Free-response – 21 out of 50 pts. (2 qns., 10 pts. each + 1)

Answer these questions in your blue-book. Show your work and intermediate steps for partial credit. If you cannot complete part of the problem, e.g. if you get stuck on some algebra, you may still earn partial credit for explaining intuitively how you would solve if you were not stuck. You can expect to see fractions in your answers. Your score will only be based on the marks in your blue-book. You will not receive any credit for anything written on your exam paper. You will receive 1 point for correctly writing your name, perm number, version (A, B, C, or D), and TA’s name on your blue-book.

1. Consider an exchange economy consisting of two people, A and B, endowed with two goods, 1 and 2. Person A is initially endowed with \( \omega^A = (4, 4) \) and person B is initially endowed with \( \omega^B = (1, 6) \). They both have preferences given by \( U(x_1, x_2) = x_1x_2^3 \). (Points split equally across parts)

(a) Write the equation of the contract curve (express \( x_2^A \) as a function of \( x_1^A \))

Answer: \( x_2^A = 2x_1^A \)

(b) Let \( p_2 = 1 \) and find the competitive equilibrium price \( p_1 \). To obtain partial credit for an incorrect answer, you should write down a condition that must hold when person A is choosing the optimal allocation given the prices she faces.

Answer: \( p_1 = \frac{2}{3} \) (and \( MRS^A = \frac{x_2^A}{x_1^A} = p_1 \))

(c) Find the competitive equilibrium allocations, \( x^A = (x_1^A, x_2^A) \) and \( x^B = (x_1^B, x_2^B) \)

Answer: \( x^A = x^B = (2.5, 5) \)

(d) Now suppose that the two people began with the same endowments, but B’s preferences instead were \( U(x_1, x_2) = 2x_1 + 3x_2 \). How would the price and allocations change?

Answer: They would not change. \( MRS^B = \frac{2}{3} \), so \( MRS^A = MRS^B = p_1 = \frac{2}{3} \). The contract curve stays the same as well. Same contract curve, same endowment, same price, same budget constraints mean that the allocation is the same as well.

(e) In which case does B gain more from trading with A— when \( U^B(x_1, x_2) = x_1x_2^3 \) or when \( U^B(x_1, x_2) = 2x_1 + 3x_2 \)? Briefly explain.

Answer: B gains more when \( U^B(x_1, x_2) = x_1x_2^3 \). When \( U^B(x_1, x_2) = 2x_1 + 3x_2 \), all the points on B’s budget constraint are on the same indifference curve, meaning that she gains nothing from trade.

2. The citizens of Quietville (population: 3) lack recreational opportunities. The town council is thinking about creating some public parks, which the town is currently lacking, but does not know how many acres to devote to parkland, if any. The total cost of converting \( x \) acres of land into a park is 18x. The town council cannot observe directly individuals’ demand for parkland. (Points indicated in brackets.)

(a) [4] Suppose that for the three residents of Quietville (A, B, and C), the total benefit of \( x \) acres of parkland is \( B^A = 30x - \frac{18x^2}{2} \) for person A, \( B^B = 4x - x^2 \) for person B, and \( B^C = 2x - \frac{x^2}{2} \) for person C. If Quietville’s leaders could observe the benefit of parkland for each person, what would be the socially optimal number of acres of parkland?

Answer: \( \sum MB = 36 - 18x = 18 = MC \), so \( x = 1 \), partial credit of 2 for incorrect answer, but correctly writing optimality condition: \( \sum MB = MC \)

(b) [2] Suppose that the town council identifies only a single, 1 acre lot and is trying to decide whether or not to build a park on that lot. Not being able directly to observe the marginal benefit to the three residents, the town council submits a plan to build a park on that one acre and split the cost of doing so (18) evenly among the three residents. What is the net valuation of this plan for
each of the three citizens?

Answer: \( v_A = 16.5, \quad v_B = -3, \quad v_C = -4.5 \)

(c) [2] If the residents voted on this plan, would it be approved by a majority? Will the residents vote to approve this plan? Who will vote for and who will vote against?

Answer: No. \( A \) will vote for and \( B \) and \( C \) will vote against.

(d) [2] Now suppose that the council use a VCG mechanism to determine whether or not to build the 1 acre park. Will the park be approved and what taxes will be paid by whom?

Answer: Yes. \( A \) will pay a tax of 7.5.