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Equilibrium
Chapter 16
Firms are ‘price-takers’ in competitive markets, but how is the market price (and quantity) determined? **competitive equilibrium**

What happens to equilibrium price and quantity when either supply or demand changes? **comparative statics**

What are the effects of taxes and subsidies on prices and quantities?

What are the welfare effects of taxes and subsidies? **deadweight loss, tax incidence**
Today

- Quantity (or excise) tax
  - Effect on $p$, $q$
  - Subsidy
  - Incidence
  - Welfare effects

- Price tax: effect on $p$, $q
Quantity Taxes

- Levied on each unit sold.
- E.g. gasoline tax: seller sets price at $2.05/gallon and gasoline tax is $0.35/gallon. Consumer must pay $p_d = 2.05 + 0.35 = 2.40$ dollars/gallon
- Seller gets $p_s = 2.05$
- Like any tax, this creates a wedge between what consumer pays and what producer receives
- The $0.35$ tax, collected by the govt., is the difference between the consumer price, $p_d$, and the producer price, $p_s$:

$$p_d - p_s = 0.35$$
Suppose gasoline tax is $t$ dollars/gallon.

- $t$ as a wedge:
  \[ p_d - p_s = t \implies p_d = p_s + t \]

- How does this affect equilibrium?
- New condition: $D(p_d) = S(p_s)$
- Rewrite as $D(p_s + t) = S(p_s)$ or $D(p_d) = S(p_d - t)$
- Can think of this as either shifting $D$ or $S$
Equilibrium with a Quantity Tax

One view: demand shifts \textit{downward}

\begin{align*}
\text{Equilibrium with Quantity Tax: Shifting Demand} \\
\text{Gas} \\
p_s \\
D(p_s + t) \\
S(p_s) \\
q^* \\
q^t \\
p^* \\
p_d \\
p_s \\
\end{align*}
Equilibrium with a Quantity Tax

Another view: supply shifts *upward*

\[ D(p_d) \]

\[ S(p_d - t) \]

\[ p_d \]
\[ p^* \]
\[ p_s \]

\[ q^t \]
\[ q^* \]
Equilibrium with a Quantity Tax

Either way: \( q^t < q^* \) and \( p_s < p^* < p_d \)
Example

- Inverse Demand: $P_d(q) = 50 + \frac{q}{2}$
- Supply: $S(p) = 10 + 7p$
- Suppose govt. imposes tax $t = 0.90$ per gallon. What is the after-tax equilibrium?
- We need to find $D(p)$ first:

$$p = 50 + \frac{D(p)}{2} \implies D(p) = 100 - 2p$$

- Equilibrium condition:

$$D(p_s + t) = S(p_s) \implies 100 - 2(p_s + 0.90) = 10 + 7p_s$$
$$\implies 9p_s = 90 - 2 \times 0.90$$
$$\implies p_s = 10 - 0.2 = 9.80$$
Example

- Consumer price:
  \[ p_d = p_s + t = 9.80 + 0.90 = 10.70 \]

- So the equilibrium quantity is
  \[ q^t = S(p_s) = 10 + 7p_s = 10 + 7 \times 9.80 = 78.6 \]

- How much tax revenue does the government collect?
  \[ R_t = tq^t = 0.90 \times 78.6 \approx 70.74 \]
Equilibrium with a Subsidy

Example

- What if govt. wants to keep gas prices low, e.g. \( p = 8 \)? A price ceiling will lead to shortages.
- An alternative is to subsidize gasoline by paying sellers $s$ per gallon.
- How large must $s$ be? Well, \( p_d + s = p_s \) so

\[
D(p_d) = S(p_s) \quad \implies \quad D(8) = S(8 + s) \\
\implies 100 - 2 \times 8 = 10 + 7 \times (8 + s) \\
\implies s = \frac{18}{7}
\]
Example

\[ p_d = 8 \]

\[ q^t = 100 - 2 \times 8 = 84 \]

\[ s = \frac{18}{7} \]
Just a *negative* tax: \( q^t > q^* \) and \( p_d < p^* < p_s \)
Who really pays this tax?

- The division of $t$ between the buyers and sellers is the *incidence* of the tax.

- Compare pre-tax equilibrium price, $p^*$, with consumer price, $p_d$, and producer price, $p_s$.

  - $p^* = 10$, $p_d = 10.70$, and $p_s = 9.80$

  - So consumer ‘pays’ $10.70 - 10 = 0.70$ per gallon and the producer ‘pays’ $10 - 9.80 = 0.20$ per gallon.
Tax Incidence and Elasticity

The incidence of a quantity tax depends upon the price-elasticities of demand and supply.

The producers pay all the tax when supply is perfectly inelastic.
The incidence of a quantity tax depends upon the price-elasticities of demand and supply.

The consumers pay all the tax when supply is perfectly elastic.

The consumers pay all the tax when supply is perfectly elastic.
Total Surplus

\[ \text{CS} = A + B + E \]
\[ \text{PS} = C + D + F \]
Total surplus = \( A + B + C + D + E + F \)
Deadweight Loss (DWL)

CS = A, PS = D, Gov’t = B+C
Total surplus with Tax = A+B+C+D
DWL = E+F

Price

Gas

S(p)

D(p)

A

B

C

D

E

F

pt

t

pt - t

q^t

q^*

pt

E

F

B

C

D

DWL = E+F
Tax example: recall that $p^* = 10$, $q^* = 80$, $p^t = 10.70$, $p_s = 9.80$, $q^t = 78.6$.

Deadweight Loss: Example with Tax
Recall from previous examples: before tax $p^* = 10$, $q^* = 80$, and after tax $p^t = 10.7$, $p_s = 9.8$, $q^t = 78.6$.

$$\text{DWL} = \frac{1}{2} \times (10.7 - 9.8) \times (80 - 78.6)$$

$$= 0.63$$
Subsidy example: recall that $p^* = 10$, $q^* = 80$, 
$p_s = 8 + \frac{18}{7} = 10.57$, $p_d = 8$, $q^t = 84$.

\[
\text{DWL} = \frac{1}{2} \times (10.57 - 8) \times (84 - 80) = 5.14
\]
A price tax is a per-dollar (as opposed to per-unit) tax.

Also known as *ad valorem* tax

Examples: sales tax, interest tax, value-added tax (VAT)
Example: Value-added Tax

- Demand: \( D(p) = 100 - 2p \), supply \( S(p) = 10 + 7p \) (borrowed from above)

- Suppose government imposes a VAT of \( t = 0.10 = 10\% \).

- With VAT \( t \), consumer pays \( p_d \), but producer only gets \( (1 - t)p_d \). So

\[
p_s = (1 - t)p_d
\]

- What is the after-tax equilibrium?
Example: Value-added Tax

The equilibrium condition:

\[ D(p_d) = S((1 - t)p_d) \]

\[ 100 - 2(p_d) = 10 + 7(1 - t)p_d \]

\[ 7(1 - t)p_d + 2p_d = 100 - 10 \]

\[ (9 - 7t)p_d = 90 \]

\[ p_d = \frac{90}{9 - 7t} \]

\[ p_d = \frac{90}{9 - 7 \times 0.10} = 10.84 \]

The producer price is

\[ p_s = (1 - t)p_d = .9(10.84) = 9.76 \]
Example: Value-added Tax

- After-tax equilibrium quantity:
  
  \[ q^t = D(p_d) = 100 - 2p_d = 100 - 2 \times 10.84 = 78.31 \]

- Government revenue?

- Government is paid \( tp_d \) for every unit sold, revenue is

  \[ tp_d q^t = 0.1 \times 10.84 \times 78.31 = 84.89. \]
Example: Value-added Tax

What is the deadweight loss with a VAT?

\[
\text{DWL} = \frac{1}{2}(10.84 - 9.76)(80 - 78.31) = 0.91
\]