dollars/gallon.

- \( t \) as a wedge:

\[
    p_d - p_s = t \implies p_d = p_s + t
\]

- How does this affect equilibrium?

- New condition: \( D(p_d) = S(p_s) \)

- Rewrite as \( D(p_s + t) = S(p_s) \) or \( D(p_d) = S(p_d - t) \)

- Can think of this as either shifting \( D \) or \( S \)
Equilibrium with a Quantity Tax

One view: demand shifts *downward*

\[
D(p_s) + t
\]

\[
S(p_s)
\]

\[
p_d \quad p^* \quad p_s
\]

\[
q^t \quad q^*
\]

\[
Gas
\]
Equilibrium with a Quantity Tax

Another view: supply shifts *upward*

\[ S(p_d - t) \]

\[ D(p_d) \]

\[ p_d \]

\[ p^* \]

\[ p_s \]

\[ q^* \]

\[ q^* \]
Equilibrium with a Quantity Tax

Either way: \( q^t < q^* \) and \( p_s < p^* < p_d \)
Inverse Demand: \( P_d(q) = 50 + \frac{q}{2} \)

Supply: \( S(p) = 10 + 7p \)

Suppose govt. imposes tax \( t = 0.90 \) per gallon. What is the after-tax equilibrium?

We need to find \( D(p) \) first:

\[
p = 50 + \frac{D(p)}{2} \implies D(p) = 100 - 2p
\]

Equilibrium condition:

\[
D(p_s + t) = S(p_s) \implies 100 - 2(p_s + 0.90) = 10 + 7p_s \\
\implies 9p_s = 90 - 2 \times 0.90 \\
\implies p_s = 10 - 0.2 = 9.80
\]
Example

- **Consumer price:**

\[ p_d = p_s + t = 9.80 + 0.90 = 10.70 \]

- So the equilibrium quantity is

\[ q^t = S(p_s) = 10 + 7p_s = 10 + 7 \times 9.80 = 78.6 \]

- How much tax revenue does the government collect?

\[ R_t = tq^t = 0.90 \times 78.6 \approx 70.74 \]
Example

Who really pays this tax?

- The division of $t$ between the buyers and sellers is the *incidence* of the tax.

- Compare pre-tax equilibrium price, $p^*$, with consumer price, $p_d$, and producer price, $p_s$.

- $p^* = 10$, $p_d = 10.70$, and $p_s = 9.80$

- So consumer ‘pays’ $10.70 - 10 = 0.70$ per gallon and the producer ‘pays’ $10 - 9.80 = 0.20$ per gallon.
The incidence of a quantity tax depends upon the price-elasticities of demand and supply.

The producers pay all the tax when supply is perfectly inelastic.

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The consumers pay all the tax when supply is perfectly elastic.
Clicker Vote: Govt will raise the most revenue when demand is

A) Elastic
B) Inelastic
C) Unit elastic
D) Elasticity doesn’t matter
Equilibrium with a Subsidy

Example

- What if govt. wants to keep gas prices low, e.g. $p = 8$? A price ceiling will lead to shortages.
- An alternative is to subsidize gasoline by paying sellers $s$ per gallon.
- How large must $s$ be? Well, $p_d + s = p_s$ so

$$D(p_d) = S(p_s) \implies D(8) = S(8 + s)$$

$$\implies 100 - 2 \times 8 = 10 + 7 \times (8 + s)$$

$$\implies s = \frac{18}{7}$$
Equilibrium with a Subsidy

Example

Gas

\[ p_d = 8 \]

\[ q^t = 100 - 2 \times 8 = 84 \]

\[ p_s = 8 + s \]

\[ s = \frac{18}{7} \]
Equilibrium with a Subsidy

Just a *negative* tax: $q^t > q^*$ and $p_d < p^* < p_s$
Total Surplus

\[ CS = A + B + E \]
\[ PS = C + D + F \]

Total surplus = A + B + C + D + E + F

Diagram showing the relationship between price, gas, supply (S(p)), and demand (D(p)) with regions A, B, C, D, E, and F highlighted to represent different surplus contributions.
Deadweight Loss (DWL)

CS = A, PS = D,  Gov’t = B+C
Total surplus with Tax = A+B+C+D
DWL = E+F

Price

Gas

A

B

C

D

S(p)

D(p)

q^t

q^*

pt

pt - t

p^t

t

E

F

0
Deadweight Loss (DWL)

Tax example: recall that $p^* = 10$, $q^* = 80$, $p^t = 10.70$, $p_s = 9.80$, $q^t = 78.6$.

\[
\text{DWL} = \frac{1}{2} \times (10.7 - 9.8) \times (80 - 78.6) = 0.63
\]
Subsidy example: recall that $p^* = 10$, $q^* = 80$, $p_s = 8 + \frac{18}{7} = 10.57$, $p_d = 8$, $q_t = 84$.

\[
\text{DWL} = \frac{1}{2} \times (10.57 - 8) \times (84 - 80) = 5.14
\]
A price tax is a per-dollar (as opposed to per-unit) tax.

Also known as *ad valorem* tax

Examples: sales tax, interest tax, value-added tax (VAT)
Example: Value-added Tax

- **Demand**: \( D(p) = 100 - 2p \), supply \( S(p) = 10 + 7p \) (borrowed from above)

- Suppose government imposes a VAT of \( t = 0.10 = 10\% \).

- With VAT \( t \), consumer pays \( p_d \), but producer only gets \( (1 - t)p_d \). So

\[
ps = (1 - t)p_d
\]

- What is the after-tax equilibrium?
Example: Value-added Tax

The equilibrium condition:

\[ D(p_d) = S((1 - t)p_d) \]

\[ \Rightarrow 100 - 2(p_d) = 10 + 7(1 - t)p_d \]

\[ \Rightarrow 7(1 - t)p_d + 2p_d = 100 - 10 \]

\[ \Rightarrow (9 - 7t)p_d = 90 \]

\[ \Rightarrow p_d = \frac{90}{9 - 7t} \]

\[ \Rightarrow p_d = \frac{90}{9 - 7 \times 0.10} = 10.84 \]

The producer price is

\[ p_s = (1 - t)p_d = .9(10.84) = 9.76 \]
Example: Value-added Tax

- After-tax equilibrium quantity:
  \[ q^t = D(p_d) = 100 - 2p_d = 100 - 2 \times 10.84 = 78.31 \]

- Government revenue?

- Government is paid \( tp_d \) for every unit sold, revenue is
  \[ tp_d q^t = 0.1 \times 10.84 \times 78.31 = 84.89. \]
What is the deadweight loss with a VAT?

Example: Value-added Tax

\[ \text{DWL} = \frac{1}{2} (10.84 - 9.76)(80 - 78.31) \]

\[ = 0.91 \]