Consumer Surplus and Welfare Measurement
Questions

Q: How can we... 

- Find a monetary measure of a consumer’s utility/happiness?
- Evaluate a consumer’s willingness to pay for a unit of a good?
- Evaluate whether or not a market maximizes welfare without government intervention?
- Quantify the effect of economic policy on consumers?

A: Use the concept of gains-from-trade
Examples:

- Suppose you had to pay to download iTunes, but once you did, you could buy as many songs as you like for $1.
- You have to pay a cover charge to get into a bar. Once you’re in, beers are $3.50 a pint.
- Costco sells cheap goods in bulk, but you have to pay a membership fee.

What is the most you would pay to enter these markets?

You would pay up to the dollar value of the gains-to-trade you would enjoy once in the market.
Q: How can we put a dollar value on

a) the welfare gains resulting from a trade, or
b) the change in consumer welfare resulting from a price/policy change?

A:

a) Consumer and Producer Surplus are monetary approximations of gains from trade for consumers & producers, respectively. (Benefits - Costs)

b) We have three measures of the effects of a policy or price change on consumer welfare:

1. Change in Consumer Surplus
2. Compensating Variation
3. Equivalent Variation

Only in special circumstances do these three measures coincide.
Q: How much would a consumer pay for a unit of a good?

A: *Reservation Price* = the *maximum* price that the consumer is willing to pay for a unit.

Example: suppose utility is *quasilinear*, i.e.

\[ U(b, d) = v(b) + d, \]

where \( b \) is the number of beers consumed and \( d \) is the amount of money (dollars) spend on other goods.

**Successive reservation prices:**

\[
\begin{align*}
    r_1 &= v(1) - v(0) \\
    r_2 &= v(2) - v(1) \\
    \vdots
\end{align*}
\]
Example: if $r_4 \leq p \leq r_3$, the consumer will demand 3 beers.

Assumption: the more you have already consumed, the lower the reservation price for the next good. (*Downsloping demand*)
Willingness to Pay for \( n \) Units

Q: How much is the consumer willing to pay for \( n \) beers?
A: \( v(n) \). Why? Use reservation prices to show:

\[
r_1 + r_2 + r_3 = v(1) - v(0) + v(2) - v(1) + v(3) - v(2) \\
= v(3) - v(0) \text{ (assume } v(0) = 0) 
\]

This is called *gross benefit* or *gross gains from trade*
Q: How much does the consumer spend for \( n \) beers?
A: Expenditure = \( pn \)
(Net) Gains from Trade = gross benefit — expenditures in other words, net gain is $v(n) - pn$.

This is the minimum amount of money the consumer would need to be paid to give up $n$ units of the good.
Gains From Trade

With continuous units (if you can drink beer straight from the tap):
Estimating the reservation-price curve is difficult.
As an approximation, we replace the reservation-price curve with the consumer’s ordinary demand curve.
Consumer Surplus

- Say what? Reservation-price curve ≠ demand curve? Why not?
- Reservation-price curve describes sequential purchases of single units
- Demand curve describes willingness-to-pay for \( q \) units purchased simultaneously?
- *But...* in our example, utility is quasilinear in income, so there are no income effects & CS is an exact measure of gains from trade.
**Consumer Surplus: Example**

Suppose that the price of a beer is $4.25.

- Q: How many beers will the consumer buy?
- A: 3
- Q: What is the consumer surplus?
- A: \((10 + 8 + 6) - (3 \times 4.25) = $11.25\)
**Consumer Surplus: Example**

What if the price increases to $5.50?

- Q: How many beers will the consumer buy?
  - A: 3
- Q: What is the consumer surplus?
  - A: $(10 + 8 + 6) - (3 \times 5.50) = $7.50
Q: More generally, how does consumer welfare change when the price changes from $p$ to $p'$?

A: Three measures:

- Change in consumer surplus
- Compensating variation
- Equivalent variation
Change in Consumer Surplus

CS when price is $p$: 

![Diagram showing the change in consumer surplus with price $p$ and $p'$, the demand curve, and the consumer surplus area shaded.](image-url)
Change in Consumer Surplus

CS when price is $p'$:
Change in Consumer Surplus

Region A = change in CS due to higher price for all units consumed
Region B = change in CS due to reduction in consumption
Compensating Variation

Before the price change: let $p_1 = p$ and $p_2 = 1$

CV: how much money we would have to give to the consumer after the price change to make her just as well off as she was before
Compensating Variation

Now $p_1$ increases to $p_1 = p'$:
Compensating Variation

CV = the amount of extra money needed to bring the budget line back up to the old indifference curve.
Equivalent Variation

EV: how much money we would have to take away from the consumer before the price change to leave her just as well off as she would be after.
Equivalence Under Quasilinear Utility

- In general, $CV$, $EV$, and $\Delta CS$ are not the same. But they do coincide if utility is quasilinear: $U(c_1, c_2) = v(c_1) + c_2$.

- With prices $(p, 1)$, if the consumer chooses $c_1$ units of good 1, her consumption of good 2 is $m - pc_1$. Total utility:

  $$U = v(c_1) + m - pc_1.$$ 

- On the other hand, with prices $(p', 1)$, total utility is

  $$U' = v(c') + m - p'c_1.$$
Equivalence Under Quasilinear Utility

- **CV** is the amount of money such that

\[
\begin{align*}
CV &= v(c_1) - v(c_1') + p'c_1' - pc_1
\end{align*}
\]

\[
\begin{align*}
\underbrace{v(c_1) + m - pc_1}_{\text{before change}} &= \underbrace{v(c') + (m + CV) - p'c_1'}_{\text{after change with compensation}} \\
CV &= v(c_1) - v(c_1') + p'c_1' - pc_1
\end{align*}
\]

- **EV** is the amount of money such that

\[
\begin{align*}
\underbrace{v(c_1) + (m - EV) - pc_1}_{\text{before change with } EV} &= \underbrace{v(c') + m - p'c_1'}_{\text{after change}} \\
EV &= v(c_1) - v(c_1') + p'c_1' - pc_1
\end{align*}
\]

- **CV = EV**
What about $\Delta CS$?

$$\Delta CS = (v(c_1) + m - pc_1) - (v(c'_1) + m - p'c'_1)$$

utility before change - utility after change

$$= v(c_1) - v(c'_1) + p'c'_1 - pc_1$$

$\Delta CS = CV = EV$

For a simple example illustrating $CV = EV$, try workout 14.7
Q: What about gains from trade for the producer?  
A: Changes in a firm’s welfare be measured in dollars as much as for the consumer  
*Producer Surplus* = the area above the supply curve and under the price line.
Q: How can we measure the gain or loss caused by market intervention/regulation?
A: Use consumer and producer surplus: total surplus = \(CS + PS\).
Our benchmark will be competitive, free-market equilibrium.

![Diagram of supply and demand with consumer surplus (CS) and producer surplus (PS)]